

Computation of Optimal Path Length for Terrestrial Line of Sight Microwave Link Using Newton–Raphson Algorithm

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Abstract: This paper presents an efficient algorithm for determination of optimal path length of terrestrial line of sight microwave communication link. The algorithm computes and adjusts the path length based on the difference between the maximum fade depth the link can accommodate and the actual fade depth that is expected in the link. The algorithm uses Newton-Raphson iteration method to adjust the path length until it arrives at the optimal path length at which the maximum fade depth the link can accommodate and the actual fade depth that is expected in the link at the given set of link parameters. A numerical example is performed for a Ku-band microwave link at 12 GHz. The results show that after 4 cycle the algorithm converged when the path length dropped from its initial value of 19.9903km to the optimal value of 5.89Km. In addition, for free space, the initial value of 140.40dB drops to a value of 129.43dB at the optimal point and maximum fade depth dropped from initial value of 104.04dB to optimal value of 30.65dB.

Keywords: Newton–Raphson, Optimal Path Length, Microwave Link, Fade Margin, Fade Depth, Rain Fading, Multipath Fading

1. Introduction

Microwave communication is based on the transmission of a microwave radio signal through the atmosphere. Generally, microwave radio system propagates in virtually straight line through Earth's atmosphere from the transmitter to the receiver spaced over a distance that constitute the path length. Generally, electromagnetic waves are attenuated while propagating between two geometrically separated points. Apart from free space pathloss, the two other types of attenuation that affect microwave links are rain and multipath attenuation [1]. Studies show that mutual relation between rain and multipath attenuation rules out the possibility that the link could be affected by both types of attenuation at the same time. As such, to determine the fade margin it is necessary to calculate both rain and multipath attenuation; the larger of the two types of attenuation determines the value of fade margin to be adopted [2-3].

In terrestrial LOS microwave communication system planning, determination of the expected transmission range or

path length is essential. Existing studies focus on computing the maximum transmission range. However, in this paper, an algorithm is developed to obtain the Optimal Path Length, $d_{(op)}$ for terrestrial LOS microwave communication link. The maximum path length can be determined from the link budget using the free space path loss and the specified fade margin. On the other hand, the optimal path length uses the free space path loss and the specified fade margin to determine the initial path length which is then evaluated to see if the system can accommodate the fade depth that can occur at the given path length. If the fade depth is larger than what the microwave link can accommodate at the given path length, the path length is reduced and the process is repeated. Otherwise, if the fade depth is smaller than what the microwave link can accommodate at the given path length, the path length is increased and the process is repeated until the path length is obtained at which the fade depth the link can accommodate is equal to the fade depth that can occur in the system. Essentially, the computation involves iterative adjustment of the path length which in this paper is performed using the

Newton–Raphson algorithm.

2. Review of Relevant Literatures

2.1. Link Budget

Link budget is a calculation involving the gain and loss factors associated with the antennas, transmitters, receivers, transmission lines, and propagation environment, used to determine the maximum distance at which a transmitter and receiver can successfully operate [4-5]. According to the Friis' law (Friis free space equation), the received power by a receiver antenna which is separated from the transmitter by a distance d is

$$P_r = P_t \cdot G_t \cdot G_r \cdot \left(\frac{\lambda}{4\pi d}\right)^2 \quad (1)$$

where

P_r is the received power in dBm

P_t is the transmitter power in dBm

G_t and G_r are the transmitter and receiver antenna gains in dBi, respectively

λ is the wavelength in metres.

d = the separation in metres between the receiver and transmitter antenna

The received power can then be related to the free space propagation taking account of the path loss [6-10] using the following relation:

$$P_R = P_T + (G_T - L_T) + (G_R - L_R) - PL(d) - L_M \quad (2)$$

Where,

PL = the free space path loss in dB

L_T = sum of losses within the transmitter in dB.

L_R = sum of losses within the receiver in dB.

