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# Minimization of Total Operation Cost and the Risk of Shedding in Microgrids

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**Abstract:** The objective of this paper is to determine and minimize the total operation cost and the risk of load shedding in a microgrid ( $\mu G$ ) composed of two areas: a generation center and a load center. The system operation is formulated as an optimization problem, where the objective function minimizes the costs of the system operation and the risk of load shedding. The constraints secure the balance between generation and load. Also generation and transmission may not exceed the available capacity. Monte Carlo simulation (*MCS*) is used for the solution of the optimization problem giving two main outputs: loss of load occasion (*LOLO*) and total operation cost (*TOC*). A variance reduction technique is used to reduce the variance of *MCS*. One other objective of the paper is to study how much the simulation efficiency can be improved by introducing variance reduction techniques. Simulation results show that, (i) the formulated optimization problem, objective function, and constraints is capable to achieve the study target, and (ii), with even a quite straightforward and simple model the proposed *MCS* methods show considerable variance reductions compared to Simple sampling in this model of the  $\mu G$ .

**Keywords:** Micro Grid, Monte Carlo Simulation, Variance Reduction Techniques, Optimization, Operation Costs, Load Shedding, Distribution System Planning, Dispersed Generation, Power System Management

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## 1. Introduction

Recent years, increasing trends on electrical supply demand, urge search for the new alternative in supplying the electrical power.  $\mu G$ s are new challenging power systems under development. A study in  $\mu G$  system with embedded Distribution Generations (*DGs*) to the system is rapidly increasing. Also the  $\mu G$  energy systems are considered to be the most cost effective power solutions to meet the load requirements of the people that live in the rural areas. Due to the intensive cost of extending the transmission and distribution lines to the remote communities, the power demands in such areas can be met by using a  $\mu G$  power system that consists of the diesel generator, PV, WTG and ESS. Moreover, the integration of  $\mu G$ s in distribution systems will offer a decentralized control of local resources for satisfying the network reliability and the power quality required by local loads.  $\mu G$  system basically is designed to operate either in islanding mode or interconnect with the main grid system. In any condition, the system must have reliable power supply and

operating at low transmission power loss. During the emergency state such as outages of power due to electrical or mechanical faults in the system, it is important for the system to shed some loads in order to maintain the system stability and security. In order to determine the total operation cost and the risk of load shedding in the  $\mu G$ , it is very important to design the  $\mu G$  optimally [1-9].

The transition of  $\mu G$  from grid-connected mode to islanded mode is usually associated with excessive load (or generation), which should be shed. Ref. [1] proposes a robust load shedding strategy for  $\mu G$  islanding transition, which takes into account the uncertainties of renewable generation in the  $\mu G$  and guarantees the balance between load and generation after islanding. A robust optimization model is formulated to minimize the total operation cost, including fuel cost and penalty for load shedding. The proposed robust load shedding strategy works as a backup plan and updates at a prescribed interval. It assures a feasible operating point after islanding given the uncertainty of renewable generation. The proposed algorithm is demonstrated on a simulated  $\mu G$  consisting of a wind turbine, a PV panel, a battery, two distributed generators

(DGs), a critical load and an interruptible load. Numerical simulation results validate the proposed algorithm.

A multi-objective optimization model is proposed in [2] to calculate best possible size of energy storage system (*ESS*) utilizing weighted sum method. Positive effects of demand response program (*DRP*) are considered in the proposed paper. The best possible solution is selected by fuzzy satisfying approach. The proposed multi-objective model includes two conflicting objective functions: 1) the first objective function is minimization of  $\mu$ G investment cost as well as the operation cost; 2) the second objective function is minimization of loss of load expectation. The local units inside the  $\mu$ G may have some unknown outages and the renewable units produce variable and unstable output, so utilization of *ESS* is essential to improve stability of  $\mu$ G. The impact of *DRP* implementation is evaluated on  $\mu$ G related costs and the results are compared to validate the proposed technique. A mixed-integer program is utilized to simulate and model the proposed stochastic *ESS* optimal sizing problem in a  $\mu$ G.

