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The Verification of Propotionality Between Time Period and Length of a Simple Pendulum Experiment Using Deep Neural Network

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Abstract: This research shows a pedagogic experimental and theoretical study of the motion of a simple pendulum, which considers the propotionality to the variables length (L) and period-time (T) of a simple harmonic motion, is presented. The study has used RELU (RECTIFIED LINEAR UNIT) activation function in deep neural network which is a branch of artificial neural network to examine the correlation between the dependent and independent variables in a simple pendulum experiment, the variables and their values was first generated from an online physics laboratory, the values and their corresponding variables were later separated into two CSV files, after which they were analyzed with the use of linear regression model in PYTHON programming language. It also applies to Physics Direct Method to represent these equations, in addition to the numerical solutions discusses. This research investigates the relationship between Length and Period using neural network models to find out a unique numerical solution by using simulation to see their behavior which shows in last part of this article. The results obtained shows that the linear approach to modeling the relationship between a scalar response and one or more variables with the RELU activation function proves their proportionality, this would be a good reference against which other results obtained from other simple harmonic motion experiments can be compared.

Keywords: Simple Pendulum, Artificial Neural Network, Deep Learning Neural Network (DNN), RELU, Python Programming

1. Introduction

A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting equilibrium position, it is subject to a restoring force due to gravity that will accelerate back toward the equilibrium position [1, 2]. The simple pendulum is one of the most popular examples analyzed in the textbooks and undergraduate courses in physics and it is perhaps the most investigated oscillatory motion in physics [6]. In a physics experiment, applying formulas and calculation processes are a part of physics language. The mathematical expression and processes are used to incorporate or deliver critical components of content; most physicists read a formula in composed writings and expressed it into physical notions. The ability to apply and calculate formulas in physics learning can improve students' concepts and knowledge through applying what they discovered in authentic context as well as scientifically communicating with others [7]. It is essential for students not only to comprehend physical and mathematical concepts independently but also understand the correlation between physical and mathematical for active learning [8]. Simple pendulums consist solely of a rod fixed at one end and attached to a weight at the other. When released, they swing. As one of the first to study pendulums, Galileo famously determined that their motion is independent of the swept angle. While observing a chandelier, he saw that simple pendulums are iso-chronic with regards to the maximum angular displacement: no matter the angle a given pendulum makes with the vertical in its swing, its period of motion will always be the same [3]. If this is true, then pendulums exhibit simple harmonic motion, which occurs when the force that pushes a system to equilibrium is proportional to the displacement from that equilibrium [4]. Since Galileo's observation, pendulums have led breakthroughs in time-measurement and science in general, at one point proving that the Earth rotates once a day. In high school physics courses, we learn to treat pendulums as iso-chronic. While modelling their motion, we follow Galileo's assumption and ignore the swept angle while calculating the period of a pendulum. In electronics engineering and related fields,[9] artificial neural networks (ANNs) are mathematical or computational models that are inspired by a human's central nervous system (in particular the brain) which is capable of machine learning as well as pattern recognition. Whereas animal's nervous system is more complex than the human so the system designed like this will be able to solve more complex problems. Artificial neural networks are generally presented as systems of highly interconnected "neurons" which can compute values from inputs. Neural networks have a remarkable ability to learn and generalize from data. This lets them excel at tasks such as image classification [10], reinforcement learning [13, 11, 12], and robotic dexterity [14, 15].

This research is going to explain the SHM as regards to the operable mode of a simple pendulum, analyse all variables with numerical solutions, apply DNN (Deep Neural Networks) algorithm to the obtained data and compare resulting graphs.

2. Modeling of the Simple Pendulum

A simple pendulum may be described ideally as a point mass suspended by a mass- less string from some point about which it is allowed to swing back and forth in a place. A simple pendulum can be approximated by a small metal sphere which has a small radius and a large mass when compared relatively to the length and mass of the light string from which it is suspended. If a pendulum is set in motion so that is swings back and forth, its motion will be periodic. The time that it takes to make one complete oscillation is defined as the period T. Another useful quantity used to describe periodic motion is the frequency of oscillation. The frequency of the oscillations is the number of oscillations that occur per unit time and is the inverse of the period, f = 1/T. Similarly, the period is the inverse of the frequency, T = l/f. The maximum distance that the mass is displaced from its equilibrium position is defined as the amplitude of the oscillation.

