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Research Article

An Improved Single Objective Optimization Approach for Double-Layered Multi-Head Weighing Process

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Abstract

In recent years, the double-layered multi-head weighers whose hoppers are arranged in two levels are widely used in the accurate and reliable weighing for packing food products. The weighing processes are mathematically modeled into a single objective optimization problems. The objective of packing problem is to minimize the total weight of combined hoppers for a package under the condition that the total weight must be no less than a specified target weight. This paper proposes a novel single objective optimization approach for double-layered multi-head weighing process. More precisely, relying on a new bound on the optimal weight, this study accurately determines the number of hoppers to be combined at each packing operation, and find the best possible hopper combination using the single-objective algorithm. This method significantly speeds up the packing process as a whole. According to the present approach, the candidate number of hoppers to be combined can be taken one or two integral values. The probability that the accurate number of hoppers to be combined becomes one integral value is explicitly calculated, which is the performance factor to the previous one. In addition, results from the numerical experiments to show the effectiveness of the proposed approach are presented.

Keywords

Double Layer, Multi-Head Weigher, Packaging Process, Optimization, Single Objective Problem

1. Introduction

The multi-head weigher (also known as combination weigher) is a weighing machine which is used to provide the accurate weight for packing food products such as confectionery, biscuits, nuts, snack foods, fresh and frozen foods, rice, pasta pieces dried fruits etc [6, 14]. The multi-head weigher was first invented and developed in the 1970s. Ever since its appearance, this machine launched into the food industry across the world. Nowadays this kind of machines, thanks to their high speed and accuracy, have widely spread in the packaging industry and are produced worldwide by a number of manufacturers. In spite of widespread use of multi-head weighers, vary few studies are known.

The double-layered multi-head weigher is the one whose hoppers are arranged in two levels. It is composed of a set of $n-$weighing hoppers, a set of $n-$booster hoppers and a

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discharge chute to the packaging machine (see Figure 1 and Figure 2). Each weighing hopper has its own highly accurate load cell. This load cell will calculate the weight of product in the weighing hopper. The booster hoppers are nothing but the ordinary hoppers without load cell. They are placed underneath of weighing hoppers uprightly or diagonally based on the constructional feature of machine, according to which distinguish the double-layered multi-head weighers upright from diagonal. The packing process begins when the food product is fed into the top of the multi-head weigher, where a dispersal system, normally a vibrating or rotating top cone, distributes the product into a series of linear or radial feeder plates. This top cone is normally equipped with a load cell which controls the feed of product to the multi-head weigher. The linear or radial feeder plates vibrate individually and deliver the food product into the set of $n-$weighing hoppers. The set of weighing hoppers send the food product into the set of $n-$booster hoppers. After each delivery to the weighing and booster hoppers, the linear or radial feeders will stop vibrating and wait until the weighing hoppers have emptied their contents into the booster hoppers before starting again.

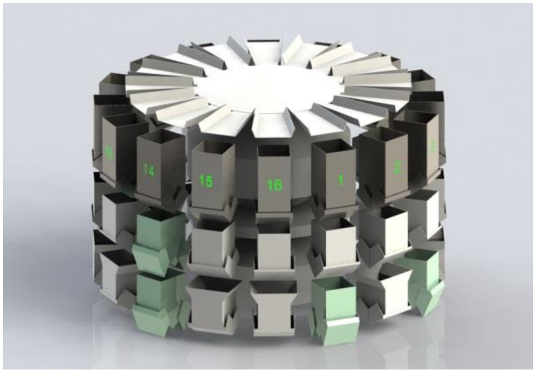

Figure 1. *Upright type double-layered multi-head weigher.*

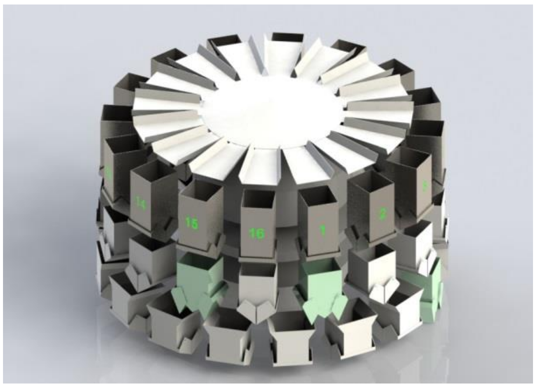

Figure 2. *Diagonal type double-layered multi-head weigher.*

The built-in computer in the multi-head weigher will then calculate the best combination of available weights from the set of $n-$weighing hoppers and $n-$booster hoppers to achieve the desired target weight T, and chooses some hoppers from the total $2n$ hoppers for a package. Once the calculation is completed, it will open the combined hoppers (hereafter, denote by H' this subset, i.e. $H' \subset H$) and discharge the accurately weighed portion into the packing system or product trays. The resulting empty weighing hoppers (i.e. $H - H'$) are supplied with next new contents of product, and such a packing operation is repeated continuously to produce a large number of food packages one by one.

