Journal of Plant Sciences
2024, Vol. 12, No. 5, pp. 138-145
https://doi.org/10.11648/j.jps.20241205.12

Methodology Article

A Mathematical Model for Estimating the Area of a Large Leaf

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Abstract

A mathematical model for estimating the area of large leaves was developed and validated with the leaves of plantain (*Musa paradisiaca*). The length of each plantain leaf was characterized by the mid-rib length which was divided into a certain number of equal parts of length. By this divisions, two major geometrical shapes were obtained, viz; trapezoidal shapes bounded by half elliptical shapes at the two ends of the leaf. In this work, the mathematical expression for the surface area of a plantain leaf was obtained by summing the areas of the trapeziums and the two half ellipses. The values of leaf areas as generated from this model for various seizes of plantain leaves converged reasonably well with true values obtained by the weighing method of same plantain leaf seizes. The correlation between these two values was carried out to ascertain the fitness of the mathematical model as developed here showed a linear relationship for the various mid-rib divisions tested in this study. It is thus concluded here that once the calculated area of the large leaf is known for a specific number of divisions with this model, the true area can be estimated. The main advantages of the new method are precision, accuracy, and applicability to determine the area of large leaves far out in the field where electrical weighing balance and large graph papers are not available.

Keywords

Large Leaf, Leaf Area, Leaf Shape, Trapezium, Ellipse

1. Introduction

Study of plant processes, such as photosynthesis and evapo-transpiration [1], light interception, water and nutrient use, crop growth [2], yield potential [3, 4] and the effect of different plant treatments [5], provide the basis for plant improvement and is tied to the leaf area. Leaf area is also used to determine the variability of these parameters with the weather variation. It is therefore essential to develop a good technique for determining the leaf true area.

The destructive and non-destructive methods are the existing methods for now, that are consistently used to determine leaf area. The destructive method which involves plucking the leaf from the plant and determining its true area by any of the following methods. The graphical method where the leaf is placed flat on a graph paper and its blade is carefully traced on it, and then the total number of "1mm^2" grids covered by the leaf inside the profile is counted, this account for the true area of the leaf [6-8]. The weighing method, the leaf surface is photocopied on a paper of known mass per unit area and the part of the paper containing the leaf template is carefully cut out along the leaf blade and weighed, making it possible to calculate the leaf area [9] The punched disc [10], the automatic planimeters based on light interception [11] or

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Received: 23 April 2024; **Accepted:** 27 August 2024; **Published:** 18 October 2024

air flow [12] and the leaf area meter methods are based on direct measurement of the leaf area using meters used for this purpose [13]. These methods are very convenient for leaves of small sizes, but have their limitations for large leaves, leaves of limited availability and for leaves where measurements have to be repeated on the same leaves' samples of a growing plant.

The non-destructive method mostly involves using the data of the area of leaves of various sizes from plants of the same species, obtained by the destructive method, to develop a mathematical model which correlates the leaf true area with one or a combination of the biometric properties of the leaf, such as the leaf length, the leaf maximum width or their combinations [14-21]. If one knows the leaf length for example, one can use the model to estimate the true area of the leaf.

A large number of mathematical models have been developed by researchers to estimate the leaf area of different crop species and varieties in different environments, [22]. A mathematical model is very useful when interest is on repeating measurements of the area of the same leaf sample during plant growth, [23], or when focus is on estimating the area of a leaf in the field where it is difficult to make measurements by destructive methods and especially where electric power supply is not available to operate measuring equipment.

Most models developed for measuring the area of leaves are for leaves of small sizes, as it is very easy to measure their true leaf areas using leaf area meters or other known destructive methods to build a model.

The mathematical models for estimating the area of large leaves are very scarce in the literature as it is very tedious to make the initial measurements of the true areas of some of the leaf samples to develop the model, [24]. The objective of this work therefore is to develop a mathematical technique that builds up the surface area of the leaf from a number of simple geometrical shapes. The large leaf used in this study is the plantain leaf. The geometrical shapes obtained from methods used in this study are: trapeziums in the middle part of the leaf and halves of ellipses at the end parts. These mathematical shapes are proposed for building up the area of the leaf.

