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Research Article

Modeling the Influence of Raleigh Numbers on Thermal and Fluidic Behaviors in a Trapezoid-shaped Cell

Windé Nongué Daniel Koumbem[1, 2, *], Bouwèreou Bignan-Kagomna[1], Noufou Bagaya[1], Wend Dolean Arsène Ilboudo[4], Pare Daouda[3], Issaka Ouédraogo[4], Sié Kam[1, 5]

[1]Renewable Thermal Energy Laboratory, Formation and Research Unit, Applied Exact Sciences, Joseph Ki-Zerbo University, Ouagadougou, Burkina Faso

[2]Formation and Research Unit in Sciences and Technologies, Kaya University center, Kaya, Burkina Faso

[3]Formation and Research Unit in Sciences and Technologies, Norbert Zongo University, Koudougou, Burkina Faso

[4]Energy Department, Research Institute of Applied Sciences and Technologies, Ouagadougou, Burkina Faso

[5]Physics department, formation and Research unit, Applied Exact Sciences, Joseph Ki-Zerbo University, Ouagadougou, Burkina Faso

Abstract

We model the influence of Raleigh numbers in a trapezoidal cavity. One wall among the sloping walls is exposed to a heat flux density Q=100 W/ m^2 and the other inclined wall is kept adiabatic. The temperature of the two horizontal walls is assumed to constant such that T_{sup}=305K is greater than T_{inf}=300K. The equations of heat and mass transfer which direct our template are described by the Navier-Stockes equation. These equations are discretized using the finite difference method and solved by the Thomas and Gauss-Seidel algorithms. Thus, we analyze the effects of the Raleigh numbers (*Ra*) on temperature profiles T = 303.15 K and speeds v = 0 m/s. For a variation of $Ra=10^3$-10^5, we note that the convective exchanges of the confined air and the different walls become preponderant with the increase in the Rayleigh number. Also, we contact that the speed of the confined air remains high along the horizontal walls for a *Ra* high number, but low near the inclined walls. These results show the effects of natural convection in this trapezoidal cavity.

Keywords

Raleigh Number, Trapezoidal Cavity, Thermal, Fluidics

1. Introduction

Like other countries in the world and in the Sahel region in particular; Burkina Faso is facing a worrying situation in the field of energy. The energy problem in the field of construction has occupied significant place in all government strategies for decades. To address the issue of population increase, housing in countries with hot and dry climates is

*Corresponding author: koumbemdaniel@gmail.com (Windé Nongué Daniel Koumbem)

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most often built without taking into account environmental constraints. To meet the energy challenge, several studies have been conducted in the field of construction to improve thermal comfort in countries with hot climates. We can cite, among others, studies on thermal comfort by improving the roof envelope [1-3], researches on the phenomena which manage convection in the nature, the effect of the nature of the systems in which it takes place (geometry) [4, 5], the physico-thermal properties of materials [6, 7] and the physico-chemical the features of the fluids which are involved. Much has been documented about the phenomena of heat and mass transfer via ordinary convection in different limited cavities [8-15]. these results highlight the effect of non-dimension numbers, namely Grashoft (Gr), Nusselt (Nu) on the transfer of heat and mass in these restricted cells. Thus, our work is devoted to the numerical study of the impact of dimensionless numbers on thermal and fluidic behaviors in a limited cavity with trapezoid shape within a warm and dry climate. One of the aims of this study is to analyze the influence of the Rayleigh number on the evolutions of temperatures and air velocity profiles inside, along the different walls of the trapezoidal cavity.

2. Explanation of the Physical Model

As exposed in Figure 1, A trapezoid-shaped restricted cavity, the upper (small base) and lower (large base) horizontal walls of which are kept at stable temperature ($T_{sup} > Tinf$). One of the sloped walls is under pressure of a heat flux density $Q = 100\ W.m^{-2}$and the other maintained adiabatic, i. e. no convective exchange with the ambient environment. The procedure of our model is that the air controlled in the trapezoid-like shape interfere with various walls inside and outside (with the room environment) [8].

Thus, mathematical equations are used for the explanation of the physical phenomena that occur there, i. e. heat and mass transfers. we also make some of the simpler suppositions [8]:

1. The flow is laminar, stationary and two-dimensional,
2. the fluid has Newtonian nature and is impenetrable,
3. exchanges of rays amid the walls are insignificant,
4. circulation of enthalpy is neglected,
5. Viscous energy dissipation is neglected,
6. The Dufour and Soret effects are insignificant,
7. no concentration on the walls.

