

Research Article

Analytical and Numerical Resolution of Viscoelastic Upper-Convected Maxwell Fluid in Couette Flow with Thermal Effects

Messaouda Guemmadi^{1,2,*} , Faiza Brahimi^{1,3} , Ahmed Ouibrahim⁴ 

¹Department of Mechanical Engineering, University of M'hamed Bougara, Boumerdes, Algeria

²Laboratory of Mechanical Energetics and Engineering, University of M'hamed Bougara, Boumerdes, Algeria

³Laboratory of Dynamic Motors and Vibro-Acoustic, University of M'hamed Bougara, Boumerdes, Algeria

⁴Laboratory of Mechanical Energetics and Materials, University of Mouloud Mammeri, Tizi- Ouzou, Algeria

Abstract

The recent development of theoretical and experimental rheology, coupled with the increasing performance of computers, now allows us to have a different approach and to envisage numerical predictions on complex geometries. Unfortunately, with current differential models, simulations of viscoelastic fluids in complex geometries still run up against the limits of memory resources and prohibitive computational times. In this study, the commercial software Fluent used in combination with a calculation code developed in C⁺⁺, via sub-programs defined by User Defined Functions and User Defined Scalars. The purpose of this study is to compare the results with the analytical solution; which makes it possible to validate the numerical results by using the code developed in C⁺⁺ and also, to give an assurance to use this code in the numerical simulation of several problems in UCM fluid, which does not exist on the data base of the Fluent software. The results obtained in this study, shows the effectiveness of the code developed in C⁺⁺.

Keywords

Viscoelastic Fluid, UCM Model, Computational Fluid Dynamic, Couette Flow, Heat Transfer

1. Introduction

The behavior of fluids in confined devices such as the between two cylinders, two cores, or two spheres, is studied the measurement of fluids. The flow between two coaxial cylinders has resulted in more than 2000 articles [15-18] Mr Couette's. Because of its simplicity, this system has a rich variety of systems and records: laminar (basic flow), instabilities and chaos. The modes of transition these registers of

rotation of the two cylinders, the characteristics of the system (ratio of rays, aspect ratio), and the properties of the fluid used (viscosity, elasticity, etc.).

The importance of thermal effects in hydrodynamic lubrication of Journal bearings (two eccentric cylinders) was highlighted in the first scientific studies, however, the consideration of these effects for the calculation of mechanisms

*Corresponding author: m.guemmadi@univ-boumerdes.dz (Messaouda Guemmadi)

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is recent [8, 18, 21, 22].

The geometry of two coaxial cylinders is named after Maurice Couette, the first researcher to use this system in 1884, whose purpose was to measure the viscosity of fluids. It rotated the outer cylinder and measured the torque of the inner cylinder suspended by a twisting wire. He observed that the viscosity remains constant as long as the speed of rotation does not exceed a critical value; this is the basic circular flow: laminar speed. Four years later, A. Mallock (1888) observed similar characteristics by rotating the inner cylinder. In 1890, Couette [9] remarked that in the situation where the inner cylinder rotates and the outer cylinder remains fixed, the speed conditions for which the movement of the fluid can be considered as purely circular are more restrictive than the mobile outer cylinder. These first authors did not then explain the phenomenon encountered in these experiments.

Viscoelastic fluids are generally obtained by adding a long molecular chain polymer to a Newtonian solvent. Subjected to shearing, the resulting solution exhibits non-Newtonian behaviors such as rheofluidification or the appearance of differences in normal stresses induced by the elongation of macromolecules in the flow. These viscoelastic properties modify the nature and characteristics of instabilities (flow regimes) observed for a Newtonian fluid.

Exact solutions of the equations of motion including the constituent equation of viscoelasticity are rare when the field of flow does not consist of homogeneous shear or when it presents elongation (i. e. normal constraints are not homogeneous). In this case, non-linearity in the constituent equation requires that the flow be determined by the numerical solution [8].