When a simple pendulum is displaced from its equilibrium position, there will be a restoring force that moves the pendulum back towards its equilibrium position. As the motion of the pendulum carries it past the equilibrium position, the restoring force changes its direction so that it is still directed towards the equilibrium position. If the restoring forcebar Fis opposite and directly proportional to the displacement X from the equilibrium position, so that it satisfies the relationship.

$$F = - k x \tag{1}$$

where F and X are vector quantities, then the motion of the pendulum will be simple harmonic motion and its period can be calculated using the equation for the period of simple harmonic motion.

$$T = 2\pi\sqrt{\frac{M}{g}} \tag{2}$$

This can be described that if the amplitude of the motion is kept small, Equation (2) will be satisfied and the motion of a simple pendulum will be simple harmonic motion, and Equation (2) can be used.

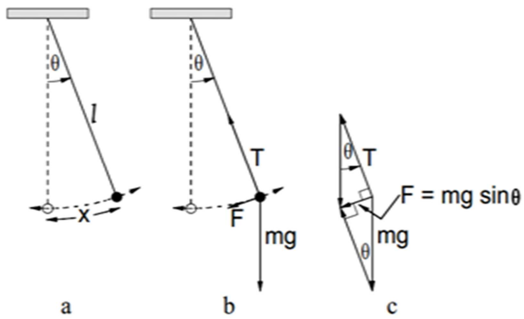


Figure 1. *Diagram illustrating the restoring force for a simple pendulum.*

The restoring force for a simple pendulum is supplied by the vector sum of the gravitational force on the mass. mg, and the tension in the string, T. The magnitude of the restoring force depends on the gravitational force and the displacement of the mass from the equilibrium position. Consider Figure 1 where a mass m is suspended by a string of length land is displaced from its equilibrium position by an angle θ and a distance x along the arc through which the mass moves. The gravitational force can be resolved into two components, one along the radial direction, away from the point of suspension, and one along the arc in the direction that the mass moves. The component of the gravitational force along the arc provides the restoring force F and is given by:

$$F = - mg \sin\theta \tag{3}$$

where g is the acceleration of gravity, θ is the angle the pendulum is displaced, and the minus sign indicates that the force is opposite to the displacement. For small amplitudes where θ is small, sinθ can be approximated by θ measured in radians so that Equation (3) can be written as

$$F = - mg\ \theta. \tag{4}$$

The angle θ in radians is x/l, the arc length divided by the length of the pendulum or the radius of the circle in which the mass moves. The restoring force is then given by

$$F = -MG\frac{X}{L} \tag{5}$$

and is directly proportional to the displacement x and is in the form of Equation (1) where

$$K = \frac{MG}{L}$$

Substituting this value of k into Equation (2), the period of a simple pendulum can be found by

$$T = 2\pi\sqrt{\frac{M}{\frac{MG}{L}}} \tag{6}$$

$$\text{And} T = 2\pi\sqrt{\frac{L}{g}} \tag{7}$$

3. Apparatus and Procedure

This research work was carried out in a virtual laboratory, where all physical parameters could be measured, the URL for this virtual environment is [5].

Below is an image of the virtual laboratory.

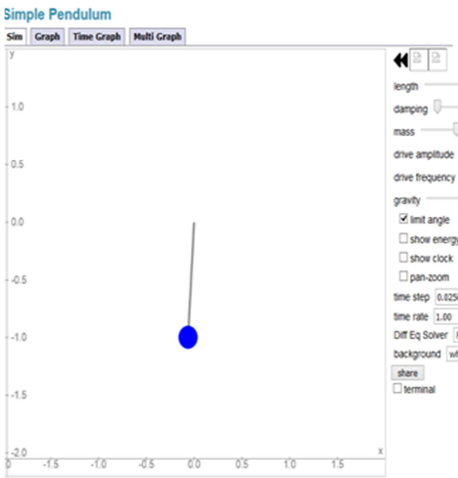
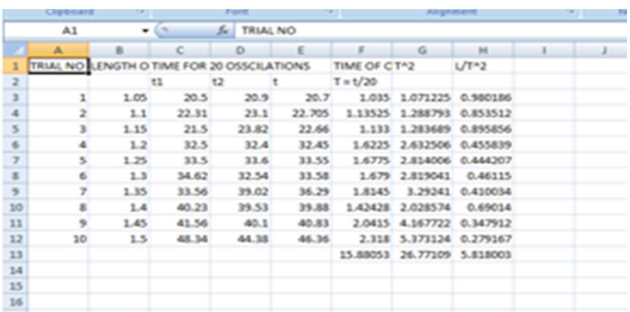


Figure 2. *The virtual physics laboratory.*

We only considered an un-damped pendulum, as indicated in Figure 2 the damping effect remained at 0, drive amplitude remained at zero, likewise the mass of the pendulum bulb remained as 1kg. But all other variables such as gravity remained constant.

Below is the result of the readings taken and recorded in an excel sheet.



