Pure and Applied Mathematics Journal
2014; 3(6-1): 1-6
Published online September 01, 2014 (http://www.sciencepublishinggroup.com/j/pamj)
doi: 10.11648/j.pamj.s.2014030601.11
ISSN: 2326-9790 (Print); ISSN: 2326-9812 (Online)

New types of chaos and non-wandering points in topological spaces

Mohammed N. Murad Kaki

Math Dept., School of Science, University of Sulaimani, Sulaimani, Iraq

Email address:
muradkakaee@yahoo.com

To cite this article:
Mohammed N. Murad Kaki. New Types of Chaos and Non-Wandering Points in Topological Spaces. *Pure and Applied Mathematics Journal.* Special Issue: Mathematical Theory and Modeling. Vol. 3, No. 6-1, 2014, pp. 1-6. doi: 10.11648/j.pamj.s.2014030601.11

Abstract: In this paper, we will define a new class of chaotic maps on locally compact Hausdorff spaces called α-type chaotic maps defined by α-type transitive maps. This new definition coincides with Devaney's definition for chaos when the topological space happens to be a metric space. Furthermore, we will study new types of non-wandering points called α-type nonwandering points. We have shown that the α-type nonwandering points imply nonwandering points but not conversely. Finally, we have defined new concepts of chaotic on topological space. Relationships with some other type of chaotic maps are given.

Keywords: Chaos, α-Type Chaotic Maps, α-Type Nonwandering Points, Transitive

1. Introduction

I studied a new type of topological transitive map called topological α - transitive [1] and investigated some of its topological dynamic properties. Further, I introduced and defined the notions of α- wandering points, and studied the notion of minimal α-open sets [2]. I have proved some new theorems and propositions associated with these new definitions. I have also shown that topologically α-type transitive maps, α-wandering points and topologically α-mixing are preserved under αr-conjugacy. Recently there has been some interest in the notion of a locally closed subset of a topological space. According to Bourbaki [3] a subset S of a space (X,τ) is called locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [4] to define the concept of LC–continuity, i.e. a function $f:(X,\tau)\to(X,\sigma)$ is LC–continuous if the inverse with respect to f of any open set in Y is closed in X[4]. The study of semi open sets and semi continuity in topological spaces was initiated by Levine [5]. Bhattacharya and Lahiri [6] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets which was introduced by Levine [7]. Throughout this paper, the word "space " will mean topological space . Let A be a subset of a space X,. Recall that a point x is said to be an α - limit point of A if for each α - open U containing x, $U\cap(A\setminus x)\neq\phi$.The set of all α - limit points of A is called the α -derived set of A and is denoted by $D_\alpha(A)$. The point x ϵ X is in the α –closure of a set A ⊂ X if α(U) ∩ A≠φ, for each open set U containing x. Then the α -closure of a set A is the intersection of all α -closed sets containing A and is denoted by $Cl_\alpha(A)$. A subset of a space X, Then the $\operatorname{int}_\alpha(A)=\cup\{U:U$ is α–open and $U\subset A\}$. In this paper, we will define some new conceptions such as: totally α-transitive, α-type hyper-cyclic maps and we proved some theorems associated with this definition. If the map $f:X\to X$ is α-irresolute. f is said to be weakly topologically α-mixing if $f\times f$ is α-type transitive , i.e. there is a positive integer n such that $f^n(U_1)\cap V_1\neq\phi$ and $f^n(U_2)\cap V_2\neq\phi$ provided that $U_1, U_2,, V_1, V_2$ are non-empty α open subsets of X. f is said to be topologicallyα- mixing if there is a positive integer N such that $f^n(U)\cap V\neq\phi$ for every n>N provided U and V are non-empty α-open subsets of X. Note that a topologically α-mixing map is weakly topologically α-mixing and a weakly topologically α-mixing map is α-type transitive which is transitive. We will also define a new types of chaotic maps called α-type chaotic and prove some new theorems associated with this definition. In [10] we defined a new class of chaotic maps on locally compact Hausdorff spaces called λ-type chaotic maps defined by λ-type transitive maps. This new definition implies John Tylar definition which coincides with Devaney's definition for chaos when the topological space happens to be a metric space. A non-degenerate topological space X is said to be:α-type chaotic, if for any two distinct points p and q of X there exists an α- open set U containing p and an α-open set V containing q such that no α-open subset of U is homeomorphic to any α-open subset of V ; and the space X is said to be strongly α-type chaotic if for any two distinct points p and q of X there exist α-open sets U containing p and V containing q respectively such that no α-open subset of U is homeomorphic to any subset of V Relationships with some other type of chaotic maps are given

2. Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let X be a topological space and $A \subset X$. The interior (resp. closure) of A is denoted by Int(A) (resp. Cl(A).

Definition 2.1. By a topological system we mean a pair (X, f) , where X is a compact Hausdorff topological space (the phase space), and $f: X \to X$ is a continuous function. The dynamics of the system is given by $x_{n+1} = f(x_n),\ x_0 \in X,\ n \in \mathbf{N}$ *and the solution passing through* x *is the sequence* $\{f(x_n)\}$ *where* $n \in \mathbf{N}$. An element $x \in X$ is called periodic point if for some $n \geq 1,\ f^n(x) = x$. The least such n is called the period of x. The set of all periodic points of f is denoted by $Per(f)$.

Definition 2.2. Let (X, f) be a topological system. Suppose that $f: X \to X$ is α-irresolute map.

1. The map f is said to be α-type transitive if there is a positive integer n such that $f^n(U) \cap V \neq \phi$. provided that U and V are non-empty α-open subsets of X [1].
2. The map f is called totally α-transitive if f^n is topologically α-type transitive for all $n \geq 1$.

Definition 2.3. [1,2] Suppose $f: X \to X$ is α-irresolute map. The map f is called topologically α-mixing if, given any nonempty α-open subsets $U, V \subseteq X$ $\exists\ N \geq 1$ such that $f^n(U) \cap V \neq \phi$ for all n>N. Clearly if f is topologically α-mixing then it is also α-type transitive but not conversely

Let (X, f) be a topological system. A point $x \in X$ "moves," its trajectory being the sequence $x, f(x), f^2(x), ...f^n(x),.....$, where f^n is the nth iteration of the map f. The point $f^n(x)$ is the position of x after n units of time. The set of points of the trajectory of x under f is called the orbit of x, denoted by $O_f(x)$.

Definition 2.4. Suppose $f: X \to X$ is α-irresolute map. The map f is said to be α-type hyper cyclic if there is a point $x \in X$ (called α-type hyper cyclic point) whose orbit under f , $O_f(x) = \{f^n(x): n \in \mathbf{N}\}$, is α-dense in X.

Definition 2.5. Let (X, f) *be a topological system, then the map* $f: X \to X$ is α-type chaotic if

1. The set of all periodic points for f is α-dense in X.
2. f is α-type hyper cyclic map.

Proposition 2.6

1. Let X be a α-compact space without isolated point, if f is α-type hyper cyclic, that is there exists x_0 ∈X such that the set $O_f(x_0)$ is α- dense then f is α-type transitive .
2. if $f: X \to X$ *and* $g: Y \to Y$ are topologically αr -conjugated by the homeomorphism $h: Y \to X$. Then g is α-type hyper cyclic (i.e. *for all* y ∈ Y *the orbit* $O_g(y)$ *is α-dense in* Y) *if and only if* f is α-type hyper cyclic *(i.e. the orbit* $O_f(h(y))$ *of* h(y) *is α-dense in* X*).*
3. *If X is separable and second category then* topologically transitive *then the map f is hyper cyclic.*