The recent development of theoretical and experimental rheology, coupled with the ever-increasing performance of computers, now allows us to have a different approach and to consider numerical predictions on complex geometries. Unfortunately, with current differential-type models, simulations of viscoelastic fluids in complex geometries [5] and [18-20] still come up against memory resource limits and prohibitive calculation times.

This work is devoted firstly to analytical and numerical studies of the Couette flow between two concentric cylinders. Secondly, comparisons between the results obtained numerically and those obtained analytically. The fluid used in these studies is a Maxwell viscoelastic fluid. The Fluent software is used to which a C++ calculation code is integrated.

The main aim of these studies is to validate the calculation code in order to use it in the field of hydrodynamic lubrication of journal bearings by viscoelastic fluids obeying the Maxwell model [8].

2. Analytical Study of Couette Flow

The fluid is between the inner cylinder of radius R_1 , rotating at speed ω , and the outer cylinder of radius R_2 is fixed, the velocity field is of the form $\vec{v}(r, \theta, z)$: $u = 0$, $v(r)$, $w = 0$.

In cylindrical coordinate system (r, θ, z) , the equations of dynamic equilibrium, strain tensor, velocity gradient tensor and stress tensor are given as follows:

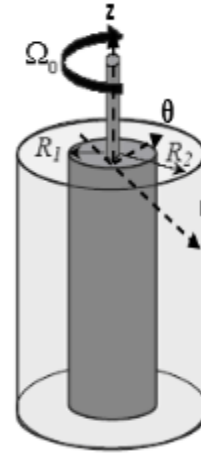


Figure 1. Couette Flow.

2.1. Constitutive Equation of Upper Convected Maxwell Fluid

The viscoelastic fluid considered is a Maxwell fluid, and its constitutive law is given by [2, 6, 7]:

$$\bar{\tau} + \lambda \frac{D\bar{\tau}}{Dt} = 2\mu \bar{D} \quad (1)$$

Or: τ_{ij} : the stress tensor. λ : the relaxation time. \bar{D} : the strain tensor $\frac{D\bar{\tau}}{Dt} = \frac{d\bar{\tau}}{dt} - ((\vec{\nabla}\vec{V}) \cdot \bar{\tau} + \bar{\tau} \cdot (\vec{\nabla}\vec{V})^T)$: is the convective derivative of the stress tensor [1].

2.2. Dynamic Equilibrium

Let the dynamic equilibrium system in the general case as follows [3]; (Flow is permanent).

$$\begin{cases} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + F_r - \rho \gamma_r = 0 \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + 2 \frac{\sigma_{r\theta}}{r} + F_{\theta} - \rho \gamma_{\theta} = 0 \\ \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} + F_z - \rho \gamma_z = 0 \end{cases} \quad (2)$$

2.3. Energy Equation

In the general case, the energy equation in cylindrical coordinates is written as follows [4]:

$$\rho C_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_{\theta}}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = - \left(\frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{1}{r} \frac{\partial q_{\theta}}{\partial \theta} + \frac{\partial q_z}{\partial z} \right) - \left[\tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \frac{1}{r} \left(\frac{\partial v_{\theta}}{\partial \theta} + v_r \right) + \tau_{zz} \frac{\partial v_z}{\partial z} \right] - \left[\tau_{r\theta} \left(r \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right) + \tau_{rz} \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right] + \tau_{\theta z} \left[\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_{\theta}}{\partial z} \right] \right] \quad (3)$$

2.4. Simplifying Assumptions

Flow is permanent.

