

# Genetic algorithm based Finite State Markov Channel modeling

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**Abstract:** Statistical properties of the error sequences produced by fading channels with memory have a strong influence over the performance of high layer protocols and error control codes. Finite State Markov Channel (FSMC) models can represent the temporal correlations of these sequences efficiently and accurately. This paper proposes a simple genetic algorithm (GA) based search for the optimum state transition matrix for a block diagonal Markov model. The burst error statistics of the GA based FSMC model with respect to Autocorrelation Function and error free interval distribution of the original error sequence are presented to validate the proposed method. The superiority of the GA approach over the semi-hidden Markov model (SHMM) based Fritchman model is exhibited in significant improvement of closeness of match and in the usage of shorter length of error sequences. Another Baum-Welch algorithm (BWA) based GA search method has been proposed and compared with the BWA and SHMM methods for the same error sequence. Again the superiority of GA approaches is recognized, especially for the smaller error lengths.

**Keywords:** Genetic Algorithm, Finite State Markov Channel, Semi-Hidden Markov Model, Baum-Welch Algorithm, Autocorrelation Functions, Error-Free Interval Distributions

## 1. Introduction

The current generation networking and communication technology which has to cater to high speed multimedia traffic is strongly dependent on the identification and availability of accurate and tractable models for the mobile/wireless radio fading channels. Fading affects the overall network performance in the physical or media access control layers, and also the designing of efficient and reliable transceivers. The radio link is highly variable due to the statistical distribution of the environmental propagation parameters. Reliable and efficient channel modeling can thus be effective to develop network protocols that mitigate or exploit fading. There are two main approaches to capture the inherent memory in fading channels: the physical (analog) or waveform level and the digital or discrete channel modeling. Physical channel modeling is based on parameters such as received signal strength, noise/interference power, speed of mobile etc. They are more appropriate for design and testing of transmitter-receiver applications and parameter optimization. Discrete Channel Modeling on the other hand places

emphasis on the statistical properties of the bursty error sequence and without modeling the physical functionality of the channel, they can characterize the temporal correlation. The correlated error sequence of fading channels with memory is reproduced by representing the channel with a finite number of states: this is more commonly referred to as Finite State Markov Channel or FSMC modeling [1]. These models have a wide application in the design and performance evaluation of error control coding schemes and higher level wireless communication protocols. In this model, fading is approximated as a discrete-time Markov process with time discretized to a given interval, typically the symbol period. Specifically, the set of all possible fading gains is related to the received signal to noise ratio (SNR) and by partitioning the SNR into a finite number of levels, each interval is associated with a state of a Markov process. The channel varies over these states at each symbol period duration, according to a set of Markov transition probabilities. The FSMC model has evolved from the two states Gilbert Elliot channel [2, 3] representing a time varying binary symmetric channel. The crossover probabilities of the channel are determined by the current state of a discrete time stationary binary Markov process.