Proof 1. Let $x_0 \in X$ is such that $O_f(x_0)$ is α-dense in X. Given any pair U, V of α-open subsets of X, by α-density there exists n such that $f^n(x_0) \in U$, but $O_f(x_0)$ is α- dense implies that $O_f(f^n(x_0))$ is α-dense, so intersects V , i.e. There exists a positive integer m such that $f^m(f^n(x_0)) \in V$. Therefore $f^{m+n}(x_0) \in f^m(U) \cap V$ that is $f^m(U) \cap V \neq \varphi$. So f is α-type transitive.

proof 2. Let $h: Y \to X$ be the αr-conjugacy. Assume that g is α-type hyper cyclic so there is $y \in Y$ such that $O_g(y)$ is α-dense and let us show that f is α-type hyper cyclic i.e. $O_f(h(y))$ is α-dense in X. For any U ⊂ X non-empty α-open set, $h^{-1}(U)$ is a α-open set in Y since h^{-1} is continuous because h is αr- homeomorphism and it is non-empty since h is surjective. By α-density of $O_g(y)$, there exists k ∈ N such that $g^k(y) \in h^{-1}(U)$ $\Leftrightarrow$ $h^{-1}(g^k(y)) \in U$ .Since h is a αr-conjugacy, $f^k \circ h = h \circ g^k$ so $f^k(h(y)) = h(g^k(y)) \in U$, therefore $O_f(h(y))$ intersects U. This holds for any non-empty α- open set U and thus shows that $O_f(h(y))$ is α-dense. The other implication follows by exchanging the role of f and g.

Suppose $f: X \to X$ is α-irresolute map. f is said to be weakly topologically α-mixing if $f \times f$ is α-type transitive , i.e. there is a positive integer n such that $f^n(U_1) \cap V_1 \neq \phi$ and $f^n(U_2) \cap V_2 \neq \phi$ provided that $U_1, U_2,, V_1, V_2$ are non-empty α open subsets of X. f is said to be topologicallyα- mixing if there is a positive integer N such that $f^n(U) \cap V \neq \phi$ for every n>N provided U and V are non-empty α-open subsets of X. It is clear that a topologically α-mixing map is weakly topologically α-mixing and a weakly topologically α-mixing map is α-type transitive which is transitive.

Definition 2.7. A non-degenerate topological space X is said to be:

(a) α-type chaotic if for any two distinct points p and q of X there exists an α- open set U containing p and an α-open set V containing q such that no α-open subset of U is homeomorphic to any α-open subset of V ;

(b) strongly α-type chaotic if for any two distinct points p and q of X there exist α-open sets U containing p and V containing q respectively such that no α-open subset of U is homeomorphic to any subset of V ;

Let (X,f) *be a topological system.* Suppose $f : X \to X$ is α-irresolute map, the α- minimality of *(X, f)* is defined by requiring that every point $x \in X$ visit every α- open set *V* in *X* (i.e. $f^n(x) \in V$ for some $n \in$ N) then, instead, one may wish to study the following concept: every nonempty α-open subset *U* of *X* visits every nonempty α-open subset *V* of *X* in the following sense: $f^n(U) \cap V \neq \phi$. for some $n \in$ N. If the system *(X, f)* has this property, then it is called *topologically α-type transitive* as we mentioned before. We also say that *f* itself is topologically α-type transitive if the system cannot be broken down or decomposed into two α-subsystems (disjoint sets with nonempty α-interiors) which do not interact under *f* , i.e. , are invariant under the map ($A \subset X$ is *f*-invariant if $f(A) \subset A$).

An α- minimal topological system is a system that has no non-trivial sub-α-system, that is, any α-closed set $A \subset X$ satisfying $f(A) \subset A$ is either empty or the whole X itself. Equivalently, (X,f) is α-minimal if the orbit of every point x in X is α-dense (i.e. $Cl_\alpha(O_f(x)) = X$) . If X itself is a minimal set we say that the system (X, *f*) is a minimal system.