The objective [3] is to minimize the total system planning cost comprising investment and operation costs of local  $\mu$ Gs, the co-optimized planning of large generating units and transmission lines, and the expected cost of unserved energy. The cost of unserved energy reflects the cost of load shedding which is added to the objective function for reliability considerations. The  $\mu$ G-based co-optimization planning problem is decomposed into a planning problem and annual reliability sub-problem. The optimal integer planning decisions calculated in the planning problem will be examined against the system reliability limits in the sub-problem and the planning decisions will be revised using proper feasibility cuts if the annual reliability limits are violated. Numerical simulations demonstrate the effectiveness of the proposed  $\mu$ G-based co-optimization planning in power systems and explore the economic and reliability merits of  $\mu$ G planning as compared to grid-based generation and transmission upgrades.

Ref. [4], aims to minimize total operation cost of  $\mu$ G in presence of Battery Energy Storage *BES* of optimal size. The  $2m$  point estimate method has been applied to model the uncertainties in load demand, market prices and available power from Renewable Energy Sources *RES* in the  $\mu$ G, as it is computationally efficient and reliable probabilistic method. Moreover, Gram-Charlier expansion is used to provide more accurate probability distribution of  $\mu$ G operation cost. Swine Influenza Model Based Optimization with Quarantine and Whale Optimization Algorithm have been applied to minimize operation cost of  $\mu$ G. Simulation results prove the effectiveness of the algorithms. The incorporation of *BES* of optimum size reduces operation cost of  $\mu$ G effectively.

In [5], a risk-constrained stochastic framework is presented to maximize the expected profit of a  $\mu$ G operator under uncertainties of renewable resources, demand load and electricity price. In the proposed model, the trade-off between maximizing the operator's expected profit and the risk of getting low profits in undesired scenarios is modeled by using conditional value at risk (*CVaR*) method. The influence of consumers' participation in *DR* programs and their emergency

load shedding for different values of lost load (*VOLL*) are then investigated on the expected profit of operator, *CVaR*, expected energy not served (*EENS*) and scheduled reserves of  $\mu$ G. Moreover, the impacts of different *VOLL* and risk aversion parameter are illustrated on the system reliability. Extensive simulation results are also presented to illustrate the impact of risk aversion on system security issues with and without *DR*. Numerical results demonstrate the advantages of customers' participation in *DR* program on the expected profit of the  $\mu$ G operator and the reliability indices.

The  $\mu$ G controller in [6] seeks to operate the local energy storage unit to minimize the risk of load shedding, and renewable energy curtailment over a finite time horizon. The problem is formulated for optimizing the operation of the storage unit as a finite stage dynamic programming problem. The multi-stage objective function of the energy storage is proved strictly convex in the state of charge of the battery at each stage. The uniqueness of the optimal decision is proven under some additional assumptions. The optimal strategy is then obtained. The effectiveness of the energy storage in decreasing load shedding and renewable energy curtailment is illustrated in simulations.

In [7], a load shedding algorithm is proposed for an optimization problem to maximize the satisfaction of system components. The proposed algorithm preferentially assigns the power to the sub demand with a high preference to maximize the satisfaction of power consumers. In addition, the algorithm assigns the power to maximize the power sale and minimize the power surplus for satisfaction of power suppliers. A multi-agent system is implemented on top of a conventional development framework and assessed the algorithm's adaptability, satisfaction metric, and running time.

Following the penetration of  $\mu$ Gs in distribution systems, frequency deviations in contingency conditions are becoming increasingly important. The article [8] develops a new load shedding method for  $\mu$ Gs considering wind speed changes. The proposed method uses a combination of frequency and voltage data for determining load-shedding amounts in each contingency condition. For this purpose, the total required load shedding is determined first by using transient stability analysis in different contingency scenarios in  $\mu$ Gs. This will establish a database for an adaptive neuro-fuzzy inference system network to determine the total required load shedding. Then a fuzzy system is used to determine the load shedding in each step dynamically based on the severity of contingencies. The proposed method capability is compared with the conventional load-shedding method, showing the effectiveness of the proposed method for  $\mu$ G control in contingency conditions.

Ref [9] proposes an adaptive optimization-based approach for under frequency load shedding in  $\mu$ Gs following an unintentional islanding. In the first step, the total amount of load curtailments is determined based on the system frequency response model. Then, the proposed mixed-integer linear programming model is executed to find the best location of load drops. The novel approach specifies the least cost load shedding scenario while satisfying network operational

limitations. A look-up table is arranged according to the specified load shedding scenario to be implemented in the network if the islanding event occurs in the  $\mu$ G. To be adapted with system real-time conditions, the look-up table is updated periodically. The efficiency of the proposed framework is thoroughly evaluated in a test  $\mu$ G with a set of illustrative case studies.