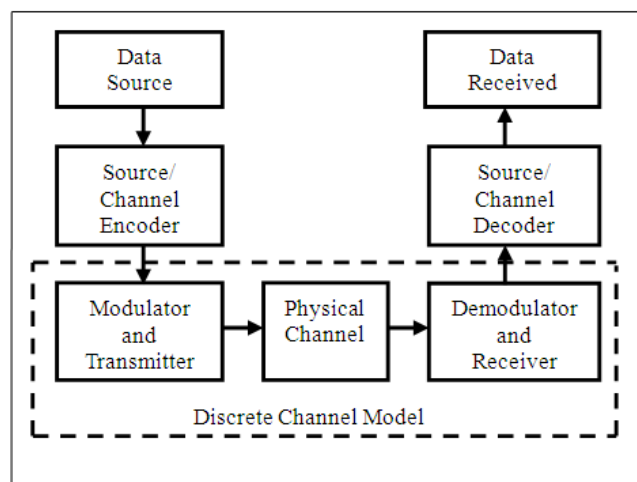
The source has two states: good or no errors and bad or burst errors. Fritchman [4] studied the Markov chains where the outputs are a deterministic function of the states based on the principle of semi-hidden Markov models. As an extension of the Hidden Markov Model (HMM), a hidden semi-Markov model (HSMM) is traditionally defined by allowing the underlying process to be a semi-Markov chain. Each state has a variable duration, which is associated with the number of observations produced while in the state [5]. Fritchman's model has received considerable attention in recent years because it is relatively easy to estimate the parameters of this model from burst error distributions. FSMC and other error modelling approaches and its features such as computational efficiency, etc have been excellently reviewed and discussed in literature [6-10]. HMMs are an important category of generative error models that can accurately represent the statistical patterns of bursty error data in fading channels [11]. The most important and difficult problem in HMMs is to estimate the model parameters that best explains the observations and training is usually performed by an iterative procedure following the Maximum likelihood criteria. Genetic Algorithms, on the other hand, are a powerful computational model having optimization capabilities that can encode a solution on a simple chromosome like data structure using techniques inspired by natural evolution such as inheritance, crossover, mutation and selection. GA's ability of global searching of better maxima without getting trapped into local maxima can offer better and computationally efficient solutions [12-16]. In this context, Genetic Algorithms (GA) for HMM training in general has already been identified to be a promising area mostly in applications related to automatic speech recognition [17], text/web information extraction [18-19], etc. Most of the GA and HMM related papers have used the chromosome structure as the complete set of  $\Gamma = \{A, B, \Pi\}$ , and the chromosomes have been evaluated for the objective function of log likelihood ratio. In the area of Discrete Channel Modeling, however, not much work has been reported on the applicability of Genetic Algorithms for reproducing error sequences following certain statistical characteristics. Zhao Zhi-Jin et al [20] have proposed a hybrid method of using (GA) and simulated annealing (SA) to train HMM for discrete channel modeling. A GA based equalization technique has been proposed [21] with much lower computational complexity for direct sequence ultra-wideband (DS-UWB) wireless communications. The Genetic Algorithm is associated with the RAKE receiver to combat the inter-symbol interference (ISI) due to the frequency selective nature of UWB channels. Another GA based novel technique has been developed to train a hidden Markov model (HMM) with for the cognitive radio channel [22]. This paper has used the idea of HMM and SHMM in proposing new GA based FSMC model which is slightly different from the papers encountered in the literature. The mean square error of the statistical properties of the error sequences has primarily been used as the fitness function in the GA for the search based estimation of the parameters of

the Markov model. Upon comparison of the validation with the analytical HMM/SHMM approach, the proposed GA methods are observed to have some significant improvement towards the closeness of match as well as in the length of training sequences required for a given accuracy.

The rest of the paper is organized as follows. Section 2 discusses the finite state channel model for discrete channel modeling. The GA based FSMC models have been proposed in section 3. The proposed models are based on BWA and SHMM approaches. The simulation results for these two proposed technique have been provided in section 4. The results are compared with the BWA and SHMM methods for the same error sequence. The superiority of GA approaches is recognized, especially for the smaller error lengths and finally conclusions are summarized in Section 5.

## 2. Finite State Channel Model

Discrete or Finite State Channel models are characterized generally by conducting waveform level simulations and/or from measurement data. The output of waveform level simulation is a time series of bit errors that are long enough to represent the statistical properties of the discrete channel. Once a DCM is developed, it eliminates the necessity of further waveform level simulations at high sampling rates. From the modulator input at the transmitter to the output at the receiver, the blocks can be clubbed as a sequence of discrete symbols as shown in Figure 1.



**Figure 1:** Basic Communication system with the Discrete Channel Model.

The relation between the output of the channel encoder at the transmitter and the input of the channel decoder at the receiver in the wireless fading propagation medium can be captured efficiently by using discrete-time Markov sequence to model the errors in the channel. In the sequence, the channel contains several different states and a set of transition probabilities and error probability matrix defining the inter-relationship between the states. Usually, we only know or can observe the input and output of a channel and therefore the error sequence, but the state sequence is not easily observed in a state channel and it is therefore called

“hidden” and the Markov model is called a hidden Markov model (HMM). Both Gilbert and Fritchman are types of hidden Markov models that for large data sets need the powerful Baum Welch algorithm for computation of the HMM parameters [23]. Long error sequences of millions of symbols may be required for accurately estimating the small values of the state transition matrix. The variables in the algorithm are computed for each symbol in the given error sequence, resulting in slow convergence and high computational burden. To improve the efficiency and speed of estimation, several approaches have been adopted. A common one is based on the fact that for a general Markov model there is a statistically equivalent Fritchman like model with  $k$  good states and  $N-k$  bad states [24]. These models also referred as block diagonal Markov models are computationally more efficient as the variables in the estimation algorithms are computed at the beginning of each burst of errors rather than once every symbol. The motivation in this work was to work towards improving the efficiency of estimation of the HMM parameters still further with respect to better characterization of the original error vector and training with shorter length of error sequence.