Definition 2.8. (Topological weak α-mixing) A topological system (X,f) is *topologically weakly α-mixing* if the product system $X \times X$ is topologically α-type transitive. If for every two non-emptyα- open sets $U,V \subset X$, all but finitely many time steps k $\in$ N satisfy $f^k(U) \cap V \neq \phi$, then the system is said to be (topologically) α- mixing. .

In between α-minimality and topologically α-type transitivity, we have the notion of strong α- transitivity.

Definition 2.9. A system is strongly α-transitive if for every point $x \in X$. the set $\bigcup_{n=0}^{\infty} f^{-n}(\{x\})$ is α-dense, or equivalently, if every non-empty α-open set $U \subset X$ satisfies $\bigcup_{n=0}^{\infty} f^n(U) = X$.

Definition 2.10. (Topologically α-Exact Map): A map f : X → X is *topologically α-type exact* if for any non-empty α-open set U ⊂ X there is appositive integer n for which $f^n(U) = X$.

Proposition 2.11. We have the following results:

Exact implies mixing implies weakly mixing implies transitive.

Topologically α-Exact Map implies topologically exact map but not conversely

A non-empty α- closed invariant set not containing proper subset which would be α-closed and invariant is called α-minimal.

Theorem 2.12. Any two α-minimal sets must have empty intersection.

Proof. Let M1 and M2 be two distinct α-minimal sets, and suppose that

$A = M_1 \cap M_2 \neq \phi$. Then A is α-closed, and for every a ∈ A and every

n ∈ N, $f^n(a) \in M_1 \cap M_2$, so A is invariant. But then A is a proper subset of both M_1 and M_2 which is α-closed, invariant and non-empty, contradicting the fact that M_1 and M_2 are α-minimal.

Remark 2.13. It is easy to see that topologically exact maps are also transitive

Remark 2.14. It is easy to see that any topologically α-exact map is also α-type transitive map which implies transitive map.

Remark 2.15. Any topologically α-exact map implies topologically exact.

Definition 2.16. Let (X,f) *be a topological system, then* $f : X \to X$ is α-type chaotic if

1. The set of all periodic points for f is α-dense in X.
2. f is α-type transitive.

Theorem 2.17. Suppose $f : X \to Y$ is α-irresolute map that is onto and suppose that D is α-dense subset of X. Then $f(D)$ is α-dense subset of Y .

Proposition 2.18. Recall that if (X,f) is a topological dynamical system, where X is a Hausdorff space. Then the following hold:

1. The set of all fixed points is a closed subset of X.
2. Orbits of any two periodic points are either identical or disjoint.
3. If a trajectory converges, it converges to a fixed point.
4. An element is eventually periodic if and only if it has a finite orbit.
5. Every orbit is an invariant set; the orbits of periodic points are minimal invariant sets.
6. A subset of X is invariant if and only if it is a union of orbits.
7. The closure of an invariant set is also invariant.
8. The set of all periodic points is an invariant set.
9. For each $A \subset X$, the set $\bigcup_{n=0}^{\infty} f^n(A)$ is the smallest invariant set containing A.

Let (*X; f*) and (*Y; g*) be two topological systems. Then a topological conjugacy h from *f* to *g* carries orbit of *f passing through x* to orbit of g passing through h(x).

Theorem 2.20. Let (X; f) and (Y; g) be two topological systems and let $h : X \to Y$ be a topological αr-conjugacy[1]. Then

1. $h^{-1} : Y \to X$ is a topological αr-conjugacy.
2. $h \circ f^n = g^n \circ h$ for all $n \in \mathbf{N}$.
3. $p \in X$ is a periodic point of f if and only if h(p) is a periodic point of g.

