Science Journal of Applied Mathematics and Statistics
2015; 3(3): 63-69
Published online April 13, 2015 (http://www.sciencepublishinggroup.com/j/sjams)
doi: 10.11648/j.sjams.20150303.11
ISSN: 2376-9491 (Print); ISSN: 2376-9513 (Online)

Fractional Dynamics of Computer Virus Propagation

Bonyah Ebenezer[1, *], Nyabadza Farai[2], Asiedu-Addo Samuel Kwesi[3]

[1]Department of Mathematics and Statistics, Kumasi Polytechnic, Kumasi, Ghana
[2]Department of Mathematical Science, University of Stellenbosch, Matieland, South Africa
[3]Department of Mathematics Education, University of Education, Winneba Ghana

Email address:
ebbonya@yahoo.com (Bonyah E.)

To cite this article:
Bonyah Ebenezer, Nyabadza Farai, Asiedu-Addo Samuel Kwesi. Fractional Dynamics of Computer Virus Propagation. *Science Journal of Applied Mathematics and Statistics.* Vol. 3, No. 3, 2015, pp. 63-69. doi: 10.11648/j.sjams.20150303.11

Abstract: This paper studies the fractional order model for computer virus in SEIR model. Firstly, the basic reproduction number R_0, which determines the threshold of the spread of the virus is determined. The stability of equilibra was also determined and studied. The Adams-Bashforth-Moulton algorithm was employed to solve and simulate the system of differential equations. The results of the simulation depicts that by small change in α led to big change in the associated numerical results.

Keywords: Nonlinear System, Fractional Calculus, Computer Virus Model

1. Introduction

While the information technology revolution has significantly improve business activities and made living comparatively easy to manage through services such as managing bank account, travelling arrangement, and buying items online, it has also come with a high cost operations and manipulations through propagation of computer virus. This computer virus does not only propagates and leads to huge losses in terms of money to companies and customers, but is also implicated for loss of important data. It is estimated that annually, millions dollars are lost by the virtue of various infection [13]. In early 1980(s) the idea of mathematical models for the study of computer virus spread became pronounced. Since 1988 Epidemic models for computer viruses have been studied. Murray [13] seems to be the first to propose the relationship between epidemiology and computer viruses, although he did not provide any specific models.

Viruses were once propagated by exchanging disk; now; global connectivity gives malicious code to propagate a farther and faster. Bad use of computer through network invasion is on the increase. Today, over 74000 different strains of computer viruses have been observed since in 1986 the first virus was identified (Symantec Security Response, 2010). The have been several challenges on cyber world due to cyber attack leading to a great defense to protect valuable information from certain malicious agents (Trojan horse, worms, virus). The spread of these dangerous agents is similar to that of spread of endemic in biological processes. Many studies have employed biological system to understand the dynamics of spread of malicious objects in a computer network and prescribe procedures for protecting the computer system [10 ,12].

The activity of malicious objects in entire network can be examined by applying epidemiological models for disease spread [2, 5, 12]. Richard et al. 2005 design an SEI (Susceptible-Exposed- Infected) model to study the propagation of computer virus. They however, did not consider the length of latency period and consider the effect of anti-virus software. Yen and Liu, 2006 also proposed SEIR that assumes that recovery hosts have a permanent immunization period with a certain probability, which is not consistent with real life situation. By obtaining solution to this obstacle, [2, 3], also propose a SEIRS model with latent and temporary immune periods, which can identify common worm propagation. Garretto et al. [29] present a model that seeks to examine the propagation of virus and worms in distinct network topologies. Zou et al. [9] propose an internet worm monitoring system that checks a worm in its initial stage of propagation employing Kalman filter. Zhu et al. [21] apply optimal control method to study the dynamics of computer virus. They take into consideration a controlled delayed model and then use an optimal control technique, by making an assumption that there is a trade off between central loss and the effect. Carla and Teneriro also present fractional dynamics of computer virus propagation to study [21] model.

In this paper we analyze the fractional order version of the integer order model proposed by Mei et al. [30] for computer virus and its dynamics. We simulate numerically the model for different values of the order of the fractional derivatives α . In this regard, the paper is arranged as follows. In Section3, we describe the model proposed for computer virus dynamics. In Section 4, we analyze several simulations of the model for different values of the fractional derivatives explain the implication of the results. In the last Section, we show the main conclusion and outline same future research figures.