## 2. The Method

In this paper the priorities of the  $\mu$ G operation compose an optimization problem. Monte Carlo Simulation (*MCS*) [10] is used for the solution this optimization problem giving the two main outputs: loss of load occasion (*LOLO*) and total operation cost (*TOC*). Variance reduction techniques are used to reduce the variance of the *MCS*. The variance reduction techniques tested on the optimization problem are [11, 12]: Two alternatives of Complementary Random number Sampling, Importance Sampling, Stratified Sampling and the Control Variates method. These methods are arguably the most commonly used techniques for increasing the probability of obtaining good estimates from *MCS*.

### 2.1. The Study $\mu$ Gs Power System

The study  $\mu$ G is a power system divided in two areas: a generation center and a load center. The generation center is dominated by large renewable power plants, but there is also a small local load, whereas the load center has most of the load in the system, but there is also some thermal power plants. The renewable power plants (hydro) in the generation center are assumed to have negligible variable operation costs and the risk of outages is also negligible. Moreover, the capacity of the renewable power plants is larger than the maximal local load; hence, the generation center always has excess power to export to the load center. However, the interconnection between the areas has a limited capacity and there is a risk of outages. There are also electrical losses on the interconnection; these losses are proportional to the square of the power injected into the interconnection. The variable costs of the thermal power plants in the load center is assumed to be directly proportional to the power output (neglecting, start-up costs and ramp rates [2]) and there is a risk of outages in these units. Finally, the load in the two areas is varying randomly. It can be assumed that the load is described by one probability distribution for the total load of the system, and another probability distribution for the share of the load that is located in the main load center.

The system is operated in such a manner that the first priority is to avoid load shedding when there is not sufficient generation capacity available in the system and the second priority is to minimize the total operation cost. Voltage and frequency control may be neglected.

### 2.2. Problem Formulation

The objective of simulating this  $\mu$ G is to determine the total operation cost and the risk of load shedding. The random

outages in thermal power plants and the interconnection between the areas, as well as random loads in the two areas composes the random inputs variables of the model. These variables reflect: share of the total load that is located in the main load center, total load in the system, available generation capacity in thermal plants, and available transmission capacity on the interconnection between the two areas. This paper is actually only interested in two outputs: the total operation cost and whether load shedding occurs or not. However, to compute these two values, some other partial results are needed. Since the partial results also depend on the values of the random inputs, they will in practice also be random outputs (i.e., random variables with unknown probability distributions depending on the probability distributions of the inputs). In this case, the model will be generating the following outputs: load in the main load center, load in the generation center, generation in the renewable power plants, transmission from the generation center to the load center, loss of load occasion (binary variable equal to 1 if load shedding is necessary and 0 otherwise), transmission from the generation center to the load center, total operation cost, and unserved load.

#### Mathematical Model

Some additional values are needed to formulate the  $\mu$ G mathematical model: variable operation cost of thermal power plant, penalty cost for unserved load, loss coefficient for the interconnection between the areas, and available generation capacity in the renewable (hydro) power plants. To compute the values of the outputs for a scenario, first, it is necessary to compute the local load in each area:

$$D_1 = cD_{tot} \quad (1)$$

$$D_2 = (1 - c)D_{tot} \quad (2)$$

The next step is to determine how the system will be operated. This can be formulated as an optimization problem, where the objective function (3) minimizes the costs of the system and the risk of load shedding in the  $\mu$ G.

$$\text{minimize } \sum_g \beta_{G_g} G_g + \beta_U U \quad (3)$$

The constraints (4), (5) conserve the balance between generation, load and import in the load center and between generations, load export in the generation center.

$$\text{subject to } \sum_g G_g + P - \gamma P^2 = D_1 - U \quad (4)$$

$$H = D_2 + P \quad (5)$$

The limits (6) ~ (8) confine generation and transmission within the available capacity.

$$0 \leq G_g \leq G_{g_{max}}, \forall g \quad (6)$$

$$0 \leq H \leq H_{max} \quad (7)$$

$$0 \leq P \leq P_{max} \quad (8)$$

The loss of load occasion as per (9).

$$0 \leq U \tag{9}$$

in Load center.