1. If p is a periodic point of f with α-neighborhood D of p, then the α-neighborhood of h(p) is h(D).
2. The periodic points of f are α-dense in X if and only if the periodic points of g are α-dense in Y .
3. f is topologically α-type transitive on X if and only if g is topologically α-type transitive on Y .
4. f is topologically α-minimal map on X if and only if g is topologically α-minimal map on Y .
5. f is topologically α-mixing map on X if and only if g is topologically α-mixing map on Y .
6. f is α-type chaotic map on X if and only if g is α-type chaotic map on Y .

Let (X,f) be a topological system. A map $f: X \to X$ is called $\alpha-type$ chaotic, if it is topological α-type transitive and, its periodic points are α-dense in X, i.e. every non-empty α-open subset of X contains a periodic point

Definition 2.21. Let (X,f) be a topological system. A point $x \in X$ is called α-recurrent if for every α- open set V containing x, there is $n \in \mathbf{N}$ such that $f^n(x) \in V$.

Proposition 2.22. Every α-recurrent point is recurrent point but not conversely.

Theorem 2.23. [1] Let (X, τ) be a topological space and $f: X \to X$ be α -irresolute map. Then the following statements are equivalent:

(1) f is topological α-transitive map

(2) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^n(U)$ is α-dense in X

(3) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is α-dense in X

(4) If $B \subset X$ is α-closed and B is f- invariant i.e. $f(B) \subset B$. then B=X or B is nowhere α-dense

(5) If U is α-open and $f^{-1}(U) \subset U$ then U=ϕ or U is α-dense in X.

For proof see [1]

3. The Product of Two Topological Systems

Now, given two maps $f: X \to X$ *and* $g: Y \to Y$ on topological spaces X and Y. respectively, consider their product $f \times g: X \times Y \to X \times Y$, $(f \times g)(x, y) = (f(x), g(y))$, with product topology on X × Y.

Lemma 3.1. Let (X,f) , (Y,g) be topological systems. The set of periodic points of $f \times g$ is α-dense in $X \times Y$ if and only if, for both of f and g , the sets of periodic points in X and Y are α-dense in X, respectively Y.

Proof: Assume that the set of periodic points of f is α-dense in X (i.e. $Cl_\alpha(Per(f)) = X$) and the set of periodic points of g is α-dense in Y (i.e. $Cl_\alpha(Per(g)) = Y$). We have to prove that the set of periodic points of $f \times g$ is α-dense in $X \times Y$. Let $W \subset X \times Y$ be any non-empty α-open set. Then there exist non-empty α-open sets $U \subset X$ *and* $V \subset Y$ with $U \times V \subset W$. By assumption, there exists a point $x \in U$ such that $f^n(x) = x$ with $n \geq 1$. Similarly, there exists $y \in V$ such that $g^m(y) = y$ with $m \geq 1$. For $p = (x, y) \in W$ and $k = mn$ we get

$$(f \times g)^k(p) = (f \times g)^k(x, y)$$

$$= ((f^k(x), g^k(y)) = (x, y) = p$$

Therefore W contains a periodic point and thus the set of periodic points of $f \times g$ is α-dense in $X \times Y$.

Conversely let $U \subset X$ *and* $V \subset Y$ be non-empty α-open subsets. Then $U \times V$ is a non-empty α-open subset of $X \times Y$. As the set of the periodic points of $f \times g$ is α-dense in $X \times Y$, there exists a point $p = (x, y) \in U \times V$ such that $(f \times g)^n(x, y) = ((f^n(x), g^n(y)) = (x, y)$ for some n. From the last equality we obtain $f^n(x) = x$ for $x \in U$ and $g^n(y) = y$ for $y \in V$.

By Lemma 3. 1, α-denseness of periodic points carry over from factors to products. But, topological α-type transitivity may not carry over to products. The converse of this situation is however true:

Lemma 3.2. Let $f: X \to X$ *and* $g: Y \to Y$ be maps and assume that the product $f \times g$ is topological α-type transitive on $X \times Y$. Then the maps f and g are both topological α-type transitive on X and Y respectively.