2. Fractional Order Calculus (FOC)

The theory of differential calculus began when Leibniz wrote about $D^{1/2}f(x)$ for generalization of the derivative operator $D^{\alpha}f(x)$ to fractional values of α, the order of the derivative.

The growth of the fractional calculus (FC) is attributable to many contributions of mathematicians for example Euler, Liouville, Riemann, and Letnikov [17].

Presently fractional calculus has been associated with long memory in the fields such as physics and engineering [18, 15]. Notwithstanding the FC applications in engineering there are ongoing effort to explore new areas of applications of FC such as the modeling of dynamical [19].

We present some of the definitions of fractional calculus. The most common applied definitions of a fractional derivative of order α are the Riemann-Liouville (RL), Gr¨unwald-Letnikov (GL), and Caputo (C) formulations. GL is stated as

$$ {}_{a}^{GL}D_{t}^{\alpha}f(t)=\lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{k=0}^{[t-a/h]}(-1)^{k}\binom{\alpha}{k}f(t-kh),\quad t>a,\alpha>0, \quad (2.1) $$

Definition 2.2. The Caputo fractional derivative of order $\alpha\in(n-1,n)$ of a continuous function $f:R^{+}\to R$ is given by

$$ D^{\alpha}f(x)=I^{n-\alpha}D^{\alpha}f(x),\quad D=\frac{d}{dt}. \quad (2.2) $$

where $\Gamma(.)$ is Euler's gamma function, $[x]$ means the integer part of x, and h is the step time increment. These expressions contain the history of the past dynamics, divergent to the integer counterpart that is a "local" operator. This property was observed in numerous phenomena and their modeling turns easier employing the FC formalism, while integer order models are often looks more complicated. We observe that the definition of time-fractional derivative of a function $f(t)$ at $t=t_n$ deals with integration and computing time-fractional derivative that demands. The concept of fractional derivative, we will implement Caputo's definition which is a variation of the Riemann- Liouville definition and has the advantage of solving initial value problems.

3. Model Formulation

By considering connection of computers, it is classified as external if connected to internet and external not connected. We subdivide the population into four classes. $S(t)$ represents the susceptible computers, that is, uninfected computers and new computers which connected to network at time t , $E(t)$ represents the exposed computers, that is, infected but not yet broken-out, $I(t)$ denotes the infectious computers and $R(t)$ the recovered computers, that is, virus-free computer having immunity. Let $S(t)$, $E(t)$, $I(t)$, $R(t)$ denote their corresponding numbers at time t, without ambiguity; $S(t)$, $E(t)$, $I(t)$, $R(t)$ will be abbreviated as S, E, I, R, respectively. The model is formulated as the following system of differential equations:

Now we introduce fractional order into the ODE model by Mei et al. [30]. The new system is described by the following set of ODE

$$ \begin{aligned} \frac{dS^{\alpha}}{dt}&=(1-q)N-\beta_1 SI-\beta_2 SE-pS-\mu S,\\ \frac{dE^{\alpha}}{dt}&=\beta_1 SI+\beta_2 SE-kE-\sigma E-\mu E,\\ \frac{dI^{\alpha}}{dt}&=\sigma E-dI-\mu I,\\ \frac{dR^{\alpha}}{dt}&=pS+kE+dI, \end{aligned} \quad (3.1) $$

$$ \frac{dN^{\alpha}}{dt}=\frac{dS^{\alpha}}{dt}+\frac{dE^{\alpha}}{dt}+\frac{dI^{\alpha}}{dt}+\frac{dR^{\alpha}}{dt} \quad (3.2) $$