L_M = the miscellaneous attenuation in dB

Often L_T , L_R and L_M are omitted from the equation [11], simplifying it to:

$$P_R = P_T + (G_T) + (G_R) - PL \quad (3)$$

P_R may also be defined with respect to the fade margin (Fm) in dB and receiver sensitivity (P_s) in dBm

$$P_R = Fm + P_s \quad (4)$$

From Friis formula, the expression for computing free space path loss is given as [12-18];

$$PL_{[dB]} = 20\log_{10} d + 20 \log f + 92.45 \quad (5)$$

Equation (5) can also be written in terms of distance as shown below

$$d_{pl} = 10^{(PL_{[dB]} - 92.45 - 20 \log f)/20} \quad (6)$$

Substituting for PL from Eq. 3 and Eq 4 into Eq. 6 gives

$$d_{pl} = 10^{(P_T + G_T + G_R - Fm_s + P_s - 92.45 - 20 \log f)/20} \quad (7)$$

Effective Total Loss (L_{etotal}): It should be noted that fade

margin is actually the additional loss expected in the microwave link due to various fade mechanisms that are prevalent in the communication path. Hence, the Effective Total Loss (C_{etotal}) the system can accommodate based on the given link parameters can be states as follows;

$$L_{etotal} = PL_e + L_T + L_M + L_R + fm_e \quad (8)$$

Where PL_e is the effective free space path loss and fm_e is the effective fade margin

When L_T , L_R and L_M are ignored, Eq 8 becomes;

$$L_{etotal} = PL_e + fm_e \quad (9)$$

In order to maintain the link availability for the given set of link parameters, the bound on L_{etotal} is as follows;

$$L_{etotal} = PL_e + fm_e = PL + fm_s \quad (10)$$

Let $L_{etunadj} = PL + fm_s$ be the effective total loss based on the specified fade margin, fm_s and the initial (unadjusted) free space path loss, FSPL.

$$L_{etunadj} = PL + fm_s \quad (11)$$

Let $L_{etadj} = PL_e + fm_e$ be the effective total loss based on the adjusted fade margin, fm_e and the adjusted free space path loss, PL_e .

$$L_{etadj} = PL_e + fm_e \quad (12)$$

Then, Equation 10 becomes

$$L_{etotal} = L_{etunadj} + L_{etadj} \quad (13)$$

In this case, fm_s is the initial fade margin specified for the link and FSPL is the initial value obtained for the free space path loss before the path length is adjusted. As stated earlier, the fade margin is added to accommodate the fading due to rain fading and multipath fading. The maximum fade depth expected from rain and multipath must be determined in order to compute the optimal path length.

2.2. Multipath Attenuation Model

Multipath Fade Depth, $A_{multipath}$ (in dB) is obtained from the expression for po (the percentage of time the specified fade depth is exceeded) as follows[19-21]:

$$A_{multipath} = 10(-0.00089(h_L)) - (10)\log\left(\frac{po}{\{K(d^{3.1})(1+|\epsilon_p|)^{-1.29(f^{0.8})}\}}\right) \quad (14)$$

where:

d is the propagation path length or distance (in km) between the transmitter and the receiver

f is frequency (GHz)

h_L is altitude of lower antenna (m)

$A_{multipath}$ is multipath fade depth (dB)

K is geoklimatic factor and can be obtained from: ϵ_p is the path inclination, (in mrad).

$dN1$ is the point refractivity gradient d is the propagation path length or distance (in km) between the transmitter and the

receiver

h_t is the transmitter antenna height

h_r is the receiver antenna height (where h_t and h_r are in meters about sea level),

$$h_L = \text{minimum}(h_t, h_r) \quad (15)$$

2.3. Rain Attenuation Model

Rain fade depth, A_R (dB) is the product of specific rain attenuation, $\gamma_{R_{po}}$ in dB/km and the propagation path length, d (km) between the transmitter and the receiver.

$$A_R = (\gamma_{R_{po}})d \text{ (dB)} \quad (16)$$

In respect of ITU-R PN.838-3 [5] recommendation, the following terms can be defined;

