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Three Quantum Particles Hardy Entanglement from the Topology of Cantorian-Fractal Spacetime and the Casimir Effect as Dark Energy – A Great Opportunity for Nanotechnology

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Abstract: The present work brings together three different fields which depend crucially upon nano hardware under the umbrella of E-infinity theoretical framework. We start by following E-infinity topological methodology by dividing Hardy's entanglement into two parts, a global 'counterfactual' part given by Φ^3 where $\Phi = 2/(1+\sqrt{5})$ and a 'local' part Φ^n where n is the number of quantum particles. For Hardy's celebrated gedankenexperiment with two quantum particles, which was moreover experimentally confirmed with high accuracy, the quantum probability is found for n = 2 to be P(2) (Hardy) = $\Phi^{3+2} = \Phi^5$ exactly as calculated by Hardy using orthodox quantum mechanics. Applying the same topological E-infinity entanglement theory to three quantum particles give a maximal Φ^6 as well as a three partite much smaller value equal $\Phi^3(1-\Phi^3)/ = 0.018033989$. We conclude by outlining the relevant and extremely timely ideas and remarks on the possible connection, via a state of the art nanotechnology, to the Casimir effect as a conjectured origin of dark energy.

Keywords: Three Quantum Particles Entanglement, Hardy's Topological Entanglement, E-Infinity, Hilbert Space, Cantorian Spacetime, Golden Mean Number System, Casimir Effect Connection to Dark Energy, Nanotechnology

1. Introduction

Hardy's quantum entanglement [1-12] is, without a trace of a doubt, all that Sir Roger Penrose considered it to be and more [6], namely an ingenious gedankenexperiment [4-8] which revealed a striking feature of nature and the building blocks of our spacetime with far reaching physical and cosmological consequences [5]. In fact it is in the meantime an understatement to call Hardy's quantum entanglement a gedankenexperiment or a model since it is generic, was verified with high accuracy in numerous experiments [3-7] and was found to explain various fundamental mysterious phenomena in nature, notably dark energy [5] as well as theoretical findings with a claim to reality. In this context we mention Hawking radiation [13-15], Unruh temperature [13], Rindler's wedge [14] in addition to the Casimir effect which may be the origin of dark energy as sophisticated nanotechnology aided experiments may eventually reveal.

In the present work we show first how this result of three Hardy particles may be found from orthodox quantum mechanics in Hilbert space using Gram-Schmidt orthonormalization procedure [1-3]. In particular we point out that interpreted as a three-partite Hardy entanglement one finds P(Hardy) to be a relative minimum equal to 'tHooft dimensional regularization order parameter $k = \phi^3(1-\phi^3)$ = 0.18033989 divided by ten, i.e. $P_{min}(H) = 0.018033989$. On the other hand for 3 actual quantum particle entanglement P(H) corresponds to a maximal quantum probability almost three times larger than $k/10$, namely P(H) = $\phi^{3+3} = \phi^6$ = 0.05572809014. This value $3+(k/2)$ = 3.0901699 is interpreted within fuzzy set theory as the fractal weight of the number of the three entangled quantum particles Immirzi parameter ϕ^6 [16] and $k = 2\phi^5$ could be drawn in to explain why we could really have many compactified dimensions [15] in addition to various other aspects of high energy physics and quantum cosmology [15-18]. As is well known, in its original form Hardy's entanglement is concerned with two quantum particles [1-4]. Using Dirac's formalisms and a number of conventional mathematical tools of orthodox quantum mechanics Hardy found that the maximal probability for entanglement is almost nine percent [1-5]. Hardy did not notice at first that this nine percent is actually ϕ^5 = 0.09016994393 and means the golden mean to the power of five that is until Prof. Mermin [1-5] and much later the present author [1-5] noticed this remarkable and intriguing fact linking the most irrational KAM theoretical number which is ϕ to quantum entanglement [19-21]. Consequently and in turn, quantum entanglement is linked directly with topological entanglement [4]. The chain-like reasoning propagated from there in a natural, predictable manner to mean that quantum entanglement [24-29] could be interpreted as a topological consequence of the zero measure topology [4] of our real spacetime geometry at the microscopic scale [19-21]. In short this became the background against which the quantum entanglement theory of E-infinity theory was developed and the simple quantum entanglement formula of n quantum particles was established [4,11], namely $P_n(H) = \phi^{3+n}$. The last five years or so saw a flurry of papers published on the topological side of Hardy's entanglement and its topological interpretation [10-19].

From the above it could not pass unnoticed that the dark energy density is equal 1 minus half of Hardy's ϕ^5 divided by 2. In addition dark energy is the energy of the quantum wave. Consequently it must be related to the Casimir effect [30-31]and a possible source of near to infinite clean energy. This extremely important technological breakthrough innovation [32-44] will also be discussed at the end of the present paper.

