Teacher Education and Curriculum Studies
2019; 4(3): 50-57
http://www.sciencepublishinggroup.com/j/tecs
doi: 10.11648/j.tecs.20190403.11
ISSN: 2575-498X (Print); ISSN: 2575-4971 (Online)

Evaluation and Thinking of "IF-Problem" of Mathematics for China's College Entrance Examination

Jiguang Yu[1, *], Yuejun Huang[2]

[1]Keqiao High School of Zhejiang, Shaoxing, China
[2]Pingdi Middle School of Shenzhen, Shenzhen, China

Email address:
yushu8488@126.com (Jiguang Yu), szuhyjin@126.com (Yuejun Huang)
[*]Corresponding author

To cite this article:
Jiguang Yu, Yuejun Huang. Evaluation and Thinking of "IF-Problem" of Mathematics for China's College Entrance Examination. *Teacher Education and Curriculum Studies*. Vol. 4, No. 3, 2019, pp. 50-57. doi: 10.11648/j.tecs.20190403.11

Received: July 15, 2019; **Accepted**: August 13, 2019; **Published**: August 26, 2019

Abstract: The Mathematics Proposition of College Entrance Examination is going through a process of deepening reform. According to the requirement of mathematics education for examinee's mathematics literacy, a lot of attempts have been made in the past mathematics teaching and proposition practice. Now the new type of questions is gradually infiltrating into the proposition of mathematics in college entrance examination."IF-Problem" is a type of mathematical problem for China's college entrance examination, It is helpful to improve the shortcomings of the past mathematics evaluation in college entrance examination, to evaluate the innovative consciousness and practical ability of candidates in mathematics thinking, and to infiltrate the core quality of mathematics in mathematics teaching. This essay focuses on researching examples of this new type in terms of its positive, reverse, strategic, and linguistic aspects from evaluation basis, score to the establishment of evaluation criteria, from its structure, its elements, test subject and content to the interpretation of innovative examples, which may pave the way for scientific evaluation of mathematics education.

Keywords: Divergent Thinking, Scientific Evaluation, Structure of Condition, Basis of Evaluation, Innovative Consciousness, Variant Practice

1. Introduction

The problems type aims to realize the test goal so it should be in line with the nature and content of the test. With the renewal of the content of courses and an increase in testing ability, it is necessary to study the corresponding types of problems so as to meet the test requirements [1]. Mathematics for China's college entrance examination is an important evaluation subject. In order to test students' ability of innovation and practice, only by multiple choices, fill in the blanks, and answer problems can no longer meet the requirements. Therefore, to achieve the goal of scientific evaluation on education, we need to introduce or innovate the types of problems. [2]

In terms of "IF-Problem, an examinee is asked to obtain information from such conditions as the known conclusions, properties, and theorems, and to sort out the information, in order to make conclusions or give specific examples that match the given information. There was a trial in the college entrance examination in Shanghai in the spring of 2007: "in the plane rectangular coordinate system of xOy, calculate the distance from the point (1, 2) to the straight line (3x+4y=0)". Try to give a meaningful "reverse" problem on the said problem and solve it. [3]

2. Methodology

The evaluation of mathematics teaching takes the form of proposition as the main body. Therefore, case studies and interpretations of the form of mathematics proposition and examination-oriented training questions in the College Entrance Examination are carried out, and the evaluation is carried out through experimental research methods, which has certain guiding significance for mathematics teaching.

3. Evaluation Basis and Score of IF-Problem

IF-Problem seems to be more like writing a short or long essay in the Chinese subject. In the past, evaluation of a text was done by human subjectively. Now it is partially done by computer. The fixed evaluation rules produce the scientific evaluation. In this way, at least two persons will attend the evaluation and the difference between the scores cannot exceed 2 points, otherwise there will be the third person, or the fourth final reviewer, which is similar to the regular evaluation.

3.1. Evaluation Purpose

With the changes in the training indicators of modern mathematics education, in order to detect students' potential to be innovative, students can not only solve problems in a passive way, but also propose valuable mathematical problems with mathematical significance and innovation. Therefore, it is more worthwhile to introduce IF-Problem. [4]

3.2. Evaluation Basis

How to evaluate the questions raised by students? The difficulty lies in evaluating whether the mathematical "meaning" of a candidate's question is achieved by mathematical thinking and reflecting the nature of mathematics. And what is a "meaningful" mathematical problem? How to judge them? How to design the standard of judgments?

3.2.1. Accuracy of a Problem - The Accuracy of the Narrative Language

To make a problem clear, especially a mathematical problem, it is necessary to highlight the accuracy of the narrative language. It is impossible to propose a "meaningful" and "valuable" question with an unclear logic or a confusing language. How to accurately use mathematical symbols and impart meaning to them? Neither can a problem be made clear without clear understanding of the mathematical concepts.

