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# Mathematical Understanding of How a Car Engine Cooling System Works

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**Abstract:** The research is related to the modeling of Newton's cooling law (in its ideal form) through the numerical, graphical and analytical approaches to the automobile heating and cooling system (CHSV) and comparing it with the model of the exponential solution of the ordinary differential equation, from starting the car engine until the fan is activated several times and then turned off, with the aim that students relate school mathematics with problem situations in their context, an environment that is not common to deal with in the subject of Mathematics for Engineering II in the Mechanical Metal Engineering career at the Technological University of the Costa Grande de Guerrero, Mexico. In this sense, the use of technology is proposed to present the student with a strengthened scenario with the MaxiDAS DS808 Scanner and Oscilloscope automotive tool, with which the data of the time and temperature variables are obtained, exported and processed with the GeoGebra software, to obtain the plot and the functions that model the CHSV. The theory that supported the study was the Ontosemiotic approach to Mathematical Cognition and Instruction (EOS). The students worked with a didactic sequence, into which the activities to be developed during the experimental phase were integrated. Due to the COVID-19 pandemic, the sequence was placed on the LMS INSTRUCTURE CANVAS platform, where the developed products, explanatory videos and the comics "In search of a mathematical model" were recorded. Videoconferences were generated in Google Meet as an interactive student-teacher and student-student medium, to provide explanations of the subject and doubts. The research was of the qualitative type and the learning processes of 21 students were studied, the evidence was reviewed and as a result it is stated that it was possible to model the heating and cooling system of the engine, as well as understand the numerical, graphical and analytical approaches, as an alternative solution to the exponential solution of Newton's law of cooling, in its ideal form.

**Keywords:** Newton's Law of Cooling, Modeling, EOS Theory, Problem Situation, Function Adjustment

## 1. Introduction

The proposal includes an everyday life problem situation involving the cooling and heating system of a vehicle (CHSV), for the purpose of enabling students to understand the existing relationship with Newton's law of cooling by way of an analytical, numerical and graphic approach [1]. This was done because examples are usually taken from books that deal with the differential equation and solution approach, i.e., they focus on the algorithmic process, leaving aside alternatives to strengthen student understanding when applying school mathematics to CHSV.

Different studies have been done [2-4] on the predominance of analytical methods over numerical and graphical approaches, pointing out that the learning achieved by students is only partial. As result, the proposal to re-conceive of an alternative to their teaching, for instance on the basis of every day problem situations [5-9], where polynomial adjustments, and graphical and numerical representations are included as tools for resolution and interpretation of solutions, with the support of Information and Communication Technologies (ICT), as described in several works [4, 10, 11].

In the study at hand students use the *MaxiDAS DS808 Automotive Scanner and Oscilloscope* automotive diagnostic tool *(Figure 1)* to communicate with the vehicle Engine Control Unit

(ECU) and monitor the coolant temperature sensor to obtain data (numerical approach). The latter data was then exported to GeoGebra, where they were graphed (graphical approach) to analyze the behavior of the CHSV and adjust the function that models it after each of the stages. As a means of validation, using the initial ambient temperature conditions of the engine coolant the students proposed and solved the differential equation in its ideal form and to compare that with the approaches obtained experimentally, which proved to be very satisfactory.


***Figure 1.*** *MaxiDAS808 oscilloscope scanner connected to the car.*

The proposal was designed for the work to be carried out in-person, but in view of the global COVID-19 pandemic the staging was redesigned and placed on the *LMS INSTRUCTURE CANVAS* platform, where course modules, the materials and the resources for this proposal were incorporated. Synchronous student-student and student-teacher activities were carried out through *Google Meet*. By using these platforms, learning was active, appealing, and student-focused.

Given students' lack of CHSV knowledge a video was used to reinforce any prior knowledge of the problem situation. In the video students were able to identify 1) the heating stage as of starting the engine; 2) followed by the periodic phase in which the engine fan comes into action to keep the coolant temperature within a range of 93°C to 101°C; and finally, 3) the cooling stage when the engine is turned off. The latter stages were analyzed to obtain the data-adjusted functions.