2. Materials and Methods

Between January 2022 and December, 2023, seventy-five (75) leaves were randomly sampled from both old and young plantain plants of the same cultivar, from the plantain farm in Ugbe Akoko, Ondo State, Nigeria. Each leaf was placed on a large sheet of brown paper, spread on a horizontal surface and its boarder edge was carefully traced with an HB pencil A line was drawn to mark the position of the midrib on the paper. The length of each leaf was measured and divided into 16 equal parts. The 15 points dividing the length of the leaf into 16 portions were marked, and a large set-square was used to draw perpendicular lines to the midrib through these marked points (see figure 1). The distance between the opposite edges of the leaf blade at the marked points were measured as the width of the leaf at those points. These widths are the lengths of the bases of the trapeziums at those points and are respectively noted as W_1, W_2... W_{15}, captioned as W_i where i = 1, 2, 3, n-1.

The true area A_t of each leaf was estimated from $A_t = \left[\frac{A_p}{W_p}\right] x\ W_t$, where A_p is the area of the large paper of weight W_p and W_t is the weight of the same paper cut to the size of the leaf template, [10]. The calculated leaf area A_n is obtained from a mathematical model equation proposed in this study when the leaf length is divided into n portions.

The relationship between the true leaf area A_t and the calculated leaf area A_n was evaluated by plotting the graph of A_t against A_n, using the linear regression procedure of SPSS (SPSS Inc. Chicago IL) graph facility.

3. The Mathematical Model Equation for Leaf Area

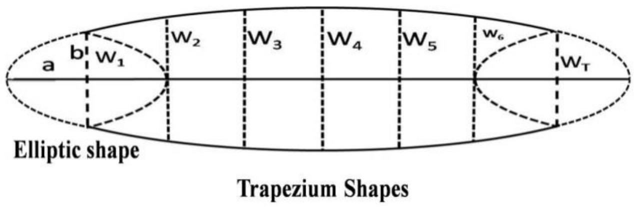

Figure 1. *Sketch of idealized Plantain Leaf with its length L divided into 8 equal parts.*

In this study, the plantain leaf was idealized into an ellipse. The length, L which is the length of the leaf mid rib of the leaf was divided into n equal portions of width h_n = L/n along the mid-rib. The end portions of the leaf were approximated to two half ellipses of semi major axes of length a = h_n and respective semi-minor axes, $b_1 = \frac{1}{2}W_1$ and $b_{(n-1)} = \frac{1}{2}W_{(n-1)}$, at points i = 1 and i = n-1. The n-2 trapeziums between the first end half ellipse and the second end half ellipse each has a height h_n .

If the area of an ellipse is πab, with a and b respectively being the lengths of the semi-major and semi-minor axes, the sum of the above two half ellipses can be written as $\frac{1}{2}\pi a\ (b_1 + b_2)$. Since $\frac{1}{2}\pi = 1+\frac{4}{7}$, $b_1 = \frac{1}{2}W_1$ and $b_2 = b_{(n-1)} = \frac{1}{2}W_{(n-1)}$, the expression for the area of the two half ellipses is $[(\frac{1}{2}\ h_n$

$W_1 + \frac{2}{7} h_n W_1) + (\frac{1}{2} h_n W_{(n-1)} + \frac{2}{7} h_n W_{(n-1)})]$. The expression for the area of all the trapeziums is also given as $[\frac{1}{2} h_n (W_1 + W_{(n-1)}) + h_n \sum_{2}^{n-2} Wi]$, where i = 1, 2, 3, …., n-2, from the first trapezium to the last one.

The calculated area A_n of the leaf is thus obtained by summing up all the areas of the trapeziums and the areas of the two end half ellipses. This summation is equal to:

$$A_n = h_n \sum_{i=1}^{n-1} Wi + \frac{2}{7} h_n \left[W_1 + W_{(n-1)}\right], \text{ i = 1, 2, 3… n-1} \quad (1)$$

The following expressions for A_n are obtained from equation (1) when h_n = L/n for the respective values of n = 2, 4, 6, 8.

Thus for n, An = $\frac{L}{n}$ $[W_1 + W_2 + W_3 + ... + W_{(n-1)}] + \frac{2L}{7n} [W_1 + W_{(n-1)}]$,.

$$n = 2, A_2 = \frac{L}{2} [W_1] + \frac{2L}{7x2} [W_1 + W_1] = 0.7857 W_1 \quad (2)$$

$$n = 4, A_4 = \frac{L}{4} [W_1 + W_2 + W_3] + \frac{2L}{7x4} [W_1 + W_3] \quad (3)$$

$$n = 8, A_8 = \frac{L}{8} [W_1 + W_2 + W_3 + ... + W_7] + \frac{2L}{7x8} [W_1 + W_7] \quad (4)$$

$$n= 16, A_{16} = \frac{L}{16} [W_1 + W_2 + W_3 + ... + W_{15}] + \frac{2L}{7x16} [W_1 + W_{15}] \quad (5)$$

Equations 2, 3, 4 and 5 are used to calculate the area of each leaf when the leaf length is divided respectively into 2, 4, 8 and 16 portions.