It can be obviously noticed that the first three equations in (3.1) are independent of the fourth equation, and hence, the fourth equation can be done away with without loss of generality. Therefore, system (3.1) can be stated as

$$ \begin{aligned} \frac{dS^{\alpha}}{dt}&=(1-p)N-\beta_1 SI-\beta_2 SE-(p+\mu)S\\ \frac{dE^{\alpha}}{dt}&=\beta_1 SI+\beta_2 SE-(k+\sigma+\mu)E,\\ \frac{dI^{\alpha}}{dt}&=\sigma E-(d+\mu)I \end{aligned} \quad (3.3) $$

where N represents the rate at which external computers are connected to the network; p represents the recovery rate of susceptible computer as a results of the anti-virus ability of network; k stands for the recovery rate of exposed computer due to the anti-virus ability of network; β_1 denotes the rate at which, when having a connection to one infected computer, one susceptible computer can turn into exposed but has not broken-out; β_2 indicates the rate of which, when having connection to one exposed computer, one susceptible computer can turn into exposed; σ symbolizes the rate of the exposed computers cannot be cured by anti-virus software and broken-out; d represents the recovery rate of

infected computers that are cured; μ indicates the rate at which one computer is removed from the network.
where $0<\alpha\leq 1, N=S+E+I,(S,E,I)\in\mathbb{R}_+^3$.

The reason for considering a fractional order system instead of its integer order counterpart is that fractional order differential equations are generalizations of integer order differential equations. Also, using fractional order differential equations can help us to reduce the errors arising from the neglected parameters in modeling real life phenomena. We should note that the system (3.1) can be reduced to an integer order system by setting $\alpha=1$.

Adding up the equations given in (3.1), we obtain

Let $\mathbb{R}_+^3=\{X\in\mathbb{R}^3 : X\geq 0\}$ and $X(t)=(S(t),E(t),I(t))^T$. For the proof of the theorem about non-negative solutions we shall require the following Lemma [20].

Lemma 3.1 (Generalized Mean Value Theorem) Let $f(x)\in C[a,b]$ and $D^\alpha f(x)\in C(a,b]$ for $0<\alpha\leq 1$. Then we state

$$f(x)=f(a)+\frac{1}{\Gamma(\alpha)}D^\alpha f\big|(\xi)(x-a)^\alpha$$

with $0\leq\xi\leq x,\forall x\in(a,b]$.

Remark 3.2 Suppose $f(x)\in C\,[0,b]$ and $D^\alpha f(x)\in(o,b]$ for $0<\alpha\leq 1$. It is obvious from the Lemma 3.1 that if $D^\alpha f(x)\geq 0$), $\forall x\in(a,b)$ then the function f is nondecreasing, and if $D^\alpha f(x)\leq 0$, $\forall x\in(a,b)$ then the function f is nonincreasing for all $x\in[0,b]$.

Theorem 3.1 There is a unique solution for the initial value problem given by (2.1) - (2.2), and the solution remains in $\mathbb{R}_+^3$

Proof. The existence and uniqueness of the solution of (2.1)-(2.2) in $(0,\infty)$ can be obtained from [10, Theorem 3.1 and Remark 3.2]. We need to show that the domain $\mathbb{R}_+^3$ is positively invariant. Since

$$\begin{aligned} D^\alpha S\big|_{S=0} &= A\geq 0,\\ D^\alpha E\big|_{E=0} &= \beta_1 SI\geq 0,\\ D^\alpha I\big|_{I=0} &= \sigma E\geq 0, \end{aligned} \tag{3.4}$$

on each hyperplane bounding the nonnegative orthant, the vector field points into $\mathbb{R}_+^3$.

It is clear that $N(t)$ also remains nonnegative. For convenience in calculations we the following system, which can be obtained from (3.1):

$$\begin{aligned} \frac{dS^\alpha}{dt} &= (1-q)N-\beta_1 SI-\beta_2 SE-(p+\mu)S\\ \frac{dE^\alpha}{dt} &= \beta_1 SI+\beta_2 SE-(k+\sigma+\mu)E,\\ \frac{dI^\alpha}{dt} &= \sigma E-(d+\mu)I \end{aligned} \tag{3.5}$$

with initial conditions

$$S(0)=S_0, E(0)=E_0, I(0)=I_0. \tag{3.6}$$

The basic reproduction number, $\mathbb{R}_0$, of the integer order model $(\alpha=1)$ is computed in [22] to be

$$\mathbb{R}_0=\frac{N(1-p)\left(\beta_1\alpha+\beta_2(r+\mu)\right)}{(p+\mu)(r+\mu)(k+\alpha+\mu)} \tag{3.7}$$

The basic reproduction number, $\mathbb{R}_0$, is expressed as the number of secondary infections owing to a single infection in a completely susceptible population. For $\mathbb{R}_0<1$ the disease-free equilibrium is globally asymptotically stable and if $\mathbb{R}_0>1$, the endemic equilibrium is globally asymptotically stable [21].