The double-layered multi-head weighing process are mathematically modeled into a single-objective optimization problem. The objective of packing problem is to minimize the total weight of combined contents of product for a package under the condition that the total weight must be no less than a specified target weight T. It is a common knowledge that the number of possible different hopper subsets to be combined is different at each packing operation. This optimization problem that minimize the difference between the combined and the target package weight is known as the NP-complete subset-sum combinatorial one [11].

Let us briefly summarize the previous research in the field of multi-head weighing process. Much efforts have been devoted to the study of single-layered multi-head weighing process (without booster layer). Barreiro et al. [1] introduced the percentage variability reduction index for the reduction and control of production process variability. Keraita and Kim [18] investigated the optimum scheme for the determination of the operation time of line feeders in automatic combination weighers. Keraita and Kim [19] developed a weighing algorithm for multi-head weighers based on bit operation. Karuno et al. [15] introduced a second objective called ``priority", and formulated the weighing process as a bi-objective optimization problem, where they proposed an lexicographic dynamic programming pseudo polynomial time algorithm. The proposed approach is primarily aimed to minimize the difference between the target weight and the combined weight for a package, and then maximized the duration time of food contents in the weigher. Karuno, Nagamochi and Wang [16] applied the pseudo-polynomial dynamic programming algorithm to the double-layered lexicographic bi-criteria combinatorial optimization problems. Imahori et al. [12, 13] and Karuno et al. [17] have studied the possibility of improving the bi-objective optimization model proposed by Karuno et al. [19], and investigated different types of actual packing operations.

Recently, some researchers presented the statistical model for the multi-head weighing process. Based on a study of real data, Beretta and Semeraro [2], Beretta et al. [3] and del Castillo et al. [5] have noted that the weights of food contents thrown into the multi-head weigher are normally distributed, i.e.

$$\{w_1, w_2, \cdots, w_n\} \sim N(\mu, \sigma),$$

where $w_1, w_2, \cdots, w_n$ are the real weights of food contents feeded in the weighing hoppers and $N(\mu,\sigma)$ denotes the normal distribution whose mean is μ and standard deviation is σ. In the sequel, this paper uses the cumulative distribution function F of normal distribution defined by

$$F(a)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{a}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}dx.$$

Pulido-Rojano, Garcia-Diaz and Giner-Bosch [22] proposed a new effective bi-objective optimization approach assuming that all the weights in the hoppers are independently filled according to the same distribution $N(\mu,\sigma)$. Since all the weights are independently and identically distributed, the total weight of randomly selected k hoppers follows a normal distribution $N(k\mu,\sqrt{k}\sigma)$. Thus, One might expect that the average package mean weight $k\mu$ equals to the target weight T, that is, the number of hoppers k to be combined at each packing operation is constant and fixed in advance, where k is determined by $k\approx\frac{T}{\mu}$. One might call such k hopper combination are valid hopper combination. Afterawards [7, 8, 23, 24] designed a set of algorithms and proposed a different filling strategy of the hoppers aimed to reducing process variability.

Recently, Garcia-Diaz et al. [9] proposed a new single objective optimization algorithm for the double-layered multi-head weighing process, where the number of hoppers k to be combined has previously been defined. However, there is an important problem for choosing k in the industrial setting. Although the weights of food product in the hoppers follow the normal distribution, the average package mean μ of food product is not the same for all the types of food products. Morever, $\frac{T}{\mu}$ is not always integer in general. For example, if one packs some food product whose target weight T is 300 and average package mean μ is 54, then the candidate number k of hoppers to be combined becomes $\frac{300}{54}\approx 5.55$. In such case, it is natural to choose $k=5$ and $k=6$. In other words, one must find the optimal weight among both 5 and 6 hopper combinations. It will reduce the computational time if it is proved that only 5 hopper or only 6 hopper combinations are valid. The computational cost of generating and evaluating all the valid hopper combinations is closely related to the accurate determination of the number of hoppers to be combined from the candidate numbers. This problem becomes more difficult when the entire number of hoppers are large, especilly when the double-layered multihead weighing process work.