Finally, once it has been determined how the power system is operated, the two main outputs, (10) and (11) can be computed from the solution to the optimization problem:

$$LOLO = \begin{cases} 0 & \text{if } U = 0, \\ 1 & \text{if } U = 1, \end{cases} \tag{10}$$

$$TOC = \sum_g \beta_{G_g} G_g + \beta_U U. \tag{11}$$

The  $\mu G$  is not connected to the grid, but has a local system of its own. The local grid is supplied by a 350kW hydro power plant in the Generation center. The variable costs of the hydro power plant are negligible and it can be considered to be 100% reliable. There is an 11kV transmission line between the Generation center and Load center. This line has a maximal capacity of 300 kW, a 99% reliability and losses equal to  $10^{-5}P^2$ , where  $P$  is the power injected on the Generation center side. In Load center proper, there are two diesel generator sets. The first unit has 200 kW capacity, the variable costs are 10  $\text{¢}/\text{kWh}$  and the availability is 90%. The second unit has 150 kW capacity, the variable costs are 12  $\text{¢}/\text{kWh}$  and the availability is 80%. The total load of the system is varying between 200 and 600 kW according to the probabilities listed in Table 1. It is 50% probability that 85% of the total load is in Load center and 50% probability that 90% of the total load is

Table 1. Probabilities of different load levels in the  $\mu G$ .

Total load [kW]	Probability [%]
200	20
300	40
400	25
500	10
600	5

The following model constants (the inputs that do not change value between scenarios) are identified:

$\beta_{G_1}$  = variable operation cost of thermal power plant  $g$ , 10  $\text{¢}/\text{kWh}$

$\beta_{G_2}$  = variable operation cost of thermal power plant  $g$ , 12  $\text{¢}/\text{kWh}$

$\gamma_L$  = loss coefficient for the interconnection between the areas,  $10^{-5}$

The capacity of diesel generator sets and the transmission line are also not changing between scenarios. However, due to the risk of outages, the *available capacity* (which is what needs to be considered in the simulation) is a random variable; hence, the maximal capacity will appear in the probability distribution of the inputs.

Also, the following frequency functions stating the probability of a certain outcome are identified in (12) ~ (15):

$$f_{G_{1max}} = \text{available generation capacity } x \text{ in the large diesel generator set} = \begin{cases} 0.1 & x = 0, \\ 0.9 & x = 200, \\ 0 & \text{all other } x, \end{cases} \tag{12}$$

$$f_{G_{2max}} = \text{available generation capacity } x \text{ in the small diesel generator set} = \begin{cases} 0.1 & x = 0, \\ 0.8 & x = 150, \\ 0 & \text{all other } x, \end{cases} \tag{13}$$

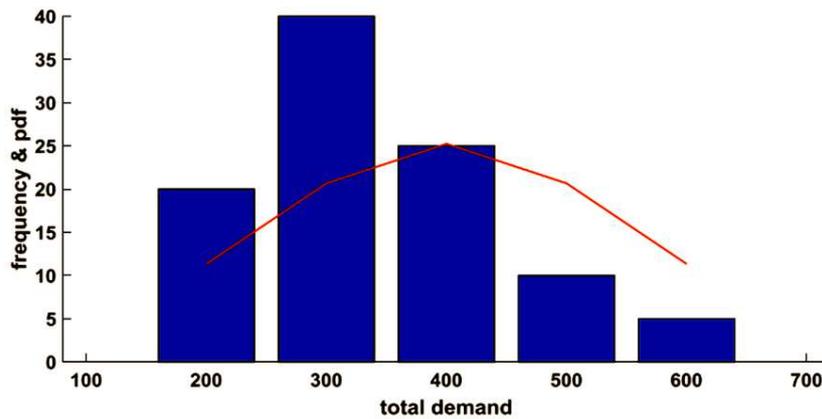


Figure 1. Frequency & Scaled Probability Distribution functions of total demand.

$$f_{P_{max}} = \text{available generation capacity } x \text{ in the tie line between Generation center and Load center} = \begin{cases} 0.01 & x = 0, \\ 0.99 & x = 300, \\ 0 & \text{all other } x, \end{cases} \tag{14}$$

$f_{D_{tot}}$  = total demand  $x$  given by Table 1 and depicted in Figure 1

$$f_c = \text{the share } x \text{ of the total demand located in Load center} = \begin{cases} 0.5 & x = 0.85, \\ 0.5 & x = 0.9, \\ 0 & \text{all other } x, \end{cases} \tag{15}$$

