



On The Prototype Solutions of Symmetric Regularized Long Wave Equation by Generalized Kudryashov Method

Hasan Bulut¹, Hacı Mehmet Baskonus², Eren Cüvelek¹

¹Department of Mathematics, Firat University, Elazığ, Turkey

²Department of Computer Engineering, Tunceli University, Tunceli, Turkey

Email address:

hbulut@firat.edu.tr (H. Bulut), hmbaskonus@gmail.com (H. M. Baskonus), erencuvelek@gmail.com (E. Cüvelek)

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Abstract: In this study, we have applied the generalized kudryashov method to the symmetric regularized long wave equation for obtaining some new analytical solutions such as trigonometric function solution, exponential function solution, complex function solution, hyperbolic function solution after giving the fundamental properties of method. Afterwards, we have observed that these analytical solutions are verified the symmetric regularized long wave equation by means of Wolfram Mathematica 9. Then, we have drawn two and three dimensional surfaces of analytical solutions. Finally, we have submitted a conclusion to literature.

Keywords: Generalized Kudryashov Method, Symmetric Regularized Long Wave Equation, Exponential Function Solution, Complex Function Solution, Hyperbolic Function Solution, Trigonometric Function Solution

1. Introduction

All over the world, some researchers have submitted to literature some analytical solutions of many nonlinear partial differential equations. Many powerful methods such as the transformation techniques, the sine-cosine technique, the exponential function method, the standard tanh and developed tanh methods, the jacobi function method, trial equation method, darboux transformation, homotopy perturbation method, sumudu transform method, kudryashov method and so on have been applied successfully [1-11].

When it comes to this paper, we have presented the general properties of generalized kudryashov method (GKM) [12] in section 2. In section 3, we have applied GKM to the symmetric regularized long wave equation (SRLW) defined by [13]

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0. \quad (1.1)$$

Finally, we have submitted a conclusion including some important remarks in section 4.

2. General Properties of Generalized Kudryashov Method

Recently, some authors have constructed the generalized kudryashov method [12]. We consider the following nonlinear

partial differential equation for a function u of two real variables, space x and time t :

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \dots) = 0. \quad (2.1)$$

The basic phases of the generalized kudryashov method are expressed as the follows:

Step 1. First of all, we must get the travelling wave solution of Eq.(2.1) as following form;

$$u(x, t) = u(\rho), \quad \rho = kx - ct, \quad (2.2)$$

where k and c are arbitrary constants. Eq.(2.1) was converted into a nonlinear ordinary differential equation of the form:

$$N(u, u', u'', u''', \dots) = 0, \quad (2.3)$$

where the prime indicates differentiation with respect to ρ .

Step 2. Suggest that the exact solutions of Eq.(2.3) can be written as the following form;

$$u(\rho) = \frac{\sum_{i=0}^N a_i Q^i(\rho)}{\sum_{j=0}^M b_j Q^j(\rho)} = \frac{A[Q(\rho)]}{B[Q(\rho)]} \quad (2.4)$$

where Q is $\frac{1}{12}e^{\rho}$. We note that the function Q is solution of equation [12]

$$Q_{\rho} = Q^2 - Q. \quad (2.5)$$

Taking into consideration Eq.(2.4), we obtain

$$u'(\rho) = \frac{A'Q'B - AB'Q'}{B^2} = Q' \left[\frac{A'B - AB'}{B^2} \right] = (Q^2 - Q) \left[\frac{A'B - AB'}{B^2} \right], \quad (2.6)$$

$$u''(\rho) = \frac{Q^2 - Q}{B^2} [(2Q - 1)(A'B - AB') + \frac{Q^2 - Q}{B} [B(A''B - AB'') - 2B'A'B + 2A(B')^2]], \quad (2.7)$$

$$u'''(\rho) = (Q^2 - Q)^3 \left[\frac{(A'''B - AB''' - 3A''B' - 3B'A'')B}{B^3} + \frac{6B(AB'' + B'A'')}{B^3} - \frac{6A(B')^3}{B^4} \right] + 3(Q^2 - Q)^2 (2Q - 1) \left[\frac{B(A''B - AB'')}{B^3} - \frac{2B'A'B + 2A(B')^2}{B^3} \right] + (Q^2 - Q)(6Q^2 - 6Q + 1) \left[\frac{A'B - AB'}{B^2} \right]. \quad (2.8)$$