By reason of symmetry relative to θ we can write: $\frac{\partial}{\partial \theta} = 0$

No flow in the z direction, so: $\frac{\partial}{\partial z} = 0$

Taking into account the simplifying assumptions and the boundary conditions. Finally, we arrive to the following expressions:

$$v(r) = \frac{\Omega_0 R_1^2}{R_2^2 - R_1^2} \left(\frac{R_2^2 - r^2}{r} \right) \quad (4)$$

$$p(r, z) = \rho \left[\frac{\Omega_0 R_1^2}{R_2^2 - R_1^2} \right]^2 \left[-\frac{R_2^4}{2r^2} - 2R_2^2 \ln(r) + \frac{r^2}{2} \right] + \left[\frac{\Omega_0 R_1^2}{R_2^2 - R_1^2} \right]^2 \left[\frac{2\mu\lambda R_2^4}{r^4} \right] - \rho g z + \text{Cst.} \quad (5)$$

$$\tau_{r\theta} = -\frac{2\mu\Omega_0 R_1^2 R_2^2}{R_2^2 - R_1^2} \left(\frac{1}{r^2} \right) \quad (6)$$

$$\tau_{\theta\theta} = \frac{8\mu\lambda\Omega_0^2 R_1^4 R_2^4}{(R_2^2 - R_1^2)^2} \frac{1}{r^4} \quad (7)$$

3. Numerical Procedure

3D flow between two coaxial cylinders is modeled by Fluent. The distance C ($R_2 - R_1$) between the cylinders is equal to 0.05 m and the range considered is of length L such as $L = 5C$. The inner cylinder rotates with a rotation speed ω .

The simulation concerns a viscoelastic fluid flow (Maxwell model) of density ρ and dynamic viscosity μ . The precision on the convergence is taken 10^{-7} and the coefficient of under relaxation of momentum equation is 0.5, [10-14].

The following table represents the different terms of the continuity equation, momentum equation, constitutive law and energy equation [8].

Table 1. The different source terms.

Continuity Equation	$\text{div}(\vec{u}) = S_M$	$S_M = 0$
Momentum Equation along x	$\text{div}(\rho u \vec{u}) = -\frac{\partial p}{\partial x} + \text{div}(\overrightarrow{\mu \text{grad} u}) + S_{Mx} \mu = 0$	$S_{Mx} = \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right]$
Momentum Equation along y	$\text{div}(\rho v \vec{u}) = -\frac{\partial p}{\partial y} + \text{div}(\overrightarrow{\mu \text{grad} v}) + S_{My} \mu = 0$	$S_{My} = \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right]$
Stress τ_{xx}	$\text{div}(\rho \tau_{xx} \vec{u}) = \text{div}(\overrightarrow{\Gamma \text{grad} \tau_{xx}}) + S_{\tau_{xx}} \Gamma = 0$	$S_{\tau_{xx}} = \rho \left(\frac{-1}{\lambda} + 2 \frac{\partial u}{\partial x} \right) \tau_{xx} + \rho \left(2 \frac{\partial u}{\partial y} \tau_{xy} \right) + \rho \left(\frac{\mu}{\lambda} \frac{\partial u}{\partial x} \right) \mu = \mu_p$
Stress τ_{yy}	$\text{div}(\rho \tau_{yy} \vec{u}) = \text{div}(\overrightarrow{\Gamma \text{grad} \tau_{yy}}) + S_{\tau_{yy}} \Gamma = 0$	$S_{\tau_{yy}} = \rho \left(\frac{-1}{\lambda} + 2 \frac{\partial v}{\partial y} \right) \tau_{yy} + \rho \left(2 \frac{\partial v}{\partial x} \tau_{xy} \right) + \rho \left(\frac{\mu}{\lambda} \frac{\partial v}{\partial y} \right) \mu = \mu_p$
Stress τ_{xy}	$\text{div}(\rho \tau_{xy} \vec{u}) = \text{div}(\overrightarrow{\Gamma \text{grad} \tau_{xy}}) + S_{\tau_{xy}} \Gamma = 0$	$S_{\tau_{xy}} = \rho \left(\frac{\partial v}{\partial x} \tau_{xx} + \frac{\partial u}{\partial y} \tau_{yy} \right) + \rho \frac{\mu}{\lambda} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - \frac{\rho}{\lambda} \tau_{xy} \mu = \mu_p$
Energy Equation	$\text{div}(\rho C_p T \vec{u}) = \text{div}(\overrightarrow{\Gamma \text{grad} T}) + S_T \Gamma = k$	$S_T = \left(\tau_{xx} \frac{\partial u}{\partial x} + \tau_{yy} \frac{\partial v}{\partial y} \right) + \tau_{xy} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$

4. Results and Discussion

The figure 2 shows the velocity vectors in the annular space. The figure shows that the speed is maximized on the inner cylinder and zero on the outer cylinder.

