

New conceptions of transitivity and minimal mappings

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Abstract: The concepts of topological δ -transitive maps, α -type transitive maps, δ -minimal and α -minimal mappings were introduced by M. Nokhas Murad Kaki. In this paper, the relationship between two different notions of transitive maps, namely topological δ -type transitive maps and topological α -type transitive maps has been studied and some of their properties in two topological spaces (X, τ^δ) and (X, τ^α) , τ^δ denotes the δ -topology (resp. τ^α denotes the α -topology) of a given topological space (X, τ) has been investigated. Also, we have proved that there exists a dense orbit in X , where X is locally compact Hausdorff space and τ has a countable basis. The main results are the following propositions: Every topologically α -type transitive map is a topologically transitive map which implies topologically δ -transitive map, but the converse not necessarily true, and every α -minimal map is a minimal map which implies δ -minimal map in topological spaces, but the converse not necessarily true. Finally, we have proved that a map which is γ -conjugated to γ -transitive (resp. γ -minimal, γ -mixing) map is γ -transitive (resp. γ -minimal, γ -mixing).

Keywords: Topologically δ -Transitive, δ -Irresolute, δ -Type Transitive, δ -Dense, γ -Dense, γ Transitive

1. Introduction

Let A be a subset of a topological space (X, τ) . The closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A of a topological space (X, τ) is said to be regular open [1] (resp. preopen [2]) if $A = Int(Cl(A))$ (resp. $A \subseteq Int(Cl(A))$). A set $A \subseteq X$ is said to be δ -open [3] if it is the union of regular open sets of a space X . The complement of a regular open (resp. δ -open) set is called regular closed (resp. δ -closed). The intersection of all δ -closed sets of (X, τ) containing A is called the δ -closure [3] of A and is denoted by $Cl_\delta(A)$. Recall that a set S is called regular closed if $S = Cl(Int(S))$. A point $x \in X$ is called a δ -cluster point [3] of S if $S \cap U \neq \emptyset$ for each regular open set U containing x . The set of all δ -cluster points of S is called the δ -closure of S and is denoted by $Cl_\delta(S)$. A subset S is called δ -closed if $\delta Cl(S) = S$. The complement of a δ -closed set is called δ -open. The family of all δ -open sets of a space X is denoted by $\mathcal{O}_\delta(X, \tau)$. The δ -interior of S is denoted by $Int_\delta(S)$ and it is defined as follows $Int_\delta(S) = \{x \in X : x \in U \subseteq Int(Cl(U)) \subseteq S\}$ for some open set U of X .

The area of Dynamical Systems where one investigates dynamical properties that can be described in topological terms is called Topological Dynamics Let X be a compact

topological space and let $f : X \rightarrow X$ be continuous. The pair (X, f) is so called topological system. The topological system (X, f) is called topologically δ -type transitive (or just δ -type transitive [4]) if for every pair of nonempty δ -open sets U and V in X there is a nonnegative integer n such that $f^n(U) \cap V \neq \emptyset$. If the space X has no isolated points, this is equivalent to the existence of a point $x \in X$ whose orbit $O_f(x) = \{x, f(x), f^2(x), \dots, f^n(x), \dots\}$ is δ -dense in X . Consequently, a topologically δ -type transitive topological system cannot be decomposed into two disjoint sets with nonempty δ -interiors. For more information on topological δ -type transitivity see, e.g. [4] and references there.

In this paper, we will study some new class of topological transitive maps called topological δ -type transitive [4], also, we will study the relationship between two types of minimal mappings, namely, δ -minimal mapping and α -minimal mapping, and we will prove that the properties of δ -type transitive, δ -mixing and δ -minimal maps are preserved under δ -conjugacy and study some of its properties.

2. Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let X be a space and $A \subset X$. The intersection (resp. closure) of A is denoted by $\text{Int}(A)$ (resp. $\text{Cl}(A)$).