### 3. The Proposed GA Based FSMC Model

Inspired from biology and based on natural genetics law of the survival of the fittest, GA has already proved to be very capable in many research areas and NP problems. The algorithm can evolve new and better solutions in the expectation of which an implementation of GA based FSMC models are being proposed here to search for the best model parameters for a block-diagonal Markov model. A three state Fritchman model with two good states and one bad state is considered, so that a known error observation matrix, say

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ denotes the third state to be the 'bad' error}$$

producing state. This model is semi hidden in the sense that if an error is produced, it is known that it has been generated by the third state. If however, no error is generated, the state cannot be identified. The implementation of this estimation algorithm based on block equivalent Markov model has been named as SHMM (semi-hidden Markov model) in the results of the current work. The state transition probability

$$\text{matrix } B_{ij}, \text{ of the 3-state channel model } \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix},$$

thus has basically six independent variables ( $P_m = 1\%$ ) to be estimated, as the sum of the rows of the  $A_{ij}$  matrix is unity.

A master error sequence  $E_M$  was made available from a waveform level simulation of an OFDM system. Another GA search method based on Baum-Welch algorithm has been proposed for 3-state channel model, with nine independent

variables  $A_{11}, A_{12}, A_{22}, A_{23}, A_{31}, A_{33}, B_{11}, B_{12}, B_{13}$ , with the sum of rows of state transition matrix  $A_{ij}$  is unity and the sum of columns of error generation matrix  $B_{ij}$  is also unity.

