

Methodology Article

Robust Detection Method of Arrival Time Difference Under Minimum Maximum Entropy Criterion

Wenhong Liu¹, Junhao Li², Niansheng Chen¹, Guangyu Fan¹¹School of Electronic and Information, Shanghai Dianji University, Shanghai, China²School of Electrical Engineering, Shanghai Dianji University, Shanghai, China**Email address:**

liuwenhong@sdju.edu.cn (Wenhong Liu)

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Abstract: Accurate measurement of arrival time difference is one of the key technologies in many areas, such as the global positioning system. Due to the effects of environmental noises around receiver, the classic methods under least mean-square error rule are the lack of robustness. In this paper, a robust method of the time difference detection is addressed based on the minimum maximum entropy, referred to MMEATD. The maximum entropy function used in this method is a smooth approximation of the L1 norm. It has robustness to large outliers, but also is differentiable. Under the minimum maximum entropy criterion, the adaptive filter weight vector will be convergence, and its peak position indicates the arrival time difference. The computer simulation experiments show the estimation performance of this algorithm under different signal and noise ratio or different impulsive noise intension. Meanwhile, its estimation performance is compared with minimum mean square error algorithm. Results show that the proposed method has a good robustness under the impulsive noise environment.

Keywords: Time Difference Detection, Impulsive Noises, Robustness, Minimum Maximum Entropy, Adaptive Filter

1. Introduction

In many areas, such as the global positioning system, arrival time difference detection accurately is one of the core technologies [1]. Noises are one of the main problems on arrival time difference detection to consider [2]. When noises are in conformity with the Gaussian distribution, traditional methods under minimum mean square error criterion have good estimation performance, and also have advantageous for the theoretical analysis. However, actual application category may have significant spike pulse noises, and their probability density functions have thick tail compared with Gaussian distribution. There is no second order moment and above. At this point, under the minimum mean square error criterion designing adaptive time difference detection algorithms, the performances appear degradation, can't even use [2, 3]. One of solutions is to use more accord with the actual noises model, choose a more robust criterion functions. A subclass of alpha stable distribution - *SaS* [4] can better describe this kind of impulsive noises. The minimum dispersion criterion is alpha

stable distribution of linear theory, a kind of optimal criterion. Minimum dispersion is equivalent to minimize the estimation error of fractional lower order moment, *SaS* fractional lower order moments and p ($0 < p < \alpha$) are directly proportional to the norm. Considering the cost function is convex and differentiable, in the actual time difference algorithms based on alpha stable distribution, usually use the minimum average p ($1 < p < \alpha$) norm criterion [3]. Selection of parameters p , however, depending on the alpha stable distribution characteristic index of alpha prior knowledge or estimates, there is inconvenient [5]. All fractional lower order moments of alpha stable distribution are equivalent, using the L1 norm criterion can reduce or avoid the estimate of the alpha value. However, the L1 norm is related to non-differentiable optimization problems, optimization methods based on the gradient are no longer applicable. Literature [6] in the L1 norm model is transformed into an equivalent form of easy to implement, with a neural network to realize with no adjustment circuit. Developed in recent years on the maximum entropy method is one way of solving

non-differentiable optimization problems [7-10]. Robust signal processing methods are currently researched, and detection methods of arrival time difference under robust criterions are considered [11-15].

Impulse noise modeling with *SaS*, moving average filter is adopted to simulate the channel delay effect of the source signals reaching the second receiver under the minimum maximum entropy. The relative time delay can be obtained. Maximum entropy function is differentiable and smooth approximation to the absolute deviation of cost function. This will be change under the minimum L1 norm criterion of adaptive time difference problems into the minimum Maximum Entropy criterion of the adaptive arrival time difference problems. This adaptive time difference detection algorithm based on minimum Maximum Entropy function is used under the computer simulation experiment compared the traditional minimum mean square error of the time difference algorithm. The proposed algorithm, referred to MMEATD, under the condition of Gaussian and non-Gaussian *SaS* noises, estimation performances are better. So the MMEATD has good robustness. It is a kind of adaptive time difference detection algorithm in wide application scope.