Step 3. Under the terms of proposed method, we suppose that the solution of Eq.(2.3) can be explained in the form of the following:

$$u(\rho) = \frac{a_0 + a_1Q + a_2Q^2 + \dots + a_NQ^N + \dots}{b_0 + b_1Q + b_2Q^2 + \dots + b_MQ^M + \dots}. \quad (2.9)$$

To calculate the values M and N in Eq.(2.9) that is the pole order for the general solution of Eq.(2.3), we progress conformably as in the classical kudryashov method on balancing the highest order nonlinear terms in Eq.(2.3) and we can determine a formula of M and N . We can receive some values of M and N .

Step 4. Replacing Eq.(2.4) into Eq.(2.3) provides a polynomial $R(Q)$ of Q . Establishing the coefficients of $R(Q)$ to zero, we acquire a system of algebraic equations. Solving this system, we can describe ρ and the variable

coefficients of $a_0, a_1, a_2, \dots, a_N, b_0, b_1, b_2, \dots, b_M$. In this way, we attain the exact solutions to Eq.(2.3).

3. Implementation of Method Proposed

For symmetric regularized long wave equation [13]

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \quad (3.1)$$

with Eq.(2.2) transformation, nonlinear differential form for Eq.(3.1) is obtained as the following;

$$(k^2 + c^2)u - \frac{ck}{2}u^2 + c^2k^2u'' = 0. \quad (3.2)$$

u^2 and u'' terms is used according to balance principle. And then, balance term is obtained as the following;

$$N = M + 2. \quad (3.3)$$

Case 1: For $M = 1$ and $N = 3$ when Eq.(2.7) is rewritten with Eq.(2.4), we can find follows;

$$u(\rho) = \frac{\sum_{i=0}^N a_i Q^i(\rho)}{\sum_{j=0}^M b_j Q^j(\rho)} = \frac{A[Q(\rho)]}{B[Q(\rho)]}, \quad (3.4)$$

$$u''(\rho) = \frac{Q^2 - Q}{B^2} [(2Q - 1)(A'B - AB') + \frac{Q^2 - Q}{B} [B(A''B - AB'') - 2B'A'B + 2A(B')^2]], \quad (3.5)$$

where $a_3 \neq 0$, $b_1 \neq 0$. If Eq.(3.4) and Eq.(3.5) are considered in Eq.(3.2), a new equation is occurred with Q . Solving this equation with Mathematica 9, it yields us the following coefficients;

Case-1.1

$$a_0 = \frac{2k^2b_0}{\sqrt{-1+k^2}}, a_1 = \frac{2k^2(6b_0 - b_1)}{\sqrt{-1+k^2}}, a_2 = \frac{12k^2(-b_0 + b_1)}{\sqrt{-1+k^2}}, a_3 = \frac{12k^2b_1}{\sqrt{-1+k^2}}, c = -\frac{k}{\sqrt{-1+k^2}}. \quad (3.6)$$

If Eq.(3.6) is written in Eq.(3.4), we can obtain the hyperbolic function solution as the following:

$$u_1(x, t) = \frac{k^2(-2 + 3 \sec h(\frac{1}{2}k(\frac{t}{\sqrt{-1+k^2}} + x))^2)}{\sqrt{-1+k^2}}. \quad (3.7)$$