3.2.2. The Simplicity of a Problem - The Simplicity of the Narrative Language

If a problem can be explained with 30 words, do not use one more word. To illustrate a mathematical problem simply can best reflect the "beauty" of mathematics and the mathematical nature of the problem. More importantly, it can better show one's mathematical thinking.

3.2.3. The Thinking of a Problem - The Breadth and Depth of Thinking

Whether a problem is meaningful or not largely depends on the breadth and depth of thinking of the problem, namely, the depth of mathematical thinking, the depth of underlying conditions and the breadth of solving the problem- the variety of the methods.

3.2.4. The Innovation of a Problem - The Angle of Posing a Problem

The "edge" of a problem is fully reflected in its innovation. Do you raise a question from a novel perspective? People will be inspired when they read it. [5]

3.3. Evaluation Rules

According to SOLO (Structure of the Observed Learning Outcome), the evaluation rules are formulated for the solution to a problem according to the multiple scoring evaluation method.

3.4. Evaluation Score

According to the reality of basic education, the scores of such innovative questions are controlled at around 10 points and their difficulty is controlled in the middle range.

4. The Structure and Elements of IF-Problem

The design direction of IF-Problem is quite extensive. For example, students are asked to propose a reverse math problem like what mentioned above, or to propose a positive mathematical problem with the given constraints, or to propose a problem in an actual context with the given mathematical conclusion or mathematical model. The key point of this problem is that an examinee can fully describe a specific problem of mathematical thinking and mathematical connotation.

4.1. The Structure of IF-Problem

In the case of giving a reverse problem, a positive problem must be provided as a guide to promote the innovative thinking; the other is to provide some information or conditions directly, requiring students to compile a solvable mathematical problem with research value and solve it. There is also a type that connects theory with reality, where an examinee is asked to create a mathematical problem by combining his own learning and life experience with the given theoretical background or mathematical models. This is a bit like writing a proposition essay.

4.2. The Elements of IF-Problem

A problem can be given high scores if it meets the given and the following conditions:

a) Its answer covers all the aspects; and,
b) It is accurately illustrated; and,
c) It is creative.

4.3. The Attribute of Capability

IF-Problem shows a student's comprehensive ability to think, describe and solve a problem. The ability to raise valuable questions with the given conditions within a short time is not obtained from a day's work but days' work. Besides, the ability to use a language is required to accurately and completely describe a problem. It is even harder as the problem raised by one should be solved by himself. This is not just to solve a problem.

5. The Test Subject and Content of IF-Problem

Opinions will vary from person to person at the beginning when IF-Problem is introduced to China's college entrance examination or the academic test for the ordinary high school students. As the test progresses and perfects, and the mathematics classroom teaching is followed, people will gradually realize its role - proposing a new meaningful mathematical problem is more important than just solving a math problem. [6]

5.1. Test Subject

Students who have studied high school mathematics can be tested. It is also feasible for the academic test or the college entrance examination.

5.2. Important Content

The key point of IF-Problem is "to give a problem", so the key content of the initial stage of a test is the mathematics knowledge which is relatively intuitive, not so abstract, nor too difficult to raise a question. In this connection, students of different levels of mathematics can raise different levels of questions and solve them.

5.3. Interpretation of Examples

5.3.1. An Example of Reverse Problems

Example1 Let ⊙C: $x^2+(y-1)^2=5$, and the line $l: mx-y+1-m=0$, if $m=\sqrt{3}$, calculate the length of the cord intercepted by ⊙C with the line l.

Solve this problem;

Propose a meaningful converse problem to this problem and solve it.

We found that students produced many meaningful questions

Case 1 (About area) Let ⊙C: $x^2+(y-1)^2=5$, and the line $l: mx-y+1-m=0$, AB is the cord intercepted by C with the line., its length is $\sqrt{17}$, the midpoint of AB is P, calculate $S_{\Delta APC} : S_{\Delta AOB}$

Case 2 (About minimum or maximum of the area) Let ⊙C: $x^2+(y-a)^2=b^2$, and the line $l: mx-y+1-m=0$, AB is the cord intercepted by C with the line., its length is $\sqrt{17}$, calculate the minimum area of ⊙C.

Case 3 (the minimum or maximum) Let ⊙C: $x^2+(y-1)^2=5$, and the line l, AB is the cord intercepted by ⊙C with the line., its length is $\sqrt{17}$, when the length of the cord intercepted by the line and y-line has minimum, find the linear equation of l,

Case 4 (About the track)Let ⊙C: $x^2+(y-1)^2=5$, and the line l, AB is the cord intercepted by ⊙C with the line., its length is $\sqrt{17}$, the midpoint of AB is P, find the equation of P and the difference of the area with the image P and ⊙C.