2. Maximum Entropy Function

The linear parameter estimation model widely used in signal processing can be described by a set of linear equations:

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{e} \quad (1)$$

here, $\mathbf{b}=[b_1, b_2, \dots, b_m]^T \in R^m$ represents the observation vector containing error; $\mathbf{A}=[a_{ij}] \in R^{m \times n} (m \geq n)$ is a matrix composed of known signal; $\mathbf{x}=[x_1, x_2, \dots, x_n]^T \in R^n$ is to estimate the parameters of the vector; $\mathbf{e}=[e_1, e_2, \dots, e_n]^T \in R^m$ for unknown error vector.

Parameter vector \mathbf{x} can be minimized the error vector L1 norm optimization, algorithm is:

$$\min_{\mathbf{x}} [\|\mathbf{e}(\mathbf{x})\|_1] = \min_{\mathbf{x}} [\|\mathbf{b} - \mathbf{A}\mathbf{x}\|_1] \quad (2)$$

Although minimal L1 norm criterion of fractional lower order *SaS* pulse data parameter estimation has better robustness, but the non differentiable character of the L1 norm limits its use widely.

Maximum entropy function method is a kind of new approximate method for solving non-differentiable optimization problem [2]. The basic idea is to change the maximum function $f(\mathbf{x}) = \max[f_i(\mathbf{x})]$, ($i=1, 2, \dots, m$) of the optimal solution to the maximal entropy function value function into:

$$F(\mathbf{x}, p_e) = \frac{1}{p_e} \ln \left(\sum_{i=1}^m \exp(p_e f_i(\mathbf{x})) \right) \quad (3)$$

here, the $p_e > 0$. The approximate optimal solution of original problem is obtained.

Maximum entropy function has the following properties [8, 10]:

(1) For any $\mathbf{x} \in R^n$, have $0 \leq F(\mathbf{x}, p_e) - f(\mathbf{x}) \leq (1/p_e) \ln(m)$, when the $p_e \rightarrow \infty$, $F(\mathbf{x}, p_e)$ is uniform convergence in $f(\mathbf{x})$.

(2) $F(\mathbf{x}, p_e)$ in term of \mathbf{x} is differentiable.

(3) Linear L1 norm of the maximum entropy function $F(\mathbf{x}, p_e)$ in term of \mathbf{x} is a convex function.

Obviously, $F(\mathbf{x}, p_e)$ is a very good approximation $f(\mathbf{x})$, and has a good nature.

The smallest L1 norm can be thought of as a minimax problem:

$$\min_{\mathbf{x}} [\|\mathbf{e}(\mathbf{x})\|_1] = \min_{\mathbf{x}} \{ \max[\mathbf{e}(\mathbf{x}), -\mathbf{e}(\mathbf{x})] \} \quad (4)$$

therefore, it can be transferred into a differentiable maximum entropy function of unconstrained optimization problems. For the following to deduce linear minimum L1 norm time difference problems, it has a unique solution.

3. Time Difference Detection Under Minimum Maximum Entropy Criterion

Two signals $x_1(n)$ and $x_2(n)$ are described the discrete signal model:

$$\begin{aligned} x_1(n) &= s(n-d_1) + v_1(n) \\ x_2(n) &= \lambda s(n-d_2) + v_2(n) \end{aligned} \quad (5)$$

$s(n-d_1)$ and $s(n-d_2)$ are the two received delay signals; d_1 and d_2 respectively are the source signals to reach the receiving end with two time delays; λ is for attenuation factor (for simple, often take $\lambda = 1$); $v_1(n)$ and $v_2(n)$ are background noises with two receiver, obeying to *SaS*. Assuming the signal and the noise, and the noise and the noise were independent of statistics. Needing to estimate the arrival time difference of two received signals true value is $D = |d_2 - d_1|$. The following derivation has assume of $d_1 < d_2$.

Two signal $x_1(n)$, $x_2(n)$ in the moving average filter modeling is followed:

$$x_2(n) = \sum_{i=0}^Q w(i) x_1(n-i) + e(n) \quad (6)$$

$e(n)$ is moving average model error; $w(i)$ ($i=1, 2, \dots, Q$) is the moving average model parameter, $Q > D$. At $i=D$, $w(i)$ have a maximum value, namely $i \neq D$, $|w(i)| < |w(D)|$.

The block diagram of arrival time difference detection using adaptive filter is shown in Figure 1.

The estimation of arrival time difference is converted to estimate of the moving average filter weight coefficient maximum position. It can be adaptive to realize under certain optimization criterion. Traditional, least mean square error is adopted. However, this criterion has the lack of robust under impulsive noises. In this paper, the minimum maximum entropy criterion is used. The optimization criterion in Fig. 1 will be changed to the minimum maximum entropy.