To address this problem, this paper proposes a new method for choosing the accurate number of hoppers to be combined, and solves the packaging problem through a novel single objective algorithm for double-layered weighers. More precisely, relying on a new bound on the optimal weight of weighing process, this paper proves a necessary condition that the optimal weight satisfies, and determine the accurate number of hoppers to be combined. The probability that the accurate number of hoppers to be combined becomes one integer instead of two values is explicitly calculated. In addition, results from the numerical experiments to show the effectiveness of the proposed approach are presented.

This study significantly improves the optimization approach of the research of Garcia-Jiménez, R et al [9], and can be applied to the single objective optimization problem.

Last but not least, Garcia-Diaz et al. [10] also treated bicriteria food packaging process optimization problems in double-layered upright and diagonal multihead weighers. Nurcahyadi et al. [21] applied Ant Colony Optimization algorithm to find the solution in the single-layered multi-head weighing process.

The rest of paper is organized as follows. Section 2 discusses the previous works on the double-layered multi-head weighing process. Section 3 and 4 propose the single objective optimization method based on the accurate determination of valid hopper combinations. Section 5 and 6 show the numerical result and conclusion of the paper.

2. Double-Layered Multi-Head Weighing Process and Single Objective Optimization Problems

The following notations are used throughout this paper.

H : Set of total $2n$ hoppers.

Q : The total number of packages needed.

ℓ : Current iteration number of packing operation.

$w_{1\ell},w_{2\ell},\cdots,w_{n\ell}$: The weights of food product in the weighing hoppers at ℓ-th packing operation.

$b_{1\ell},b_{2\ell},\cdots,b_{n\ell}$: The weights of food product in the booster hoppers at ℓ-th packing operation.

H_ℓ : Set of all hopper combinations at ℓ-th packing operation.

W_ℓ : The total weight calculated as the sum of the weight in H_ℓ hoppers at ℓ-th packing operation.

T : Target weight.

A weighing hopper sends its food product to the following booster, whenever the booster becomes empty. The booster receives it as the next content. Then, the weighing hopper is filled with a new food product since it becomes empty after sending its content to the following booster. If a weighing hopper also becomes empty when the following booster becomes empty (such a situation occurs in a double-layered food packing system of upright type), it sends a new content to the following booster after it is filled. According to the constructional feature of two layers, a double-layered food packing system classified into upright type if it cannot choose the food product in a weighing hopper for a package unless it chooses the current food product in the following booster, while a double-layered system is classified into diagonal type if it can choose the food content in a weighing hopper for a package no matter whether it chooses the food product in the following booster or not.

It was proved in [9] that the total number N_U of combinations for the upright type double-layered weighing process when the fixed k hoppers are combined is calculated by

$$N_U = \sum_{i=0}^{[k/2]} C_n^i C_{n-i}^{k-2i}, \tag{1}$$

where $[k/2]$ denotes the integer part of $k/2$. It was also proved in [9] that the total number N_D of combinations for the diagonal type double-layered weighing process when the fixed k hoppers are combined is computed by

$$N_D = \sum_{i=0}^{k} C_n^i C_{n-i}^{k-i}. \tag{2}$$

The double-layered multi-head weighing processes are mathematically modeled into a single objective optimization problems. This weighing process seek to find the best combination $H_\ell \subset H$ at every packing operation such that some weight W_ℓ is minimized under the condition that $W_\ell \geq T$. The $2n$ -dimensional binary vectors $(x, y) = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_n)$ is defined as follows.

$$x_i = \begin{cases} 1 & \text{if i}-\text{th weighing hopper is selected,} \\ 0 & \text{otherwise,} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if i}-\text{th booster hopper is selected,} \\ 0 & \text{otherwise,} \end{cases}$$

Then, by using the above vectors, the upright type double-layered food packing problem $Package-U$ is formulated as follows.

$$\min \leftarrow f(x, y) = \sum_{i=1}^{n} w_{i\ell} x_i + \sum_{i=1}^{n} b_{i\ell} y_i, \tag{3}$$

$$\sum_{i=1}^{n} w_{i\ell} x_i + \sum_{i=1}^{n} b_{i\ell} y_i \geq T, \tag{4}$$

$$x_i - y_i \leq 0, i = 1, \cdots, n, \tag{5}$$

$$x_i, y_i \in \{0,1\}, i = 1, \cdots, n. \tag{6}$$

The objective f of (3) aims at attaining the total weight of selected hoppers as close to the target weight T as possible, under the condition (4) (i.e., the target weight constraint). The equation (5) reflects the constructional feature of upright type that it cannot choose the i -th weighing hopper unless it chooses the i -th booster hopper. A solution (x, y) satisfying (4)-(6) is referred to as a feasible solution of the problem $Package-U$. Moreover, for the problem $Package-U$, T^* is denoted by the minimum of the total weight of a feasible solution (x, y). An optimal solution $(x, y) = (x^*, y^*)$ is defined as a feasible solution satisfying $T^* = f(x^*, y^*)$.