The temperature distribution in the annular space is shown in Figure 3 and Figure 7. Note that the temperature decreases from $T_a = 127^\circ \text{C}$ (the temperature of the inner cylinder) to the temperature of the outer cylinder (T_c).

Figure 4 shows the evolution of fluid velocity in the radial direction, obtained by analytical and numerical calculations under Fluent. Note that the speed is maximum at the inner

cylinder and decreases to zero at the fixed outer cylinder.

Figure 5 shows the evolution of the shear stress as a function of r. It is noted that the variation is non-linear which is confirmed by the analytical solution obtained from equation (eq. 6).

Figure 6 shows the evolution of the analytically and numerically calculated normal stress as a function of r. It is noted that the normal stress varies not linearly according to r and decreases as a function of the radius of the inner cylinder towards the outer cylinder.

The results shown in figures 4, 5, 6 and 7 display a conformity between the analytical solution and the numerical solution obtained by the Fluent software.

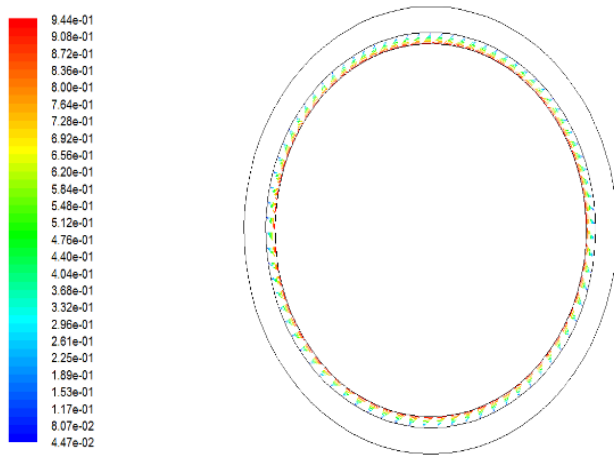


Figure 2. Velocity vectors in annular space (m/s).

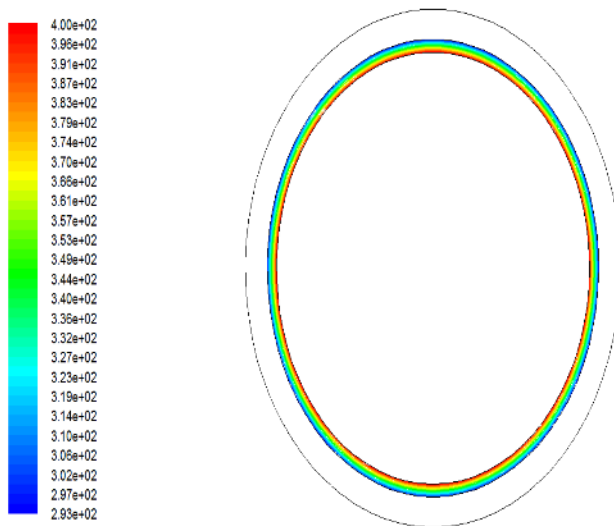


Figure 3. Static fluid temperature contours (K).

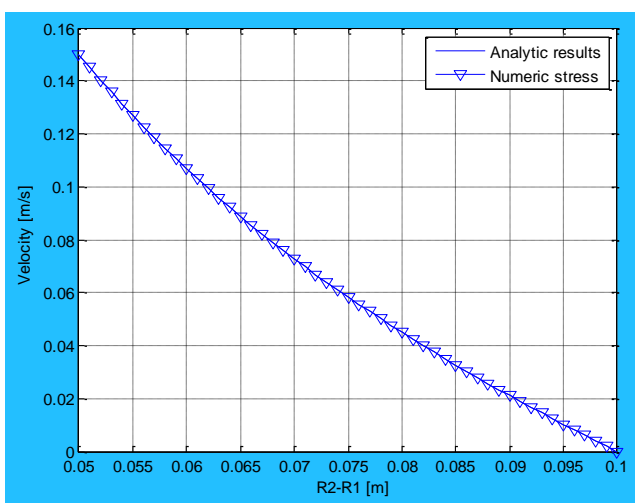


Figure 4. Evolution of velocity as a function of radius.