The research was carried out at the Educational Program for Metal-Mechanical Engineering of the Technological University of the Guerrero Costa Grande (UTCGG). The research involved the participation of 17 students studying their eighth term, and its purpose was to determine the effect produced on learning and understanding Newton's law of cooling, from the numerical, graphic and analytical approaches of the CHSV, as well as assess the work done by students on the *LMS INSTRUCTURE CANVAS platform,* interaction with Google Meet, use of the GeoGebra program [12] and, last but not least, student satisfaction pertaining to their experiences with the didactic proposal.

Qualitative research was performed given that the study primarily focused on learning processes in student interaction and experience during development of the proposed activities, student motivation as they worked with a problem situation of the context in which students develop -mediated by way of the activities planned- making it possible for them to identify the relationship that exists between the data collected via the *MAXIDAS DS808* Scanner and Oscilloscope and the mathematical object.

## 2. The Ontosemiotic Approach to Mathematics Knowledge and Instruction

According to Godino [13], in the Ontosemiotic Approach (EOS) to mathematical knowledge and instruction a mathematical object is defined as everything that can be indicated, everything that can be pointed out or referred to when mathematics are done, communicated or learned (p. 434).

Therefore when referring to the mathematical object in this paper, the CHSV and its relationship with application of Newton's law of cooling will be understood as the numerical, graphic and analytical approach. The fact that Newton's law of cooling applies in its ideal form to CHSV is also taken into account, as such a system is too complex to be described via modeling of a differential equation of separable variables.

The problem situation refers to tasks that lead to mathematical activity, such as this proposal, in which aspects related to CHSV are analyzed. A problem situation tends to foster an opening of minds so that functional representations arise that guide students to attain a mathematical understanding of the mathematics object in question [14, 15].

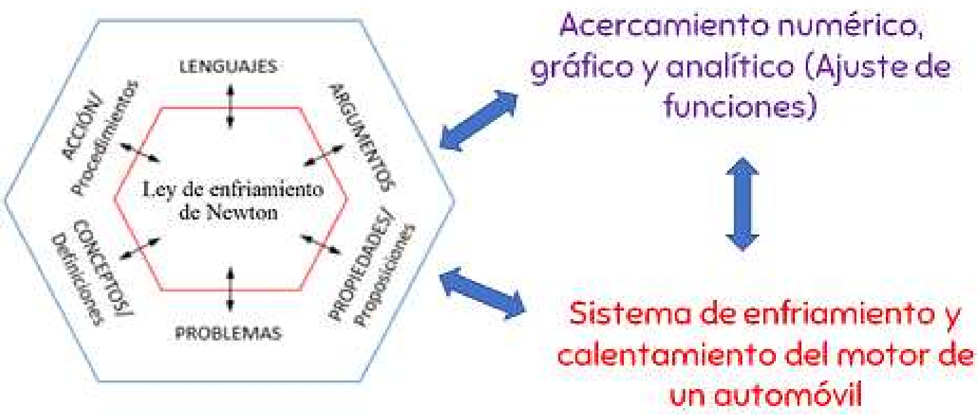



***Figure 2.*** *Objects involved in the practices from which the mathematical object emerges.*

In order to promote interpretation of the parameters of the problem situation, six basic or primary mathematical entities were proposed in the OSA. They are developed based on the solution to the CHSV, namely problem situation, language, concepts, properties, procedure and argument. The primary objects involved in mathematical practices are shown in Figure 2 [13, 16].

These primary objects are not independent, but rather, are related to each other, forming configurations that can be seen as networks of objects in which some are involved in and others emerge from the systems of practices.

The experimentation phase was carried out in three sessions with a group of 17 students studying their eighth term -January to April 2021. The first two sessions lasted two hours and the last session lasted one hour. At the end a clinical interview and a survey were applied, in which the participants expressed their ideas and conclusions concerning their experiences with the activities carried out in the workshop.

