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Stochastic differential equation driven by the Wiener process in a Banach space, existence and uniqueness of the generalized solution

Badri Mamporia

Niko Muskhelishvili Institute of Computational Mathematics, Technical University of Georgia, Tbilisi, Georgia

Email address:
badrimamporia@yahoo.com

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Abstract: In this paper the stochastic differential equation in a Banach space is considered for the case when the Wiener process in the equation is Banach space valued and the integrand non-anticipating function is operator-valued. At first the stochastic differential equation for the generalized random process is introduced and developed existence and uniqueness of solutions as the generalized random process. The corresponding results for the stochastic differential equation in a Banach space is given. In [5] we consider the stochastic differential equation in a Banach space in the case, when the Wiener process is one dimensional and the integrand non-anticipating function is Banach space valued.

Keywords: Covariance Operators, Ito Stochastic Integrals and Stochastic Differential Equations in a Banach Space, Wiener Process in a Banach Space

1. Introduction and Preliminaries

The main problem in developing the stochastic differential equation in a Banach space is the construction of the Ito stochastic integral. The traditional finite dimensional methods allow to construct the stochastic integral in Banach spaces with special geometrical properties (see [1-3]). In an arbitrary Banach space it is possible to define the stochastic integral only in case, when the integrand function is non-random (see [4]). We define the generalized stochastic integral for a wide class of operator-valued non-anticipating random processes which is a generalized random element (a random linear function), and if there exists the corresponding random element, that is, if this generalized random element is decomposable by the random element, then we say that this random element is the stochastic integral. Thus, the problem of existence of the stochastic integral is reduced to the well known problem of decomposability of the random linear function. Another problem to develop the existence and uniqueness of the solution of the stochastic differential equation is to estimate the stochastic integral in a Banach space which is impossible by traditional methods. We introduce the stochastic differential equation for the generalized random process; here it is possible to use traditional methods to develop the problem of existence and uniqueness of a solution as a generalized random process. Afterward, from the main stochastic differential equation in a Banach space we receive the equation for a generalized random process, and the solution as a generalized random process. Thus, we reduced the problem of the existence of the solution to the problem of decomposability of the generalized random process. In [5] we consider the stochastic differential equation in the case when the Wiener process is one dimensional and the integrand function is Banach space valued, and we give some sufficient conditions of decomposabilility of the generalized random process.

Let X be a real separable Banach space, X^* its conjugate, $B(X)$ the Borel σ-algebra in X. $(\Omega, \mathrm{B}, \mathrm{P})$ a probability space. A measureble map $\xi : \Omega \to X$ is called a weak second order random element if $E\langle \xi, x^* \rangle^2 < \infty$ for all $x^* \in X^*$. A linear operator $L : X^* \to L_2(\Omega, \mathrm{B}, \mathrm{P})$ is called the generalized random element (sometimes it is used the terms: a random linear function or a cylindrical random element). Denote by $M_1 := L(X^*, L_2(\Omega, \mathrm{B}, \mathrm{P}))$ the Banach space of the generalized random elements (GRE) with the norm

$\|T\| = \sup_{\|x^*\|\le 1}(E(Tx^*)^2)^{1/2}$. Every weak second order random element generates the GRE $T_\xi : X^* \to L_2(\Omega,\mathrm{B},\mathrm{P})$, defined by the equality $T_\xi x^* = \langle \xi, x^* \rangle$ for all $x^* \in X^*$ but not conversely, if X is infinite dimensional, then every generalized random element may not be generated by a random element. Denote by M_2 the normed space of weak second order random elements with the norm $\|\xi\| = \|T_\xi\|$. Consequently, $M_2 \subset M_1$. A generalized random element generated by a random element is called decomposable. The decomposability problem of generalized random elements is a well known problem. It is equivalent to the problem of the extension of a weak second order finitely additive measure to the countable additive measure. The correlation operator of $T \in M_1$ is defined as $R_T : X^* \to X^{**}$, $R_T = T^*T$. R_T is a positive and symmetric linear operator. If $T = T_\xi \in M_2$, then R_T maps X^* to X (see [6], th. 3.2.1). For $R : X^* \to X$ positive and symmetric linear operator there exist $(x^*_k)_{k\in N} \subset X^*$ and $(x_k)_{k\in N} \subset X$ such that $\langle Rx_k^*, x_j^* \rangle = \delta_{kj}$, $Rx_k^* = x_k$ and for $x^* \in X^*$, $Rx^* = \sum_{k=1}^{\infty} \langle x_k, x^* \rangle x_k$ (see [6], lemma 3.1.1). In general, if $\operatorname{Im} T \subset L_2(\Omega,\mathrm{B},\mathrm{P})$ is separable, there exist $(x^*_k)_{k\in N} \subset X^*$ and $(x^{**}_k)_{k\in N} \subset X^{**}$ such that $\langle Rx_k^*, x_j^* \rangle = \delta_{kj}$, $Rx_k^* = x_k^{**}$ and $Rx^* = \sum_{k=1}^{\infty} \langle x_k^{**}, x^* \rangle x_k^{**}$.