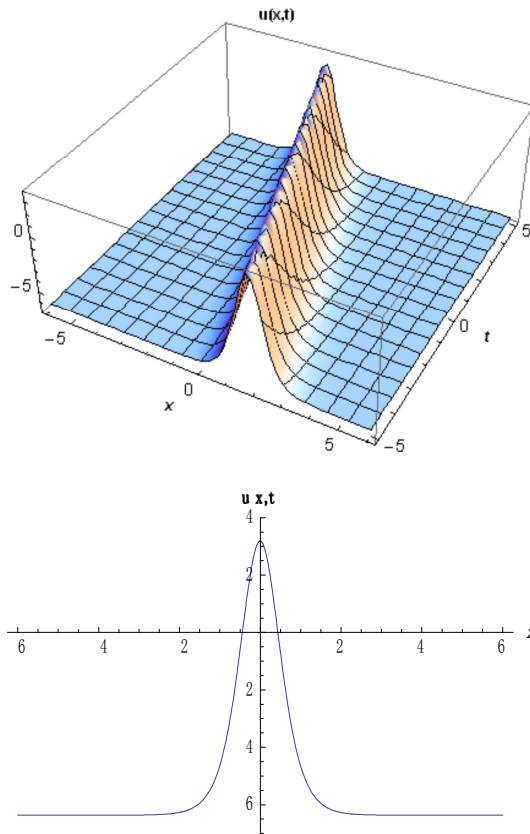


Figure 1. Two and three dimensional surfaces of (3.7) hyperbolic function solutions for $k=3$ and $t=0.01$ for two dimensional surfaces.

Case-1.2

$$\begin{aligned} a_0 &= 0, a_1 = -6i\sqrt{2}b_0, a_2 = 6i\sqrt{2}(b_0 - b_1), \\ a_3 &= 6i\sqrt{2}b_1, k = -1, c = -\frac{i}{\sqrt{2}}. \end{aligned} \quad (3.8)$$

If Eq.(3.8) is written in Eq.(3.4), we can obtain being complex trigonometric function solution as the following:

$$u_2(x,t) = -\frac{3i\sqrt{2}}{1 + \cos\left(\frac{t}{\sqrt{2}} + ix\right)}. \quad (3.9)$$

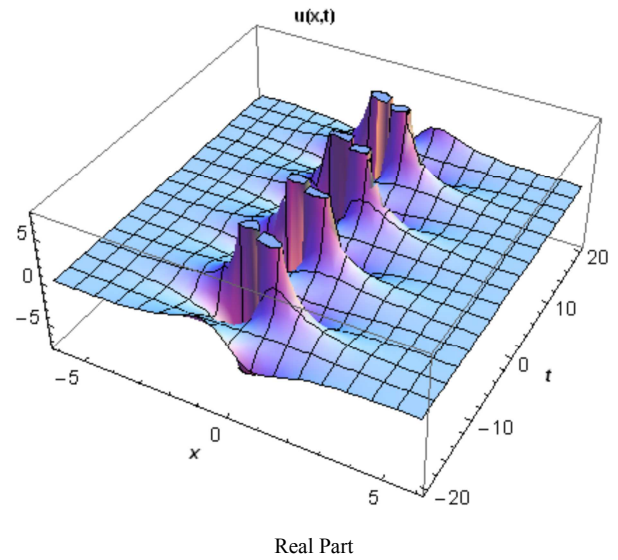
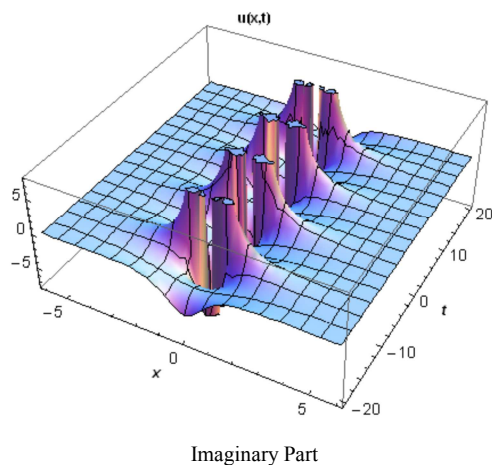


Figure 2. Three dimensional surfaces of (3.9) the being complex trigonometric function solutions for $t=0.01$.