Interpretation: The given condition is the positional relationship between a straight line and a circle. The four cases involve a wide range of aspects. The nature of the problem is profoundly understood after the original problem is solved, then ask such a question though reverse thinking. This requires the ability obtained from study of mathematical research.

Evaluation rules: (I) According to SOLO, the solution process is divided into parts with the corresponding score;

(II) According to the multiple scoring evaluation method, it is divided into three levels: in the first-level, the problem is described completely, but it is easy to solve it without much thinking; in the second-level, the problem is described completely and it requires much thinking to do so. But it does not have any obvious edge; in the third-level, the problem is described completely and it requires much more thinking to do so. What's more, it has an obvious edge. According to the said levels, Case 1, 2 can be judged as the second-level, Case 3, 4 may be as the third-level. In the actual situation, as it is reviewed by a computer, it can be confirmed through discussion if the difference is large.

5.3.2. An Example of Positive Problems

Example 2 As shown in the figure 1, AC is the diameter of C, B is a point of ⊙C which does not coincide with A and C.PA is perpendicular to the plane created by ⊙C, connect PB, PC, AB, BC, AN⊥PB on B, AS⊥PC on S, connect SN. [7]

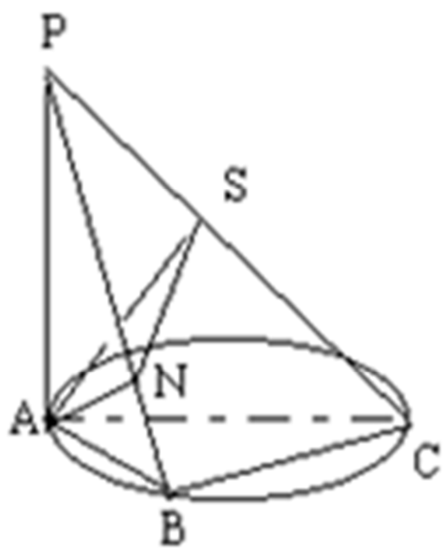

Figure 1. *An example of positive problems.*

Proof: plane ANS plane PBC

If the length of the diameter of ⊙C is 1, and PCA=45°, please take an example about "the angle created by two different surface straight lines or planes, dihedral angel", then solve it.

Students give many valuable questions:

Case 1 if $\angle$ACB=30°, calculate the cosine of angle which created by PB and AC;

Case 2 if $\angle$ACB=30°, calculate the sine of angle which created by PB and plane PAC;

Case 3 Let B is a point of $\odot$C, calculate the maximum of the sine of angle which created by PB and plane PAC;

Case 4 if $\angle$ACB=30°, calculate the sine of the dihedral angel A-PC-B;

Case 5 if $\angle$ACB=30°, calculate the cosine of the dihedral angel which created by plane ASN and plane PAB.

Interpretation: It is not difficult to raise a question according to the background of this question. But it is difficult to raise a question with difficulty in thinking. Among the said five cases, Case 3 is the most challenging as the problem is relatively difficult, which needs much knowledge. Also, there are drawings, proofs, calculations which include trigonometric functions and basic inequalities. The other four are basically similar to regular problems students solve.

Let BD$\perp$AC, connect PD

$$\left.\begin{matrix} PA\perp planeABC \\ PA\subset planePAC \end{matrix}\right\} \Rightarrow \left.\begin{matrix} planePAC\perp planeABC \\ planePAC\cap planeABC=AC \\ BD\perp AC \end{matrix}\right\} \Rightarrow \text{BD plane PAC}$$

Hence, BPD is the angle which created by PB and plane ABC, denoted by θ,

Let $\angle ACB=\alpha$, $0<\alpha<\frac{\pi}{2}$,

We have BD=$2\cos\alpha\sin\alpha$, $\angle$PCA=45°,

$$PB=\sqrt{AB^2+PA^2}=2\sqrt{1+\sin^2\alpha}$$

$$\sin\theta=\frac{BD}{PB}=\frac{\sin\alpha\cos\alpha}{\sqrt{1+\sin^2\alpha}}=\sqrt{\frac{\sin^2\alpha(1-\sin^2\alpha)}{1+\sin^2\alpha}}$$

$$=\sqrt{-(1+\sin^2\alpha)-\frac{2}{1+\sin^2\alpha}+3}\leqslant\sqrt{3-2\sqrt{2}}=\sqrt{2}-1$$

If and only if $\sin^2\alpha+1=\frac{2}{\sin^2\alpha+1}$, when $\sin^2\alpha=\sqrt{2}-1$, $PB=\sqrt{2}-1$.

Hence the maximum of $\sin\theta$ is $\sqrt{2}-1$.

When Example 2 is designed, IF-Problem is limited to a classic figure (a triangular pyramid), which not only allows for the students' divergent thinking, but also enables the problem to be judged due to limitations. This should be focused on in designing IF-Problem.