Definition 1. Let X be a separable Banach space. The random process $(W_t)_{t\in[0,1]}$, $W_t : \Omega \to X$, is called a (homogeneous) Wiener process if 1) $W_0 = 0$ almost surely (a. s.); 2) $W_{t_{i+1}} - W_{t_i}$, $(i = 0,1,\cdots,n-1)$ are independent random elements for every $0 \le t_0 < t_1 < \cdots t_n \le 1$; 3) for every t from $[0,1]$, W_t is a Gaussian random element with the covariance operator tR, where $R : X^* \to X$ is a fixed Gaussian covariance.

If X is finite dimensional and R is the identity operator, then our definition of a Wiener process coincides with the definition of the standard Wiener process. When X is an infinite dimensional Hilbert space, then no Wiener process exists for which R is the identity operator. Our definition is a direct extension of the definition of a Wiener process for the Hilbert space case ([7], p. 113). If $(W_t)_{t\in[0,1]}$ is a Wiener process in a separable Banach space, then it has a. s. continuous sample pats, and there exists the representations of the Wiener process by the uniformly for t a. s. convergence sums of one dimensional Wiener processes with the coefficients from X and by independent Gaussian random elements with the covariance operators R and corresponding real valued coefficients (see [8-11]). Denote by $(F_t)_{t\in[0,1]}$ the increasing family of σ-algebras, $F_t \subset \mathrm{B}$, such that W_t is F_t-measurable and for all $s > t$, $W_s - W_t$ is independent to F_t. In this case we say that $(W_t)_{t\in[0,1]}$ is adapted to the family of the σ-algebra $(F_t)_{t\in[0,1]}$. For many purposes we need F_0 to contain all P-null sets in B.

Denote by $G_R(X^*)$ the linear space of weakly measurable random functions $\phi : \Omega \to X^*$ such that $\tau^2_R(\phi) \equiv \int_\Omega \langle R\phi(\omega), \phi(\omega) \rangle d\mathrm{P} < \infty$. $\tau_R(\phi)$ is a pseudonorm in $G_R(X^*)$.

We use the following proposition to prove the existence of a solution of the linear stochastic differential equation.

Proposition 1. If $\phi \in G_R(X^*)$ and $\int_\Omega \langle \phi(\omega), x \rangle^2 d\mathrm{P} < \infty$ for all $x \in X$, then there exists $K > 0$ such that

$$\int_\Omega \langle R\phi(\omega), \phi(\omega) \rangle d\mathrm{P} \le K \sup_{\|x\|\le 1} \int_\Omega \langle \phi(\omega), x \rangle^2 d\mathrm{P}.$$