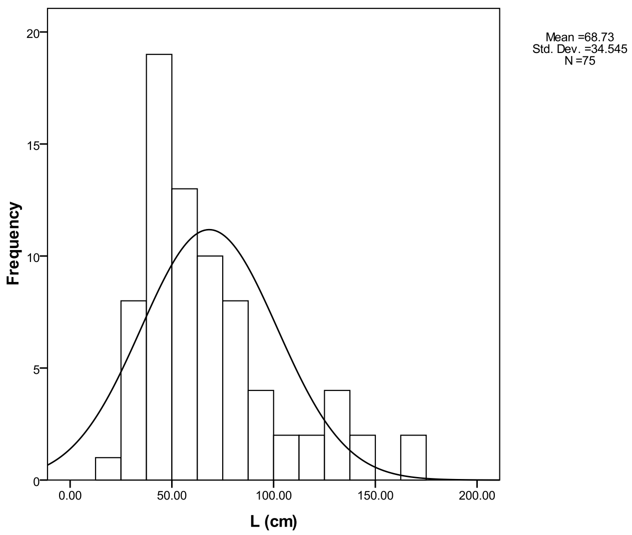

Figure 2. *Frequency distribution of the length, L (cm) of plantain leaves.*

4. Results

4.1. Range of Leaf Lengths

The distribution of the lengths of the plantain leaves used for this report is given by the normal curve plotted on the histogram of the frequency against the leaf lengths shown in figure 2. The figure shows that the data of the leaf lengths is roughly normally distributed.

4.2. Relation Between the True Leaf Area and the Calculated Leaf Area

Table 1 contains the regression equations when the true leaf area (At) is plotted against the respective calculated leaf area (An), for the respective values of n= 2, 4, 8, 16 where n is the number of equal portions into which the length of the mid-rib of the leaf is divided. The table also contains the values of the regression equation coefficients a and b, the goodness of fit r^2 and the root mean square error (RMSE) of their scatter about their best straight lines. The plot of the calculated leaf area An against the true leaf area At for the respective values of n= 2, 4, 8, 16, is shown in Figure 3.

***Table 1**. The regression relation between true leaf area A_t and calculated areas A_n for the respective values of n = 2, 4, 8, 16.*

Regression Equation	Regression Parameters			
	a	*b*	r^2	RMSE
$A_t = a + bA_2$	-101.48±16..31	1.056±0.006	0.998	104.9
$A_t = a + bA_4$	-43.87±14.24	1.005±0.005	0.998	92.9
$A_t = a + bA_8$	9.53±12.78	1.003±0.005	0.998	84.0
$A_t = a + bA_{16}$	0.36±13.46	0.999±0.005	0.998	88.7