Evaluation rules: (I) According to SOLO, the solution process is divided into parts with the corresponding score;(II) According to the multiple scoring evaluation method, it is divided into three levels: in the first-level, the problem is described completely, but it is easy to solve it without much thinking; in the second-level, the problem is described completely and it requires much thinking to do so. But it doe not have any obvious edge; in the third-level, the problem is described completely and it requires much thinking to do so. What’s more, it has an obvious edge. According to the said levels, Case 1, 2 can be judged as the first-level, Case 3 may be as the third-level, Case 4, 5 as the second-level. In the actual situation, as it is reviewed by a computer, it can be confirmed through discussions if the difference is large. [8]

5.3.3. An Example of Strategic Problems

Example 3: There is a “WeChat Base Station” tower near the school. It is required to measure the height of the tower and the horizontal distance from the tower to the tester with a tape measure, a set square and a protractor on the school field. Give an example of your measurement process, calculation method and results. [9]

Case 1: choose three points P, Q, R on the playground’s racetrack (P, Q, R are on the same line), PR is the measured data, Q is the midpoint of PR, the elevation angle of AB are α,β,γ, sketch is mapped by the measured data (as shown in the figure 2), calculate the height of AB and the length of PA.

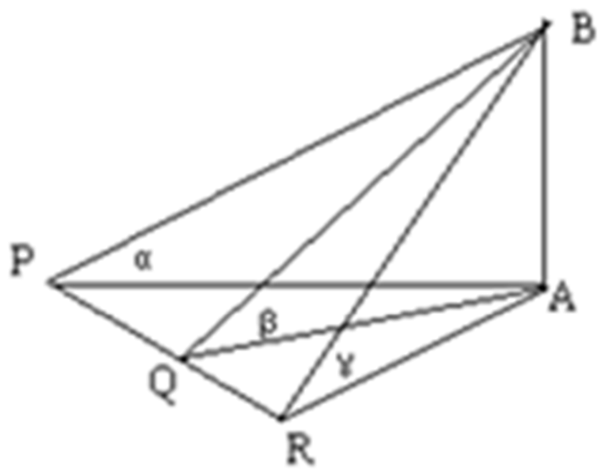

Figure 2. *An example of strategic problems (Case 1).*

Let AB=x, PA=y, then y=$\frac{x}{\tan\alpha}$, $AQ=\frac{x}{\tan\beta}$, $AR=\frac{x}{\tan\gamma}$ in ΔAPR, according to the center line Formula, we have

$$AQ=\sqrt{\frac{PA^2+AR^2-\frac{PR^2}{2}}{2}}$$

Input the data, hence

$$x=\frac{PR}{\sqrt{\frac{2}{\tan^2\alpha}-\frac{4}{\tan^2\beta}+\frac{2}{\tan^2\gamma}}},\quad y=\frac{\frac{PR}{\tan\alpha}}{\sqrt{\frac{2}{\tan^2\alpha}-\frac{4}{\tan^2\beta}+\frac{2}{\tan^2\gamma}}}$$

Case 2: As shown in the figure 3, P is on the playground, we measure the length of PC and PQ, the elevation angle are α,β. Let $AB=x$, $PA=y$, we have

$$x=PC+y\tan\alpha,\quad x-PC=(y+PC)\tan\beta$$

Hence $x=PC+\frac{PQ\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$, $y=\frac{PQ\tan\beta}{\tan\alpha-\tan\beta}$.

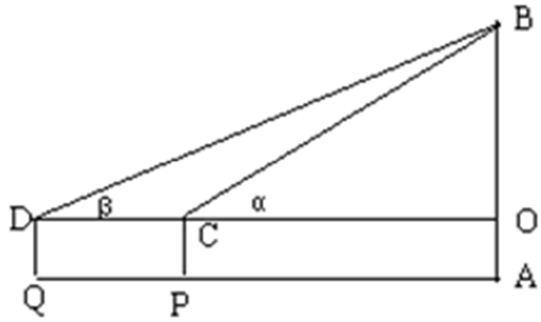

Figure 3. *An example of strategic problems (Case 2).*

Case 3: if the building stands on P (as shown in the figure 4), we can choose two measuring position C, D, which on a perpendicular line, measure the length of PC and CD, α, β ,

Let $AB = x$, $PA = y$, we have

$$\frac{x - PC}{y} = \tan\alpha, \quad \frac{x - PC - CD}{y} = \tan\beta$$

hence, $x = PC + \dfrac{CD\tan\alpha}{\tan\alpha - \tan\beta}$, $y = \dfrac{CD}{\tan\alpha - \tan\beta}$