	A	B	C	D	E	F	G	H
1	TRIAL NO	LENGTH O	TIME FOR 20 OSSCILATIONS			TIME OF C	T^2	L/T^2
2			t1	t2	t	T = t/20		
3	1	1.05	20.5	20.9	20.7	1.035	1.071225	0.980186
4	2	1.1	22.31	23.1	22.705	1.13525	1.288793	0.853512
5	3	1.15	21.5	23.82	22.66	1.133	1.283689	0.895856
6	4	1.2	32.5	32.4	32.45	1.6225	2.632506	0.455839
7	5	1.25	33.5	33.6	33.55	1.6775	2.814006	0.444207
8	6	1.3	34.62	32.54	33.58	1.679	2.819041	0.46115
9	7	1.35	33.56	39.02	36.29	1.8145	3.29241	0.410034
10	8	1.4	40.23	39.53	39.88	1.42428	2.028574	0.69014
11	9	1.45	41.56	40.1	40.83	2.0415	4.167722	0.347912
12	10	1.5	48.34	44.38	46.36	2.318	5.373124	0.279167
13						15.88053	26.77109	5.818003

Figure 3. *Result of the reading from the virtual lab.*

Ten (10) trials, length of thread was fixed at 1, time for 20 0 sscilations (t1 and t2) using a stop clock in the virtual environment, the period T was obtained by getting the average of t1, t2, the square of the period was also recorded.

4. Methodology

The major aim of this research is to understand the relationship between the variables (both dependent and independent), basically from previous experiments in this research area it is observed that the time- period of the pendulum is a function of length, which depicts that length is the independent variable.

Deep learning is a subset of machine learning (AI) that has networks capable of learning unsupervised from data that is unstructured or unlabel edit is also known as deep neural network.

Artificial neural network is a parallel computational models that originated from the biological neuron and neural structures, in which every neural takes in multiple unique inputs and produces one output.

This research would be using tools like Python to analyze the obtained data using regression model which is a type of deep neural network. In python we would use the RELU activation function the linear activation function in particular. The function is linear of values greater than zero, meaning it has a lot of the desirable properties for a linear activation function when training a neural network using back-propagation.

The data was separated into two parts that contains (length) of the pendulum thread as input and (period, period square) as targets or outputs. Below is an algorithm showing how the data was trained.

```
# Import input (x) and output (y) data, and asign these to dfl and dfl
df1 = pd.read_csv('PEND.csv')
df2 = pd.read_csv('S_PEND.csv')
# Split the data into input (x) training and testing data, and ouput (y) training and testing data,
# with training data being 80% of the data, and testing data being the remaining 20% of the data
X_train, X_test, y_train, y_test = train_test_split(df1, df2, test_size=0.2)
# Scale both training and testing input data
X_train = preprocessing.scale(X_train)
X_test = preprocessing.scale(X_test)
# Plots the results of a learning rate of 100, 1000, and 10000 respectively, with all other parameters constant
LR = [100,1000,10000]
for i in LR:
#Defines linear regression model and its structure
model = Sequential()
model.add(Dense(3, input_shape=(5,)))
#Compiles model
model.compile(Adam(lr=i), 'mean_squared_error')
#Fits model
history = model.fit(X_train, y_train, epochs = 50, validation_split = 0.1, verbose = 0)
history_dict=history.history
```

Below are the result obtained from running the above code. Graphically:

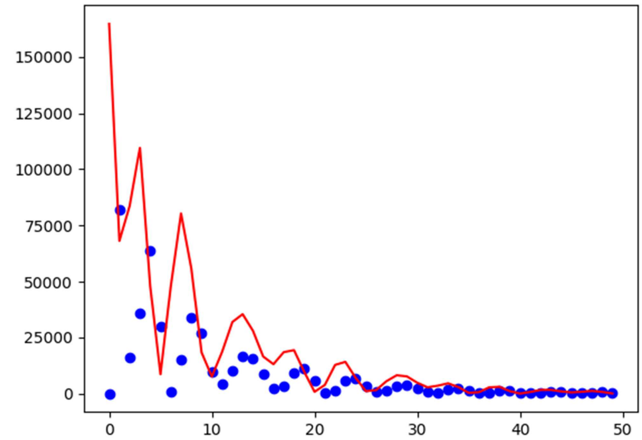


Figure 4. *Describes the training of the modelloss and validation of the loss/cost of the models which was separated into training and test set with the accuracy of about 0.2, which means the training set would be 80% and test set would be 20% for efficiency.*