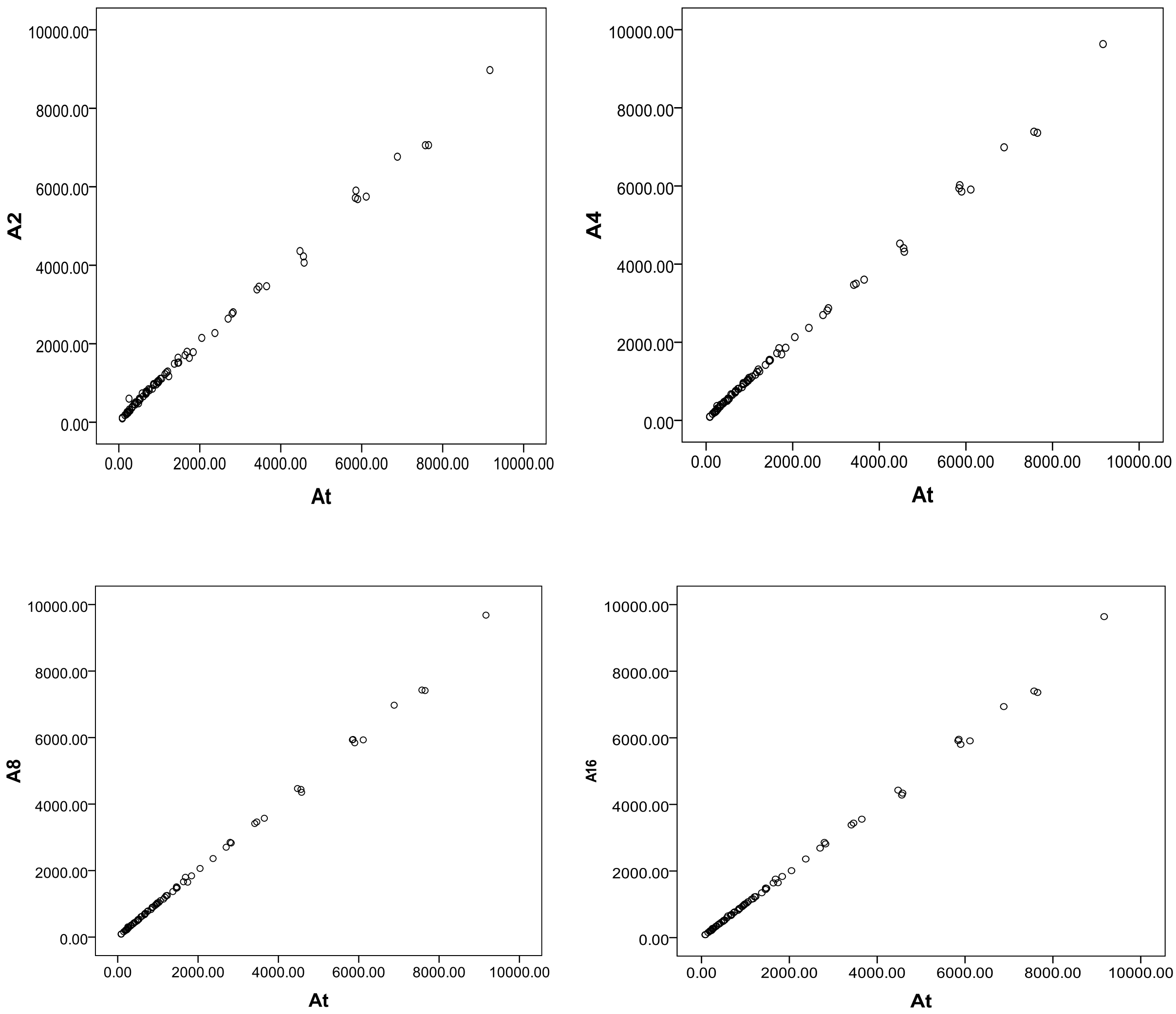

***Figure 3**. Linear relationship between and the predicted leaf area A_n ., for n = 2, 4, 8, 16 respectively.*

4.3. Percentage Difference (PD) Between the Corresponding Calculated and the True Areas of a Leaf

Table 2 contains the values of the calculated areas A_2, A_4, A_8 and A_{16} for the same leaf when the leaf length was respectively divided into n = 2, 4, 8, 16 parts for some randomly selected leaves of plantain with lengths ranging between 29 and 170 cm. The true leaf areas (A_t) and the % differences (PD) between the corresponding pair of the true area and the calculated area are also shown in the table. The plot of the percentage differences (PD) between the true and the calculated areas for the respective values of n against the true leaf area At is shown in Figure 4.

Table 2. The percentage difference (PD) between the corresponding true leaf area A_t and the calculated leaf areas, A_n .

leaf length	Calculated Area A_n (cm^2)				True Area (cm2)	PD (% difference from A_t) (cm^2)			
L (cm)	A_2	A_4	A_8	A_{16}	A_t	A_2	A_4	A_8	A_{16}
29.3	96.69	92.92	87.95	86.41	85.58	12.99	8.59	2.78	0.97
31.0	199.73	203.05	191.87	187.16	185.51	7.67	9.46	3.43	0.89
35.6	464.33	421.22	390.33	387.98	377.29	23.07	11.64	3.46	2.85
41.5	720.62	710.84	690.53	681.82	665.99	8.20	6.73	3.68	2.38
50.0	754.29	751.07	713.48	695.04	685.84	9.98	9.51	4.03	1.34
63.0	841.50	806.40	667.47	750.32	734.82	14.52	9.74	4.44	2.11
90.0	1640.56	1690.71	1651.18	1648.61	1741.04	-5.77	-2.89	-5.16	-5.31
169.2	8973.64	9631.71	9681.56	9639.57	9166.78	-2.11	5.07	5.62	5.16