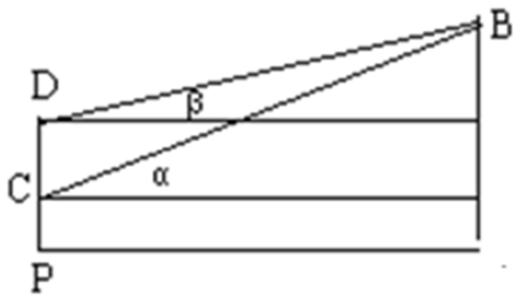

Figure 4. *An example of strategic problems (Case 3).*

Case 4: Party A with the height of 1.65 meters is moving on a straight line (OC), and Party B is crawling at point B with his eyes looking at the top of the tower. When the top of the tower that Party B sees coincides with the top of Party A's head, immediately set Party A to point E. At this time, Party C measures the linear distance between the top of Party A's head (D1) and Party B's eyes (B) and Party B is crawling at point C. When Party A is at point F, the top of the tower coincides with the top of Party A's head, and Party C measures the length of CD and BC, and finally calculates the height of the tower height. As shown in the figure 5

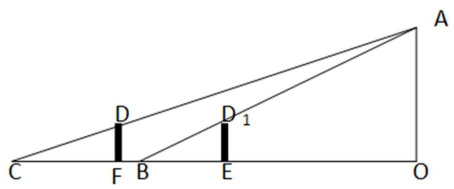

Figure 5. *An example of strategic problems (Case 4).*

Interpretation: The first three cases are the basic methods introduced in the mathematics textbook "Solution of Triangles". Case 4 only uses a measuring tape to complete this activity by skillfully utilizing the height of the person. Three people work together and benefit by mutual discussion. Despite no specific results, the algorithm is simple which fully reflects the students' creative thinking.

Evaluation rules: According to the multiple scoring evaluation method, it is divided into three levels: in the first-level, the problem is described completely, but it is easy to solve it without much thinking; in the second-level, the problem is described completely and it requires much thinking to do so. But it does not have any obvious edge; in the third-level, the problem is described completely and it requires much thinking to do so. What's more, it has an obvious edge. According to the said levels, Case 1, 2 can be judged as the first-level, Case 3 may be as the second-level,

Case 4 as the third-level if a complete result is given. In the actual situation, as it is reviewed by a computer, it can be confirmed through discussion if the difference is large.

5.3.4. An Example of a Problem with Mathematical Language

In mathematical science, the mathematical languages often include mathematical words, mathematical symbols, and mathematical graphic. Mathematical words have the unique nature of mathematics--relational attributes. For example, the mathematical words "pairwise" refers to a kind of relationship among three or more things, "pairwise intersect" "pairwise mutually exclusive" "pairwise perpendicular", such as

Example 4 $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors, which are perpendicular to one another, if the vector $\vec{d}$ satisfies $(\vec{a} - \vec{d})(\vec{b} - \vec{d}) = 0$, $(\vec{a} - \vec{d})(\vec{c} - \vec{d}) = 0$, $(\vec{c} - \vec{d})(\vec{b} - \vec{d}) = 0$, then the maximum of $|\vec{d}|$ is"

This condition includes "are perpendicular to one another". Please try to take a valuable question, which must include "one another", solve it or explain it.

The following are various questions that may have mathematical significance:

Case 1: there are three lines that parallel to one another, A plane can be determined by choosing two straight lines. How many planes can these three straight lines determine?

Explain: Three lines that parallel to one another, the position relationship among three lines can be shown in the following figure 6 and figure 7.

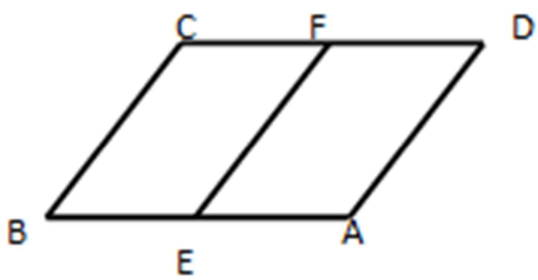

Figure 6. *Three parallel lines are coplanar.*

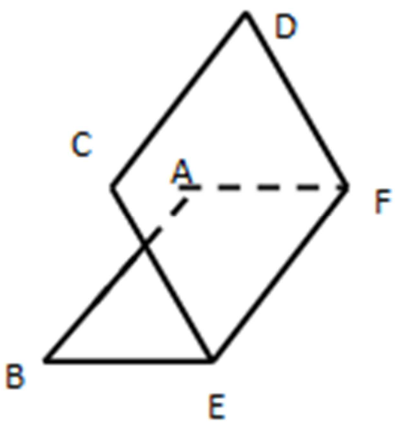

Figure 7. Three parallel lines are not coplanar.

Case 2: There are three lines that intersect one another, A plane can be determined by choosing two straight lines. How many planes can these three straight lines determine?