$A_{R/h}$ is the rain fade depth (attenuation) for horizontal polarization

$A_{R/v}$ is the rain fade depth (attenuation) for vertical polarization

A_{Rain} is the effective rain fade depth (attenuation) considering both horizontal and vertical polarization.

d is the propagation path length or distance (in km) between the transmitter and the receiver (in this case, $d = d_{pl}$)

Hence,

$$\begin{cases} A_{R/h} = d[\gamma_{R_{po+h}}] = K_h[R_{po}]^{\alpha_h} \times d \\ A_{R/v} = d[\gamma_{R_{po+v}}] = K_v[R_{po}]^{\alpha_v} \times d \\ A_R = (\gamma_{R_{po}})d = \text{maximum}(d[\gamma_{R_{po+h}}], d[\gamma_{R_{po+v}}]) \end{cases} \quad (17)$$

2.4. Maximum Fade Depth (fd_m)

In the determination of the maximum fade margin for terrestrial microwave links, it is necessary to calculate both rain and multipath attenuation (or fading); the larger of the two fading is the dominant fading and the value is the maximum fading the link can accommodate. Given that fd_m is defined as the link maximum fade depth in dB, hence;

When A_R is $> A_F$, then A_R becomes the dominating attenuation and the later becomes zero

Else when A_F is $> A_R$, then A_F becomes the dominating attenuation and the later becomes zero.

2.5. The Optimal Path Length

The optimal path length $d_{(op)}$ is obtained when the following conditions are fulfilled;

$$\begin{cases} fm_e = fd_m = P_R - P_S \\ L_{etotal} = PL_e + fm_e = PL + fm_s \\ \text{and} \\ fm_e = fd_m = (PL + fm_s) - PL_e \end{cases} \quad (18)$$

Now, it has been shown that;

$$fm_e = P_T + G_T + G_R - 92.45 - 20 \log f - 20 \log(d_e) - P_S \quad (19)$$

$$A_R = (\gamma_{R_{po}})(d_e) \quad (20)$$

$$A_{multipath} = 10(-0.00089h_L) - (10)\log\left(\frac{p_o}{\{K((d_e)^{3.1})(1+|\epsilon_p|)^{-1.29}(f^{0.8})\}}\right) \quad (21)$$

For high frequencies microwaves link in the tropical region (with high rain rates) the rain fading dominates, that means, $A_R > A_{multipath}$ and hence, $fd_m = A_R$. Then, the optimal path length ($d_{(op)}$) can be computed as follows:

$$fm_e = P_T + G_T + G_R - 92.45 - 20 \log f - 20 \log(d_e) - P_S \quad (22)$$

$$fm_e = P_T + G_T + G_R - 92.45 - 20 \log f - P_S - 20 \log(d_e) \quad (23)$$

$$\text{Let } Kfm = P_T + G_T + G_R - 92.45 - 20 \log f - P_S$$

$$fm_e = Kfm - 20 \log(d_e) \quad (24)$$

$$fd_m = A_R = (\gamma_{R_{po}})(d_e) \quad (25)$$

Specific multipath attenuation, $\gamma_{M_{po}}$ is defined as

$$\gamma_{M_{po}} = \frac{A_{multipath}}{d_e}(d_e) \quad (26)$$

Hence;

$$A_{multipath} = (\gamma_{M_{po}})d_e \quad (27)$$

Thus,

$$fd_m = \text{MAXIMUM}([(\gamma_{M_{po}})d_e], [(\gamma_{R_{po}})(d_e)]) \quad (28)$$

And

$$\gamma_{po} = \text{MAXIMUM}(\gamma_{M_{po}}, \gamma_{R_{po}}) \quad (28)$$

When at optimal point $fd_m = fm_e = Kfm - 20 \log(d_e)$ and

$$d_e = d_{(op)} \quad (30)$$

Hence,

$$Kfm - 20 \log d_{(op)} = (\gamma_{R_{po}})d_{(op)} \quad (31)$$

$$fd_{(op)} = (\gamma_{po})d_{(op)} + 20 \log d_{(op)} - Kfm = 0 \quad (32)$$

where $d_{(op)}$ is the Optimal Path Length in km

The equation is of the form

$$f(x) = ax + b \log(x) - c = 0 \quad (33)$$

Approximate solution for the equation can be obtained graphically. However, more accurate solution can be obtained by solving the equation with the Lambert W function [22-23], also referred to as the omega function or the product log function. Furthermore, accurate solutions can also be obtained by solving the equation using any of the existing fixed point iteration (numerical) methods. In this research, the fixed point iteration methods employed is the numeric approaches called Newton-Raphson Method