Definition 2.1. Let (X, τ) be a space. A subset A of X is called dense in X if $\text{Cl}(A) = X$.

Definition 2.2(i) A space X is said to be 2nd countable if it has a countable basis.

(ii) X is said to be of First Category if it is a countable union of nowhere dense subsets of X . It is of second Category if it is not of First Category.

Theorem 2.3 Let X be a non-empty locally compact Hausdorff space. Then the intersection of a countable collection of open dense subsets of X is dense in X . Moreover, X is of second Category.

Definition 2.4 Let (X, τ) be a topological space. X is second countable if and only if the topology of X has a countable basis.

Definition 2.5 Recall that a space X is said to be separable if X contains a countable dense subset.

Corollary 2.6 A subset A of a space (X, τ) is dense if and only if $A \cap U \neq \emptyset$ for all $U \in \tau$ other than $U = \emptyset$.

Definition 2.7 Let (X, τ) be a topological space, $f: X \rightarrow X$ be a continuous map then f is said to be topologically transitive if every pair of non-empty open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \emptyset$.

The purpose of the following theorem is to prove that topological transitivity implies dense orbits in a space X where X is a non-empty locally compact Hausdorff topological space.

Theorem 2.8 Let (X, f) be a topological system where X is a non-empty locally compact Hausdorff topological space and $f: X \rightarrow X$ is a continuous map and that X is separable. Suppose that f is topologically transitive. Then there is $x \in X$ such that the orbit $O_f(x) = \{x, f(x), f^2(x), \dots, f^n(x), \dots\}$ is dense in X .

Proof: Let $B = \{U_i\}$ $i = 1, 2, 3, \dots$ be a countable basis for the topology of X . For each i , let $O_i = \{x \in X : f^n(x) \in U_i \text{ for some } n \geq 0\}$

Then, clearly O_i is open and dense. It is open since f is continuous, so, $O_i = \bigcup_{n=0}^{\infty} f^{-n}(U_i)$ is open and dense since f is topological transitive map. Further, for every open set V , there is a positive integer n such that $f^n(V) \cap U_i \neq \emptyset$.

Now, apply theorem 2.3 to the countable dense sets $\{O_i\}$

to say that $\bigcap_{i=0}^{\infty} O_i$ is dense and so non-empty. Let $y \in \bigcap_{i=0}^{\infty} O_i$.

This means that, for each i , there is a positive integer n such that $f^n(y) \in U_i$ for every i . By corollary 2.6 this implies

that $O_f(y)$ is dense in X

Definition 2.9. If for $x \in X$ the set $\{f^n(x) : n \in \mathbb{N}\}$ is dense in X then x is said to have a dense orbit. If there exists such an $x \in X$, then f is said to have a dense orbit.

Definition 2.10. A function $f: X \rightarrow X$ is called γ -homeomorphism if f is γ -irresolute bijective and $f^{-1}: X \rightarrow X$ is γ -irresolute.

Definition 2.11 [19] Two topological systems $f: X \rightarrow X$, $x_{n+1} = f(x_n)$ and $g: Y \rightarrow Y$, $y_{n+1} = g(y_n)$ are said to be topologically γ -conjugate if there is γ -homeomorphism $h: X \rightarrow Y$ such that $h \circ f = g \circ h$ (i.e. $h(f(x)) = g(h(x))$). We will call h a topological γ -conjugacy.

Remark 2.12 [19] If $\{x_0, x_1, x_2, \dots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{y_0 = h(x_0), y_1 = h(x_1), y_2 = h(x_2), \dots\}$ yields an orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$. In particular, h maps periodic orbits of f onto periodic orbits of g .

In [19], we introduced and defined the new type of transitive called γ -type transitive in such a way that it is preserved under topologically γ -conjugation. It means; we have proved that a map which is γ -conjugated to γ -transitive (resp. γ -minimal, γ -mixing) map is γ -transitive (resp. γ -minimal, γ -mixing).