Proof. We prove the α-transitivity of f ; the α-transitivity of g can be proved similarly. Let U_1, V_1 be non-empty α-open sets in X. Then the sets $U = U_1 \times Y$ *and* $V = V_1 \times Y$ are α-open in $X \times Y$. As $f \times g$ is α-type transitive, there exists a positive integer n such that $(f\ x\ g)^n(U) \cap V \neq \phi$. From the equalities:

$$(f \times g)^n(U) \cap V = [f^n(U_1) \times g^n(Y)] \cap [V_1 \times Y]$$
$$= [f^n(U_1) \cap V_1] \times [g^n(Y) \cap Y] \neq \phi.$$

So $f^n(U_1) \cap V_1 \neq \phi$. Thus f is topological α-type transitive.

Definition 3.3. Let $f: X \to X$ be a map on the topological space X. If for every nonempty α-open subsets $U, V \subset X$ there exists a positive integer n_0 such that for every $n \geq n_0, f^n(U,) \cap V \neq \phi$ then f is called topologically α-type mixing.

It is clear that topological α-type mixing implies topological α-type transitive.

There is an even stronger notion that implies topological α-type mixing.

Definition 3.4. Let $f: X \to X$ be a map on the topological space X. If for every nonempty α-open subset $U \subset X$ there is a positive integer n_0 such that for every $n \geq n_0, f^n(U) = X$, then f is called locally α-type eventually onto.

Lemma 3.5. The product of two topologically α-type mixing maps is topologically α-type mixing.

Proof: Let (X,f) , (Y,g) be topological systems and f, g be topologically α-type mixing maps. Given $W_1, W_2 \subset X \times Y$, there exists α-open sets $U_1, U_2 \subset X$ *and* $V_1, V_2 \subset Y$, such that $U_1 \times V_1 \subset W_1$ and $U_2 \times V_2 \subset W_2$. By assumption there exist n_1 *and* n_2 such that

$$f^k(U_1) \cap U_2 \neq \phi \quad for \quad n \geq n_1 \quad and \quad g^k(V_1) \cap V_2 \neq \phi \quad for \quad n \geq n_{12}.$$

$$For \ n \geq n_0 = \max\{n_1 , n_2\}$$

we get

$$[(f \, x \, g)^k(U_1 \ x \ V_1)] \cap (U_2 \ x \ V_2) = [f^k(U_1) \ x \ g^k(V_1)] \cap (U_2 \ x \ V_2)$$
$$= [f^k(U_1) \cap U_2] \ x \ [g^k(V_1) \cap V_2] \neq \phi$$

Which means that f × g is topologically α-type mixing.

Now, we give some sufficient conditions for a product map to be α-type chaotic.

Theorem 3.6. Let $f : X \to X$ *and* $g : Y \to Y$ be α-type chaotic and topologically α-type mixing maps on topological spaces X and Y. Then $f \times g : X \times Y \to X \times Y$ is α-type chaotic.

Proof.: The map $f \times g$ has α-dense periodic points by Lemma 3.1 and it is topologically α-type mixing by Lemma 3.5 and hence topologically α-type transitive. Thus the two conditions of α-type chaos are satisfied.

4. α-Minimal Maps and α-Non-Wandering Points

In the study of the dynamics of α-irresolute map $f: X \to X$ of α- compact space X into itself, a central role is played by the various low recursive properties of the points of X. One of the important such properties is α-non-wanderingness. A point x ϵ X is called α-wandering if there exists α-open set U containing x such that for all n > 0, $f^n(U) \cap U = \phi$. A point is α- nonwandering if it is not a α-wandering point. The α-non-wandering set is the complement of the set of α-wandering points. We will prove that the α-wandering set is α-open and the α-nonwandering set is α-closed. It is easy to show that the α- non-wandering set $\alpha\Omega(f)$ is a non-empty α- closed invariant subset of X. A non-wandering set of a topological system has the property that an orbit starting at any point of the set comes arbitrarily close arbitrarily often to the set. Examples of α-non-wandering sets are fixed points, limit cycles, invariant sets.