Proof. Consider the linear operator $T : X \to L_2(\Omega,\mathrm{B},\mathrm{P})$, $Tx = \langle \phi(\omega), x \rangle$. By the closed graph theorem, T is a bounded operator, therefore $\sup_{\|x\|\le 1} \int_\Omega \langle \phi(\omega), x \rangle^2 d\mathrm{P} < \infty$. As R is a Gaussian covariance, by the Kwapien-Szymanski theorem (see [12], [4] p. 262) , there exists $(x_n)_{n\in N} \subset X$ and $(x_n^*)_{n\in N} \subset X^*$ such that $\langle x_n^*, x_k \rangle = \delta_{nk}$, $Rx^* = \sum_{k=1}^{\infty} \langle x_k, x^* \rangle x_k$, $x^* \in X^*$ and $\sum_{k=1}^{\infty} \|x_k\|^2 < \infty$. We have

$$\int_\Omega \langle R\phi(\omega), \phi(\omega) \rangle d\mathrm{P} = \int_\Omega \sum_{k=1}^{\infty} \langle x_k, \phi(\omega) \rangle^2 d\mathrm{P} = \sum_{k=1}^{\infty} \|x_k\|^2 \int_\Omega \langle \frac{x_k}{\|x_k\|}, \phi(\omega) \rangle^2 d\mathrm{P} \le$$

$$\sum_{k=1}^{\infty} \|x_k\|^2 \sup_{\|x\|\le 1} \int_\Omega \langle x, \phi(\omega) \rangle^2 d\mathrm{P}.$$

Definition 1. A function $\phi : [0,1]\times\Omega \to X^*$ is called non-anticipating with respect to $(F_t)_{t\in[0,1]}$ if the function $(t,\omega) \to \langle \phi(t,\omega), x \rangle$ from $([0,1]\times\Omega, \mathrm{B}[0,1]\times\mathrm{B})$ into $(R^1, \mathrm{B}(R^1))$ is measurable for all $x \in X$, and the function $\omega \to \langle \phi(t,\omega), x \rangle$ is F_t-measurable for all $t \in [0,1]$.

By $TG_R(X^*)$ we define the class of nonparticipating random function ϕ, for which $P^2_R(\phi) \equiv (\int_o^1 \int_\Omega \langle R\phi(t,\omega), \phi(t,\omega) \rangle dt d\mathrm{P} < \infty$. $TG_R(X^*)$ is a linear space and P_R is a pseudonorm in it. We use the following proposition to prove the existence of the solution of the stochastic differential equation.

Proposition 2(see [13]). If $\phi : [0,1]\times\Omega \to X^*$ is non-anticipating and for all $x \in X$ $\int_0^1 \int_\Omega \langle \phi(t,\omega), x \rangle^2 dt d\mathrm{P} < \infty$, then $\phi \in TG_R(X^*)$ and

$$\int_o^1 \int_\Omega \langle R\phi(t,\omega), \phi(t,\omega) \rangle dt d\mathrm{P} \le K \sup_{\|x\|\le 1} \int_0^1 \int_\Omega \langle \phi(t,\omega), x \rangle^2 dt d\mathrm{P} < \infty.$$

The proof of this proposition is analogous to the proof of the proposition 1.

If $\phi\in TG_R(X^*)$ is a step-function $\phi(t,\omega)=\sum_{i=0}^{n-1}\phi_{t_i}(\omega)\chi_{[t_i,t_{i+1})}(t)$, $0=t_0<t_1<\cdots<t_n=1$, $i=0,\cdots,n-1$, then the stochastic integral of ϕ with respect to $(W_t)_{t\in[0,1]}$ is naturally defined by the equality

$$\int_0^1\phi(t,\omega)dW_t=\sum_{i=0}^{n-1}\langle\phi_{t_i}(\omega),W_{t_i+1}-W_{t_i}\rangle .$$

The following lemma is true

Lemma 1 ([8]). For an arbitrary $\phi\in TG_R(X^*)$ there exists a sequence of step-functions $(\phi_n)_{n\in N}\subset TG_R(X^*)$ such that $\phi_n\xrightarrow{P_R}\phi$ and $\int_0^1\phi_n dW_t$ converges in $L_2(\Omega,\mathrm{B},\mathrm{P})$.

Definition 2 ([8]). Let $\phi\in TG_R(X^*)$ and $(\phi_n)_{n\in N}\subset TG_R(X^*)$ be step-functions such that $\phi_n\xrightarrow{P_R}\phi$ and $\int_0^1\phi_n dW_t$ converges in $L_2(\Omega,\mathrm{B},\mathrm{P})$. The limit of the sequence $\int_0^1\phi_n dW_t$ is called the stochastic integral of a random function $\phi\in TG_R(X^*)$ with respect to the Wiener process $(W_t)_{t\in[0,1]}$ and is denoted by $\int_0^1\phi dW_t$.