### 2.3. Variance Reduction in MCS for Minimization of Operation Cost and Load Shedding Risk in $\mu G$

MCS is used for the solution to the optimization problem giving the two main outputs: loss of load occasion (LOLO) and total operation cost (TOC). Variance reduction technique is used to reduce the variance of a MCS. In this paper five of the most commonly used variance reduction techniques are tested to estimate the avoidance of load shedding and minimization of the total operation cost in the  $\mu G$ . The variance reduction techniques [10] tested on the optimization problem are: Two alternatives of Complementary Random number Sampling, Importance Sampling, Stratified Sampling and the Control Variates method. These methods are arguably the most commonly used techniques for increasing the probability of obtaining good estimates from MCSs.

### 2.4. Simulation Algorithms with Different Variance Reduction Techniques

Below follows explanation of the algorithms used for Matlab coding for MCS with different Variance Reduction Techniques for the  $\mu G$ .

*Scenario:* This is the core of the simulation of the detailed mathematical model of the  $\mu G$ . It takes a scenario: ( $G_{1max}$  and  $G_{2max}$  are the available capacities in the diesel generator sets,  $P_{max}$  is the available transmission capacity,  $D_{tot}$ ,  $D_1$  and  $D_2$  are the total load, load in the generation center and load in the load center respectively) as inputs and computes the total operation cost as well as the binary loss of load occasion variables.

*Simple Sampling:* This algorithm executes one simulation using simple sampling. The principle is straightforward: generate a scenario by applying the inverse transform method [11] on each input, analyze the scenario using the mathematical model and store the results.

#### 2.4.1. Complementary Random Number 2 Sampling

This algorithm executes one simulation using complementary random numbers. In this variant each original scenario, ( $G_{tot}$ ,  $D$ ), generates one complementary scenario, ( $G_{tot}^1$ ,  $D^1$ ). The principle is the same as for simple sampling, except for the additional code that switches between generating original and complementary random numbers. However, as we can see in table 2, this tiny extra code increases the average simulation time by approximately 35%. Simulation of the  $\mu G$  uses in this algorithm complementary random numbers for total load and total generation capacity (1 complementary scenario).

#### 2.4.2. Complementary Random Number 4 Sampling

This script executes one simulation using complementary random numbers. In this variant each original scenario, ( $G_{tot}$ ,  $D$ ), generates three complementary scenarios, ( $G_{tot}^1$ ,  $D^1$ ),

( $G_{tot}^1$ ,  $D^1$ ) and ( $G_{tot}^1$ ,  $D^1$ ) respectively. The principle is the same as for simple sampling, except for the additional code that switches between generating original and complementary random numbers. This variant require only 75% less pseudorandom numbers to generate the values of the generation capacity and the total load, which compensates for the extra time to switch between generating original and complementary random numbers; therefore the average simulation time is more or less the same as for simple sampling. Simulation of the  $\mu G$  uses complementary random numbers for total load and total generation capacity (3 complementary scenario).

#### 2.4.3. Control Variates Sampling

This When using the control variates method our knowledge of the system is used to get a better estimate [12, 13]. This script executes one simulation using control variates. First, a probabilistic production cost simulation is carried out using some script\*. Then, simple sampling is applied to the difference between the output of the detailed model of the  $\mu G$  and the simplified model. Finally, the expectation values of the control variates are added to the estimated differences.

#### 2.4.4. Importance Sampling

In this code the probability measure of the random variables is changed to reduce the variance of the MCS. To get an unbiased estimate the resulting randomized cases must then be weighted before taking the mean [12, 13]. This script executes one simulation using importance sampling. The principle is mostly the same as for simple sampling, except that scenarios are randomized according to the importance sampling functions and that each observation is multiplied by a weight factor. The weight factors for each input are assigned at the same time as the random numbers are computed.

#### 2.4.5. Stratified Sampling

If there is a natural way to partition the sample space into  $m$  subsets, then the variance will be reduced if we divide the sampling procedure into  $m$  different MCSs in which we constrain the *r.v.* to live on the  $m$  subsets respectively [12, 13]. This script executes one simulation using importance sampling. First, the stratum weights are computed and analytical results are assigned to those strata where it is possible. The next step is to run a pilot study; the number of samples are the same for all strata for which there is no analytical result. The randomization of the input values is modified to take into account the probability distribution of the inputs for the corresponding strata. After the pilot study, estimates of the variance for all sampled strata are calculated and the sample allocation for the next batch of scenarios is computed. Scenarios are then generated in the same way as in the pilot study. The results for each stratum are stored and a new batch is generated according to an updated sample allocation. At the end of the simulation, the results are weighted together according to the stratum weights.