3.1. Equilibrium Points and Stability

We consider the initial value problem (3.5)-(3.6) with α satisfying $0<\alpha\leq 1$ in order to estimate the equilibrium points of (3.5), let

$$\begin{cases} D^\alpha S=0,\\ D^\alpha E=0,\\ D^\alpha I=0. \end{cases}$$

Then the equilibrium points are $E_0=(1,0,0)$ and $E_1=(S^*,E^*,I^*)$, where

$$\begin{aligned} S^* &= \frac{A}{a\mathbb{R}_0},\\ E^* &= \frac{A(\mathbb{R}_0-1)}{b\mathbb{R}_0},\\ I^* &= \frac{A\alpha(\mathbb{R}_0-1)}{bc\mathbb{R}_0} \end{aligned}$$

The Jacobian matrix $J(E_0)$ for the system given by (2.6), computed at the disease free equilibrium is below

$$J(E_0)=\begin{pmatrix} -(p+\mu) & -\beta_2 & -\beta_1\\ 0 & (\beta_2-(k+\alpha+\mu)) & \beta_1\\ 0 & \alpha & -(r+\mu) \end{pmatrix}$$

Theorem 3.2 Disease free equilibrium of the system (3.5) is asymptotically stable if

$$\frac{N(1-p)\left(\beta_1\alpha+\beta_2(r+\mu)\right)}{(p+\mu)(r+\mu)(k+\alpha+\mu)}<1$$

Proof. Disease free equilibrium is asymptotically stable if all of the eigenvalues, $\lambda_i=\ i=1,2,3,$ of $J(E_0)$ satisfy the following conditions [26, 22]:

$$\left|\arg\lambda_i\right| > \alpha\frac{\pi}{2}. \quad (3.8)$$

These eigenvalues can be obtained by solving the characteristic equation $\det(J(E_0)-\lambda I)=0$

Hence, we obtain the following algebraic equation:

$$(\lambda+(p+\mu))[\lambda^2-((A+B)-\beta_2)\lambda-(ABC)]=0$$

where

$$A=(k+\alpha+\mu),$$

$$B=r+\mu),$$
$$C=(\mathbb{R}_0-1),$$

If $AB>C$, then the condition given by (3.8) is met.

We now examine the asymptotic stability of the endemic (positive) equilibrium of the system given by (3.1). The Jacobian matrix $J(E_1)$ determined at the endemic equilibrium is expressed as

$$(E_1)=\begin{pmatrix} -(p+\mu)\mathbb{R}_0 & -\beta_2 S^* & -\beta_1 S^* \\ (p+\mu)(\mathbb{R}_0-1) & \beta_2 S^*-(k+\alpha+\mu) & \beta_1 S^* \\ 0 & \alpha & -(r+\mu) \end{pmatrix}$$

Thus, the characteristic equation of the linearized system is expressed of the form

$$\lambda^3+c_1\lambda^2+c_2\lambda+c_3=0,$$

$$c_1=(p+\mu)\mathbb{R}_0-(\beta_2 S^*-(k+\alpha+\mu)-(r+\mu)),$$

$$c_2=(p+\mu)(k+\alpha+\mu)(r+\mu)(\mathbb{R}_0-1),$$

$$c_3=(p+\mu)(k+\alpha+\mu)(r+\mu)(\mathbb{R}_0-1),$$

Let $D(\phi)$ stands for the discriminant of a polynomial f. If $\phi(x)=x^3+c_1x^2+c_2x+c_3$ then Denote

$$D(\phi)=-\begin{vmatrix} 1 & c_1 & c_2 & c_3 & 0 \\ 0 & 1 & c_1 & c_2 & c_3 \\ 3 & 2c_1 & c_2 & 0 & 0 \\ 0 & 3 & c_1 & c_2 & 0 \\ 0 & 0 & 3 & 2c_1 & c_2 \end{vmatrix}=18c_1c_2c_3+(c_1c_2)^2-4c_3c_1^3-4c_2^3-27c_3^2.$$