The stochastic integral $\int_0^1\phi dW_t$ is a random variable with mean 0 and variance $\int_o^1\int_\Omega\langle R\phi(t,\omega),\phi(t,\omega)\rangle dtd\mathrm{P}$.

Consider now the linear bounded operator $\phi: X^*\to G_R(X^*)$, for all $x^*\in X^*$, ϕx^* is the map $\Omega\to X^*$. Denote by $M_1^G\equiv L(X^*,G_R(X^*))$ the space of such operators with the property: $\tau^2{}_R(M_1^G)\equiv \sup_{\|x^*\|\le 1}E\langle R\phi x^*,\phi x^*\rangle<\infty$. $\tau_R(M_1^G)$ is a pseudonorm in M_1^G. Consider now the family of linear bounded operators $(T_t)_{t\in[0,1]}$, $T_t:X^*\to G_R(X^*)$, such that for all $x^*\in X^*$, the random process T_tx^* is nonanticipating and $\tau^2{}_R(TM_1^G)\equiv \sup_{\|x^*\|\le 1}\int_0^1\int_\Omega\langle RT_tx^*,T_tx^*\rangle dtd\mathrm{P}<\infty$. Denote by TM_1^G the space of such family of operators.

We can naturally define the stochastic integral from $(T_t)_{t\in[0,1]}\in TM_1^G$ which is the GRE defined by the equality $I(T_t)_{t\in[0,1]}x^*=\int_0^1T_tx^*dW_t$. Accordingly, we have the isometrical operator I: $TM_1^G\to M_1$, $I(T_t)_{t\in[0,1]}x^*=\int_0^1T_tx^*dW_t$.

Let now X be a separable Banach space and $L(X,X)$ be the space of bounded linear operators from X to X.

Definition 3. The random process $\xi_t:\Omega\to L(X,X)$ is nonanticipating with respect to the familly of the σ-algebra $(F_t)_{t\in[0,1]}$ if for all $x\in X$, $\xi_t(\omega)x:[0,1]\times\Omega\to X$ is measurable and for all $t\in[0,1]$, the random element $\xi_tx:\Omega\to X$ is F_t-measurable.

Definition 4. We say that the non anticipating random process $(\xi_t)_{t\in[0,1]}$, $\xi_t:\Omega\to L(X,X)$ belongs to the class $TG(L(X,X))$ if

$$\tau^2(\xi)\equiv\sup_{\|x^*\|\le 1}\int_0^1\int_\omega\langle R\xi_t^*(\omega)x^*,\xi_t^*(\omega)x^*\rangle dtd\mathrm{P}<\infty ,$$

where $\xi^*{}_t(\omega)$ is the linear operator, conjugate to the operator $\xi_t(\omega)$. $TG(L(X,X))$ is a linear space with the pseudonorm $\tau(\xi)$.

Let $\xi\in TG(L(X,X))$ and $x^*\in X^*$. $\xi^*x^*:[0,1]\times\Omega\to X^*$ be non anticipating and $\int_0^1\int_\omega\langle R\xi_t^*(\omega)x^*,\xi_t^*(\omega)x^*\rangle dtd\mathrm{P}<\infty$. We can define the stochastic integral $\int_0^1\xi^*{}_t(\omega)x^*dW_t$, which is the random variable with mean 0 and variance $\int_0^1\int_\omega\langle R\xi_t^*(\omega)x^*,\xi_t^*(\omega)x^*\rangle dtd\mathrm{P}$. Therefore, we can consider the GRE $I_\xi:X^*\to L_2(\Omega,\mathrm{B},\mathrm{P})$, $I_\xi x^*=\int_0^1\xi^*{}_t(\omega)x^*dW_t$.

We have the isometrical operator $I:TG(L(X,X))\to M_1$, $I(\xi)x^*=I_\xi x^*$.