Both the concepts involved during the process of solving the problem situation and those that arose as a result were considered (Table 1).

***Table 1.*** *Concepts involved and that arose related to the institutional practice system.*

| | |
|---|---|
| Concepts involved | 1. Operation of the CHSV and the variables involved in the phenomenon.<br>2. The function adjustment for a temperature versus time data set from the CHSV.<br>3. Analysis of graphs from each of the CHSV phases. |
| Emerging concepts | 1. Analysis of the data graph and seeking the relationship with CHSV operation.<br>2. Determining CHSV cycle-adjusted functions.<br>3. Understanding the numerical, graphic and analytical approaches as a solution for the CHSV differential equation. |

## 3. Analysis of Results

During the application of three pilot tests, we identified aspects that had to be corrected prior to applying the experimental phase. For example, it was clear that the students lacked information on modeling the problem situation. Consequently, information was included in that regard, by designing the cartoon entitled *"in search of a mathematical model"* and including explanatory videos in which the method for separating variables and evaluating functions is explained. The worksheets were also modified so as to be applied while the experimental phase was in the staging process.

The worksheets for each of the CHSV study phases are made up of sections that include the concepts, properties and procedures for development of mathematical practice, deemed essential for learning and understanding mathematical objects. Different forms of evaluation were incorporated in order to assess the interaction with respect to their learning and understanding at different levels of knowledge and, in turn, promote the emergence of other primary mathematical objects in accordance with EOS theory. An applet was built in *GeoGebra* to look at engine heating and cooling issues so that the intended learning could actually be attainable for students and so that absence of previous knowledge would not be an impediment to learning.

The students pointed out that they found the use of a problem situation related to their professional training interesting as a means of learning mathematics. With the numerical, graphic and analytical approaches to the CHSV, encouragement is given to student arguments regarding each of the activities that were carried out in each phase of the CHSV operability, The starting point for the foregoing was an aspect of automotive mechanics, and with it, students related their previous knowledge with newly emerged knowledge.

As regards student attitude and willingness during development of the online course-workshop, they were very attentive as was observed through all of their work with the activities published in the *LMS INSTRUCTURE CANVAS* platform modules, as well as in the group sessions to generate discussion around the findings and team arguments. Moreover, in synchronous sessions on Google Meet the work teams freely expressed their doubts.

From the onset we intended for the students to develop the process using a scanner, but due to the COVID-19 pandemic the course professor obtained the data and presented it to the students via video. Once the students obtained the data, they were asked to graph it and specify the coordinates where any changes occur, which is an activity related to identification of the three phases (Figure 3) of the CHSV operation.

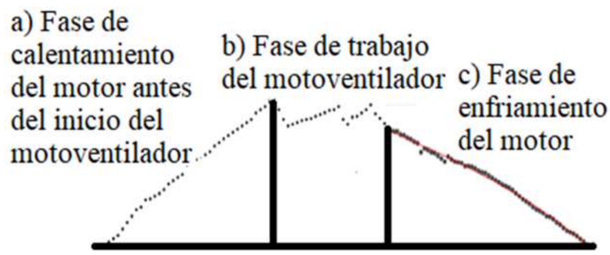



***Figure 3.*** *CHSV Phases.*

In the graphs drawn in *GeoGebra*, one can observe that there is a mismatch when the coolant reaches a certain temperature, after which the data remain within a range where they fail to fit within any particular function. With the foregoing, students are asked to specify the corresponding time $t_1$ and $t_2$ on the graph, as well as what temperatures correspond to each time $T_1$ and $T_2$, and they were required to specify them in the numerical data table.

Five teams correctly completed the graph (Figure 4), by identifying the data that corresponds to when the engine fan is activated during each stage of the heating phase, according to the information provided to them, and they adjusted the data correctly.