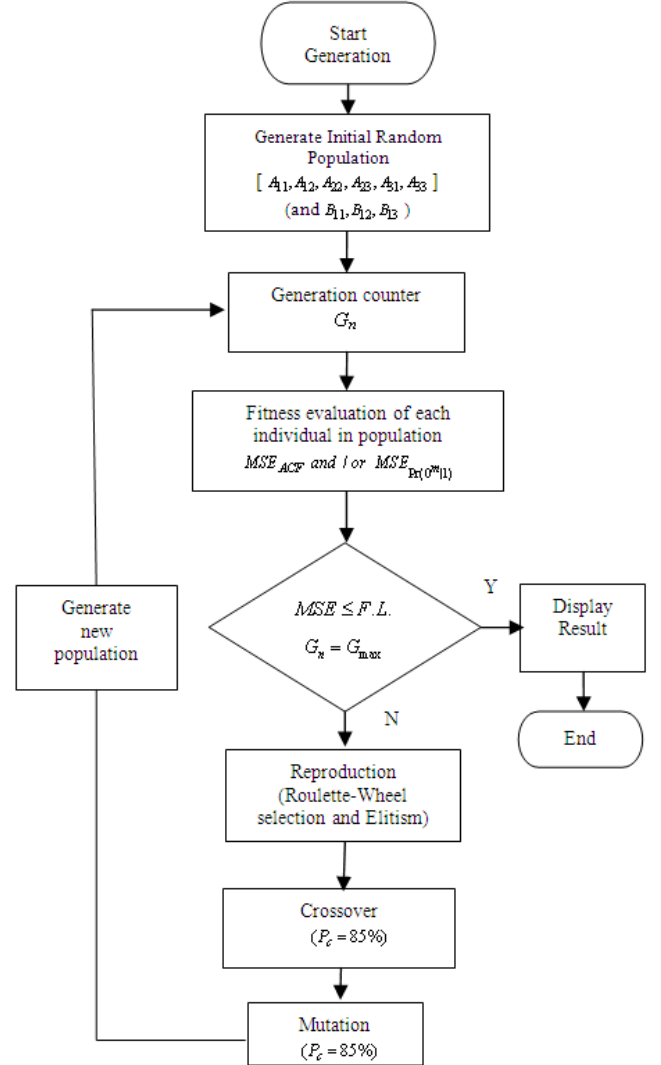


Figure 2: Block diagram of the proposed GA model

The overall block diagram of the proposed GA based model has been illustrated in Figure 2. The GA was initialized with a suitable population size, wherein the independent variables were encoded into a string of real numbers ranging from 0 to 1 on each chromosome. Each chromosome was used to generate an error sequence and the Autocorrelation Function (ACF) comparisons of the original  $E_M$  sequence and the GA generated sequence are used to evaluate the fitness function of the chromosomes. ACF comparison is one of the most popular techniques for performance evaluation of HMM and other FSMC for finding the closeness between the original and DCM generated data. The MSE (Mean Square Error) of the ACFs of the two error sequences are used as a measure of the fitness function of the chromosomes in the proposed GA based FSMC.

$$MSE_{ACF} = \frac{\sum_{i=1}^N (A_{Oi} - A_{Gi})}{N} \quad (1)$$

where,

$N$  = total number of elements of Autocorrelation function,

$A_{Oi}$  = elements of ACFs of the original error sequences,

$A_{Gi}$  = elements of ACFs of the GA-generated error sequences.

The GA is made to minimize this error with a targeted value of  $5 \times 10^{-7}$ . This approach seems to be quite promising as seen from the results of the extensive simulations carried out. The distribution of error-free intervals is also a popular metric for validating the accuracy of DCMs. It is commonly denoted as,  $\Pr(0^m | 1)$ , where,  $m$  is the length of intervals and  $(0^m | 1)$  denotes the event of observing  $m$  or more consecutive errors-free transmission followed by an error. Here both ACF and  $\Pr(0^m | 1)$  have been used for validating the performance of our proposed GA method with the SHMM method. The roulette wheel selection method is capable of effectively eliminating the weaker chromosomes in preference to fitter solutions. The standard genetic operators of cross-over (two point crossovers have been

used) and mutation has been applied. The Elite count denoting the number of individuals that are assured to survive to the next generation is taken as four.

## 4. Results and Discussions

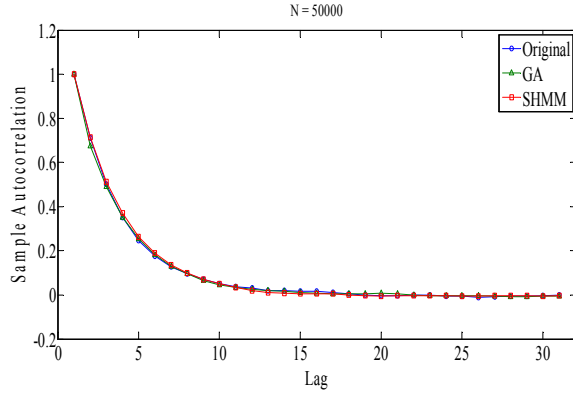
In this section the SHMM and BWA based GA approaches are discussed for channel modeling. Waveform level simulations of three different lengths, 50000, 10000, and 1000 bits of error have been used to estimate the transition probability matrix by the proposed GA approach and then the same has been compared with the popular SHMM technique for discrete channel modelling. The proposed GA algorithm was experimented upon to give a reasonably good match using a crossover fraction of 0.85 and an optimal population size of 30 for the different length of error sequences used. We can consider the smaller population size, but this may lead to missing of global fitness value. While larger population size, will results in increased simulation time. The variations in the values of MSE of ACFs with different population size, and with different crossover probabilities have been shown in Table 1 for  $N = 1000$  and 10000. From the table, it is clear that population size of 30 and crossover fraction of 0.85 is most suitable for implementation of Genetic Algorithm for different length of error sequences.

**Table 1:** Variations of MSE of ACFs with the population size, for a particular value of crossover fraction, for  $N$  (length of error sequence) = 1000, and 10000.