<sup>1</sup>This script applies probabilistic production cost simulation to the  $\mu G$ . The equivalent load duration curves ( $D_1$ ,  $D_2$  and  $D_{tot}$ ) are stored as row vectors, where the first element represents the value of the duration curve for  $0 \leq x < 50$ , the second element is for  $50 \leq x < 100$ , etc.

### 2.5. Comparing Variance Reduction Techniques

To compare different variance reduction techniques to one another [10] used the measure "time taken by the simulation – by- variance of the resulting estimate". This measure is reasonable since variance decays as  $1/n$  and  $t \propto n$ , where  $n$  is amount of samples and  $t$  is the time taken by the simulation. However, in this paper all the methods tested take approximately the same time when using the same  $n$ .

Table 2. Compilation of simulation results for the  $\mu G$ .

Simulation method	Average Simulation time [ms]	Total operation cost			Loss of load occasion		
		Mean	Var.	Eff.	Mean	Var.	Eff.
Simple sampling	309.675	573.5745	327.7999	101.010	0.0475	0.0468	0.0145
Complementary random numbers2: $(G, D)$ , $(G^*, D^*)$	419.562	447.1123	191.6767	80.6452	0.0384	0.0346	0.0145
Complementary random numbers4: $(G, D)$ , $(G, D^*)$ , $(G, D^*)$ , $(G^*, D^*)$	875.653	898.5258	972.3420	833.333	0.0649	0.0310	0.0271
Control Variates	328.008	442.0618	392.7000	128.205	0.0346	0.0210	0.0069
Importance sampling	369.412	440.7199	141.3033	52.0833	0.0347	0.0097	0.0036
Stratified sampling	213.588	443.6519	6656.0	1428.6	0.0331	1.6122e-05	3.4434e-06

## 3. Simulation Results

This script is used to test and compare the results of different simulation method. The same number of samples is collected in each MCS simulation, and number of simulations are the same for all methods. Moreover, each simulation run uses the same seed for the random number generator for all methods. This means that two methods that generate scenarios in exactly the same way (such as simple sampling and control variates) will use the same scenarios in each simulation run. Time of each simulation run is measured using the built-in Matlab timer (tic and toc). Notice that the execution time will vary slightly every time this script is run, depending on which other tasks are running on the same computer, how much memory that is available, etc. All in all five set of methods were tested, beside the Simple sampling method as depicted in Table 2.

The proposed MCS methods show, in Table 2 considerable variance reductions. Due to space limitation some examples of simulation convergence (Simple sampling, Complementary random numbers2 sampling, Complementary random numbers4 sampling, Control Variates sampling, Importance sampling) are depicted in the following figures.

Figure 2 Compares LOLP determined by MCS with different Variance Reduction methods. Figure 3 Compares TOC determined by MCS with different Variance Reduction methods, Figure 3 shows that Simple sampling, Complementary random numbers2 sampling, and Complementary random numbers4 have same convergence trends and values. Figure 4 shows that Simple sampling, Complementary random numbers2 sampling, Control Variates sampling, and Importance sampling have same convergence trends and values.

Therefore, we use the inverse of the above measure to select the variance reduction technique securing minimum product of the simulation time and resulting variance. While Importance Sampling, Stratified Sampling and are methods that affect our way of generating random variables, the Control Variates method uses the same way of randomizing the state variables as Simple Sampling. All in all five set of methods were tested, as depicted in Table 2 below.

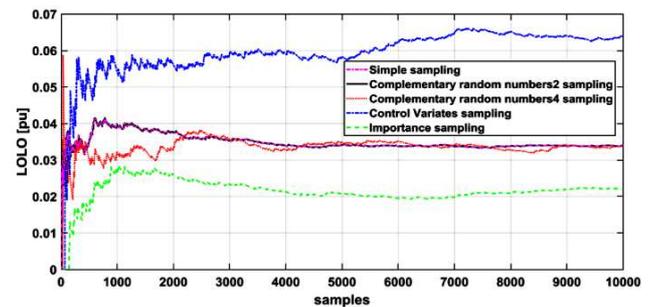


Figure 2. Comparison of LOLP from different Variance Reduction MCS methods.