Definition 5. The generalized random element $I_\xi x^*=\int_0^1\xi^*{}_t(\omega)x^*dW_t$ is called the generalized stochastic integral from the random process $\xi\in TG(L(X,X))$. If there exists the random element $\eta:\Omega\to X$ such that $\langle\eta,x^*\rangle=I_\xi x^*=\int_0^1\xi^*{}_t(\omega)x^*dW_t$ for all $x^*\in X^*$, then we say that there exists the stochastic integral from the operator valued non anticipating random process $(\xi_t)_{t\in[0,1]}$, $\xi_t:\Omega\to L(X,X)$ by the Wiener process in a Banach space X and then we write $\eta=\int_0^1\xi_t(\omega)dW_t$.

2. Main Results

Consider now the stochastic differential equation for generalized random process

$$dT_t = a(t,T_t)dt + B(t,T_t)dW_t, \qquad (1)$$

Where $a:[0,1]\times M_1 \to M_1$ and $B:[0,1]\times M_1 \to TM_1^G$

Definition 6. A GRP $(T_t)_{t\in[0,1]}$ is called the strong generalized solution of the equation (1) with the F_0- measurable initial condition $T_0 = L$, if the following assertions are true:

For all $x^*\in X^*$, $a(t,T_t)x^*$ and $B(t,T_t)x^*$ are $[0,1]\times F_1$ measurable,

$E\int_0^1 (a(t,T_t)x^*)^2 dt + E\int_0^1 \langle RB(t,T_t)x^*, B(t,T_t)x^*\rangle dt < \infty$; $T_t x^*$ is continuous, F_t- adapted and for each $t\in[0,1]$ and $x^*\in X^*$

$$T_t x^* = T_0 x^* + \int_0^t a(s,T_s)x^* ds + \int_0^t B(s,T_s)x^* dW_s \text{ a.s.}$$

Definition 7. We say that the stochastic differential equation (1) with the initial condition $T_0 = L$ has an unique strong generalized solution , if $(T_t)_{t\in[0,1]}$ and $(\overline{T_t})_{t\in[0,1]}$ are two solutions, then for each $x^*\in X^*$,

$$\mathrm{P}\{\omega : T_t x^*(\omega) = \overline{T}_t x^*(\omega) \text{ for all } x^*\in X^*\} = 1.$$

The following theorem gives the sufficient conditions of existence and uniqueness of a strong generalized solution to a stochastic differential equation for GRP.

Theorem 1. Suppose that the coefficients of the stochastic differential equation (1) satisfies the following conditions:

1. $\|a(t,T)\|_{M_1}^2 + \tau_R^2(M_1^G)(B(t,T)) \le K^2(1+\|T\|_{M_1}^2),$

2. $\|a(t,T)-a(t,S)\|_{M_1}^2 + \tau_R^2(M_1^G)(B(t,T)-B(t,S)) \le K^2\|T-S\|_{M_1}^2$

For all T,S from M_1.

Then there exists an unique strong generalized solution to (1) with the initial condition

$T_0 = L$, where , for all $x^*\in X^*$, Lx^* is F_0- measurable $\|L\|_{M_1}^2 < \infty$, $T:[0,1]\to M_1$ is continuous.

Proof. For all t define $T_t^0 = L$ and for any $x^*\in X^*$ let

$$T_t^{(n)}x^* = T_t^{(0)}x^* + \int_0^t a(s,T_s^{(n-1)})x^* ds + \int_0^t B(s,T_s^{(n-1)})x^* dW_s \,. \qquad (2)$$

$$\|T_t^{(n+1)} - T_t^{(n)}\|_{M_1}^2 \le 2\sup_{\|x^*\|\le 1} E(\int_0^t a(s,T_s^{(n)}) - a(s,T_s^{(n-1)})x^* ds)^2 +$$

$$2\sup_{\|x^*\|\le 1} E(\int_0^t B(s,T_s^{(n)}) - B(s,T_s^{(n-1)})x^* dW_s)^2 \le$$