Following [22; 23]. we arrive at the proposition

Proposition 3.2. One assume that E_1 exists in $\mathbb{R}_+^3$

1) If the discriminant of $\phi(x)$, $D(\phi)$ *is positive and Routh-Hurwitz are satisfied, that is,* $D(\phi)>0, c_1>0, c_3>0, c_1c_2>c_3$, then E_1 *is locally asymptotically stable.*
2) *If* $D(\phi)<0, c_1>0, c_2>0, c_1c_2=c_3, \alpha\in[0,1)$ then E_1 *is locally asymptotically stable.*
3) *If* $D(\phi)<0, c_1<0, c_2<0, \alpha>2/3,$ *then* E_1 *is unstable.*

4. Numerical Methods and Simulations

In view of the fact that most of the fractional-order differential equations hardly have exact analytic solutions, approximation and numerical techniques is the most effective way of solving such systems. Numerous analytical and numerical methods have been developed to solve the fractional order differential equations. For numerical solutions of system (3.1), one can apply the generalized Adams-Bashforth- Moulton method. In order to give the approximate solution by means of this algorithm, we consider the following nonlinear fractional differential equation [24, 25].

$$D_t^\alpha y(t)=f(t,y(t)),\quad 0\le t\le T$$

$y^k(0)=y_0^k,\ k=0,1,2,\ldots m-1$ where $y^k(0)=y_0^k,\ k=0,1,2,\ldots m-1$

$$D_t^\alpha y(t)=f(t,y(t)),\quad 0\le t\le T$$
$$y^k(0)=y_0^k,\qquad k=0,1,2,\ldots m-1$$

This equation corresponds to the Volterra integral equation

$$y(t)=\sum_{k=0}^{m-1}y_0^k\frac{t^k}{k!}+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,y(s))ds. \quad (4.1)$$

Diethelm et al. employed the predictor-correctors scheme [24, 25], depended on the Adams-Bashforth- Moulton algorithm to integrate Eq. (4.1). By employing this scheme to the fractional-order model for computer virus, and putting $h=\frac{T}{N}, t_n=nh, n=0,1,2,\ldots,N\in Z^+$, Eq. (4.1) can be discretized as follows [24, 25]:

$$S_{n+1}=S_0+\frac{h^\alpha}{\Gamma(\alpha+2)}\left((1-q)N-\beta_1S_{n+1}^pI_{n+1}^p-\beta_2S_{n+1}^pE_{n+1}^p-(q+\mu)S_{n+1}^p\right)+$$
$$\frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n}a_j,n+1\left((1-q)N-\beta_1S_jI_j-\beta_2S_jE_j-(q+\mu)S_j\right),$$

$$E_{n+1} = E_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left(\beta_1 S_{n+1}^p I_{n+1}^p + \beta_2 S_{n+1}^p E_{n+1}^p - (k+\alpha+\mu)E_{n+1}^p\right) + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_j, n+1\left(\beta_1 S_j I_j + \beta_2 S_j E_j - (k+\alpha+\mu)E_j\right),$$

$$I_{n+1} = I_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left(\sigma E_{n+1}^p - (d+\mu)I_{n+1}^p\right) + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_j, n+1\left(\sigma E_j - (d+\mu)I_j\right),$$

where

$$S_{n+1} = S_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_j, n+1\left((1-q)N - \beta_1 S_j I_j - \beta_2 S_j E_j - (q+\mu)S_j\right),$$

$$E_{n+1} = E_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_j, n+1\left(\beta_1 S_j I_j + \beta_2 S_j E_j - (k+\alpha+\mu)E_j\right),$$

$$I_{n+1} = I_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_j, n+1\left(\sigma E_j - (d+\mu)I_j\right)$$

$$a_j, n+1 \begin{cases} n^{\alpha+1} - (n-\alpha)(n-j), & j-0, \\ (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1} & 1 \le j \le n, \\ 1 & n+1, \end{cases}$$

$$b_j, n+1 = \frac{h^\alpha}{\alpha}\left(\left(n-j+1\right)^\alpha - \left(n-j\right)^\alpha\right), \quad 0 \le j \le n.$$

The parameter values used for the simulations were, $N = 10$, $p = 0.7$, $\beta_1 = 0.002$, $\beta_2 = 0.003$, $\mu = 002$, $\sigma = 0.09$, $r = 0.04$ and the following set of $\alpha = (1, 0.95, 0.90, 0.85)$ for each compartment. Now we take into account the initial population include susceptible nodes $S(0) = 10$, exposed to infected nodes $E(0) = 1$, infected nodes $1(0) = 1$ for numerical simulation.