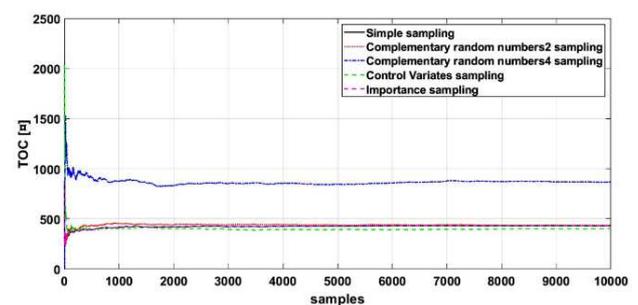


Figure 3. Comparison of TOC from different Variance Reduction MCS methods.

## 4. Discussion

The  $\mu G$  operation is formulated as an optimization problem, where the objective function minimizes the costs of the system operation and the risk of load shedding. The constraints secure the balance between generation and load. Also, generation and transmission may not exceed the available capacity.

*MCS* is used for the solution of the optimization problem giving two main outputs: loss of load occasion (*LOLO*) and total operation cost (*TOC*). Variance reduction techniques are used to reduce the variance of *MCS*. One other objective of the paper is to study how much the simulation efficiency can be improved by introducing variance reduction techniques.

## 5. Conclusion

The  $\mu$ G operation is formulated as an optimization problem, where the objective function minimizes the costs of the system operation and the risk of load shedding. The constraints secure the balance between generation and load. Also, generation and transmission may not exceed the available capacity.

*MCS* is used for the solution of the optimization problem giving two main outputs: loss of load occasion (*LOLO*) and total operation cost (*TOC*). Variance reduction techniques are used to reduce the variance of *MCS*. One other objective of the paper is to study how much the simulation efficiency can be improved by introducing variance reduction techniques.

Simulation results shows that, (i) the formulated optimization problem, objective function, and constraints is capable to achieve the study target, and (ii), with even a quite straightforward and simple model the proposed *MCS* methods show considerable variance reductions in this model of the  $\mu$ G compared to Simple sampling in this model of the  $\mu$ G.

Also, it is clear that a good choice of variance reduction technique can dramatically increase the efficiency of the simulation. A poor choice might, however, not give that much of an improvement. Some techniques, like Simple Sampling, give a result that seems very good but actually miss some very important scenarios, yielding a cardinal error. For our case study, using Control Variates sampling gave the best results for our problem. These techniques can, of course, be used on many different problems in power system analysis.

## 6. Recommendations

*MCS* is preferable in solving the minimization of the costs of the system operation and the risk of load shedding. Chose a good variance reduction technique to increase the efficiency of the simulation. For cases like our case study, using Control Variates sampling gave the best results for our problem. These techniques can, of course, be used on many different problems in power system analysis.

## Author Contributions

*MCS* is used for the solution of the optimization problem giving two main outputs: loss of load occasion (*LOLO*) and total operation cost (*TOC*). One other objective of the paper is to study how much the simulation efficiency can be improved by introducing variance reduction techniques.

## Conflict of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Future Work

This work will be further extended to address the problem of optimal operation cost and the risk of load shedding in a microgrid ( $\mu$ G) using Multi-objective Optimization both Pareto-optimal weighted sum Solutions.

## Symbols

$c$	Share of the total load that is located in the main load center,
$D_{tot}$	Total load in the system,
$G_g, G_{gmax}$	Generation and available capacity in thermal plants $g$ ,
$H, H_{max}$	Generation and available generation capacity in the renewable (hydro) power plants
$P, P_{max}$	Transmission on and available capacity of the interconnection between the two areas (centers),
$D_1$	Load in the main load center,
$D_2$	Load in the generation center,
$\beta_{G_g}$	Variable operation cost of thermal power plant $g$ ,
$\beta_U$	Penalty cost for unserved load,
$\gamma_L$	Loss coefficient for the interconnection between the areas,
LOLO	Loss of load occasion (binary variable equal to 1 if load shedding is necessary and 0 otherwise),
TOC	total operation cost,
U	unserved load,

## Acronyms

MCS	Monte Carlo Simulation,
$\mu$ G	Microgrid,
r.v.	Random variable
¤	Currency sign

Table 3. Captions of Figures.