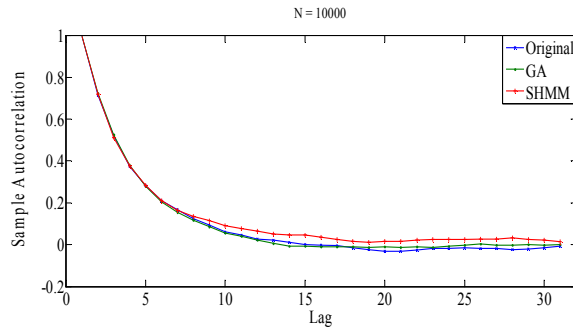
Crossover Fraction →	N = 1000				N = 10000			
Population ↓	0.75	0.80	0.85	0.90	0.75	0.80	0.85	0.90
10	0.0016	0.0014	0.0008	0.0021	0.0440	0.0018	0.0010	0.0018
20	0.0013	0.0012	0.0014	0.0007	0.0341	0.0001	0.0330	0.0001
30	<b>0.0015</b>	<b>0.0017</b>	<b>0.0012</b>	<b>0.0021</b>	<b>0.0023</b>	<b>0.0001</b>	<b>0.0001</b>	<b>0.0095</b>
40	0.0013	0.0013	0.0014	0.0013	0.0214	0.0007	0.0246	0.0002
50	0.0009	0.0013	0.0009	0.0011	0.0026	0.0002	0.0001	0.0003
60	0.0014	0.0012	0.0007	0.0007	0.0271	0.0001	0.0002	0.0001
70	0.0011	0.0012	0.0005	0.0012	0.0001	0.0001	0.0379	0.0001

Figure 3 shows the ACF comparison of original training sequence error data along with that of the GA generated and SHMM generated error data, using a training sequence of length 50,000. Figure 4 presents the similar ACF comparisons for much shorter error sequences of length 10,000. From the figure, the superiority of the GA method over the SHMM method is clearly established as the former is seen to have better matching capability even in the face of a short length error sequence. This is indeed a significant result in context to discrete channel modeling. The statistical parameter  $\Pr(0^m | 1)$ , is also used to judge the performance comparison of GA over SHMM. Figure 5 shows the error-free interval distribution comparisons and again it is

conclusively observed that for lower length of error sequence, GA indeed has better performance in comparison to SHMM. The GA experimentation was repeated with several other error sequences of length 1000, generated arbitrarily from different state transition matrices and when compared to the SHMM method, in general it was found that for shorter sequences GA is capable of producing better match especially with respect to the error free interval distribution. Figure 5 shows one of the simulation results establishing this fact.

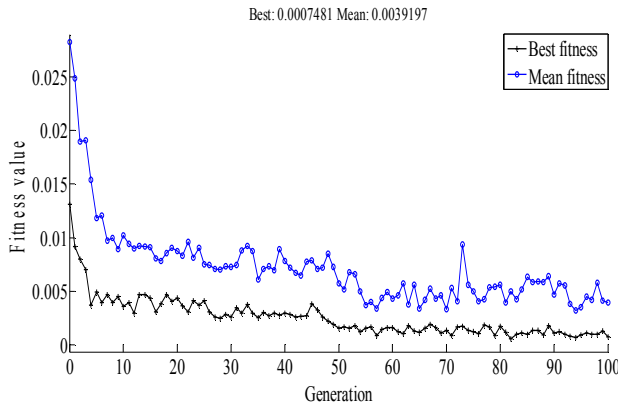


**Figure 3:** ACF comparisons of GA-generated error data and SHMM-generated error sequence with the original channel error statistics for  $N = 50000$ .



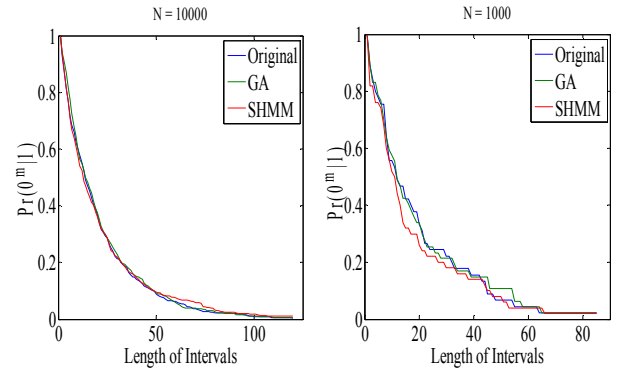
**Figure 4:** ACF comparison of GA and SHMM methods for  $N = 10000$ .

The values of best fitness and mean fitness obtained during the GA experimentations for several generations have been plotted in the Figure 7 to depict the nature of the convergence in successive generations as the algorithm progresses towards the terminating criteria.