We proceed to prove the following important proposition:

Proposition 2.13 [19] Let (X, f) and (Y, g) be two topological systems, if $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are topologically γ -conjugate. Then

- (1) f is topologically γ -transitive if and only if g is topologically γ -transitive;
- (2) f is γ -minimal if and only if g is γ -minimal;
- (3) f is topologically γ -mixing if and only if g is topologically γ -mixing.

Proof (1)

Assume that $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are topologically γ -conjugated by $h: X \rightarrow Y$. Suppose f is γ -type transitive. Let A, B be γ -open subsets of Y (to show $g^n(A) \cap B \neq \emptyset$ for some $n > 0$).

$U = h^{-1}(A)$ and $V = h^{-1}(B)$ are γ -open subsets of X since h is an γ -irresolute

Then there exists some $n > 0$ such that $f^n(U) \cap V \neq \emptyset$ since f is γ -type transitive. Thus (as $f \circ h^{-1} = h^{-1} \circ g$ implies $f^n \circ h^{-1} = h^{-1} \circ g^n$),

$$\emptyset \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B)$$

Therefore, $h^{-1}(g^n(A) \cap B) \neq \emptyset$ implies $g^n(A) \cap B \neq \emptyset$ since h^{-1} is invertible.

Proof (2)

Assume that $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are topological systems, which are topologically γ -conjugated by $h: Y \rightarrow X$. Thus, h is γ -homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are γ -irresolute) and $h \circ g = f \circ h$, that is, the following diagram commutes:

$$\begin{array}{ccc} Y & \xrightarrow{g} & Y \\ h \downarrow & & \downarrow h \\ X & \xrightarrow{f} & X \end{array}$$

We show that if g is γ -minimal, then f is γ -minimal. We want to show that for any $x \in X$, $O_f(x)$ is γ -dense. Since h is surjective, there exists $y \in Y$ such that $y = h^{-1}(x)$. Since g is γ -minimal, $O_g(y)$ is γ -dense. For any non-empty γ -open subset U of X , $h^{-1}(U)$ is a γ -open subset of Y since h^{-1} is γ -irresolute because the map h is γ -homeomorphism and it is non-empty since h is invertible map. By γ -density of $O_g(y)$ there exist $k \in \mathbb{N}$ such that $g^k(y) \in h^{-1}(U) \Leftrightarrow h(g^k(y)) \in U$. Since h is γ -conjugacy, as $f \circ h = h \circ g$ implies $f^k \circ h = h \circ g^k$ so $f^k(h(y)) = h(g^k(y)) \in U$ thus $O_f(h(y))$ intersects U . This holds for any non-empty γ -open set U and thus shows that $O_f(x) = O_f(h(y))$ is γ -dense.

Proof (3)

We only prove that if g is topologically γ -mixing then f is also topologically γ -mixing. Let U, V be two γ -open subsets of X . We have to show that there is $N > 0$ such that for any $n > N$, $f^n(U) \cap V \neq \emptyset$.

$h^{-1}(U)$ and $h^{-1}(V)$ are two γ -open sets since the map h is γ -irresolute. If g is topologically γ -mixing then there is $N > 0$ such that for any $n > N$, $g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \emptyset$. Therefore there exists $x \in g^n(h^{-1}(U)) \cap h^{-1}(V)$. That is, $x \in g^n(h^{-1}(U))$ and $x \in h^{-1}(V)$ if and only if $x = g^n(y)$ for $y \in h^{-1}(U)$ and $h(x) \in V$.

Thus, since $h \circ g^n = f^n \circ h$, so that, $h(x) = h(g^n(y)) = f^n(h(y)) \in f^n(U)$ and we have $h(x) \in V$ that is $f^n(U) \cap V \neq \emptyset$.

So, f is γ -mixing.