No.	Caption
Figure 1	Frequency & Scaled Probability Distribution functions of total demand
Figure 2	Comparison of LOLP from different Variance Reduction MCS methods
Figure 3	Comparison of TOC from different Variance Reduction MCS methods

## References

- [1] Guodong Liu, Bailu Xiao, Michael Starke, Oguzhan Ceylan, and Kevin Tomsovic, A robust load shedding strategy for microgrid islanding transition, Conference: 2016 IEEE/PES Transmission and Distribution Conference and Exposition (T&D), May 2016, DOI: 10.1109/TDC.2016.7520055.

- [2] Sayyad Nojavan, Majid Majidi, and Kazem Zare, Stochastic multi-objective model for optimal sizing of energy storage system in a microgrid under demand response program considering reliability: A weighted sum method and fuzzy satisfying approach, Manuscript received May 10, 2017; revised June 20; accepted July 1, 2017. Paper no. JEMT-1705-1015.
- [3] Amin Khodaei, and Mohammad Shahidehpour, Microgrid-based Co-optimization of Generation and Transmission Planning in Power Systems, [mysite.du.edu/.../Microgrid-based%20Co-Optimization%20of%20Generation%20and...](http://mysite.du.edu/.../Microgrid-based%20Co-Optimization%20of%20Generation%20and...)
- [4] Sai Krishna Kanth Hari, Kaarthik Sundar, Harsha Nagarajan, Russell Bent, Scott Backhaus, Hierarchical Predictive Control Algorithms for Optimal Design and Operation of Microgrids, Cornell university library, arXiv: 1803.06705v1 [math.OC] 18 Mar 2018. Conference: 2018 Power Systems Computation Conference (PSCC), DOI: 10.23919/PSCC.2018.8442977.
- [5] Mostafa Vahedipour-Dahraie, Amjad Anvari-Moghaddam, and Josep M. Guerrero, Evaluation of Reliability in Risk-Constrained Scheduling of Autonomous Microgrids with Demand Response and Renewable Resources, IET Renewable Power Generation, 2018, pp. 1-13.
- [6] Ashkan Zeinalzadeh and Vijay Gupta, Minimizing Risk of Load Shedding and Renewable Energy Curtailment in a Microgrid with Energy Storage, 2017 American Control Conference, Sheraton Seattle Hotel, May 24–26, 2017, Seattle, USA, pp. 3412-3417.
- [7] Yeongho Choi, Yujin Lim, and Hak-Man Kim, Optimal Load Shedding for Maximizing Satisfaction in an Islanded Microgrid, *Energies* 2017, 10, 45 pp. 1-13; doi: 10.3390/en10010045.
- [8] Habib Amooshahi, Rahmat-Allah Hooshmand, Amin Khodabakhshian, and Majid Moazzami, A New Load-shedding Approach for Microgrids in the Presence of Wind Turbines, *Electric Power Components and Systems*, 00 (00): 1–11, 2016, DOI: 10.1080/15325008.2015.1131761.
- [9] Amin Gholami, Tohid Shekari, and Xu Andy Sun, An Adaptive Optimization-Based Load Shedding Scheme in Microgrids, *Proceedings of the 51st Hawaii International Conference on System Sciences*, 2018, pp. 2660-2669.
- [10] Magnus Perninge ; Mikael Amelin ; Valerijs Knazkins, Comparing Variance Reduction Techniques for Monte Carlo Simulation of Trading and Security in a Three-Area Power System, 2008 IEEE/PES Transmission and Distribution Conference and Exposition: Latin America, 13-15 Aug. 2008, DOI: 10.1109/TDC-LA.2008.4641763.
- [11] Matt Bonakdarpour, Inverse Transform Sampling, 2016-02-02, last updated: 2017-03-06. [https://stephens999.github.io/fiveMinuteStats/inverse\\_transform\\_sampling.html](https://stephens999.github.io/fiveMinuteStats/inverse_transform_sampling.html)
- [12] Jack P. C. Kleijnen, Ad A. N. Ridder and Reuven Y. Rubinstein, Variance Reduction Techniques in Monte Carlo Methods, pp. 1-17. <https://pdfs.semanticscholar.org/f260/6f334bd4865b105005b887478003ce9bd3e2.pdf>
- [13] Reuven Y. Rubinstein and Dirk P. Kroese, *Simulation and the Monte Carlo Method*, Third Edition, 2017 John Wiley & Sons; DOI: 10.1002/9781118631980.