3. Review of Relevant Literatures

3.1. Review of Newton-Raphson Method of Solving NONLINEAR Equation

The Newton-Raphson method [24] is based on the principle that if the initial guess of the root of $f(x) = 0$ is at x_i , then if one draws the tangent to the curve at $f(x_i)$, the point x_{i+1} where the tangent crosses the x -axis is an improved estimate of the root [18]. Using the definition of the slope of a function, at $x = x_i$

$$f'(x_i) = \tan \theta = \frac{f(x_i) - 0}{x_i - x_{i+1}}, \quad (34)$$

which gives

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (35)$$

Equation (35) is called the Newton-Raphson formula for solving nonlinear equations of the form $f(x) = 0$. So starting with an initial guess, x_i , one can find the next guess, x_{i+1} , by using Equation (35). One can repeat this process until one finds the root within a desirable tolerance.

3.2. Application of Newton-Raphson Algorithm for Calculating the Optimal Path Length

In other to arrive at the optimal path length based on the condition in Eq 19, the value of d_e is adjusted by an adjustment value (d_{adj}) and the values of PL_e , fd_m , fm_e , L_{etadj} , $L_{etunadj}$ and P_R are re-computed. The process is repeated until the optimal path length condition in Eq 19 is satisfied. In the Newton–Raphson Method, the adjustment value for the i^{th} iteration is defined as $d_{adj}(i)$ and can be obtained as follows;

$$d_{adj} = \frac{f(d_{e(i)})}{f'(d_{e(i)})} \quad (36)$$

$$d_{e(i)} = d_{e(i)} + d_{adj(i)} \quad (37)$$

The iteration is continued until $|d_{e(i+1)} - d_{e(i)}| < 0.001$. At this point, the optimal path length, (d_{op}) condition is fulfilled. Then, the optimal fade margin (fm_{op}) and the Optimal Free Space Path Loss (PL_{op}) are given as follows;

$$fm_{op} = fd_m = fm_c = P_R - P_S \quad (38)$$

$$FSPL_{op} = 92.45 + 20 \log f + 20 \log d_{(op)} \quad (39)$$

$$FSPL_{op} = 92.45 + 20 \log f + 20 \log d_{(op)} \quad (40)$$

4. Results and Discussions for the Fade Depth with Path Length Adjustment Using Newton–Raphson Method

The Newton–Raphson algorithm is used to determine the optimal path length for a sample fixed point terrestrial LOS microwave link with the following link parameters: Frequency (f) = 12 GHz; Transmit power (P_T) = 10dBm; Transmitter Antenna Gain (G_T) = 35 dBi; Receiver Antenna Gain (G_R) = 35 dBi; Fade Margin (fm_s) = 20dB; Receiver Sensitivity (P_S) = -80dBm; Rain Zone = N; Point Refractivity Gradient ($dN1$) = -400; Link Percentage Outage (po) = 0.01% ; Rain Fade Constants ; $k_h = 0.01217$, $\alpha_h = 1.2571$, $k_v = 0.01129$, $\alpha_v = 1.2156$; $R_{po} = 95\text{mm/h}$; $h_t = 295\text{m}$; and $h_r = 320\text{m}$. For each simulation run, the convergence cycle (n) at which the optimal path length is obtained is noted along with other relevant performance parameters.