Similarly, the diagonal type double-layered food packing problem $Package-D$ is formulated as follows.

$$\min \leftarrow f(x, y) = \sum_{i=1}^{n} w_{i\ell} x_i + \sum_{i=1}^{n} b_{i\ell} y_i, \tag{7}$$

$$\sum_{i=1}^{n} w_{i\ell} x_i + \sum_{i=1}^{n} b_{i\ell} y_i \geq T, \tag{8}$$

$$x_i + y_i \leq 1, i = 1, \cdots, n, \tag{9}$$

$$x_i, y_i \in \{0,1\}, i = 1, \cdots, n. \tag{10}$$

The objective f of (7) aims at attaining the total weight of selected hoppers as close to the target weight T as possible, under the condition (8) (i.e., the target weight constraint). The equation (9) reflects the constructional feature of diagonal type that it cannot choose the i -th weighing hopper unless it chooses the i -th booster hopper. A solution (x, y) satisfying (8)-(10) is referred to as a feasible solution of the problem $Package-D$. Moreover, for the problem $Package-D$, T^* is denoted the minimum of the total weight of a feasible solution (x, y). An optimal solution $(x, y) = (x^*, y^*)$ is defined as a feasible solution satisfying $T^* = f(x^*, y^*)$.

An important property of optimal solution for both packaging problems is proved in the following. This can be regarded as an improvement of Lemma 3 in [16].

Theorem 1. Let (x^*, y^*) be the optimal solution that satisfies $T^* = f(x^*, y^*)$ for both problems $Package-U$ and $Package-D$. Then it holds that

$$T \leq T^* < T + \min\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\}. \tag{11}$$

Proof of Theorem 1. First, assertion for the problem $Package-U$ is proved. Since the optimal solution (x^*, y^*) is a feasible one with the target weight constraint (4), the left hand side inequality of (11) holds. By the optimality of (x^*, y^*), it holds that

$$f(x^*, y^*) - b_{k\ell} < T \tag{12}$$

for any vector $(x_k^*, y_k^*) = (0,1)$. If the converse inequality $f(x^*, y^*) - b_{k\ell} \geq T$ holds, then it implies that deleting $(x_k^*, y_k^*) = (0,1)$ from the optimal solution becomes another feasible solution, and it contradicts the optimality of (x^*, y^*). Similarly, it also holds that

$$f(x^*, y^*) - w_{k\ell} < T \tag{13}$$

for any vector $(x_k^*, y_k^*) = (1,1)$. Therefore, the right hand side inequality of (11) holds for the problem $Package-U$.

Next, the assertion for the problem $Package-D$ is proved. Since the optimal solution (x^*, y^*) is a feasible one with the target weight constraint (8), the left hand side inequality of (11) also holds for the problem $Package-D$. By the optimality of (x^*, y^*), it holds that

$$f(x^*, y^*) - w_{k\ell} < T \tag{14}$$

for any vector $(x_k^*, y_k^*) = (1,0)$. If the converse inequality $f(x^*, y^*) - b_{k\ell} \geq T$ holds, then it implies that deleting $(x_k^*, y_k^*) = (1,0)$ from the optimal solution becomes another feasible solution for the problem $Package-D$, and it contradicts the optimality of (x^*, y^*). In the similar fashion as above, it also holds that

$$f(x^*, y^*) - b_{k\ell} < T \tag{15}$$

for any vector $(x_k^*, y_k^*) = (0,1)$. Therefore, the right hand side inequality of (11) holds for the problem $Package-D$. Finally, the theorem is proved.