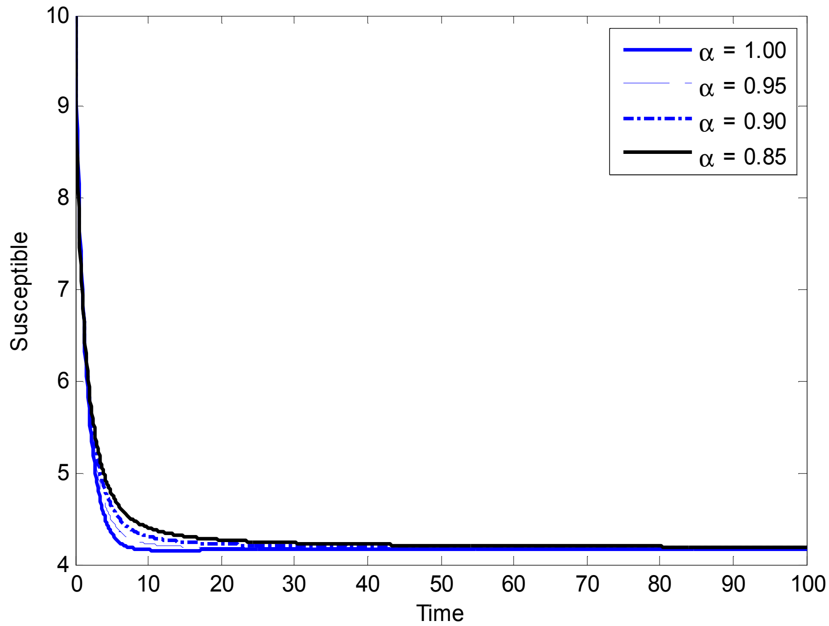

Figure 1. *Dynamics of the susceptible computers S versus time t, of system (7), for $\alpha \in \{0.1, \ldots, 1\}$. Parameter values and initial conditions are those stated.*

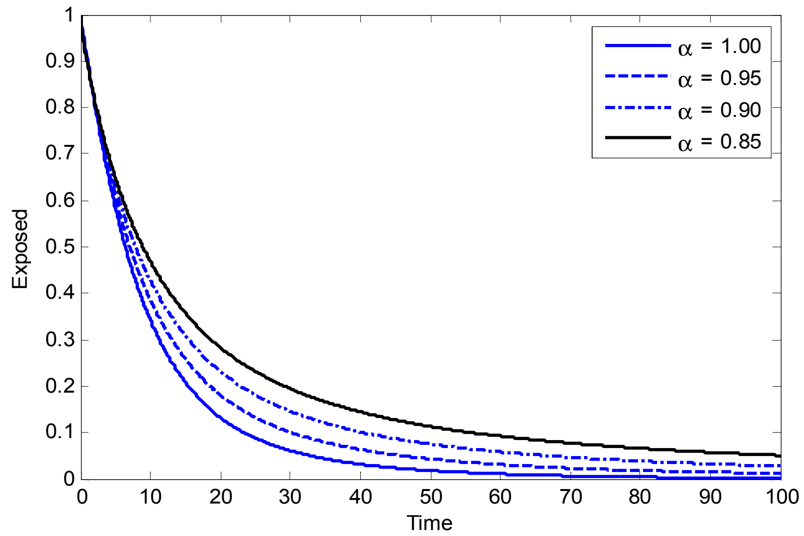

Figure 2. *Dynamics of the exposed computers I versus time t, of system (7), for $\alpha \in \{0.1, \ldots, 1\}$. Parameter values and initial conditions are those stated.*