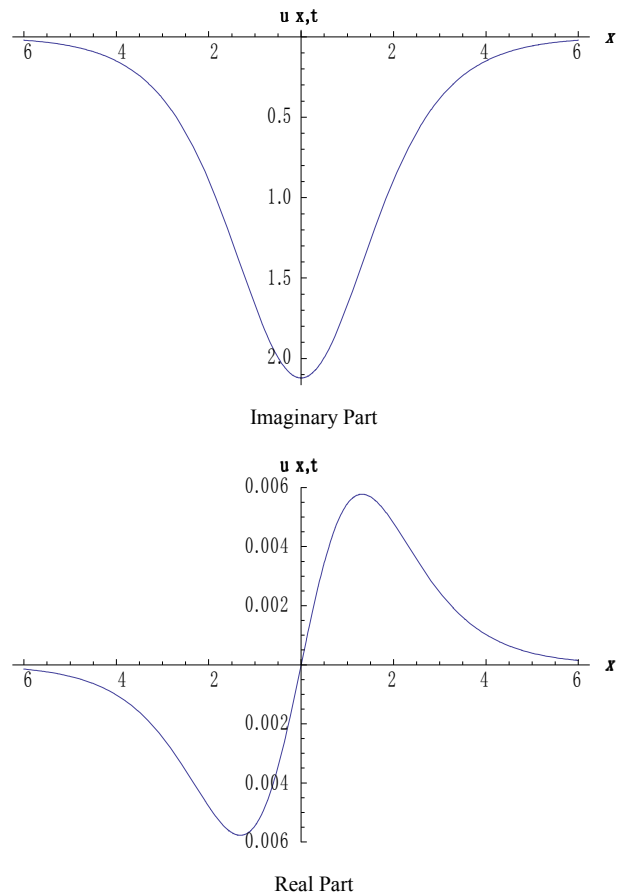


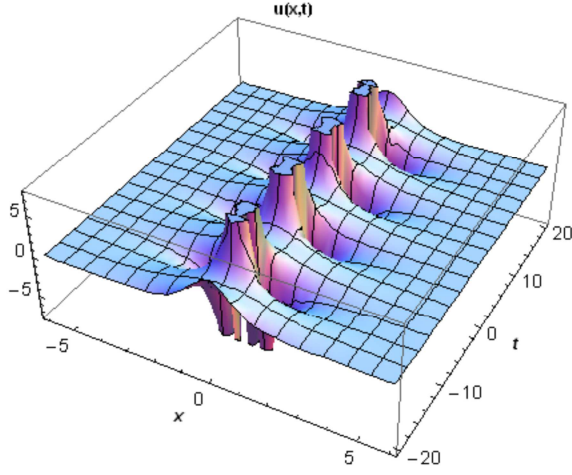
Figure 3. Two dimensional surfaces of (3.9) being complex trigonometric function solutions for $t=0.01$.

Case-1.3

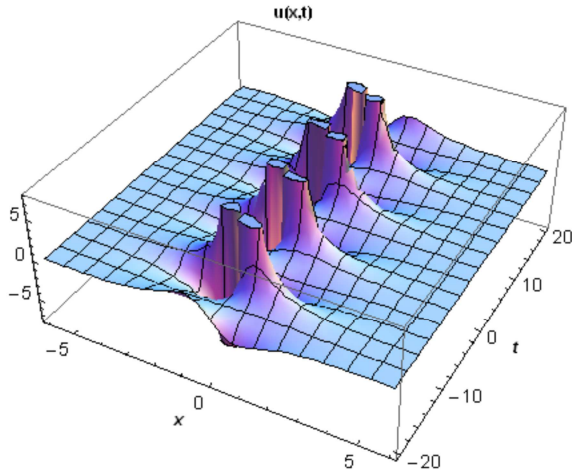
$$\begin{aligned} a_0 &= 0, a_1 = 6i\sqrt{2}b_0, a_2 = 6i\sqrt{2}(-b_0 + b_1), \\ a_3 &= -6i\sqrt{2}b_1, k = -1, c = \frac{i}{\sqrt{2}}. \end{aligned} \quad (3.10)$$

If Eq.(3.10) is written in Eq.(3.4), we can obtain being complex trigonometric function solution as the following:

$$u_3(x, t) = \frac{3i\sqrt{2}}{1 + \cos(\frac{t}{\sqrt{2}} - ix)} \quad (3.11)$$