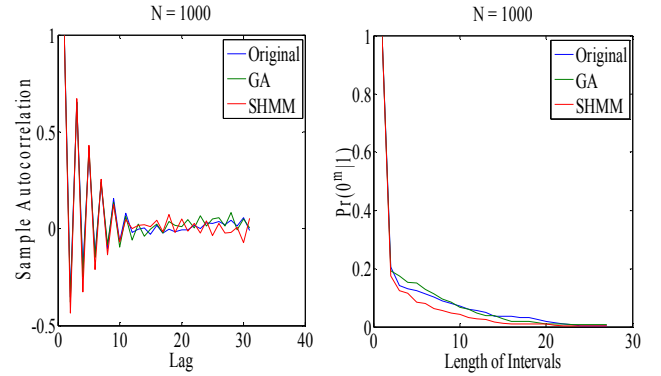


**Figure 7:** A typical nature of the GA performance for a 1000 length error sequence.

The MSE of the autocorrelation functions for different lengths of error sequences have been plotted in Figure 8 to compare the performance of the GA and SHMM methods. It

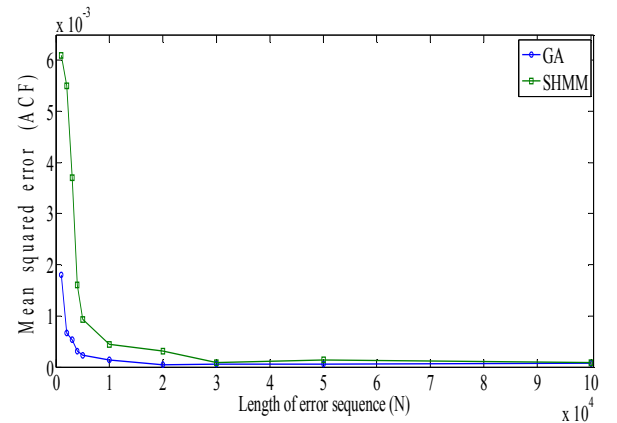


**Figure 5:** Comparison of error-free interval distribution of the original error data with error-free interval distributions of GA-generated and SHMM-generated error sequences, for  $N = 10000$  and  $1000$ .



**Figure 6:** ACFs and error-free interval distribution comparison for arbitrary error sequences with length of error sequence = 1000.

shows clearly that GA performs better in comparison to SHMM for lower lengths of error sequences up to about 30,000 while for larger length of error sequences both approaches have almost similar performance. Table 2 shows the values of MSEs of ACFs and probability of distribution of error-free intervals in testing the superiority of GA method over the SHMM method for different lengths of sequence.



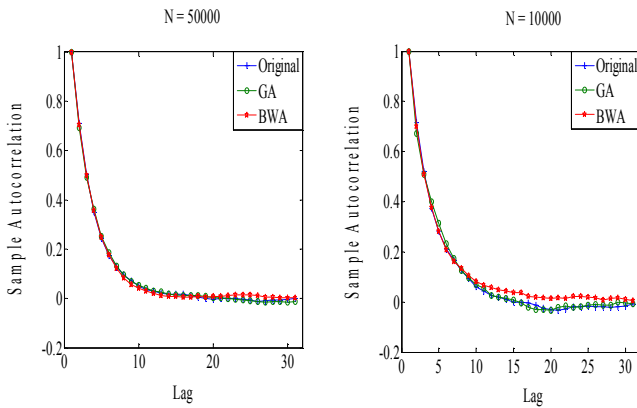
**Figure 8:** MSE of ACF with the Length of error sequences ( $N$ ) for GA and SHMM approaches.