2. The Three Quantum Particles Problem

From E-infinity theory [4,7][20-23] it is a well established result that Hardy's probability of quantum entanglement would be decomposed into two parts as mentioned earlier on. The first is the global part due to the core of spacetime. This part is exactly equal to the inverse of the Hausdorff dimension of the Cantor spacetime core [19-21], i.e. $1/(4+\phi^3)=\phi^3$. The second part is the local topological entanglement of two quantum particles, each with a zero set Hausdorff dimension ϕ. That way the total local probability is ϕ^2 [4,11]. Consequently Hardy's probability is the multiplication of the global ϕ^3 with the local ϕ^2 giving the gross total. This is in this case equal to ϕ^5 as should be [4,11]. However as far as the author is aware none of the previous papers dealt explicitly with the case of three particles which in view of its classical non-integrable analogue and the findings of quantum chaos theory [19-21] could harbour some surprises [6,7]. For these reasons the present paper is devoted to analysing explicitly Hardy's entanglement for three particles.

We may start from the outset that the present analysis confirmed the validity of the E-infinity general formula mentioned earlier [4,11] as a maximal quantum probability [4-11]. However a minor discovery was waiting for us in store, namely a minimal quantum probability relevant to quantum information science problems which require the division of the E-infinity probability by the fractal weight of the number of quantum particles concerned as will be explained in the following sections.

It is needless to reiterate that Hardy obtained the same results reported here for two particles by using conventional quantum mechanics [4] and we will now proceed in the same way for the case of 3 particles. As for the global part, it remains of course as it should be, namely ϕ^3. On the other hand for 3 particles the local part changes to $\phi\ \phi\ \phi=\phi^3$. Consequently the total Hardy entanglement is no more ϕ^5 but $\phi^3\phi^3=\phi^6$ which is the same value behind the Immirzi parameter reconciling the result of the blackhole entropy found using superstring theory with that found using loop quantum gravity [16]. It is now a nontrivial question to ask oneself if a smaller value than ϕ^6 could be realized in an actual situation. The answer is somewhat non-standard and is "yes, we can" but only when we introduce a twist to the entire set up by looking at the problem not as three 'geometrical-topological' quantum particles but as three-partite Hardy entanglement. The minimum in this case means simply the share of each particle from the union of all the three 'particles', that is to say a Hardy probability of exactly ϕ^6 divided by $3+(k/2)$ where k is 'tHooft's order parameter of dimensional regularization which is given by $k=\phi^3(1-\phi^3)$= 0.18033980 [22]. In this sense $3+(k/2)$ = 3.090169945 is a fractal weight for the number of the three particles and $k/2$ = ϕ^5 is the Hardy entanglement for two particles. The final result is thus

$$\text{P(Hardy)}=\phi^6/(3+\phi^5)=k/10=0.018033989. \quad (1)$$

3. Solution in Hilbert Space

Now we could have worked everything backwards and started from a three-partite Hardy setting and solved the problem using conventional Hilbert space techniques [4,11]. This leads us to various conditional probability densities which upon close examination could be extremized by letting the first derivative vanish and finding that

$$\text{P(Hardy)} = 0.018033989. \quad (2)$$

Subsequently by examining the second derivative we find that the value is a relative maximum but could be described as a minimum with respect to the first result, namely ϕ^6 where k is 'tHooft's order parameter of dimensional regularization. Applying the same strategy to one particle only one finds P(Hardy)_{min} = ϕ^3. That means P(Hardy)_{min} becomes equal to the global entanglement of spacetime. Going one step further and asking what is the minimal probability for no particles at all would mean division by zero [9] and leads to the minimal Hardy probability to become infinite. There is a deep meaning for this superficially nonsensical result. This limiting result means that spacetime at the quantum resolution is completely interconnected and nonlocal [23].

4. The Casimir Effect Behind Dark Energy and the Role of Nanotechnology

Topological quantum entanglement seen from E-infinity and noncommutative geometry perspective could be the solution to many hard theoretical problems ranging from wormholes to how we could obtain a near to infinite amount of clean energy. We start by recalling our result about the ordinary energy density of the cosmos

$$E(O) = \left(\phi^5 / 2\right) mc^2 \cong mc^2 / 22. \quad (3)$$

We note that $\gamma(O)$ is simply the probability for quantum entanglement for one particle only of the two particles Hardy set up, i.e.