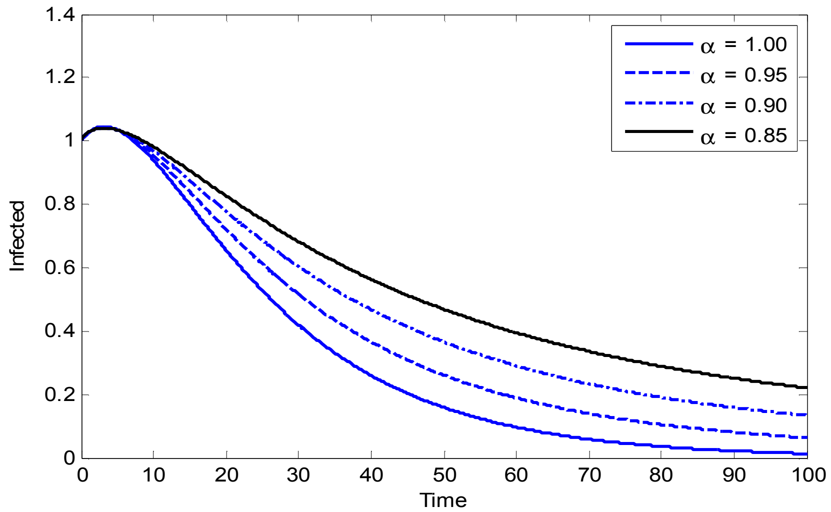

Figure 3. *Dynamics of the infected computers I versus time t, of system (7), for $\alpha \in \{0.1, \ldots, 1\}$. Parameter values and initial conditions are those stated.*

5. Discussion

In this paper, we have considered a fractional calculus model for computer virus propagation. From the numerical results in Figures 1,2 and 3, it is obvious that the approximate solutions depend continuously on the fractional derivative α.The approximate solutions $S(t)$, $E(t)$, and $I(t)$ are shown in Figures 1,2 and 3 with four different values of α. In each figure four different values of α are taken into account. Given = 1, system (3.3) is termed as the classical integer-order system (3.1). We showed in Figure 1, the variation of $S(t)$ versus time t with varying values of α = 1, 0.95, 0.90, 0.85 by fixing other parameters. It is observed that $S(t)$ does not drop sharply in a relatively small period of time for small values of α. Figure 2 depicts $E(t)$ versus time t and Figure 3 also shows $I(t)$ versus time t. However, variation in α for Figure 2 and 3 is more apparent than that of Figure 1. This buttresses the sensitive nature of fractional order models. Following [27 , 28], one observes that both steady states of integer order and fractional order turn to point to the fixed point given a long period of time. In dealing with situations in real life, data collected can help determine the order of the system. One also needs to mention that when dealing with real life problems, the order of the system can be determined by using the collected data.

6. Conclusion

In this paper, it is assumed that the virus process has a latent period and computers infected by the virus have also infectivity. An SEIR compartmental model for transmission of virus in computer network is formulated and studied. The threshold parameter $\mathbb{R}_0$ is determined. The steady states of the model is also derived and analyzed in order to determine the stability of the system. Adams-Bashforth-Moulton method is used to carry out the numerical simulations of the fraction order model. The simulation results show that by varying α small result in a big change in the associated numerical results. By transforming classical into a fractional order type gives impetus to the transformed model to be more sensitive to order of differentiation α.

References

[1] Zhu, Q.,Yang X., Yang L.X., Zhang C. Optimal control of computer virus under a delayed model," Applied Mathematics and Computation, vol. 218, no. 23,(2012) pp 11613–11619.

[2] Mishra B.K., Saini D. K. SEIRS epidemic model with delay for transmission of malicious objects in computer network," Applied Mathematics, and Computation, vol. 188, no. 2,(2007) pp. 1476-1482.

[3] Mishra B.K., Saini D.K. SEIRS epidemic model with delay for transmission of malicious objects in computer network," Applied Mathematics, and Computation, vol. 188, no. 2,(2007) pp. 1476-1482.

[4] Mishra B.K., Saini D. Mathematical models on computer virus, Applied Mathematics, and Computation," vol. 187, no. 2,(2007) pp. 929-936.

[5] Mishra B.K., N. Jha N. Fixed period of temporary immunity after run of anti-malicious software on computer nodes," Applied Mathematics and Computation, vol. 190, no. 2,(2007) pp. 1207-1212.

[6] Forest S., Hofmeyr S., Somayaji A., Longsta T. Self-nonself discrimination in a computer," Proceedings of IEEE Symposium on Computer Security, and Privacy,(1994) pp. 202-212.

[7] Symantec Security Response-Definitions, (http://www.symantec.com/avcenter/d(2010).

[8] Richard W. T., Mark J. C. Modeling virus propagation in peer-to-peer networks," IEEE International Conference on Information, Communications, and Signal Processing, ICICS 2005, pp. 981- 985.