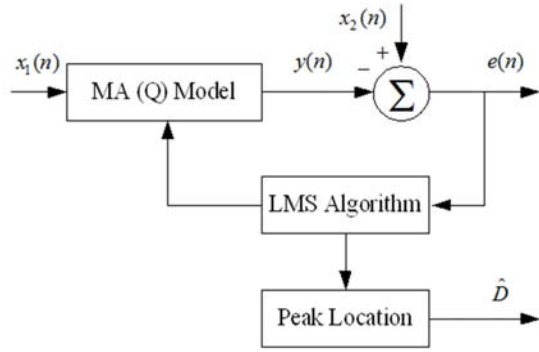


Figure 1. The principle block for arrival time difference using adaptive filter.

The MMEATD is derived. Considering cost function of adaptive systems:

$$J = \|e(\mathbf{w}(n))\|_1 = \|x_2(n) - \mathbf{w}^T(n)\mathbf{x}_1(n)\|_1 \quad (7)$$

It can be expressed in the maximum function:

$$J = \max[e(\mathbf{w}(n)), -e(\mathbf{w}(n))] \quad (8)$$

Maximum entropy function has differentiable structure:

$$F(\mathbf{w}(n), p_e) = (1/p_e) \ln(\exp(p_e e(n)) + \exp(-p_e e(n))) \quad (9)$$

First, convert the least absolute deviation time difference to minimum maximum entropy time difference. Then, according to the linear L1 norm of the maximum entropy function is convex, using the steepest descent method to solve moving average filter weight coefficient of the optimal solution. Iterative formula is:

$$F(\mathbf{w}(n), p_e) = (1/p_e) \ln(\exp(p_e e(n)) + \exp(-p_e e(n))) \quad (10)$$

here, $e(\mathbf{w}(n)) = x_2(n) - \mathbf{w}^T(n)\mathbf{x}_1(n)$; μ is adaptive filter convergence factor, often taken a smaller number.

Adaptive filter is used in the process of adaptive iteration, and gradually achieve moving average model simulation of

channel delay effect. When adaptive filter convergence, $x_2(n)$ and $y(n)$ to minimize the absolute deviation, namely the actual $s(n-D)$ and estimate of $s(n-D)$ similarity of the biggest, the adaptive filter weight vector $\mathbf{w}(n)$ becomes a copy of moving average model, and its position of the maximum value dictates the arrival time difference estimation.

4. Computer Simulations

The computer simulation experiments validate the estimation performances of the MMETDD in Gaussian and non-Gaussian *SaS* noises, and the traditional algorithm is compared with.

According to Equation (5), two received signals are modeled, which is the band-limited flat spectrum source signal $s(n)$ made from white Gaussian signal through the 0.2 bandwidth of low-pass filter with 10 orders length. The impulsive noises are in compliance to simulated non-Gaussian *SaS* signal. Mixed signal-to-noise ratio is the $\text{MSNR} = 10 \lg(\sigma_s^2/\gamma_v)$ [3], here, the σ_s^2 represents the source signal variance, γ_v is the dispersion coefficient of noise. Dispersion coefficient is half of the variance with Gaussian noise. Assume that the signal and the noise, the noise and the noise are not relevant. Set two received signals arrival time difference value of $D=10T_s$, T_s is signal sampling interval in time domain. The length of signal sequence is $n=5000$ points, select the order number $m=20$ of filter, set filter weight vector of the initial value of $\mathbf{w}(0)=[0 \dots 0]$. In the MMEATD, the selection of maximum entropy function approximation parameter $p_e=1$. Iterative step is $\mu=0.0001$ in the traditional algorithm and the proposed algorithm. The following results are 100 times independent experiment statistics.

First, in $\alpha=1.5$, under different MSNR, compare estimation precisions of the traditional algorithm and the proposed algorithm. MSNR is from -6 dB changes to 3 dB at 3 dB intervals. Root mean square errors (referred to RMSE) of estimated time differences of two algorithms are shown in Figure 2.