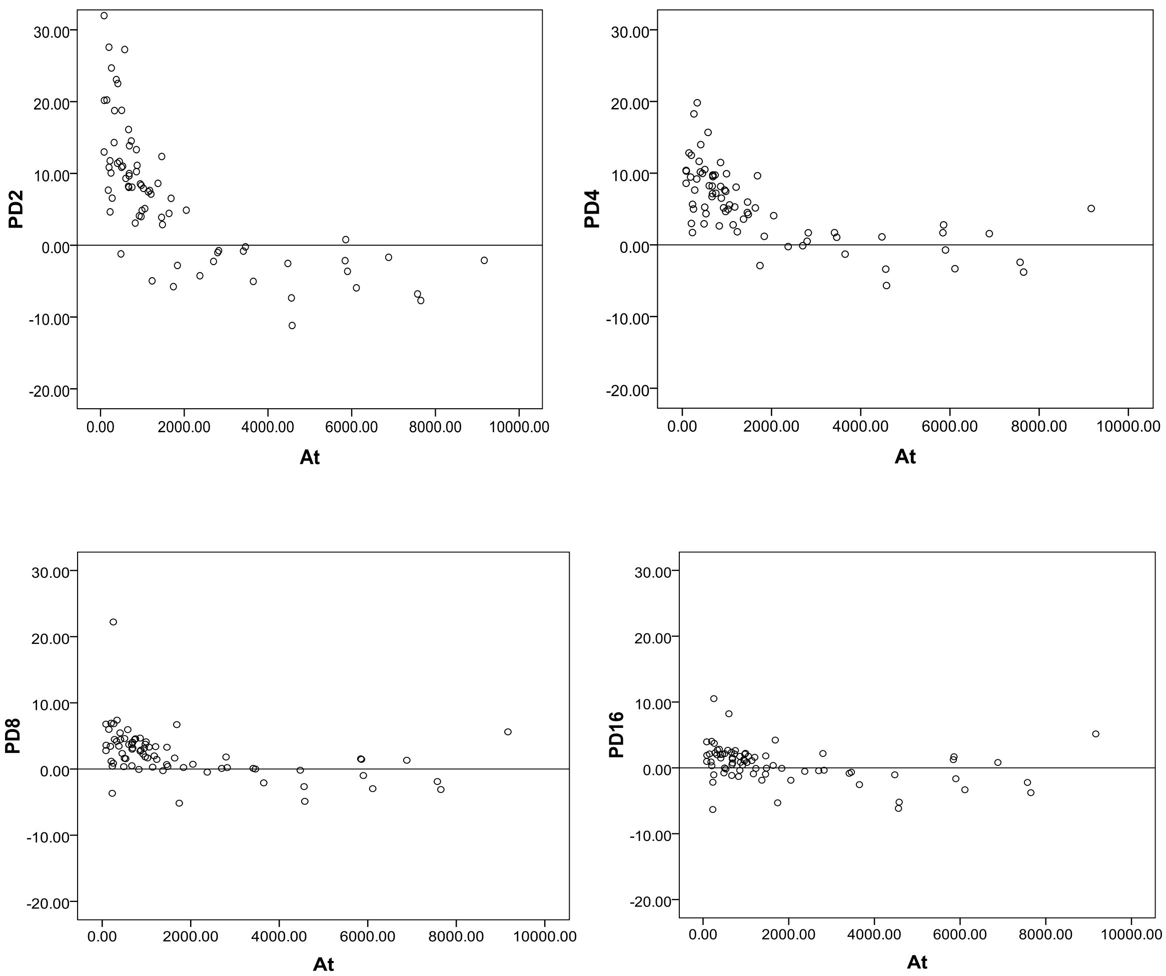

Figure 4. The % differences (PD) between the true area A_t and the calculated areas A_n for the respective values of n = 2, 4, 8, 16.

5. Discussion

Leaf area is important as it is related to plant development and growth. It is therefore necessary that any model equation advanced for estimating its value should predict it to a high level of accuracy, such that the results obtained from the model equation will compare favourably well with results obtained other standard methods.

In this study, the calculated area A_n from the mathematical equation was compared with the true area A_t of the plantain leaves obtained by weighing method [9]. The area of each of the seventy-five leaves used in this study, was estimated using the mathematical model equations (2) to (5) in turn when the mid-rib of each leaf was divided into 2, 4, 8 and 16 portions respectively. The linear relationship between true leaf area A_t and the respective calculated areas $A_{2,}$ A_4, A_8 and A_{16} is shown in figure 3. This figure shows that there is a high degree of correlation between the values of the calculated area and those of the true area of the leaves (Table 1). The correlation between A_n and A_t was further observed by calculating the percentage difference between the calculated area and the true area of the leaf.