[9] Zou C.C., Gong, W., Towsley D., Gao L. Themonitoring and early detection of internet worms," IEEE/ACMTransactions on Networking, vol. 13, no. 5,(2005) pp. 961–974.

[10] ChenT., Jamil N. Effectiveness of quarantine in worm epidemics," IEEE International Conference on Communications,(2006) pp. 2142-2147.

[11] Cohen F. Computer virus, theory, and experiments," Proceedings of the 7th DOD/NBS Computer and Security Conference,(1987) pp. 22-35.

[12] Mishra B.K., Saini D.K. SEIRS epidemic model with delay for transmission of malicious objects in computer network," Applied Mathematics and Computation, vol. 188, no. 2, (2007) pp. 1476-1482.

[13] Bimal K. M., Gholam M. A. Differential Epidemic Model of Virus and Worms in Computer Network, International Journal of Network Security, Vol.14, No.3,(2012) PP. 149-155.

[14] Murray, W. The application of epidemiology to computer viruses. Comput. Security 7,(1988) 139–150.

[15] Podlubny. Fractional Differential Equations, vol. 198 of Mathematics in Science and Engineering, Academic Press, San Diego, Calif, USA (1999).

[16] Kilbas A.A., Srivastava H.M., Trujillo J.J. Theory and Applications of Fractional Differential

[17] Equations, vol. 204 of North-Holland Mathematics Studies, Elsevier, Amsterdam, The Netherlands (2006).

[18] Oldham K. B., Spanier J. TheFractionalCalculus:Theory and Application of Differentiation and Integration to ArbitraryOrder, Academic Press, New York, NY, USA (1974).

[19] Ionescu.C.M. The Human Respiratory System: An Analysis of the Interplay between Anatomy, Structure, Breathing and Fractal Dynamics, Series in BioEngineering, Springer, London, UK (2013).

[20] Tenreiro J.A., Machado Galhano Oliveira A.M., Tar J.K. Approximating fractional derivatives through the generalized mean, Communications in Nonlinear Science and Numerical Simulation, vol. 14, no. 11,(2009) pp. 3723–3730.

[21] Odibat, Z.M., Shawagfeh, N.T. Generalized Taylor's formula, Applied Mathematics and Computation 186,(2007) 286–293.

[22] Q. Zhu Q., Yang X., Ren. J. Modeling and analysis of the spread of computer virus," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 12,(2012) pp. 5117–5124.

[23] Ahmed E., El-Sayed A. M. A., El-Saka H. A. A. On some Routh-Hurwitz conditions for fractional order differential equations and their applications in Lorenz, R¨ossler, Chua and Chen systems," Physics Letters A, vol. 358, no. 1,(2006) pp. 1–4.

[24] DingY., H. Ye H. A fractional-order differential equation model of HIV infection of CD4_T-cells,"Mathematical and Computer Modelling, vol. 50, no. 3-4,(2009) pp. 386–392.

[25] Diethelm, K., Ford. N.J. Analysis of fractional differential equations,J Math Anal Appl, 256 (2002) 229–248.

[26] Diethelm K., Ford N. J., Freed A.D. A predictor-corrector approach for the numerical solution of fractional differential equations, Nonlinear Dyn, 29 (2002), 3–22.

[27] Matignon D . Stability results for fractional differential equations with applications to control processing, in: Computational Eng. in Sys. Appl. 2 (Lille, France), (1996) pp. 963.

[28] E. Demirci, A. Unal, and N. ¨Ozalp, "A fractional order SEIR model with density dependent death rate," Hacettepe Journal of Mathematics and Statistics, vol. 40, no. 2,(2011) pp. 287–295.

[29] N. ¨ Ozalp and E.Demi¨orci¨o, "A fractional order SEIR model with vertical transmission," Mathematical and Computer Modelling, vol. 54, no. 1-2,(2011) pp. 1–6.

[30] M. Garetto, W. Gong, and D. Towsley, "Modeling malware spreading dynamics," in Proc. IEEE Infocom, San Francisco, CA, Mar. 2003.

[31] P. Mei X. He, J. Huang, and T. Dong. Modeling Computer Virus and Its Dynamics. Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2013, 5 (842614).