3. Accurate Determination of Valid Hopper Combinations

This section proposes the accurate determination method for valid hopper combination from the candidate number of hoppers, and solve the single-objective optimization problem. Assume as in [9] that all the weights in the $2n$ hoppers follow independently and identically a normal distribution

$$\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\} \sim N(\mu, \sigma), \tag{16}$$

where μ is the average package weight and σ is the standard deviation. Garcia-Diaz et al. [9] proposed an optimization approach in which the number of hoppers k to be combined at each packing operation is constant and fixed in advance, where k is determined by $k = \dfrac{T}{\mu}$ and the set of all possible k-hopper combinations becomes valid hopper combinations. As already mentiond in section 1, $k = \dfrac{T}{\mu}$ is not always integer for any μ and T. It is hard to fix μ such that $k = \dfrac{T}{\mu}$ becomes an integer in the real industrial settings.

The present paper shows a novel method for the accurate determination of hoppers to be combined. It follows from Theorem 1 that the following weight bound holds.

$$T \leq W < T + \min\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\}.$$

Thus, the accurate number of hoppers to be combined is determined by

$$\frac{T}{\mu} \leq k < \frac{T + \min\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\}}{\mu}. \tag{17}$$

This number of hoppers k is defined in detail.

Setting $k = \left[\dfrac{T}{\mu}\right]$, if

$$\left[\frac{T}{\mu}\right] = \left[\frac{T + \min\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\}}{\mu}\right]. \tag{18}$$

Otherwise, setting $k = \left[\dfrac{T}{\mu}\right]$ and $k = \left[\dfrac{T}{\mu}\right] + 1$, if

$$\left[\frac{T}{\mu}\right] + 1 = \left[\frac{T + \min\{w_{1\ell}, w_{2\ell}, \cdots, w_{n\ell}, b_{1\ell}, b_{2\ell}, \cdots, b_{n\ell}\}}{\mu}\right], \tag{19}$$

where $\left[\dfrac{T}{\mu}\right]$ denotes the integer part of $\dfrac{T}{\mu}$.

Now, the probability that the number of hoppers to be combined becomes one integral value $k = \left[\dfrac{T}{\mu}\right]$ is explicitly calculated. This probability is computed by (18) and using the property of extreme distribution [4]:

$$\begin{aligned}P\left(k=\left[\frac{T}{\mu}\right]\right) &= P\left(\left[\frac{T}{\mu}\right]=\left[\frac{T+\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}}{\mu}\right]\right)\\
&= P\left(\frac{\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}}{\mu}<1-\left\{\frac{T}{\mu}\right\}\right)\\
&= P\left(\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}<\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\\
&= 1-P\left(\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}\ge\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\\
&= 1-P\left(w_{1\ell}\ge\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\cdots P\left(b_{n\ell}\ge\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\\
&= 1-\left(1-P\left(w_{1\ell}<\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\right)\cdots\left(1-P\left(b_{n\ell}\ge\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\right)\\
&= 1-\left(1-F\left(\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)\right)\right)^{2n},\end{aligned}\tag{20}$$

where F is the cumulative distribution function of normal distribution. From (20), one concludes that this probability increases when the total number of hoppers $2n$ increases.

The paper gives some example for special values of $\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)$.

If $\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)=\mu-3\sigma$, then one has from Figure 3 that

$$P\left(\left[\frac{T}{\mu}\right]=\left[\frac{T+\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}}{\mu}\right]\right) \tag{21}$$
$$=1-\left(1-F\left(\mu-3\sigma\right)\right)^{2n}=1-\left(1-0.00135\right)^{2n}.$$

If $\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)=\mu-2\sigma$, then one has from Figure 3 that

$$P\left(\left[\frac{T}{\mu}\right]=\left[\frac{T+\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}}{\mu}\right]\right) \tag{22}$$
$$=1-\left(1-F\left(\mu-2\sigma\right)\right)^{2n}=1-\left(1-0.0227\right)^{2n}.$$

If $\mu\left(1-\left\{\frac{T}{\mu}\right\}\right)=\mu-\sigma$, then one has from Figure 3 that

$$P\left(\left[\frac{T}{\mu}\right]=\left[\frac{T+\min\{w_{1\ell},w_{2\ell},\cdots,w_{n\ell},b_{1\ell},b_{2\ell},\cdots,b_{n\ell}\}}{\mu}\right]\right) \tag{23}$$
$$=1-\left(1-F\left(\mu-\sigma\right)\right)^{2n}=1-\left(1-0.1587\right)^{2n}.$$

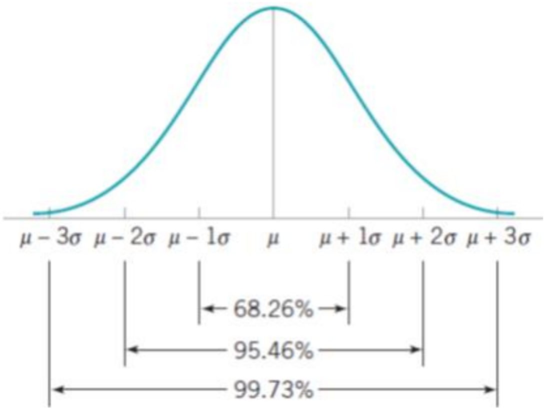

Figure 3. *Normal distribution [20].*

4. Single objective optimization approach

This section gives an improved single objective algorithm.