$$\int_0^t \|a(s,T_s^{(n)}) - a(s,T_s^{(n-1)})\|_{M_1}^2 ds + 2\int_0^t \tau_R^2(M_1^G)(B(s,T_s^{(n)}) - B(s,T_s^{(n-1)}))ds \le$$

$$2K^2\int_0^t \|T_s^{(n)} - T_s^{(n-1)}\|_{M_1}^2 ds \le (2K^2)^{n-1}\int_0^t \frac{(t-s)^{n-1}}{(n-1)!}\|T_s^{(1)} - T_s^{(0)}\|_{M_1}^2 ds$$

$$\|T_s^{(1)} - T_s^{(0)}\|_{M_1}^2 \le 2\left\|\int_0^t a(s,T_s^{(0)})ds\right\|_{M_1}^2 + 2\left\|\int_0^t B(s,T_s^{(0)})dW_s\right\|_{M_1}^2 \le 2K^2(1+\|T_0\|_{M_1}^2).$$

Therefore $\|T_t^{(n+1)} - T_t^{(n)}\|_{M_1}^2 \le \dfrac{pC^n}{n!}$ for some positive p and C .

For any fix $x^*\in X^*$,

$$E\sup_{0\le t\le 1}\left|(T_t^{(n+1)} - T_t^{(n)})x^*\right|^2 \le 2E\sup_{0\le t\le 1}\int_0^t ((a(s,T_s^{(n)}) - a(s,T_s^{(n-1)}))x^*)^2 ds +$$

$$2E\sup_{0\le t\le 1}\left|\int_0^t (B(s,T_s^{(n)} - B(s,T_s^{(n-1)}))x^* dW_s\right|^2 \le 2\int_0^1 \|a(s,T_s^{(n)}) - a(s,T_s^{(n-1)})\|_{M_1}^2 ds +$$

$$8\int_0^1 \tau_R^2(M_1^G)(B(s,T_s^{(n)} - B(s,T_s^{(n-1)}))ds \le \frac{10pC^{n-1}}{(n-1)!}.$$

Then we Have

$$\sum_{n=1}^{\infty} P(\sup_{0\le t\le 1}\left|(T_t^{(n+1)} - T_t^{(n)})\right|x^* > 1/_{n^2}) \le \sum_{n=1}^{\infty} n^4 E(\sup_{0\le t\le 1}\left|(T_t^{(n+1)} - T_t^{(n)})x^*\right|^2) \le .\ 10p\sum_{n=1}^{\infty} n^4 C^{n-1}/_{(n-1)!}.$$

By the Borel-Cantelli lemma, the series

$$T_t^{(0)}x^*(\omega) + \sum_{m=1}^{\infty}(T_t^{n}(\omega) - T_t^{(n-1)}(\omega))x^*$$

converges uniformly on t ($P-$ a.s.) to the continuous random process, which we denote by $T_t x^*, x^* \in X^*$. From equation (2) we obtain

$$T_t x^* = Lx^* + \int_0^t a(s,T_s)x^* ds + \int_0^t B(s,T_s)x^* dW_s \text{ a. s..}$$

Therefore, the GRP $(T_t)_{t\in[0,1]}$ is a strong generalized solution of the equation (1).

Uniqueness of the solution and continuity of $T:[0,1]\to M_1$ we can prove by the same way (see [5] th.6).

Let now consider the stochastic differential equation in an arbitrary Banach space

$$d\xi_t = a(t,\xi_t)dt + B(t,\xi_t)dW_t, \qquad (3)$$

where $a:[0,1]\times X \to X$ and $B:[0,1]\times X \to L(X,X)$ are such functions, that

1'. $\|a(t,\xi)\|_{M_1}^{2} + \tau_R^2(M_1^G)\left(B^*(t,\xi)\right) \le K^2(1+\|\xi\|_{M_1}^{2})$

2'. $\|a(t,\xi)-a(t,\eta)\|_{M_1}^{2} + \tau_R^2(M_1^G)(B^*(t,\xi)-B^*(t,\eta)) \le K^2\|\xi-\eta\|_{M_1}^{2}$,