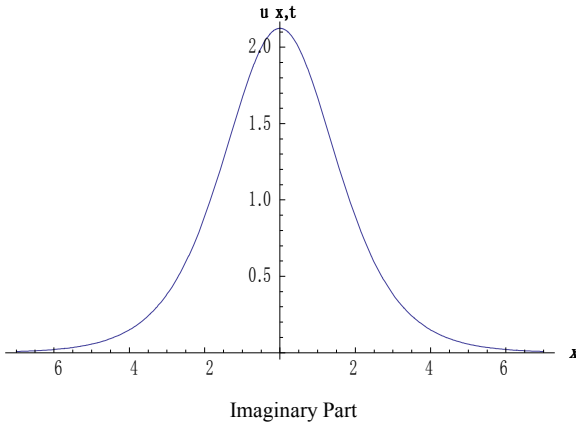


Imaginary Part

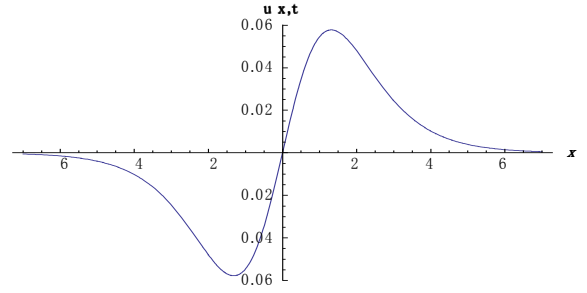


Real Part

Figure 4. Three dimensional surfaces of (3.11) being complex trigonometric function solutions for $t = 0.01$.



Imaginary Part



Real Part

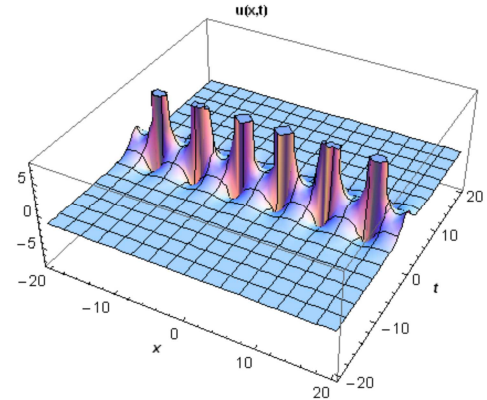
Figure 5. Two dimensional surfaces of (3.11) being complex trigonometric function solutions for $t = 0.01$.

Case-1.4

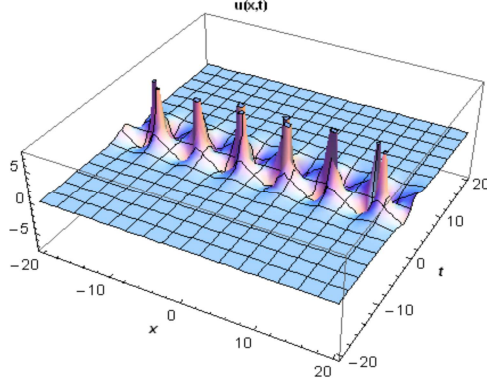
$$\begin{aligned} a_0 &= 0, a_1 = -i\sqrt{2}b_1, a_2 = 6i\sqrt{2}b_1, \\ a_3 &= -6i\sqrt{2}b_1, b_0 = 0, k = i, c = -\frac{1}{\sqrt{2}}. \end{aligned} \quad (3.12)$$

If Eq.(3.12) is written in Eq.(3.4), we can obtain complex hyperbolic function solution as the following:

$$u_4(x, t) = \frac{i(-2 + 3 \sec h(\frac{1}{4}(\sqrt{2}t + 2ix))^2)}{\sqrt{2}} \quad (3.13)$$



Imaginary Part



Real Part

Figure 6. Three dimensional surfaces of (3.13) complex hyperbolic function solutions for $t = 0.01$.

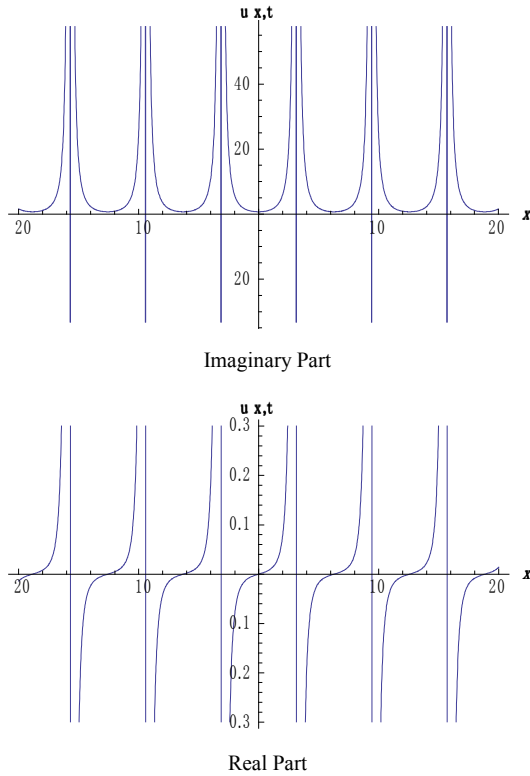


Figure 7. Two dimensional surfaces of (3.13) complex hyperbolic function solutions for $t = 0.01$.