In Table 1 to Table 3 , as well as Figure 1 to Figure 3 , the frequency is 12 GHz and the rain zone is N, with percentage availability of 99.99% or link percentage Outage (po) of 0.01% . The convergence cycle is 4. That means, as shown in Table 1, Table 2, and Table 3, (as well as, Figure 1, Figure 2, and Figure 3), The Newton–Raphson algorithm is iterated for 4 times before the optimal path length is obtained. Also, the optimal path length is 5.89 km, the optimal Free Space Path Loss is 129.43 dB, and the optimal Fade Margin the system can accommodate is 30.57 dB while the optimal Fade Depth is 30.65 dB. In essence, at the optimal path length, a maximum fade depth of 30.57 dB can be accommodated by the link. However, the maximum fade depth the rain and multipath fading can present at the optimal path length of 5.87 km is 30.57 dB which is 0.08 dB in excess of the optimal fade margin.

From Table 1 and Figure 1, it will be notices that the rain fading is equal to the effective fade depth. In essence, for the given frequency, rain zone and percentage availability, the rain fading is greater than the multipath fading and hence, determines the effective fade depth that will be experienced in the link.

It can be recalled from Table 2 and Figure 2 that the initial fade margin specified for the system is 19.60 dB, (actually, 20 dB). At this initial point, in Table 2 and Figure 2, the initial maximum path length is 19.9903 km, the initial path loss is 140.40 dB, the initial fade depth is 104.04 dB while the received signal power is -60.04 dB. At the optimal point, the path maximum path loss has reduced by 10.61 dB to a value of 129.43 dB while the received signal power has increased the same value of 10.61 dB to a value of -49.43 dB

Table 1. Newton–Raphson Method: Rain Fading, Multipath Fading, Free Space Path Loss, Effective Fade Margin, Effective Maximum Depth and Effective Path Length.

Number of Iterations (n)	Effective Rain Fading	Multipath Fading	Free Space Path Loss	Effective Fade Margin	Effective Fade Depth	Effective Path Length
0	104.03	26.59	140.04	19.96	104.03	19.99
1	26.51	2.78	128.17	31.83	26.51	5.09
2	30.58	5.36	129.41	30.59	30.58	5.88

Number of Iterations (n)	Effective Rain Fading	Multipath Fading	Free Space Path Loss	Effective Fade Margin	Effective Fade Depth	Effective Path Length
3	30.65	5.40	129.43	30.57	30.65	5.89
4	30.65	5.40	129.43	30.57	30.65	5.89
5	30.65	5.40	129.43	30.57	30.65	5.89
6	30.65	5.40	129.43	30.57	30.65	5.89
7	30.65	5.40	129.43	30.57	30.65	5.89
8	30.65	5.40	129.43	30.57	30.65	5.89
9	30.65	5.40	129.43	30.57	30.65	5.89
10	30.65	5.40	129.43	30.57	30.65	5.89

vs Number of Iterations (n)