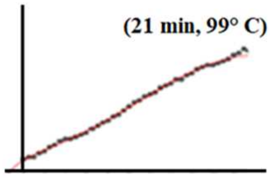
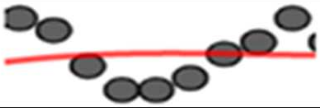
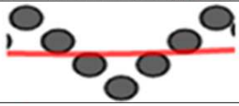
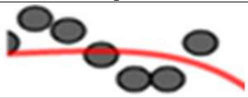
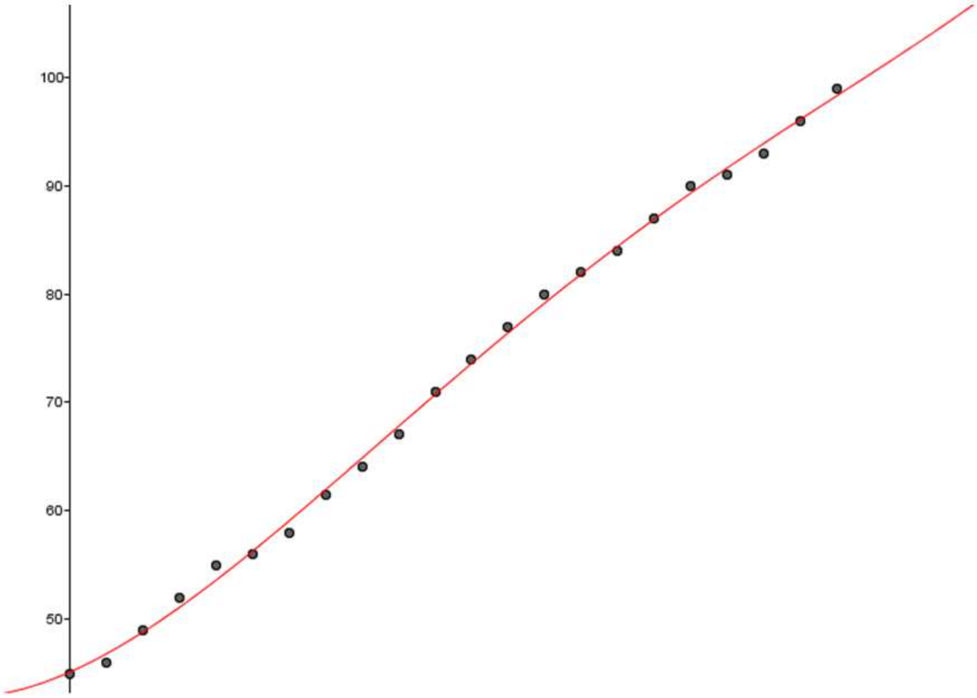
<table>
<tr><td rowspan="6"><br>At this point, from the initial time to 21 minutes, the engine reached the temperature of 99°C, at which point the motor fan starts.</td><td></td></tr>
<tr><td>From minute 21.5 to 25, the temperature decreased and the temperature cycle began again until reaching 99°C.</td></tr>
<tr><td></td></tr>
<tr><td>Again, from minute 25.5 to minute 28, the temperature was increased until the cycle is repeated.</td></tr>
<tr><td></td></tr>
<tr><td>To finish with the graph of the data, the same behavior is followed from minute 28.5 to 30.5, in which the engine is turned off and the cooling stage begins.</td></tr>
</table>

***Figure 4.*** *Description of points when the engine fan is activated and the temperature begins to increase.*

At the end of what was raised in the worksheet, a synchronous activity was carried out in which the teams were asked different questions about the practice carried out so as to gain insight on their arguments. For some questions students were asked to share the screen from their cell phones or computer devices in order to explain their answers. Figure 5 is used as a benchmark reference for their responses, where the intention is to delve into the degree to which students understood the mathematical object.