Explain: Three lines that intersect one another, according to the Plane Axiom 2, we know that these lines determine a plane.

Case 3: there are three lines that are perpendicular to one another, A plane can be determined by choosing two straight lines. How many planes can these three straight lines determine?

Explain: Three lines are perpendicular to one another; It is impossible for three straight lines to be in one plane. There are two kinds of situations: concurrent or un concurrent. The concurrent situation, such as the model of "Corner", The un concurrent situation, which we can find easily in Cube.

Case 4: There are three lines that intersect one another, How many lines do they intersect? Please draft a map to show your conclusion.

Explain: Three lines that intersect one another, the position relationship can be shown in the following figures (figure 8, figure 9 and figure 10).

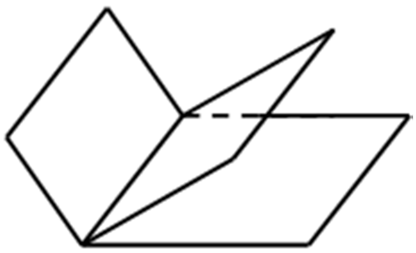

Figure 8. Intersect on an intersection line.

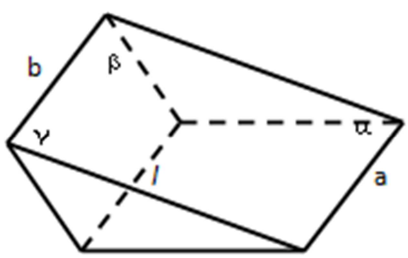

Figure 9. Three intersecting lines.

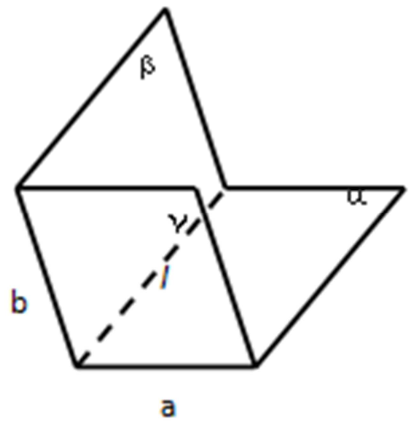

Figure 10. Three intersecting lines.

Case 5: There are three planes that intersect one another, and the angles created y these planes are equal, calculating these angles.

Explain: As the shown figures, three planes that intersect one another, and the angles created by these planes are equal, there are three kinds of situations. Intersect on an intersection line, the degree of the angle is 120°; Three intersecting lines, the degree of the angle is 60°, and the third figure, the degree of the angle is 90°.

Case 6: There are three planes that intersect one another, how many parts can they divide space into?

Explain: the best models of three planes intersect one another is the model of "Corner", As shown on the right.

Case 7: [10] Let $f(x)$、$g(x)$、$h(x)$ be three functions which their domain is R, the following propositions:

(1) If the addition one another of three functions are increasing function, then there are at least one increasing function in $f(x)$, $g(x)$, $h(x)$.

(2) If the addition one another of three functions are periodic function with length T, then $f(x)$、$g(x)$、$h(x)$ are periodic function. which of the following propositions is correct? A, (1) and (2) are correct B, (1) and (2) are incorrect C, (1) is correct, (2) is incorrect D, (1) is incorrect, (2) is correct:

Explain: $f(x)=\frac{1}{2}(f(x)+g(x))+\frac{(f(x)+h(x))-(g(x)+h(x))}{2}$ must be a periodic function with length T, hence (2) is correct; the difference of two increasing function is not necessarily increasing function, hence (1) is not necessarily true. The option D is correct.

As is well known, mathematics is a discipline that studies "spatial form" and "quantity relationship". The quantitative relationship is simply referred to as "number", and the spatial form as "form"; the objects of "number" include natural numbers, complex numbers, vectors, matrices, functions and probability; objects of "shape" include curves, graphs, spaces and manifolds. If the mathematical research object is only summarized by "number" and "shape", something cannot be summarized, such as mathematical languages and various computer languages which are based on mathematical principles. Can these languages be "form" or "number"? They belong to neither of them. Within the discipline of mathematics, the mathematical languages often occur including mathematical words, mathematical symbols, and mathematical graphic. Mathematical words have the unique nature of mathematics--relational attributes. For example, the mathematical words "two-two" refers to a kind of relationship among three or more things, "pairwise intersect" "pairwise mutually exclusive" and "pairwise perpendicular". When teaching the students in daily life, they often ask about the meaning of "pairwise XX". It can be seen that students do not clearly understand such mathematical languages. There are problems with their understanding which should be paid attention to in mathematics teaching.