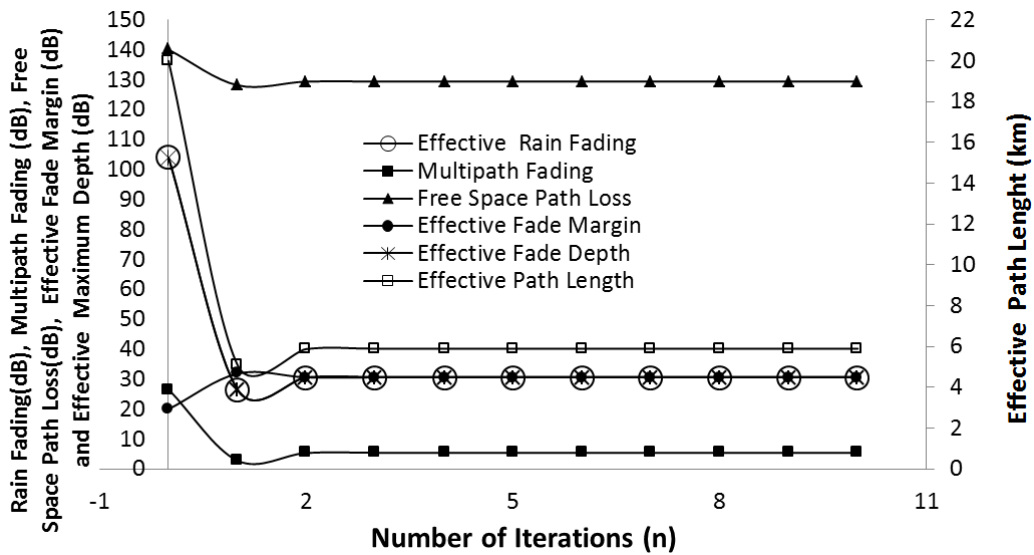


Figure 1. Newton–Raphson Method: Rain Fading(dB), Multipath Fading (dB), Free Space Path Loss(dB) , Effective Fade Margin (dB), Effective Maximum Depth (dB) and Effective Path Length vs Number of Iterations (n).

Table 2. Newton–Raphson Method: Initial and Optimal Values For Free Space Path Loss, Fade Depth, Fade Margin, Received Power, Path Length and Convergence Cycle.

	n	Free Space Path Loss (in dB)	Fade Depth (in dB)	Fade Margin (in dB)	Received Power (in dBm)	Path Length (in km)
Initial Value	0	140.04	104.03	19.96	-60.04	19.9903
Optimal Value	4	129.43	30.65	30.57	-49.43	5.8905

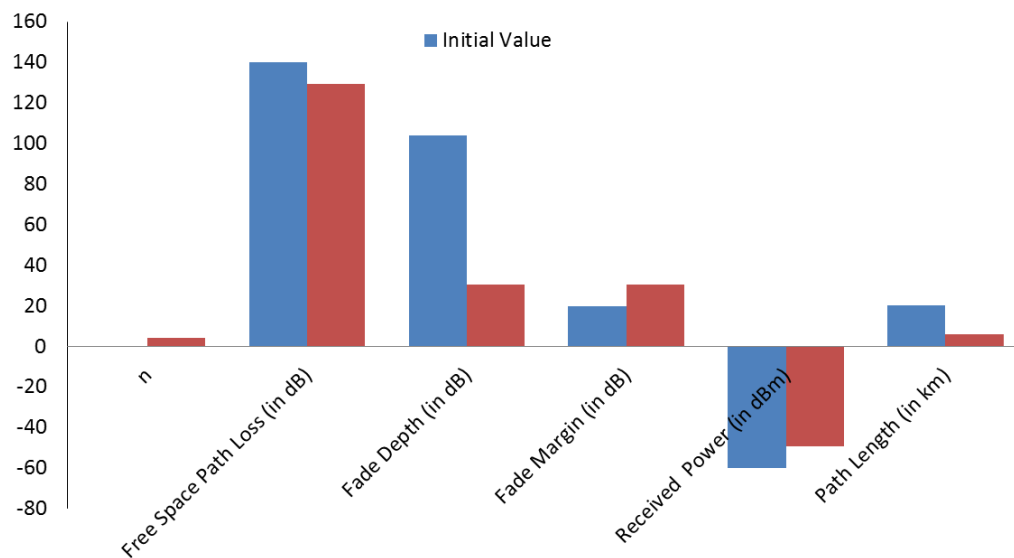
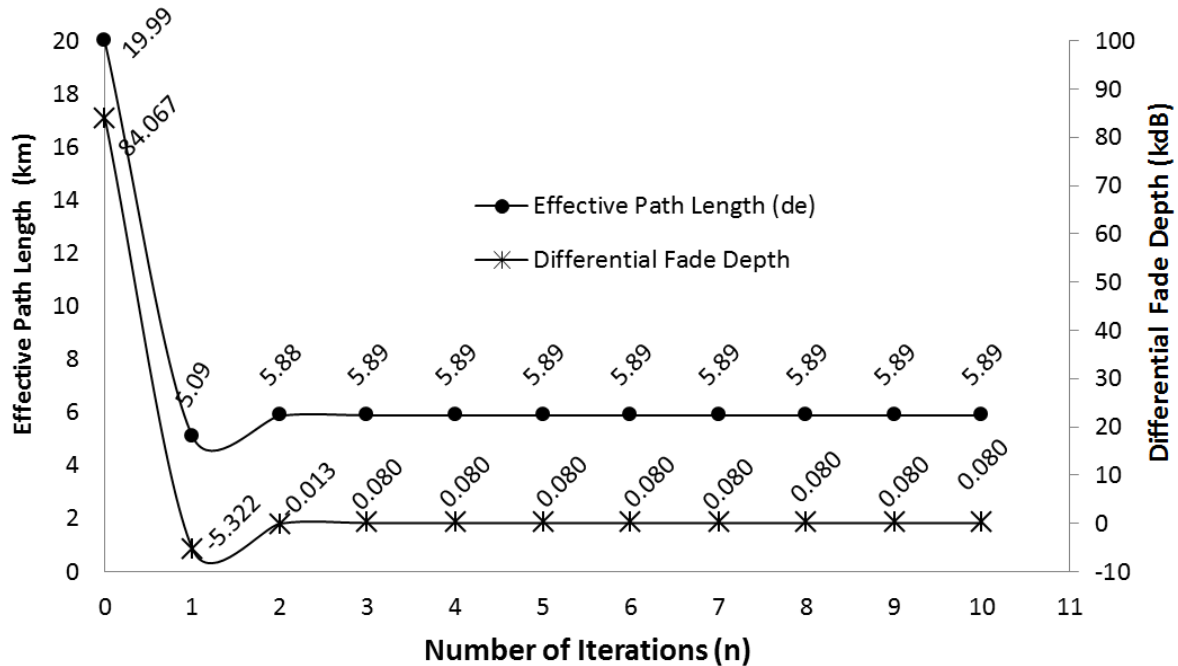