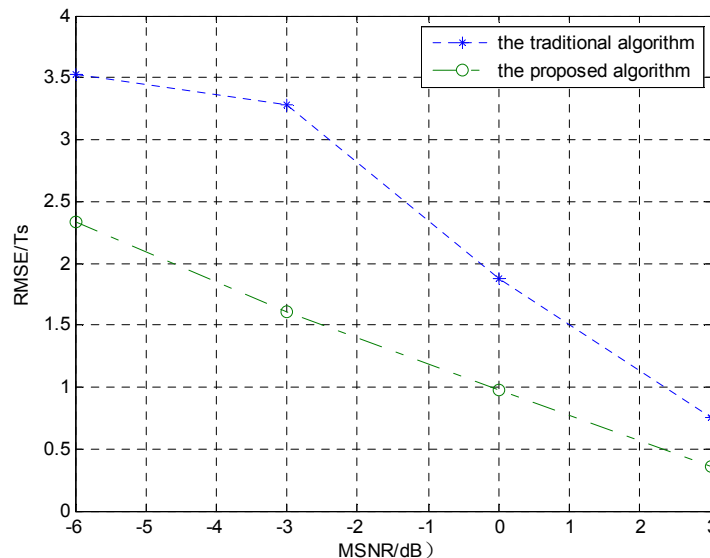


Figure 2. The RMSE of the traditional algorithm and the proposed algorithm with $\alpha=1.5$ impulsive noises at different MSNRs.

As you can see in Figure 2, in different MSNR cases, RMSE of the proposed algorithm were less than the proposed algorithm's.

Then, setting the MSNR=0dB, under different α value, α changes from 1.4 to 2 with the interval of 0.2, time difference RMSEs of the two algorithms are also shown in Figure 3.

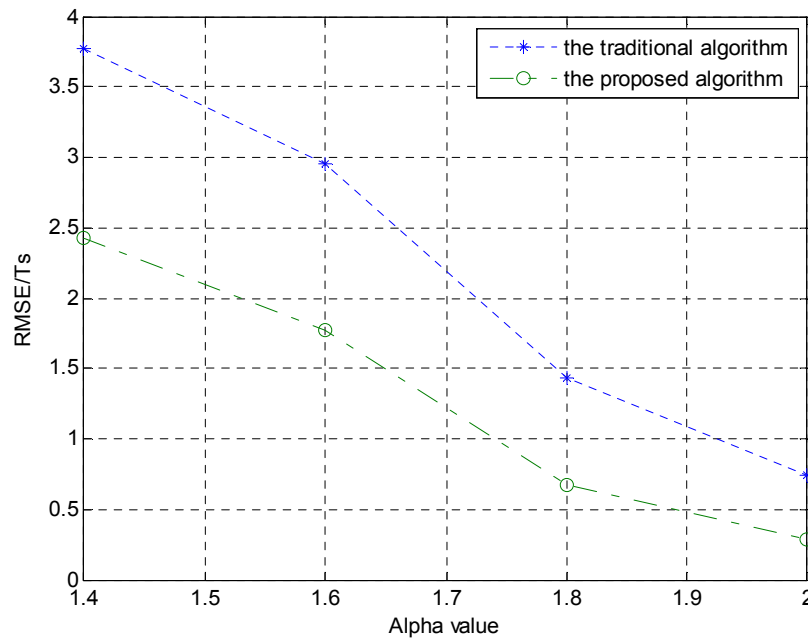


Figure 3. The time difference RMSEs of the two algorithms with MSNR=0dB at different impulsive noises α values.

As you can see in Figure 3, the MSNR=0dB, α value is not same, the RMSE of the proposed algorithm were less than the traditional algorithm in Gaussian and non-Gaussian distribution noise cases, so the MMEATD has high estimation precision.

The computer simulations show that the traditional algorithm is only applicable to Gaussian noise environment, while the proposed algorithm under Gaussian noise and non-Gaussian has good estimation performance. The MMEATD is a kind of robust time difference detection method, and has a wider scope of application than the traditional algorithm.

5. Conclusions

Minimum L1 norm criterion has robustness, but is not differentiable. The maximum entropy function is a kind of smooth approximation of L1 norm, so it can transfer a L1 norm optimization problem into a differentiable maximum entropy function optimization problem, thus the steepest descent method can be used. In this paper, Iterative formula of the moving mean filter parameter is deduced and obtained based on minimal maximum entropy criterion in proposed method.

The computer simulation verifies the MMEATD has good estimation performance both in Gaussian and in non-Gaussian alpha stable distribution noise condition, compared with the traditional algorithm. As a result, when the signal to noise conditions does not conform to the Gaussian distribution or unknown circumstances, the MMEATD is a better choice. In

the future, more practical data would be further used to demonstrate the better performance of the MMEATD.

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