Case-1.5

$$\begin{aligned} a_0 &= 0, a_1 = -\frac{12k^2b_0}{\sqrt{-1-k^2}}, a_2 = \frac{12k^2(b_0-b_1)}{\sqrt{-1-k^2}}, \\ a_3 &= \frac{12k^2b_1}{\sqrt{-1-k^2}}, c = \frac{k}{\sqrt{-1-k^2}}. \end{aligned} \quad (3.14)$$

If Eq.(3.14) is written in Eq.(3.4), we can obtain complex exponential function solution as the following:

$$u_5(x,t) = -\frac{12e^{\frac{k(-\frac{t}{\sqrt{-1-k^2}}+x)}{k^2}}}{(e^{\frac{kt}{\sqrt{-1-k^2}}} + e^{kx})^2 \sqrt{-1-k^2}}. \quad (3.15)$$

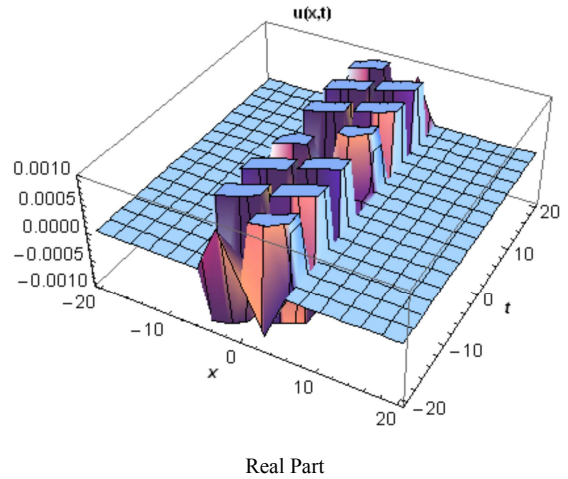
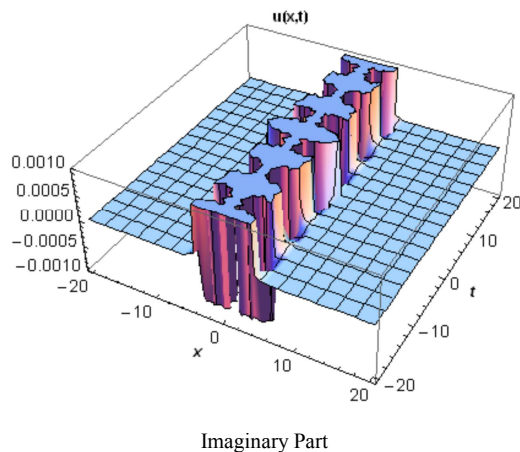


Figure 8. Three dimensional surfaces of (3.15) complex exponential function solutions for $k = 3, t = 0.01$.

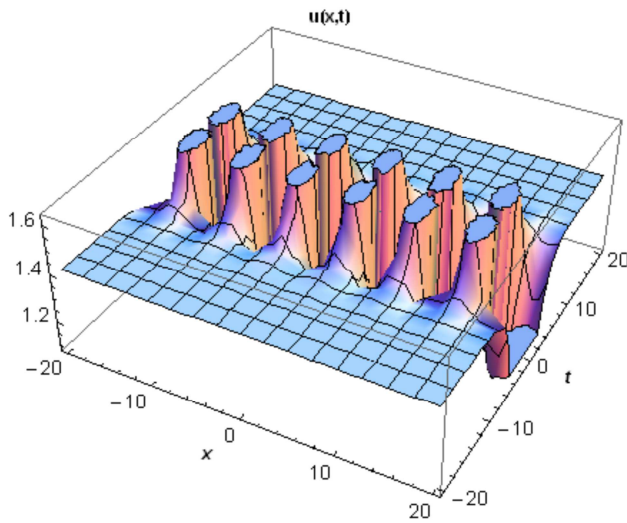
Figure 9. Two dimensional surfaces of (3.15) complex exponential function solutions for $k = 3, t = 0.01$.