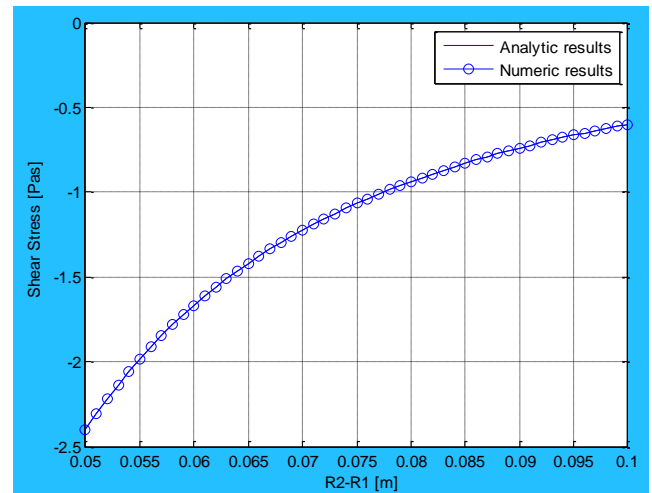


Figure 5. Evolution of the shear stress as a function of the radius r .

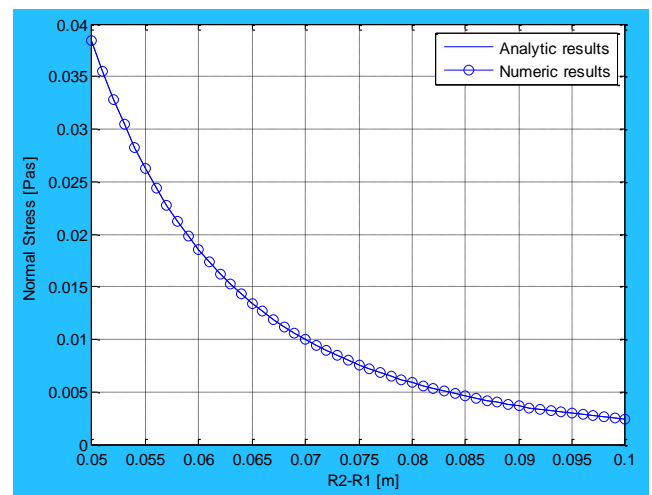


Figure 6. Evolution of the normal stress as a function of the radius r .

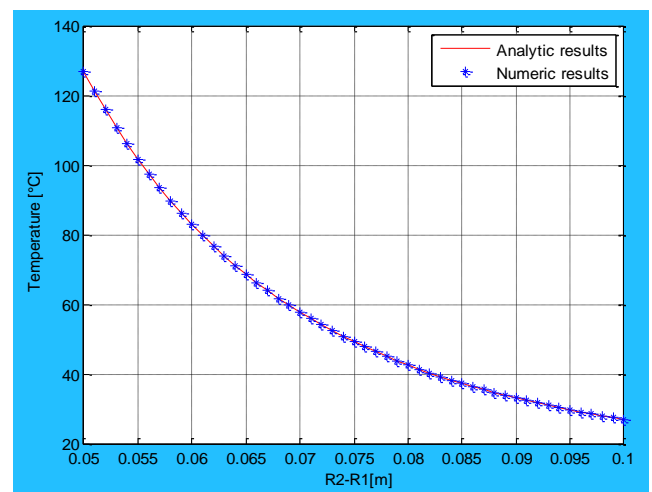


Figure 7. Evolution of the temperature in annular space as a function of the radius r .