One of the goals of dynamical system theory is to decompose the α-nonwandering set in to disjoint α- closed subsets, called α-type basic sets, which have α-dense orbits, when this can be done , the entire phase space X can be partitioned into the α-stable sets of the α-type basic sets. The α-stable set of a α-type basic set is the set of points whose w-limit is in the α-type basic set. But, the α-unstable set of α-type basic set is the set of points with α-limit set in the α-type basic set. If X is a compact space then every limit set is nonempty.

Proposition 4.1. If $f : X \to X$ *and* $g : Y \to Y$ are topologically αr -conjugate. Then

(1) f is α-type transitive if and only if g is α-type transitive;
(2) f is α-minimal if and only if g is α-minimal;
(3) f is topologically α-mixing if and only if g is topologically α-mixing.

Definition 4.2. Let $f : X \to X$ be α-irresolute self-map of a topological space X. A fundamental α-type domain for f is α-open subset $D \subset X$ such that every orbit of f intersect D in at most one point and intersect $Cl_\alpha(D)$ in at least one point.

Proposition 4.3. Let $f : X \to X$ *and* $g : Y \to Y$ be two α-irresolute self-maps. Assume that there are a fundamental α-type domain $D_f \subset X$ for f , a fundamental α-type domain $D_g \subset Y$ for g and a αr-homeomorphism $h : Cl_\alpha(D_f) \to Cl_\alpha(D_g)$ such that $g \circ h = h \circ f$ on $f^{-1}(Cl_\alpha(D_f)) \cap Cl_\alpha(D_f)$. Then f and g are topologically αr-conjugated.

Definition 4.4. Let (X; f) be a topological system. A point x ϵ X is α-type non-wandering if for any α-open set U containing x there is N > 0 such that $f^N(U) \cap U \neq \phi$. The set of all α-type non-wandering point is denoted by $NW_\alpha(f)$ or $\alpha\Omega(f)$. A point which is not α-type non-wandering is called α-type wandering or α-wandering for short.

Proposition 4.5. Let (X,f) be a topological system every α-type non-wandering point in X is non-wandering point, but not conversely.

Theorem 4.6. Let (X,f) be a topological system on α-Hausdorff space X. Then:

(1) $NW_\alpha(f)$ is α-closed.
(2) $NW_\alpha(f)$ is f -invariant.
(3) If f is invertible, then $NW_\alpha(f^{-1}) = NW_\alpha(f)$.
(4) If X is α-compact then $NW_\alpha(f) \neq \phi$.
(5) If x is α-type non-wandering point in X, then for every α-open set U containing x and $n_0 \in N$ there is $n > n_0$ such that $f^n(U) \cap U \neq \phi$.

5. Conclusion

There are the main results:

Definition 5.1. Suppose $f : X \to X$ is α-irresolute map. The map f is said to be α-type hyper cyclic if there is a point $x \in X$ (called α-type hyper cyclic point) whose orbit under f , $O_f(x) = \{f^n(x) : n \in \mathbf{N}\}$, is α-dense in X.

Definition 5.2. Let (X,f) be a topological system, then the map $f : X \to X$ is α-type chaotic if

1. The set of all periodic points for f is α-dense in X.

2. f is α-type hyper cyclic map.

Theorem 5.3. Suppose $f: X \to Y$ is α-irresolute map that is onto and suppose that D is α-dense subset of X. Then $f(D)$ is α-dense subset of Y .

Definition 5.4. Let (X,f) be a topological system. A point $x \in X$ is called α-recurrent if for every α- open set V containing x, there is $n \in \mathbf{N}$ such that $f^n(x) \in V$.

Proposition 5.5. Every α-recurrent point is recurrent point but not conversely.

Theorem 5.6. Any two α-minimal sets must have empty intersection.