Input

$2n$ -Total number of hoppers,

k -Number of hoppers involved in each packing operation ($2\le k\le n$),

T -Target weight ($T>0$),

Q -Total number of packages to be produced,

ℓ -Iteration number of packing ($1\le\ell\le Q$),

μ -Average package weight of food product,

σ -Standard deviation of the weights supplied to each hopper.

Step 1. Initialize the all data.

Set $\ell=0$, and $w_{1\ell}=\cdots=w_{n\ell}, b_{1\ell}=\cdots=b_{n\ell}=0$ for all

$1 \le i \le n$.

Step 2. New packaging operation.

Set $H_\ell{}' = \emptyset, H_{\min}^{\ell'} = \emptyset, k = 0$.

Step 3. Fill all empty hoppers. For all the weighing and booster hoppers, the random values chosen from the normal distribution $N(\mu, \sigma)$ is assigned.

Step 4. Determine the accurate number k of hoppers to be combined as follows.

Set $k = \left[\frac{T}{\mu}\right]$, if (18) holds. Otherwise, set $k = \left[\frac{T}{\mu}\right], \left[\frac{T}{\mu}\right] + 1$

Step 5. Combine all valid combinations.

The hoppers that satisfies the (5) or (9) are combined for the upright or diagonal machines, respectively.

Step 6. Evaluate all the valid combinations.

Calculate the sum of the weights for each combination of all the k hopper combinations. If sum weight is less than the target weight T, then delete this combination from H_ℓ

Step 7. Find the best possible combination among H_ℓ.

The hopper combination whose difference with T is minimal is selected.

Step 8. Discharge and pack the food product.

The food product from the best hopper combination is discharged and packed.

Step 9. Update the number of packages produced and check whether the process is complete. If the required number of Q packets has been completed, the process ends. Otherwise, it returns to Step 3.

5. Experimental Results

This section presents the experimental result. These results show that the present new approach is most effective in the double-layered multihead weighing process. Application's user interface for the double-layered upright and diagonal machine are presented in Figure 4 and Figure 5.

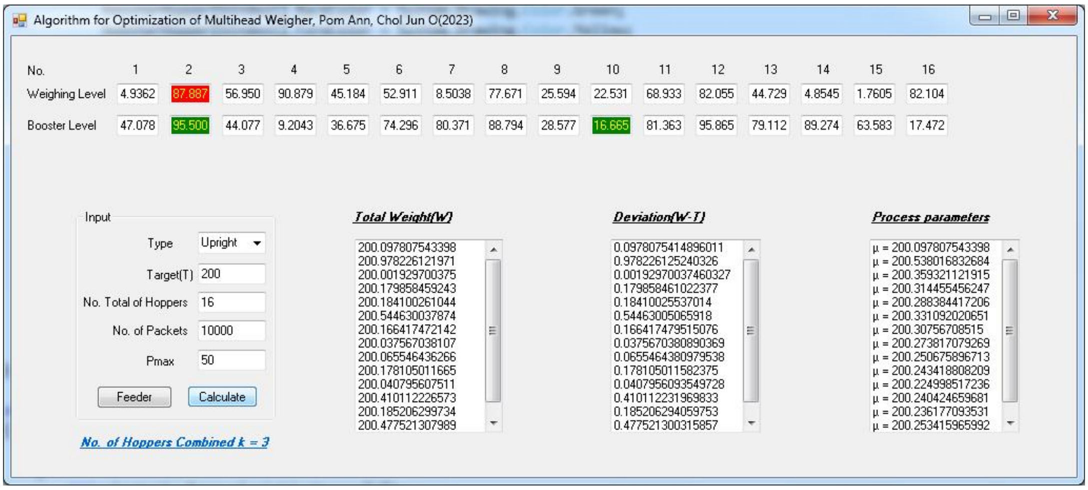

Figure 4. Experimental result for Upright type.