Case-1.6

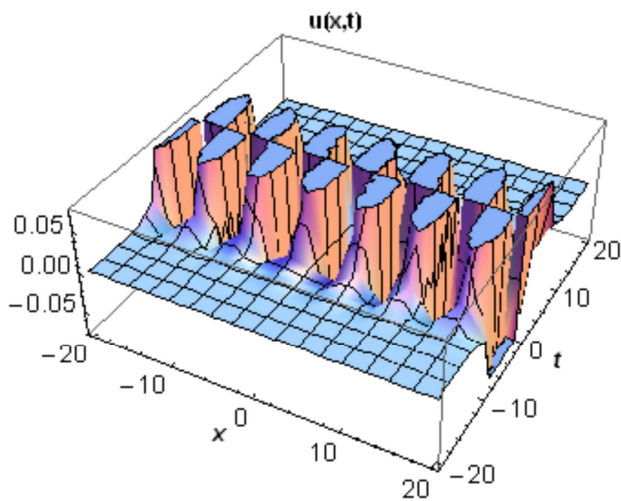
$$\begin{aligned} a_0 &= i\sqrt{2}b_0, a_1 = i\sqrt{2}(-6b_0 + b_1), a_2 = 6i\sqrt{2}(b_0 - b_1), \\ a_3 &= 6i\sqrt{2}b_1, k = -i, c = -\frac{1}{\sqrt{2}}. \end{aligned} \quad (3.16)$$

If Eq.(3.16) is written in Eq.(3.4), we can obtain complex hyperbolic function solution as the following:

$$u_6(x, t) = \frac{i(2 - 3 \operatorname{sech}(\frac{1}{4}(\sqrt{2}t - 2ix))^2)}{\sqrt{2}}. \quad (3.17)$$

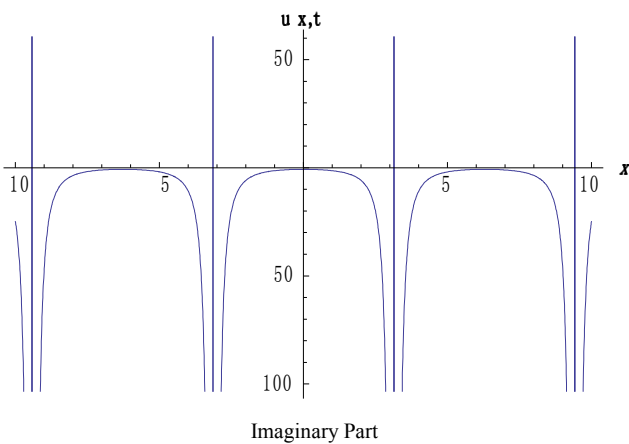


Imaginary Part

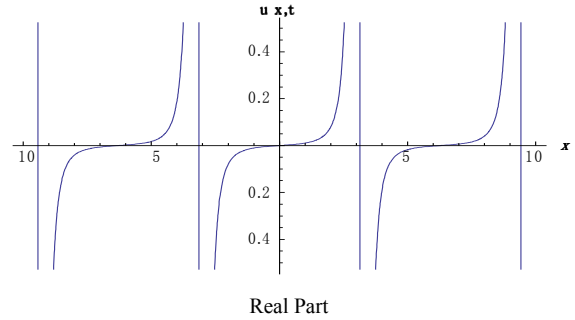


Real Part

Figure 10. Three dimensional surfaces of (3.17) complex hyperbolic function solutions for $t = 0.01$.



Imaginary Part



Real Part

Figure 11. Two dimensional surfaces of (3.17) complex hyperbolic function solutions for $t = 0.01$.

4. Conclusions

In this paper, symmetric regularized long wave equation has been solved by using GKM. We have obtained the prototype solutions such as complex function, trigonometric function, exponential function and hyperbolic function solutions. It has been observed that these analytical solutions have verified to the SRLW Eq.(1.1) by using Wolfram Mathematica 9.

These analytical solutions obtained by using GKM in this paper are new complex, trigonometric, exponential and hyperbolic function solutions when we compare these solutions with analytical solutions obtained by Jalil Manafian and Isa Zamanpour [13]. Then, two and three dimensional surfaces of these analytical solutions have been plotted by using Wolfram Mathematica 9.

According to solutions and graphics, one can see that GKM is a powerful tool for obtaining some new analytical solutions for such problems.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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