Lemma 5.7. Let (X,f) , (Y,g) be topological systems. The set of periodic points of $f \times g$ is α-dense in $X \times Y$ if and only if, for both of f and g , the sets of periodic points in X and Y are α-dense in X, respectively Y.

Lemma 5.8. Let $f: X \to X$ *and* $g: Y \to Y$ be maps and assume that the product $f \times g$ is topological α-type transitive on $X \times Y$. Then the maps f and g are both topological α-type transitive on X and Y respectively.

Lemma 5.9. The product of two topologically α-type mixing maps is topologically α-type mixing.

Definition 5.10. Let (X; f) be a topological system. A point x ϵ X is α-type non-wandering if for any α-open set U containing x there is N > 0 such that $f^N(U) \cap U \neq \phi$. The set of all α-type non-wandering point is denoted by $NW_\alpha(f)$ or $\alpha\Omega(f)$. A point which is not α-type non-wandering is called α-type wandering or α-wandering for short.

Proposition 5.11. Let (X,f) be a topological system every α-type non-wandering point in X is non-wandering point, but not conversely.

Proposition 5.12 Let (X, f) be a topological system on α-Hausdorff space X. Then:

(1) $NW_\alpha(f)$ is α-closed.

(2) $NW_\alpha(f)$ is f-invariant.

(3) If f is invertible, then $NW_\alpha(f^{-1}) = NW_\alpha(f)$.

(4) If X is α-compact then $NW_\alpha(f) \neq \phi$.

(5) If x is α-type non-wandering point in X, then for every α-open set U containing x and n_0 ϵ N there is $n>n_0$ such that $f^n(U) \cap U \neq \phi$.

References

[1] Mohammed N. Murad Kaki, Topologically α - Transitive Maps and Minimal Systems *Gen. Math. Notes, ISSN 2219-7184; Copyright © ICSRS Vol. 10, No. 2,* (2012) *pp. 43-53*

[2] Mohammed N. Murad Kaki. Topologically α- Type Maps and Minimal α-Open Sets Canadian Journal on Computing in Mathematics, Natural Sciences, Engineering and Medicine Vol. 4 No. 2, (2013)

[3] N. Bourbaki, *General Topology* Part 1, Addison Wesley, Reading, Mass. 1966.

[4] M. Ganster and I.L. Reilly, *Locally closed sets and LC-continuous functions*, Internet. J. Math. Math. Sci. 12 (3) (1989), 417–424.

[5] Levine N., *Semi open sets and semi continuity in topological spaces*. Amer. Math. Monthly. 70(1963). 36-41.

[6] Bhattacharya P., and Lahiri K.B., *Semi-generalized closed sets in topology*. Indian J. Math. 29 (1987). 376-382.

[7] Levine N., *Generalized closed sets in topology*, Rend. Cire. Math. Paler no. (2) 19 (1970). 89-96.

[8] Ogata N., *On some classes of nearly open sets*, Pacific J. Math. 15(1965). 961-970.

[9] Andrijevie D., *Some properties of the topology of α-sets,* Math. Vesnik, (1994). 1 -10

[10] Mohammed N. Murad Kaki, New types of chaotic maps on topological spaces, International J. of Electrical and Electronic Science(AASCIT), Published online March 20, 2014, 1(1): 1-5

[11] F. Nakaoka and N. Oda: Some applications of minimal open sets, Internet Jr. Math. Math. Sci., 27(2001), no. 8 , 471-476.

[12] Kasahara S., *Operation-compact spaces. Mathematica Japonica*. 24 (1979).97-105.

[13] Rosas E., Vielina J., *Operator- compact and Operator-connected spaces*. Scientific Mathematica 1(2)(1998). 203-208.

[14] Maheshwari N. S., and Thakur S. S., *On α-irresolute mappings*, Tamkang J. Math. 11 (1980). 209- 214.

[15] http://www.scholarpedia.org/article/Minimal_dynamical_systems