Figure 2. Newton–Raphson Method: Initial and Optimal Values For Free Space Path Loss, Fade Depth, Fade Margin, Received Power, Path Length and Convergence Cycle.

Table 3. Newton–Raphson Method: Differential Fade Depth and Effective Path Length vs Number of Iterations (*n*).

Number Of Iterations (<i>n</i>)	Differential Fade Depth	Effective Path Length (de)
0	84.067	19.99
1	-5.322	5.09
2	-0.013	5.88
3	0.080	5.89
4	0.080	5.89
5	0.080	5.89
6	0.080	5.89
7	0.080	5.89
8	0.080	5.89
9	0.080	5.89
10	0.080	5.89

**Figure 3.** Newton–Raphson Method: Differential Fade Depth and Effective Path Length vs Number of Iterations (*n*).

5. Conclusion

In this research, Newton–Raphson algorithm is used to adjust the maximum path length based on the difference between the maximum fade depth and the maximum fade margin the system can accommodate. The adjusted maximum path length is used to recalculate the free space path loss, the maximum fade depth and the maximum fade margin until the maximum path length is found at which the maximum fade depth is equal to the maximum fade margin the system can accommodate. Only the rain and multipath fading are considered in the determination of the effective fade depth in the link. In some cases, other fade mechanisms may severely affect the fade depth, especially at higher frequencies above 45 GHz. Also, free space path loss is also used as input for the path loss model. In practice, other path loss models can be used for more accurate prediction of the path loss. In all these, additional studies are required to incorporate other path loss models and fade mechanism.

It has also been observed that the algorithms generally have high convergence cycle whenever the multipath fading

dominates. Further work is required to reduce the high convergence cycle due to multipath fading.

References

- [1] Shoewu, O., & Edeko, F. O. (2011). Microwave signal attenuation at 7.2 GHz in Rain and Harmattan Weather. *American Journal of Scientific Research*, 2(3), 332-345.
- [2] Thorvaldsen, P., & Henne, I. Propagation measurements on a line-of-sight over-water radio link in Norway. *Radio Science*, 49(7), 531-548(2014).
- [3] Angueira, P., & Romo, J. *Microwave Line of Sight Link Engineering*. John Wiley & Sons (2012).
- [4] Lehpamer, Harvey. *Microwave transmission networks: planning, design, and deployment*. New York: McGraw-Hill (2010).
- [5] ITU-R838 ITU-R Recommendation p.838-3. Specific Attenuation Model for Rain for Use in Prediction Methods. International Telecommunication Union, Geneva (2005a).
- [6] Radio Signal Propagation, Breeze Wireless Technology Limited, Israel (2004).