5.4. (The Latest Research Progress) The Way to Train Students to Break Through "IF-Problem"

By 2021, Zhejiang Province will take the lead in the new college entrance examination where new types of problems such as "multiple choice questions", "IF-Problem" and "logical questions" will be introduced [11]. In order to adapt to this change, mathematics teaching will surely have a new look. To break through "IF-Problem" in the mathematics of college entrance examination, such abilities should be cultivated in the usual teaching process: To stimulate students' enthusiasm about creation; to cultivate their ability to express their thoughts in mathematics; to enrich their practice in giving "examples" in mathematics.[12]

5.4.1. In Mathematical Modeling, Students Should Be Guided to Present Examples of Mathematical Models

Similar to the mathematical modeling in Example 3, the students propose some innovative ideas and methods to solve the problems in the process of modeling by thinking in the activity. The ideas are proposed after improvement in mathematical learning;

5.4.2. Students Should Be Guided to Propose Extensions and Variants of Related Problems

After the college entrance examination every year, some highlights will be found in the propositions of mathematics experts. These highlights will be expanded, allowing students to ask new questions or give some counterexamples. Teachers will evaluate them according to the said evaluation criteria. [13]

Example 5 (2019 the college entrance examination of Zhejiang) The length of a square ABCD is 1. when each λ_i ($i=1,2,3,4,5,6$) take over ± 1 , the maximum of $|\lambda_1\overrightarrow{AB}+\lambda_2\overrightarrow{BC}+\lambda_3\overrightarrow{CD}+\lambda_4\overrightarrow{DA}+\lambda_5\overrightarrow{AC}+\lambda_6\overrightarrow{BD}|$ is, the minimum is.

The general solution to this problem is to use the data analysis method. Because the calculation process involves a large amount of data calculation, it tests the students' perseverance and patience, care and self-confidence, and these are to cultivate their basic abilities of scientific research. After solving the problem, students ask many new questions and solve them with data analysis.

Case 1: Isosceles triangle $Rt\Delta ABC$, the length of the opposite side is 1, when each λ_i ($i=1,2,3$) take over ± 1, the maximum of $|\lambda_1\overrightarrow{AB}+\lambda_2\overrightarrow{BC}+\lambda_3\overrightarrow{AC}|$ is, the minimum is.

Case 2: Equilateral triangle ΔABC , the length of its side is 1, when each λ_i ($i=1,2,3$) take over ± 1, the maximum of $|\lambda_1\overrightarrow{AB}+\lambda_2\overrightarrow{BC}+\lambda_3\overrightarrow{AC}|$ is, the minimum is.

Case 3: Regular hexagon $A_1A_2A_3A_4A_5A_6$, the length of its side is 1, when each λ_i ($i=1,2,3,4,5,6$) take over ± 1 , the maximum of $|\lambda_1\overrightarrow{A_1A_2}+\lambda_2\overrightarrow{A_2A_3}+\lambda_3\overrightarrow{A_3A_4}+\lambda_4\overrightarrow{A_4A_5}+\lambda_5\overrightarrow{A_5A_6}+\lambda_6\overrightarrow{A_6A_1}|$ is, the minimum is.

Case 4: Tetrahedron $A_1A_2A_3A_4$, the length of its edge is 1, when each λ_i ($i=1,2,3,4,5,6$) take over ± 1 , the maximum of $|\lambda_1\overrightarrow{A_1A_2}+\lambda_2\overrightarrow{A_1A_3}+\lambda_3\overrightarrow{A_1A_4}+\lambda_4\overrightarrow{A_2A_3}+\lambda_5\overrightarrow{A_2A_4}+\lambda_6\overrightarrow{A_3A_4}|$ is, the minimum is.

Case 5: Tetrahedron with right angles denoted by $A_1A_2A_3A_4$, A_1A_2, A_1A_3, A_1A_4 are perpendicular to one another, its length is 1, when each λ_i ($i=1,2,3,4,5,6$) takes either 1 or -1 , the maximum of $|\lambda_1\overrightarrow{A_1A_2}+\lambda_2\overrightarrow{A_1A_3}+\lambda_3\overrightarrow{A_1A_4}+\lambda_4\overrightarrow{A_2A_3}+\lambda_5\overrightarrow{A_2A_4}+\lambda_6\overrightarrow{A_3A_4}|$ is, the minimum is.

5.4.3. Students Should Be Guided to Present New Examples in a Micro-course

In teaching basic inequality of a micro-course, to teach the double hook function through variation and theory-practice combination in order to overcome the difficulties. [14]. A teacher designs the structure of a course while the students supplement the examples. In this way, both the teacher and the students will find it fun to "give examples" in teaching.

Example 6 [15] Discuss the monotonicity and the extremum property of function $y=x+\frac{1}{x}$ $(x>0)$.

Variant 1 (variant based on definition) if $x<0$, What are the properties of $y=x+\frac{1}{x}$?