$$\gamma(O) = P(H)/2 = \phi^5 / 2. \quad (4)$$

Consequently the dark energy density is give by

$$\gamma(D) = 1 - \gamma(O) = 1 - \left(\phi^5 / 2\right) = \left(5\phi^2\right)/2 \quad (5)$$

where ϕ^2 is the Hausdorff dimension of the empty set modelling the quantum wave. We also notice that for n interpreted as internal dimensions there is a vital duality between $\gamma(O)$ and $\gamma(D)$. To show this we compare

$$\gamma(O) = \phi^n / 2 \quad (6)$$

with

$$\gamma(D) = n\phi^2 / 2 \quad (7)$$

for very large n. It is clear from equations 6 and 7 that for $n \to \infty$ we have $\gamma(O) \to 0$ and $\gamma(D) \to \infty$. This leads us to make an educated guess to scientifically speculate on the possibility that the Casimir effect and dark energy are intimately connected or may be one and the same thing. It would seem that while our result that 95.5% of the energy of the cosmos is confirmed from the endophysical experiment [45,46] involving the entire universe, i.e. COBE, WMAP and Planck [10-19], the small amount of Casimir energy which we were able to measure is found by contrast from local exophysical experiments [45,46]. For this reason we think it is more than feasible to initiate a large scientific program utilizing nanotechnology [32-44] as well as E-infinity and K-theory to find out all what we can about how to utilize the Casimir-dark energy reservoir of vacuum energy [30,31] for the benefit of humanity and that way solve forever the colossal energy problems threatening our civilization.

5. Nanotechnology as the Key to the Infinite Reservoir of Empty Spacetime Clean Energy

It is curious or even more than curious that gravity within E-infinity theory was conceived from the very beginning as a fluctuation phenomena [13] caused by the "passing" of fractal time [47,48]. The idea was inspired by an old side remark due to R. Feynman and is analogous to Van der Waals forces as noted by the author as well as Professor M. Agop [49]. From this perspective the so called Feynman-El Naschie conjecture [47-49] could be extended to mean that the Casimir effect is nothing but a strong van der Waals effect and in turn dark energy is nothing but again the amplification of Casimir quantum wave forces brought about by the intricate topology and geometry of a high dimensional Banach spaces [47-54] where measure concentration of Dvoretzky's theorem [54] and holography are operative. Even more general than that, and as is obvious from the E-infinity topological conception of Hardy's quantum entanglement ϕ^5, ordinary energy density $\phi^5 / 2$ and dark energy density $5\phi^2 / 2$ one could easily conclude that these are all quantum vacuum fluctuation phenomena and include but are not restricted to Unruh temperature and Hawking's various effects [13,50]. That way we must recognize empty spacetime as the most abundant 'substance' in the universe and the origin of everything starting from energy. The scientific question which borders on science fiction and suggests itself whether we find it realistic or not, is if nanotechnology could bring us near to harnessing this vacuum energy in an efficient and economical way? Our answer to this spacetime question is a definite scientific-engineering yes. To motive our ideas we have to first recall that the Casimir forces are essentially the difference between the "quantum wave" density between the two Casimir plates and the same density outside in the surrounding space with dependence on the plate surface, material and shape. Now the universe in its entirety could be seen as analogous to the tiny space within the Casimir plates while the immensely infinite "empty set space" constituting the outside of our universe is analogous to the outside unbounded space in the Casimir classical experiments. In other words our cosmos is a one sided Casimir set up, i.e. we have Casimir pressure inside and there is no outside with outside pressure to balance it. This is essentially what we call dark energy. It is produced by the fact that the empty boundary of our universe is like a complex one sided so called Mobius strip and an extension of J.A. Wheeler's boundary of a boundary is zero principles. From that we could have an initial hunch on how nanotechnology and material surfaces technologies could simulate a holographic boundary for Casimir-dark energy experiments. We can also imagine performing experiments resembling classical reactor designs to create a gradient of Casimir-dark energy. Vaguely speaking at this very early stage, we guess that we might need to create an inverted Faraday cage so to speak in order to experiment with phenomena of vacuum fluctuation but it is fair to say that at this point, everything is in flux due to the novelty and the excitement of a completely new existence not bounded with traditional ways of thinking about matter and energy. One thing is however certain in this brand new world of post modernistic theoretical physics; nanotechnology is the key and interface between the classical world of man and the said post modernistic physical world of vacuum energy of empty space.

6. Conclusion

Computer information science, cosmological measurement in dark energy and experimenting with the Casimir effect are three different fields which depend crucially upon nanotechnology hardware. In the present work we bring all these fields together under the theoretical framework of E-infinity theory and the technical possibilities offered by nanotechnology. In the present short paper we looked first upon the three-partite quantum Hardy problem as a topological entanglement. We applied two solution strategies with different subtle interpretations. In the first we see the problem as that of three quantum particles and obtain a maximal quantum entanglement probability using E-infinity Cantorian spacetime theory amounting to $P(\text{Hardy})_{max} = \phi^6$. The second strategy is to start from a Hilbert space setting for a three-partite Hardy gedankenexperiment and find a minimal probability of about one third of ϕ^6. The exact result in the three partite case is $P(\text{Hardy})_{min} = k/10$. Motivated and encouraged by these results linking entanglement with dark energy we made a few proposals regarding the connection between the Casimir effect and dark energy as well as the role of nanotechnology in harnessing both the Casimir energy and/or dark energy which may be different sides of the same coin [30,31]. With the new possibilities offered by nanotechnology, the door is open now to utilize dark energy and Casimir energy as an unheard of source of free energy.

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