- [7] Sun, S., MacCartney, G. R., Samimi, M. K., & Rappaport, T. S. (2015, December). Synthesizing omnidirectional antenna patterns, received power and path loss from directional antennas for 5G millimeter-wave communications. In 2015 IEEE Global Communications Conference (GLOBECOM) (pp. 1-7). IEEE.
- [8] Peter, M., Keusgen, W., & Weiler, R. J. (2015, May). On path loss measurement and modeling for millimeter-wave 5G. In 2015 9th European Conference on Antennas and Propagation (EuCAP) (pp. 1-5). IEEE.
- [9] Bai, T., & Heath, R. W. (2015). Coverage and rate analysis for millimeter-wave cellular networks. *IEEE Transactions on Wireless Communications*, 14(2), 1100-1114.
- [10] Bekkali, A., Zou, S., Kadri, A., Crisp, M., & Pentty, R. V. (2015). Performance analysis of passive UHF RFID systems under cascaded fading channels and interference effects. *IEEE Transactions on Wireless Communications*, 14(3), 1421-1433.
- [11] Xu, Hao, Vikas Kukshya, and Theodore S. Rappaport. "Spatial and temporal characteristics of 60-GHz indoor channels." *IEEE Journal on selected areas in communications* 20.3 (2002): 620-630.
- [12] Patwari, N. ECE 5325/6325: Wireless Communication Systems Lecture Notes, Spring 2010.
- [13] Hall, P. S., & Hao, Y. (2006, November). Antennas and propagation for body centric communications. In *2006 First European Conference on Antennas and Propagation* (pp. 1-7). IEEE.
- [14] Basu, A., Banerjee, P., & Sen, S. (2016). OPTIMIZATION OF RF PROPAGATION MODELS FOR COGNITIVE RADIO.
- [15] Vickberg, M. E., Sainati, R. A., & Gilbert, B. K. (2016). Design Considerations for Indoor Wireless Transmission between a Body-Worn Physiological Monitoring Device and a Gateway in a Home Environment. *American Journal of Biomedical Engineering*, 6(4), 95-114.
- [16] Héctor, J., Sturm, C., & Pontes, J. (2015). Radio Channel Fundamentals and Antennas. In *Radio Systems Engineering* (pp. 81-106). Springer International Publishing.
- [17] Almeida, K. S., Santos, R. T., Silva, E., Cardoso, C. C., & Oliveira, I. M. (2015, November). UHF signal measurements and prediction using propagation models. In *Microwave and Optoelectronics Conference (IMOC), 2015 SBMO/IEEE MTT-S International* (pp. 1-6). IEEE.
- [18] Yun, Y., Kim, N., & Kim, Y. (2016). A Novel RSS-Ratio Indoor Positioning scheme in WLAN environments. *International Journal of Applied Engineering Research*, 11(9), 6720-6724.
- [19] International Telecommunications Union (ITU-R), "Propagation data and prediction methods required for the design of terrestrial line-of-sight systems," Recommendation ITU-R P.530-11, (2005b).
- [20] Freeman, R. L. *Radio system design for telecommunication* (Vol. 98). John Wiley & Sons (2006).
- [21] Haykin, S. S., Moher, M., & Koilpillai, D. *Modern wireless communications*. Pearson Education India (2011).
- [22] Weisstein, E. W. (2002). Lambert W-function.
- [23] Roberts, K., & Valluri, S. R. (2016). Tutorial: The quantum finite square well and the Lambert W function. *Canadian Journal of Physics*, (999), 1-6.
- [24] He, C. H., & He, C. H. (2016). An introduction to an ancient Chinese algorithm and its modification. *International Journal of Numerical Methods for Heat & Fluid Flow*, 26(8), 2486-2491.