5. Effect of Relaxation Time on Temperature and Normal Stress

The analytical calculation shows through the expression (eq. 8) that the temperature does not depend on the relaxation time or that the normal stress varies according to λ . The numerical results shown in figures 8 and 9 are confirmed by the analytical results. The analytical and numerical results show that the viscoelastic fluid studied does not play a major role; as regards the dissipation of the heat of this fluid that flows between two concentric cylinders, whose temperature varies according to the viscosity, the speed of rotation of the inner cylinder as well as the radius r . The conformity between the results obtained numerically and analytically shows well the efficiency of the code developed in C⁺⁺.

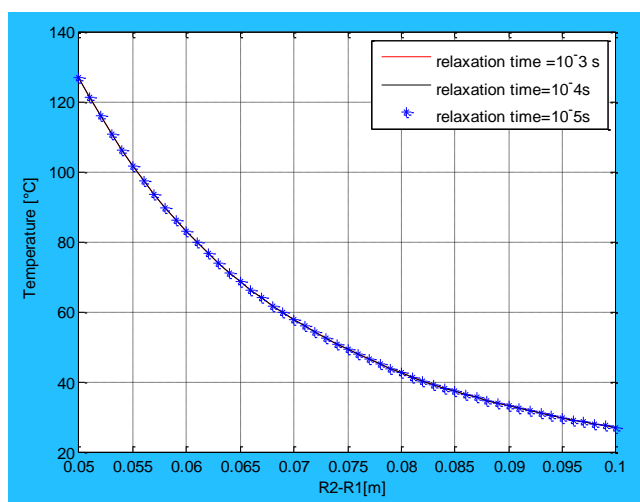


Figure 8. Temperature evolution as a function of r to $\theta=0$ for different values of λ .

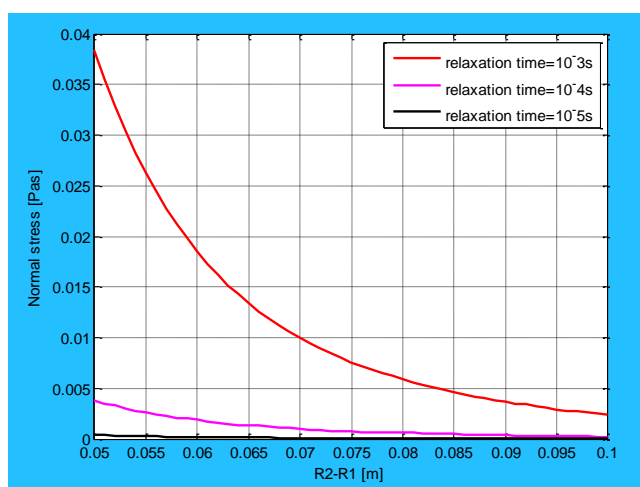


Figure 9. Evolution of Normal stress as a function of r to $\theta=0$ for different values of λ .

6. Conclusion

The study presented in this article aims to adapt Fluent with viscoelastic fluid presented by an Upper Convected Maxwell model that is not integrated into the database of this software.

To check the validity of the code developed in C⁺⁺ and the integrated under Fluent, we calculated the velocity field, the pressure field, the normal stress and the shear stress in the case of viscoelastic fluid flow between two concentric cylinders (Couette flow). The results of the numerical simulation are compared by analytic computation. Then, we studied the thermal effect by integrating the energy equation during the numerical simulation, and we found that the results obtained show the efficiency of the calculation code developed in C⁺⁺. It was also found that in the concentric case, the relaxation time has no effect on the fluid temperature.

Abbreviations

CFD	Computational Fluid Dynamic
UCM	Upper Convected Maxwell
UDSs	User Define Scalars
UDFs	User Define Functions

Author Contributions

Messaouda Guemmedi: Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing

Faiza Brahimi: Methodology, Supervision, Visualization

Ahmed Ouibrahim: Supervision

Conflicts of Interest

The authors declare no conflicts of interest.

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