| | A | B | C |
|---|---|---|---|
| 1 | 0 | 45 | |
| 2 | 1 | 46 | |
| 3 | 2 | 49 | |
| 4 | 3 | 52 | |
| 5 | 4 | 55 | |
| 6 | 5 | 56 | |
| 7 | 6 | 58 | |
| 8 | 7 | 61.5 | |
| 9 | 8 | 64 | |
| 10 | 9 | 67 | |
| 11 | 10 | 71 | |
| 12 | 11 | 74 | |
| 13 | 12 | 77 | |
| 14 | 13 | 80 | |
| 15 | 14 | 82 | |
| 16 | 15 | 84 | |
| 17 | 16 | 87 | |
| 18 | 17 | 90 | |
| 19 | 18 | 91 | |
| 20 | 19 | 93 | |
| 21 | 20 | 96 | |
| 22 | 21 | 99 | |

$$P(x) = 0.00019x^4 - 0.01105x^3 + 0.1995x^2 + 1.48365\mathrm{x} + 45.09315$$

***Figure 5.*** *Graph of the first phase of the CHSV.*

*Researcher:* According to the numerical data collected, critical points were considered in the engine heating and cooling phases. What are their coordinates?

*Team 2*: *First, GeoGebra shows us that the temperature of the engine rises as time progresses... thus reaching a temperature that activates the fan. This happens... when: x=18.5 (time in minutes) y=101 (temperature in°C).*

The answer given by the student is supported using the graph that he had already built in GeoGebra with the data (Figure 5). He only shows the representative portion of the first phase, i.e., the heating of the engine until its fan is activated, and he identifies the coordinates (18.5, 101).

*Researcher:* What is the mathematical interpretation of the graph?

*Team 2: The behavior that was observed was that the engine fan is activated when its coolant reaches a temperature* *of 101°C, to reduce the temperature to 93. 5°C. It is an event that takes place constantly (the temperature rises and the fan takes care of reducing it) optimizing the engine. "maximum temperature that is reached as of 45°C up to to 101°C, when the fan is automatically activated".*

Part of the answer does correspond to the interpretation of the CHSV, but there is an error of appreciation as it has the engine temperature starting at 0°C, which is incorrect. This means that they manage to correctly relate the mathematical aspect to operation of the CHSV, for which several factors must be taken into account, including the ambient temperature of 35°C, which is the value that is extrapolated for a time t = 0 and the scanner marks the initial engine temperature of 45°C -the value that was taken for the polynomial fit.

*Team 2: In GeoGebra when we adjust the data where we believe the fan is activated, we obtained a polynomial of degree 3,* $p(x) = 0.0001x^3 - 0.7774x^2 + 3.665x^2 + 54.2362$ which is an approximate fit. *The horizontal axis is time and the vertical axis is temperature (°C). We can see that as soon as the vehicle reaches 101°C, the temperature begins to drop because the engine fan is activated, and the temperature stays constant between 95 and 101°C. The graphs are similar at these points, but the functions are different as we first have an algebraic function and then we have an exponential function when the calculations are done. However, in our opinion the study was carried out at certain intervals as a result of which two mathematical models are obtained -the first adapts to the data obtained through the scanner, and the second does not adapt as per the theory because there are factors or variables involved, such as some mechanical or electrical component of the system or of the coolant itself.*

We observe that team 2 student interprets and identifies a relationship between adjusting a function via the numerical approach to the problem's underlying equation and the numerical and graphical approach that models body heating. However the team realizes that the functions are different because the numerical data of the fit are third degree polynomials and phase 2 has a periodic function.

*Researcher:* What type of analytical approach is used for the point at which you believe the engine fan is activated?

*Team 5: The temperature changes are plotted by sections adapting to a regression model (polynomial at 4 degrees) that reaches the first point.*

*Researcher:* At what point on the graph does the temperature begin to drop?

*Team 5: On the way to point B, we believe the temperature begins to drop as soon as the engine fan is activated. Point B has coordinates of (18.5, 101) and point C of (21, 94).*

*Researcher:* What do those coordinates represent?

*Team 5: Time and temperature, time is in relation to the x-axis and it is the independent variable, and temperature is vertical on the y-axis. That is to say, in time 18.5 minutes after starting the engine, the coolant reaches its optimum operating temperature and the engine fan activates to extract the heat.*

*Researcher:* After the points specified, what is the behavior of the system in this phase of engine fan activation?