3. Transitive and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we

mean a topological space X together with a continuous map $f: X \rightarrow X$. The space X is sometimes called the phase space of the system. A set $A \subseteq X$ is called f -invariant if $f(A) \subseteq A$.

A topological system (X, f) is called *minimal* if X does not contain any non-empty, proper, closed f -invariant subset. In such a case we also say that the map f itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them. Given a point x in X , $O_f(x) = \{x, f(x), f^2(x), \dots\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω -limit set, i.e. the set of limit points of the sequence $x, f(x), f^2(x), \dots$. The following conditions are equivalent:

- (X, f) is α -minimal (resp. θ -minimal),
- every orbit is α -dense (resp. θ -dense) in X ,
- $\omega_f(x) = X$ for every $x \in X$.

A minimal map f is necessarily surjective if X is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system (X, f) , a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|_A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f) .

Let (X, f) be a topological system, and $f: X \rightarrow X$ α -homeomorphism of X onto itself. For A and B subsets of X , we let $N(A, B) = \{n \in \mathbb{Z}: f^n(A) \cap B \neq \emptyset\}$.

We write $N(A, B) = N(x, B)$ for a singleton $A = \{x\}$ thus $N(x, B) = \{n \in \mathbb{Z}: f^n(x) \in B\}$.

For a point $x \in X$ we write $O_f(x) = \{f^n(x): n \in \mathbb{Z}\}$ for the orbit of x and $Cl_\alpha(O_f(x))$ for the α -closure of $O_f(x)$. We say that the topological system (X, f) is α -type point transitive if there is a point $x \in X$ with $O_f(x)$ α -dense. Such a point is called α -type transitive. We say that the topological systems (X, f) is topologically α -type transitive (or just α -type transitive) if the set $N(U, V)$ is nonempty for every pair U and V of nonempty α -open subsets of X .

3.1. Topologically α -Transitive Maps

In [11], we introduced and defined a new class of transitive maps that are called topologically α -transitive maps on a topological space (X, τ) , and we studied some of their properties and proved some results associated with

these new definitions. We also defined and introduced a new class of α -minimal maps. In this paper we discuss the relationship between topologically α -transitive maps and θ -transitive maps. On the other hand, we discuss the relationship between α -minimal and θ -minimal in topological systems.

Definition 3.1.1 Let (X, τ) be a topological space. A subset A of X is called α -dense in X if $Cl_\alpha(A) = X$.

Note that, in general topology, for any subset A of the space X , $A \subset Cl_\alpha(A) \subset Cl(A)$, therefore if A is α -dense, in X , then A is dense in X .

Remark 3.1.2 Any α -dense subset in X intersects any α -open set in X .

Proof: Let A be an α -dense subset in X , then by definition, $Cl_\alpha(A) = X$, and let U be a non-empty α -open set in X . Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is α -closed and $A \subset U^c = B$. So $Cl_\alpha(A) \subset Cl_\alpha(B)$, i.e. $Cl_\alpha(A) \subset B$, but $Cl_\alpha(A) = X$, so $X \subset B$, this contradicts that $U \neq \emptyset$.

Definition 3.1.3 [12] A map $f: X \rightarrow Y$ is called α -irresolute if for every α -open set H of Y , $f^{-1}(H)$ is α -open in X .

Example 3.1.4 [11] Let (X, τ) be a topological space such that $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, b\}, \{b\}\}$. We have the set of all α -open sets is $\alpha(X, \tau) = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$ and the set of all α -closed sets is $\alpha C(X, \tau) = \{\emptyset, X, \{c, d\}, \{a, c, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}\}$. Then define the map $f: X \rightarrow X$ as follows $f(a) = a$, $f(b) = b$, $f(c) = d$, $f(d) = c$, we have f is α -irresolute because $\{b\}$ is α -open and $f^{-1}(\{b\}) = \{b\}$ is α -open; $\{a, b\}$ is α -open and $f^{-1}(\{a, b\}) = \{a, b\}$ is α -open; $\{b, c\}$ is α -open and $f^{-1}(\{b, c\}) = \{b, d\}$ is α -open; $\{a, b, c\}$ is α -open and $f^{-1}(\{a, b, c\}) = \{a, b, d\}$ is α -open; $\{a, b, d\}$ is α -open and $f^{-1}(\{a, b, d\}) = \{a, b, c\}$ is α -open so f is α -irresolute.