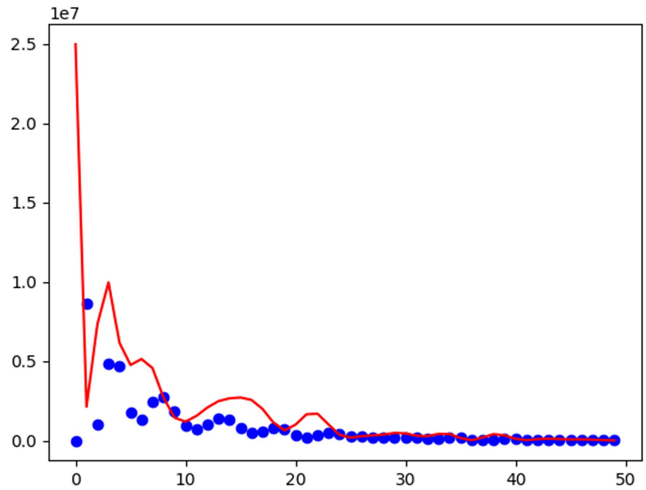


Figure 5. *This graph shows the performance of the model from a learning rate of about 1000, but with the RELU activation function and an epoch of 50.*

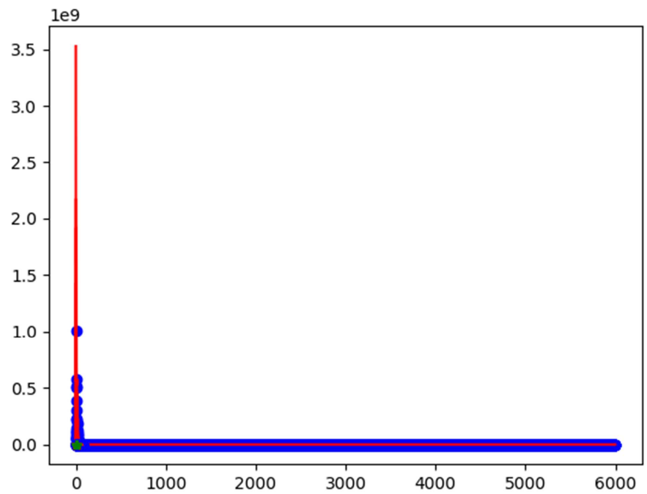


Figure 6. *This image describes the current weight on the training and test data set (dependent variables). It also defines the deep model structure of the datasetusing over 2000 epoch for smoothing the network curve to obtain the best fitting.*

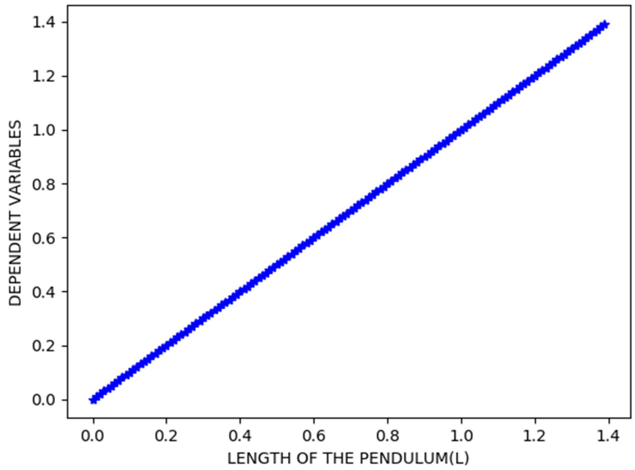


Figure 7. *This graph describes the result of the trained data set which has learnt from previous models at varying iterations of the epoch.*

The image in Figure 7 explicitly shows that the regression model is continuous and for any input we can get an output, that if the length of the bulb is directly proportional to other dependent variables. This means there is a linear relationship between length and other dependent variables. The trend line feature to draw a smooth curve that best fits our input data.

Mathematically

If we calculate using the obtained values we would get a straight line graph verifying that length of the pendulum is an independent variable, validating $T^2 \propto L$ and$T \propto L$.

$T^2 = \frac{4\pi^2}{g}$ L is of the form equation of a straight line where,

y = ax + b where$Y = T^2$. $a = \frac{4\pi^2}{g}$while x = l, and b = 0.

5. Conclusion

The linear regression model which is a linear approach to modelling the relation between a scalar response and one or more explanatory variables with the RELU activation function has proven the linear relationship that validating $T^2 \propto L$and$T \propto L$, both numerically and in simulation.

Its therefore been observed that the magnitude of values obtained in this experiment for length, time-period were in 2,3 significant figure due to the limitation of the online laboratory, if this experiment was done practically would be large considering all other variables such as amplitude, frequency, gravity, damping effect, etc. The above algorithm can be extended to analyze such data-set.

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