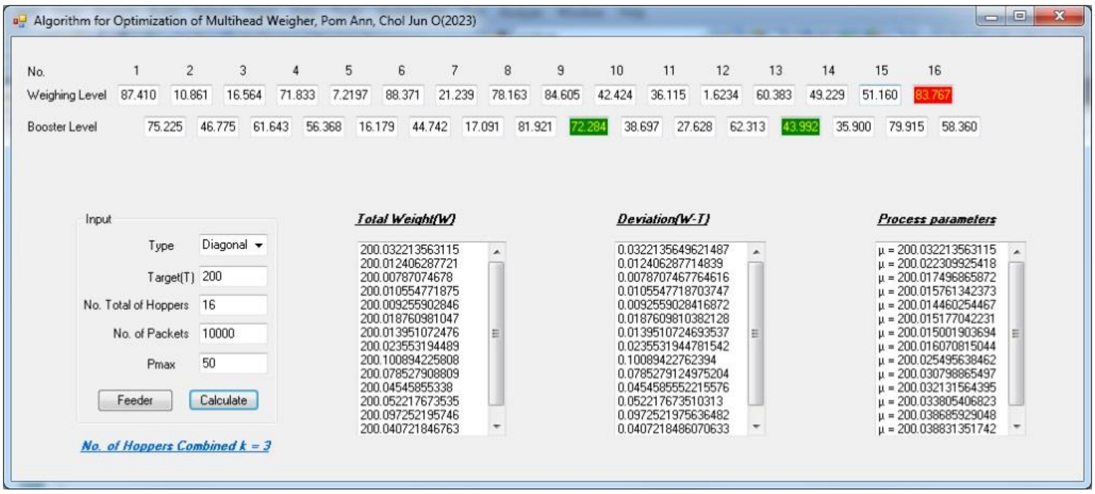

Figure 5. Experimental result for Diagonal type.

In the experiment, taking the total number of hoppers $n=16$, the total weight $T=200$, the average package mean $\mu=58.5$ and the total number of packages to be produced $Q=10000$. Then the candidate number of valid hopper k is calculated by $k=\frac{200}{58.5}=3.(418803)$.

Therefore, the valid number of hoppers to be combined is as follows.

$$k=\left[\frac{200}{58.5}\right]=3, k=\left[\frac{200}{58.5}\right]+1=4.$$

According to the previous method, one must find the optimal value from all the 3-hopper combinations and 4-hopper combinations at every packing, i.e. the total number of combinations is computed by (1) and (2), respectively.

In contrast to the previous one, the present method decreases this total number of combinations significantly because (18) is satisfied at some packing operations. Thus, the probability that the valid number of hoppers to be combined becomes one integer is calculated in terms of (20) as follows.

$$P(k=3)=1-\left(1-F\left(58.5\left(1-\left\{\frac{200}{58.5}\right\}\right)\right)\right)^{32}>0, \tag{24}$$

which represents the performance factor of the present method to the approach in [9].

6. Conclusion

As stated above, the present paper proposed an improved single objective optimization approach for double-layered multi-head weighing process. The innovative finding is an accurate determination method for the number of hoppers to be combined at each packing operation relying on a new bound on the optimal weight This method significantly speeds up the packing process as a whole. According to the present approach, the candidate number of hoppers to be combined can be taken one or two integral values. The probability that the accurate number of hoppers to be combined becomes one integral value is explicitly calculated, which is the performance factor to the previous one. The experimental result reveals the effectiveness of the proposed approach.

Abbreviations

min	Minimize

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Author Contributions

All authors contributed to this work in collaboration. Pom An, Chol-Jun Hong and Ryong-Yon Ri designed the study, conceptual approach and carried out the experiment. Chol-Jun Yu made the comparison between the theoretical and experimental results and analyzed the data. Chol-Jun O proposed the idea and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Barreiro, J. J., González, C., Salicrú, M., Optimization of multiweighing packing proceeding, Top, 1998, 6 (1), 37-44. https://doi.org/10.1007/BF02564797

[2] Beretta, A. and Semeraro, Q. On a RSM approach to the multihead weigher configuration, in the 11th biennial Conference on Engineering Systems Design and Analysis (ASME 2012), Nantes, France, 225-233.

[3] Beretta, A., Sraro, Q., Del Castillo, E., On the Multihead Weigher Machine Setup Problem, Packag. Technol. Sci., 2016, 29, 175-188.

[4] Dekking, F. M., Kraaikamp, C., Lopuhaä, H. P., Meester, L. E., A Modern Introduction to Probability and Statistics, Springer, Berlin, 2005.