**Table 2:** MSEs of ACFs and Error-free interval distributions for different length sequences.

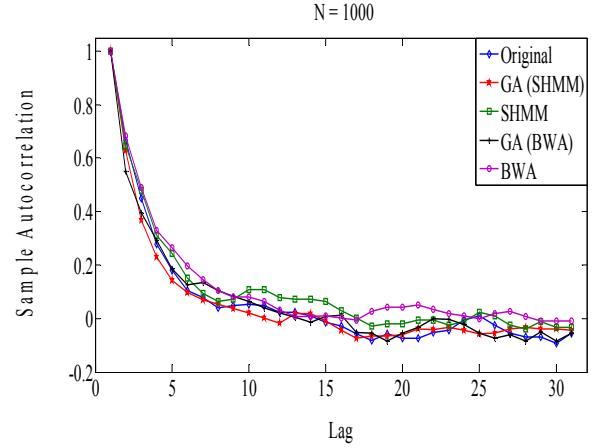
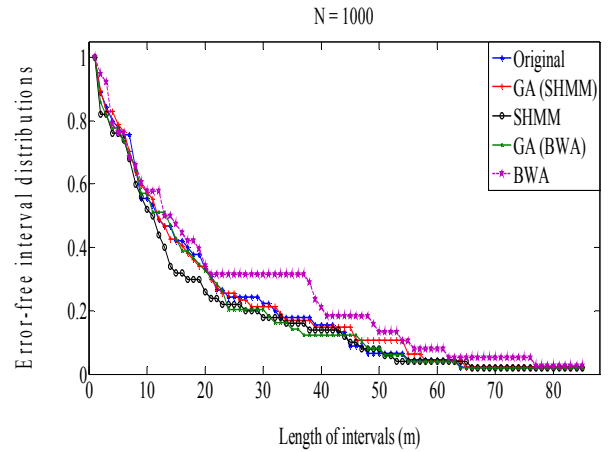
Length of error sequence	MSE of ACF and $\Pr(0^m 1)$					
	50,000		10,000		1000	
	GA	SHMM	GA	SHMM	GA	SHMM
$MSE_{ACF}$	0.000057	0.000063	0.00012	0.0011	0.0010	0.0020
$MSE_{\Pr(0^m 1)}$	0.0000081	0.000019	0.000088	0.00015	0.00034	0.0014

Another BWA based GA approach for the same error statistics has been provided next. The experiments show that GA formulations with the previous values of crossover fraction, population size, elite count, etc. can provide reasonably good fitting. Figure 9 shows the ACF comparison of original error data with the GA-generated and BWA-generated error data of lengths 50000 and 10000. The closeness of the curves establishes the superiority of GA approach over BWA method, especially for smaller lengths of error sequence.

**Figure 9:** Comparison of Autocorrelation function of original error sequence with autocorrelation functions of GA-generated and SHMM-generated error sequences, for the same error data with lengths of 50000 and 10000.

Now the performances of both the proposed GA techniques have been compared with the BWA and SHMM techniques for the smaller lengths of error statistics ( $N = 1000$ ). The autocorrelation function of the original error data has been compared with the ACFs of the BWA-generated, SHMM-generated and GA-generated error data in Figure 10. It illustrates that error sequences generated by both the proposed GA approaches have very close match with the original error data in comparison to the regenerated error data by the existing BWA and SHMM techniques. Figure 11 shows the similar performance comparison in terms of error-free interval distribution and again the

advantage of proposed GA approach for discrete channel modeling has been recognized. Therefore, for the shorter lengths of error data the proposed GA search techniques have significantly improved performance over the BWA and SHMM approaches.

**Figure 10:** Comparison of ACF of original error data with ACFs of GA-generated and SHMM-generated, and BWA-generated error sequences, for length of error data = 1000.**Figure 11:** Comparison of  $\Pr(0^m|1)$  of original error data with  $\Pr(0^m|1)$  of the error data regenerated by proposed GAs, SHMM and BWA for length of error data = 1000.

## 5. Conclusion

Research studies have shown that the performance evaluation of high layer protocols, error control codes, interleavers, voice coders, actually become computationally very expensive using waveform level simulations. Efficiently designed accurate FSCMs that can reproduce the statistical properties of an error sequence can eliminate the necessity of further waveform level simulations at high sampling rates. This paper has proposed new GA based FSMC models for estimating the parameters of the state transition matrix of a block diagonal Markov model. Upon comparison of the validation with SHMM and BWA approach, the proposed method is observed to have some significant improvement towards the closeness of match.

Also, GA performs better in comparison to SHMM and BWA for lower lengths of error sequences, which is a significant result in context to discrete channel modeling. With HMM training, the variance of several random variables or estimated values of DCM is increased by decreasing the length of error sequence, while with the proposed GA method, these variations are very less. The training with shorter sequence is important for DCM, as it reduces the simulation run-time as well as computational burden of a particular training algorithm. Also in order to model the channel error-statistics dynamically/online, one cannot wait for the entire error sequence to be received at receiver and in that case small trace of error sequence plays a significant role in analysis and modeling of channel error-bursts.

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