*Team 5: It becomes repetitive. First of all, according to the scanner data the engine does not have any heating problems. We refer to the points that my classmate mentioned and we realize that the process repeats itself, that is, it becomes cyclical. It reaches its highest temperature and drops again each time the engine fan is activated, and we have specified these points in the adjustment as point D, E and F.*

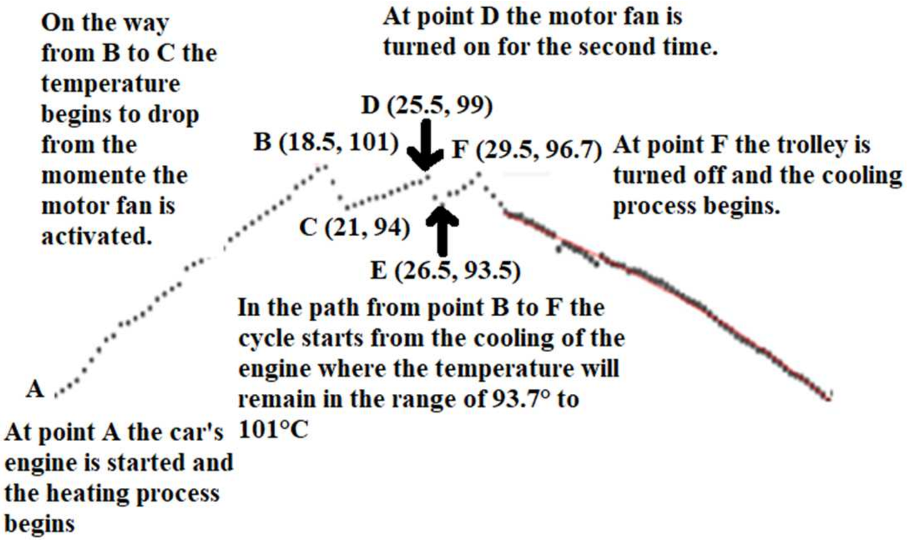



***Figure 6.*** *Location of the points at which the engine fan is activated (graph of student 1).*

In the synchronous session, student 1 mathematically represents phase 1 (from A to B) using the polynomial (Figure 6), but when the fan is activated, it is difficult for him to make an adjustment to those data because he simply refers to the points where "*the temperature decreases and rises again*" and relates it to different polynomials. It does not occur to him to use a sinusoidal adjustment. He simply states that a cyclic function can be adjusted and particular models can be obtained for each phase of the CHSV, where the engine stays within a temperature range and displays it in the synchronous session.

The different approaches to the CHSV fostered the intervention of the primary objects that make it up, from a situation of daily life with numerical, graphic and analytical aspects that guided students to developing a learning and understanding of the mathematical object. The conclusion reached is that the proposal influenced the students' learning and understanding by fostering the relationship between the different mathematical objects that make up the mathematical object EDO solution and that they were able to describe it numerically, graphically and analytically.

## 4. Conclusions

The didactic proposal to learn the mathematical object through the graphical, numerical and analytical approach of the CHSV, with the support of the *GeoGebra* program and MaxisDAS DS808 automotive diagnostic equipment, was positive for pilot test and experimental phase participants. They stated that the work in this didactic proposal was a good alternative for gaining knowledge and enhancing motivation for mathematics that make it possible to solve a context problem.

Prior mathematical knowledge and use of technological tools are two very important factors in this didactic proposal, since they allowed for relating the different mathematical objects involved in practice with a problem situation in its context.

During the proposal design phase it is important that different perspectives are considered and that teaching and learning are not restricted to algorithmic methods alone, because students would be deprived of other approaches that would allow them to enhance motivation for applying school mathematics to everyday life. In this case the work done to guide student learning about understanding the mathematical object was relevant.

The CHSV numerical, graphic and analytical approaches were helpful for the activities proposed for students, starting from a familiar event that directly influenced student motivation and interest in the study of the mathematical object.

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