Variant 2 (variant based on operation) What are the properties of $y=x-\frac{1}{x}$?

Variant 3 (variant based on constant) What are the properties of $y=x+\frac{a}{x}$ $(x>0,\ a>0)$?

Variant 4 (variant based on exponentiation) What are the properties of $y=x^2+\frac{a}{x^2}(a>0)$?

Variant 5 More generally, what are the properties of $y=x^n+\frac{a}{x^n}(a>0, n\in N^*)$?

Variant 6 Discuss the extremum property of function y= $\sin x+\frac{a}{\sin x}$, $x\neq k\pi$.

Variant 7 The function $y=x+\frac{a}{x}$ has the following properties: if $a>0$, then it is a decreasing function on

$[0,\sqrt{a}]$ and increasing function on $[\sqrt{a},+\infty)$.

If the range of $y = x + \frac{2^b}{x}$ (x>0) is $[6,+\infty)$, find the value of b.

Discuss the monotonicity of $y = x^2 + \frac{c}{x^2}$ (c is a constant, and c>0).

Give the extensions of $y = x + \frac{a}{x}$ and $y = x^2 + \frac{a}{x^2}$ (a is a constant, and a>0), such that these functions are the special cases of functions that you generalize. Discuss the monotonicity of functions that you generalize. Find the maximum and minimum of $F(x) = (x^2 + \frac{1}{x})^n + (x + \frac{1}{x^2})^n$ (n is positive integer) on the domain $[\frac{1}{2}, 2]$.

Teachers give evaluations according to the quality of students' examples in class. Students continue to improve after the evaluations.

6. Conclusion

On the basis of summarizing mathematics teaching practice and proposition research thinking, we believe that " IF-Problem " can help to improve the shortcomings in the past mathematics evaluation of college entrance examination, help to evaluate the innovative consciousness and practical ability of candidates in mathematics thinking, help to infiltrate the core literacy of mathematics in mathematics teaching and help to cultivate students' core literacy of mathematics through variant teaching. IF-Problem is one of the propositions for evaluating students' core literacy of mathematics, which can be nurtured by means of mathematical variant teaching, it will play an important role in guiding the proposition of mathematics in college entrance examination.

Acknowledgements

The authors would like to think the editor and the anonymous reviews for their in-depth comments, suggestions, and corrections on this paper.

References

[1] Jiguang Yu, Choosing Fill-in and Its Educational Function [J], Middle School Mathematics Research, 2001 (5): 35-37.

[2] ZiZhao Ren, Jianshi Zhang, Ang Chen, Research on New Mathematics Question Type Test in College Entrance Examination [J], Journal of Mathematics Education, 2015 (24-1): 21-25.

[3] Jiguang Yu, Learn to ask a meaningful mathematical question [J], Middle School Mathematics Teaching, 2007 (10): 24-25.

[4] Jianyue Zhang, Pursuing the Nature of Mathematics Classroom [J], Mathematics Education in China, 2009 (4): 2-5.

[5] Jingqiu Xie, Practice of Classroom Structure in Open Teaching of Mathematics in Senior High School [J], Mathematics Education in China, 2017 (12): 15-17.

[6] Xiaojua Zhang, Jiguang Yu, Nurturing Middle School Students'Mathematical Abstraction Literacy [J], Mathematical Communication, 2016 (7): 4-8.

[7] Jiguang Yu. Teaching Cases of Solid Geometry in a Space Model. Mathematics Teaching, 2003, Issue 4: 11-14.

[8] Jiguang Yu, Let students from "have" to "more" teaching ideas [J], Mathematical Communication, 2018 (1): 17-20.

[9] Jiguang Yu. Value and Design of Internship Education in Mathematics of New Curriculum. Mathematics Bulletin, 2010, Issue 12: 23-27.

[10] Xuejun Pan, Two Case Studies on Functional Inquiry Assignments [J], Mathematics Teaching, 2009 (4): 27-30.

[11] Jiguang Yu, Thoughts on the Proposition of "Multiple Choices" in Mathematics for College Entrance Examination [J], 2018 (4): 56-58.

[12] Xiaoyu He, Jiguang Yu, Thoughts on "Chewing Questions" after Solving a Problem [J], Mathematical Communication, 2016 (12): 17-20.

[13] Jiguang Yu, Question Series, Variant Series, Solution Series——Probe into Mathematics Teaching Model of New Course [J], Mathematics in Primary and Secondary Schools (Senior High School), 2011 (1-2): 34-38.

[14] Xiaoyu He, Jiguang Yu, Focus on "Intelligence Points" in Mathematical Operations [J], Mathematics Teaching, 2015 (9): 27-29.

[15] Jiguang Yu, Structure and Design Strategy of Mathematical Variant Problem Strings [J], Mathematics Education in China, 2012 (1-2): 40-42.