It is also observed in Table 2 that the respective calculated areas $A_{2,}$ A_4, $A_{8,}$ A_{16} of the leaf reduces in value towards the true leaf area $A_{t,,}$ i.e. as n increases from 2 to 16. As A_n decreases towards A_t, the % difference (PD) between the true area and the calculated area reduces to the minimum value. This shows that the present method can predict the area of a large leaf reasonably well when the number of potions into which the leaf length is divided is large.

The Table 2 shows that the % difference generally reduces as the number of portions into which the leaf is divided increases, and to within about 5% when the number of divisions of the leaf is 8 or above. The relevance of this observation is shown in figure 4 which shows the plot of PD from the true area against the true leaf area A_t for the respective calculated leaf areas $A_{2,}$ A_4, A_8 and A_{16} for leaves of different sizes. For each graph, the PD decreases toward zero as the true area A_t of the leaf increases; a pointer to the fact that the calculated area of the leaf tends to be equal to its true area as the leaf becomes larger.

The closeness of the calculated leaf area (An) to the true area (At) was further ascertained by the coefficient of determination (r^2) and the root mean square error (RMSE). A high value of r^2 shows that there is a high correlation between the two variables, and a low value of r^2 shows a poor correlation; as it is the r^2 that gives the percentage of the true area that is related by the linear dimension. Also a large value of RMSE indicates a large scatter of data around the best straight line, showing a poor fit. Also a low value of RMSE shows that the scatter of data points about the best straight line is small; showing that there is a good fit. Thus a combination of r^2 and RMSE are used in the decision process to know when the calculated area of the leaf obtained by model equation is very close to the true area of the leaf. It is therefore concluded from Table 1 that the calculated area of the leaf (An) is very close to the true area (At) when the leaf length is divided into at least eight or more portions. It is equally observed that as the number of portions into which a leaf is divided increases, the root mean-square error (RMSE) of estimate decreases even though the coefficient of determination (r^2) remains large. This fairly shows that the quality of the regression equations are good and that the calculated values of the leaf area becomes closer to the true area as the number of portions into which the leaf is divided increases.

In this study, the regression constant (a) increases from negative values -101.48 ± 16.31 when $n = 2$ and 4 through -43.87 ± 14.24, when $n = 4$, and -9.53 ± 12.78 when $n = 8$ to 0.36 when n=16. The corresponding values of slope (b) equally decreases from 1.056 ± 0.006 when $n = 2$ to 0.999 ± 0.005 when $n = 16$.

The mathematical model equations results for calculating the leaf area of plantain leaves obtained in this study and the regression equations obtained from reference [6], are put together in table 3 for comparison purposes. It may be seen in the table that when n=2, our mathematical model equation results reasonably agrees with the regression model equation from reference [6], suggesting that the maximum width is equal to the width of the leaf at half its length. From the Tables 1 and 2, it may be seen that equation 5.0 for calculating A_t at n= 16 gives a better result of the leaf area with $r^2 = 0.998$ and RMSE = 88.7 than for lower values of n.

Table 3. *Model equations for calculation plantain and banana leaf area.*

MODELEQUATION S OF LEAF AREA			Statistics	Reference
Leaf	**Variety**	**Model Equation**	**r^2**	
Plantain	Local	$LA = \frac{L}{2}[W_1] + \frac{2L}{7x2}[W_1+W_1] = 0.7857LW_1$	0.998	(Present Equation 2, n=2)
Plantain	Local	$LA = 0.7854LW_m$	0.73	[6]

MODELEQUATION S OF LEAF AREA			Statistics	Reference
Leaf	Variety	Model Equation	r^2	
Plantain	Local	$LA = \frac{L}{16}\sum_{n=1}^{15} Wn + \frac{2L}{7x16}(W_1+W_{15})$	0.998	(Present equation 5, n=16)

W_m = maximum width of the leaf, W_1 = width of leaf at half– leaf length when n=2.

6. Conclusion

Apart from the fact that the leaf area meter, weighing machine, the grid count, gravimetric and plannimeter methods are destructive, they are laborious and time wasting especially when estimating the area of large leaves. The present study provides a less laborious way of obtaining the area of large leaves. Once the initial measurements of the areas of a number of large leaves of different sizes and of the same species are obtained, a mathematical model equation can be developed for measuring the area of other large leaves of the same species, non-destructively.

Conflicts of Interest

The authors declare no conflicts of interest.

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