[5] Del Castillo, E., Beretta, A., Semeraro, Q., Optimal setup of a multihead weighing machine, Eur. J. Oper. Res., 2017, 259, 384-393.

[6] Erdogdu, F., Optimization in Food Engineering, Taylor & Francis, London, 2009.

[7] Garcia-Diaz, J. C., Pulido-Rojano, A., Giner-Bosch, V., Bi-objective optimisation of a multihead weighing process, Eur. J. Ind. Eng., 2017, 11, 403-423.

[8] Garica-Diaz, J. C., Pulido-Rojano, A., Monitoring and control of the multihead weighing process through a modified control chart, DYNA, 2017, 84, 135-142.

[9] Garcia-Jiménez, R., Garica-Diaz, J. C., Pulido-Rojano, A. D., Packaging Process Optimization in Multihead Weighers with Double-Layered Upright and Diagonal Systems, Mathematics, 2021, 9, 1039, 1-20. https://doi.org/10.3390/math9091039

[10] Garcia-Jiménez, R., Garica-Diaz, J. C., Pulido-Rojano, A. D., Bicriteria food packaging process optimization in double-layered upright and diagonal multihead weighers, J. Comput. Appl. Math., 2023, 428, 115168. https://doi.org/10.1016/j.cam2023.115168

[11] Garey, M. R. and Johnson, D. S., Computers and Intractability: A guide to the theory of NP-completeness, WH Freeman & Company, 1979.

[12] Imahori, S., Karuno, Y., Nagamochi, H., Wang, X., Kansei engineering, humans and computers: Efficient dynamic programming algorithms for combinatorial food packing problems, Int. J. Biomet., 2011, 3, 228-245. https://doi.org/10.1504/IJBM.2011.040817

[13] Imahori, S., Karuno, Y., Tateishi, K., Dynamic programming algorithms for producing food mixture packages by automatic combination weighers, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 8, No. 5, (2014), https://doi.org/10.1299/jamdsm.2014jamdsm0065

[14] Ishida Co., Ltd., Products (Total System Solutions), Weighing and Packaging, 2015, Available from: http://www.ishida.com/products/

[15] Karuno, Y., Nagamochi, H., Wang, X., Bi-criteria food packing by dynamic programming, J. Oper. Res. Soc. Jpn., 2007, 50, 376-389.

[16] Karuno, Y., Nagamochi, H., Wang, X., Optimization Problems and Algorithms in Double-layered Food Packing Systems, J. Adv. Mech. Des. Syst. Manuf., 2010, 4, 605-615. https://doi.org/10.1299/jamdsm.4.605

[17] Karuno, Y., Takahashi, K., Yamada, A., Dynamic Programming Algorithms with Data Rounding for Combinatorial Food Packing Problems, J. Adv. Mech. Des. Syst. Manuf., 2013, 7, 233-243. https://doi.org/10.1299/jamdsm.7.233

[18] Keraita, J. N., Kim, K. H., A study on the optimum scheme for determination of operation time of line feeders in automatic combination weighers, J. Mech. Sci. Technol., 2006, 20, 1567-1575. https://doi.org/10.1007/BF02916261

[19] Keraita, J. N., Kim, K. H., A Weighing Algorithm for Multihead Weighers, Int. J. Precis. Eng. Manuf., 2007, 8, 21-26.

[20] Montgomery, D. C., Introduction to Statistical Quality Control, 6th ed., John Wiley & Sons, 2009.

[21] Nurcahyadi, T., Blum, C., Ant colony optimization for packing process optimization in multi-head weigher machines, Aip conf. Proc., 2023, 2837, 02001. https://doi.org/10.10635/50150450

[22] Pulido-Rojano, A., Garcia-Diaz, J. C., Giner-Bosch, V., A multiobjective approach for optimization of the multihead weighing process, Proceedings of the International Conference on Industrial Engineering and Systems Management, Seville, Spain, 2015, 426-434. https://doi.org/10.1109/IESM.2015.7380192

[23] Pulido-Rojano, A. and Garcia-Daz, J. C.. Analysis of the filling setting in the multihead weighing process, in Proceedings of the International Joint Conference, CIO-ICIEOM-IIE-AIM (IJC 2016), San Sebastián, Spain, 2016, 521-528.

[24] Pulido-Rojano, A., Garcia-Daz, J. C., Optimisation algorithms for improvement of a multihead weighing process, Int. J. Prod. Qual. Manag., 2020, 29, 109-125.