Definition 3.1.5 A subset A of a topological space (X, τ) is said to be nowhere α -dense, if its α -closure has an empty α -interior, that is, $\text{int}_\alpha(Cl_\alpha(A)) = \emptyset$.

Definition 3.1.6 [11] Let (X, τ) be a topological space, $f: X \rightarrow X$ be α -irresolute map then f is said to be topological α -transitive if every pair of non-empty α -open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \emptyset$. In the forgoing example 3.1.4: we have f is α -transitive because b belongs to any non-empty α -open set V and also belongs to $f(U)$ for any α -open set it means that $f(U) \cap V \neq \emptyset$ so f is α -transitive.

Example 3.1.7 [11] Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, X\}$. Then the set of all α -open sets is $\tau^\alpha = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$. Define $f: X \rightarrow X$ as follows $f(a) = b$, $f(b) = b$, $f(c) = c$. Clearly f is continuous because $\{a\}$ is open and $f(\{a\}) = \emptyset$ is open. Note that f is transitive because $f(\{a\}) = \{b\}$ implies that $f(\{a\}) \cap \{b\} \neq \emptyset$. But f is not α -transitive because for each n in \mathbb{N} , $f^n(\{a\}) \cap \{a, c\} = \emptyset$; since $f^n(\{a\}) = \{b\}$ for every $n \in \mathbb{N}$, and $\{b\} \cap \{a, c\} = \emptyset$. So we have f is not α -transitive, so we show

that transitivity not implies α -transitivity.

Definition 3.1.8 Let (X, τ) be a topological space. A subset A of X is called θ -dense in X if $Cl_\theta(A) = X$.

Remark 3.1.9 Any θ -dense subset in X intersects any θ -open set in X .

Proof: Let A be a θ -dense subset in X , then by definition, $Cl_\theta(A) = X$, and let U be a non-empty θ -open set in X . Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is θ -closed because B is the complement of θ -open and $A \subset U^c = B$. So $Cl_\theta(A) \subset Cl_\theta(B)$, i.e. $Cl_\theta(A) \subset B$, but $Cl_\theta(A) = X$, so $X \subset B$, this contradicts that $U \neq \emptyset$.

Definition 3.1.10 [14] A function $f: X \rightarrow X$ is called θ -irresolute if the inverse image of each θ -open set is a θ -open set in X .

Definition 3.1.11 A subset A of a topological space (X, τ) is said to be nowhere θ -dense, if its θ -closure has an empty θ -interior, that is, $\text{int}_\theta(Cl_\theta(A)) = \emptyset$.

Definition 3.1.12 [15] Let (X, τ) be a topological space, and $f: X \rightarrow X$ θ -irresolute map, then f is said to be topologically θ -type transitive map if for every pair of θ -open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \emptyset$.

Associated with this new definition we can prove the following new theorem.

Theorem 3.1.13 [11]: Let (X, τ) be a topological space and $f: X \rightarrow X$ be α -irresolute map. Then the following statements are equivalent:

- (1) f is topological α -transitive map
- (2) For every nonempty α -open set U in X , $\bigcup_{n=0}^{\infty} f^n(U)$ is α -dense in X
- (3) For every nonempty α -open set U in X , $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is α -dense in X
- (4) If $B \subset X$ is α -closed and B is f -invariant i.e. $f(B) \subset B$, then $B = X$ or B is nowhere α -dense.

(5) If U is α -open and $f^{-1}(U) \subset U$ then U is either empty set or α -dense in X .