ξ,η are weak second order X-valued random elements. We can extend the coefficients a and B^* on $\bar{M}_2 \subseteq M_1$: Let $T\in\bar{M}_2$, then there exists $(\xi_n)_{n\in N}\in M_2$, such that $\|\xi_n - T\|_{M_1}\to 0$. Then $\|a(t,\xi_n)-a(t,\xi_m)\|_{M_1}\le K^2\|\xi_n-\xi_m\|_{M_1}\to 0$, $\tau_R^2(M_1^G)(B^*(t,\xi_n)-B^*(t,\xi_m))\le K^2\|\xi_n-\xi_m\|_{M_1}\to 0$. Therefore, we can define $a(t,T)=\lim_{n\to\infty}a(t,\xi_n)$ and $B^*(t,T)=\lim_{n\to\infty}B^*(t,\xi_n)$. They will satisfy the conditions 1 and 2 of the Theorem 1 with the initial condition $T_0x^*=\langle\xi_0,x^*\rangle$, therefore, we receive from the equation (3) the stochastic differential equation for GRP.

Theorem 2. If the coefficients of the equation (3) satisfy the conditions 1' and 2' and for all $\xi\in M_2$, $a(\cdot,\xi)$ from $[0,1]$ to M_1 and $B^*(\cdot,\xi)$ from $[0,1]$ to M_1^G are continuous then the stochastic differential equation (2) possesses an unique strong generalized solution with initial condition $T_0x^*=\langle\xi_0,x^*\rangle$.

Consider now a linear stochastic differential equation in a separable Banach space.

$$d\xi_t = A(t)\xi_t dt + B(t)\xi_t dW_t, \qquad (4)$$

where $A:[0,1]\to L(X,X)$ and $B:[0,1]\to L(X,L(X,X))$ are continuous. Therefore, $\max_{t\in[0,1]}(\|A(t)\|,\|B(t)\|\le M$ for some $M>0$. Then

Where

$$\|A(t)\xi - A(t)\eta\|_{M_1}^{2} = \sup_{\|x^*\|\le 1}E\langle A(t)(\xi-\eta),x^*\rangle^2 = \sup_{\|x^*\|\le 1}E\langle(\xi-\eta),A^*(t)x^*\rangle^2 =$$

$$\|A^*(t)\|^2\sup_{\|x^*\|\le 1}E\langle(\xi-\eta),\frac{A^*(t)}{\|A^*(t)\|}x^*\rangle^2 \le M^2\sup_{\|x^*\|\le 1}E\langle(\xi-\eta,x^*\rangle^2 = M^2\|\xi-\eta\|_{M_1}^2;$$

Using the proposition 1, we have

$$\tau_R^2(M_1^G)(B(t)\xi - B(t)\eta) = \sup_{\|x^*\|\le 1}E\langle R(B(t)(\xi-\eta))^*x^*,(B(t)(\xi-\eta))^*x^*\rangle$$

$$\sup_{\|x^*\|\le 1}\sup_{\|x\|\le 1}E\langle(B(t)(\xi-\eta))^*x^*,x\rangle^2 = \sup_{\|x^*\|\le 1}\sup_{\|x\|\le 1}E\langle B(t)(\xi-\eta)x,x^*\rangle^2 =$$

$$\|B^*(t)\|^2\sup_{\|x^*\|\le 1}\sup_{\|x\|\le 1}E\langle(\xi-\eta),\frac{B^*(t)}{\|B^*(t)\|}\delta(x,x^*)\rangle^2 \le M^2\sup_{\|x^*\|\le 1}E\langle(\xi-\eta),x^*\rangle^2 =$$

$$M^2\|(\xi-\eta)\|_{M_1}^{2},$$

Where $\delta(x,x^*)$ is an element of the Banach space $(L(X,X))^*$, Therefore, the equation (4) satisfies the conditions 1' and 2'. That is, by the theorem 2, the linear stochastic differential equation (4) has an unique generalized solution.

Hence, by the theorems 2, we prove the existence of solutions as a GRP (family of random linear functions).We reduce the problem of the existence of the random process as a solution to the well known problem of decomposability of GRE (random linear function). Using the corresponding results, we can receive sufficient conditions for existence of the solutions. Some sufficient conditions were received in [5].

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