Theorem 3.1.14 [4] Let (X, τ) be a topological space and $f: X \rightarrow X$ be θ -irresolute map. Then the following statements are equivalent:

- (1) f is θ -type transitive map
- (2) $\bigcup_{n=0}^{\infty} f^n(D)$ is θ -dense in X , with D is θ -open set in X .
- (3) $\bigcup_{n=0}^{\infty} f^{-n}(D)$ is θ -dense in X with D is θ -open set in X
- (4) If $B \subset X$ is θ -closed and $f(B) \subset B$, then $B = X$ or B is nowhere θ -dense
- (5) If $f^{-1}(D) \subset D$ and D is θ -open in X then $D = \emptyset$ or D is θ -dense in X .

4. Minimal Functions

We introduced a new definition on α -minimal[11] (resp. δ -minimal[4]) maps and studied some new theorems associated with these definitions.

Given a topological space X , we ask whether there exists α -irresolute (resp. θ -irresolute) map on X such that the set $\{f^n(x): n \geq 0\}$, called the orbit of x and denoted by $O_f(x)$, is α -dense (resp. δ -dense) in X for each $x \in X$. A partial answer will be given in this section. Let us begin with a new definition.

Definition 4.1 (α -minimal) Let X be a topological space and f be α -irresolute map on X with α -regular operator associated with the topology on X . Then the dynamical system (X, f) is called α -minimal system (or f is called α -minimal map on X) if one of the three equivalent conditions hold[11]:

- 1) The orbit of each point of X is α -dense in X .
- 2) $Cl_\alpha(O_f(x)) = X$ for each $x \in X$
- 3) Given $x \in X$ and a nonempty α -open U in X , there exists $n \in \mathbb{N}$ such that $f^n(x) \in U$

A system (X, f) is called δ -minimal if X does not contain any non-empty, proper, δ -closed f -invariant subset. In such a case we also say that the map f itself is δ -minimal. Another definition of minimal function is that if the orbit of every point x in X is dense in X then the map f is said to be minimal.

Theorem 4.2[4] For (X, f) the following statements are equivalent:

- (1) f is an δ -minimal map.
- (2) If E is an δ -closed subset of X with $f(E) \subset E$, we say E is invariant. Then $E = \emptyset$ or $E = X$.
- (3) If U is a nonempty δ -open subset of X , then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

5. Topological Systems and Conjugacy

Definition 5.1[4] A map $h: Y \rightarrow X$ is said to be δ -homeomorphism if h is bijective and thus invertible and both h and h^{-1} are δ -irresolute.

Definition 5.2 Let (X, f) and (Y, g) be topological systems, then $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are said to be topologically δ -conjugate if there is δ -homeomorphism $h: X \rightarrow Y$ such that $h \circ f = g \circ h$. We will call h a topological δ -conjugacy. Thus, the two topological systems with their respective function acting on them share the same dynamics.

Associated with these definitions we have the following theorem:

Theorem 5.3 [4] Let (X, f) and (Y, g) be two systems, if $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are topologically δ -conjugate. Then

- (1) f is topologically δ -transitive if and only if g is topologically δ -transitive;
- (2) f is δ -minimal if and only if g is δ -minimal;
- (3) f is topologically δ -mixing if and only if g is topologically δ -mixing.

6. Conclusion

The main results are the following:

Proposition 6.1 Every topologically α -type transitive map is a topologically transitive map which implies topologically δ -transitive map, but the converse not necessarily true.

Proposition 6.2 Every α -minimal map is a minimal map which implies δ -minimal map in topological spaces, but the converse not necessarily true.

Theorem 6.3 Let (X, f) be a topological system where X is a non-empty locally compact Hausdorff topological space and X is separable. Suppose that f is topologically transitive. Then there is $x \in X$ such that the orbit $O_f(x) = \{x, f(x), f^2(x), \dots, f^n(x), \dots\}$ is dense in X .

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