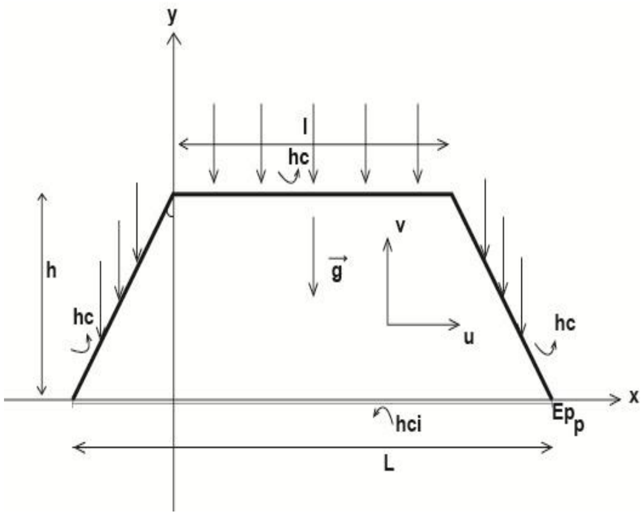

Figure 1. *Schematic of the physical model of the trapezoidal cavity.*

3. Mathematical Formulation of the Physical Model Studied

The heat and mass transfer equations applied to the model are written [1-5]:

-continuity equation:

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \tag{1}$$

calculation of the motion extent

n accordance with the x axis:

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = -\frac{1}{\rho_i} \frac{\partial p_i}{\partial x} + \nu_i \left(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right) \tag{2}$$

Along the y axis:

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial x} = -\frac{1}{\rho_i} \frac{\partial p_i}{\partial y} + \nu_i \left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} \right) - g[\beta_T (T_i - T_a) + \beta_C (C_i - C_a)\,] \tag{3}$$

-heat equation

$$\frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial x} + v_i \frac{\partial T_i}{\partial y} = \frac{\lambda_i}{\rho_i C p_i} \left(\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right) \tag{4}$$

-diffusion equation

$$\frac{\partial C_i}{\partial t} + u_i \frac{\partial C_i}{\partial x} + v_i \frac{\partial C_i}{\partial y} = D_i \left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} \right) \tag{5}$$

In order to restrict the number of parameters in the calculations of conservation of the level of motion, energy and, facilitate their resolution, we proceed to the adimensionalization of these equations.

Introducing the dimensionless parameters into equations (1-5) gives:

$$\frac{\partial U_i^*}{\partial X} + \frac{\partial V_i^*}{\partial Y} = 0 \tag{6}$$

$$\frac{\partial U_i^*}{\partial \tau} + U_i^* \left(\frac{\partial U_i^*}{\partial X}\right) + V_i^* \left(\frac{\partial U_i^*}{\partial Y}\right) = -\frac{\partial P_i^*}{\partial X} + \left(\frac{\partial^2 U_i^*}{\partial X^2} + \frac{\partial^2 U_i^*}{\partial Y^2}\right) \quad (7)$$

$$\frac{\partial V_i^*}{\partial \tau} + U_i^* \left(\frac{\partial V_i^*}{\partial X}\right) + V_i^* \left(\frac{\partial V_i^*}{\partial Y}\right) = -\frac{\partial P_i^*}{\partial Y} + \left(\frac{\partial^2 V_i^*}{\partial X^2} + \frac{\partial^2 V_i^*}{\partial Y^2}\right) - T_i^* - C_i^* \quad (8)$$

$$\frac{\partial T_i^*}{\partial \tau} + U_i^* \frac{\partial T_i^*}{\partial X} + V_i^* \frac{\partial T_i^*}{\partial Y} = \frac{1}{Pr_i}\left(\frac{\partial^2 T_i^*}{\partial X^2} + \frac{\partial^2 T_i^*}{\partial Y^2}\right) \quad (9)$$

$$\frac{\partial C_i^*}{\partial \tau} + U_i^* \frac{\partial C_i^*}{\partial X} + V_i^* \frac{\partial C_i^*}{\partial Y} = \frac{1}{Sc_i}\left(\frac{\partial^2 C_i^*}{\partial X^2} + \frac{\partial^2 C_i^*}{\partial Y^2}\right) \quad (10)$$

4. Dimensionless Boundary Conditions

For:

$$0 \leq X \leq \frac{l}{H} et\ Y = 0{,}06$$

$$\mathrm{U}_i^* = \mathrm{V}_i^* = 0$$

$$\frac{\partial T_i^*}{\partial Y} = 0.479\left(\frac{l}{H}\right)^{3/4} \frac{\lambda_a v_a^2}{\lambda_i v_i^2} T_i^{*5/4}, \frac{\partial \mathrm{C}_i^*}{\partial \mathrm{Y}} = 0 \quad (11)$$

$$-\tan\varphi \leq X \leq 0;\ l/H \leq X \leq l/H + \tan\varphi\ \ et\ Y = 0$$

$$\mathrm{U}_i^* = \mathrm{V}_i^* = 0$$

$$\frac{\partial T_i^*}{\partial Y} = 0.479(\tan\varphi)^{3/4} \frac{\lambda_a v_a^2}{\lambda_i v_i^2} T_i^{*5/4}, \frac{\partial \mathrm{C}_i^*}{\partial \mathrm{Y}} = 0 \quad (12)$$

$$0 \leq \mathrm{X} \leq l/H \text{ et } \mathrm{Y} = 0;$$

$$\mathrm{U}_i^* = \mathrm{V}_i^* = 0$$

$$\frac{\partial T_i^*}{\partial Y} = 0.479\left(\frac{l}{H}\right)^{3/4} \frac{\lambda_a v_a^2}{\lambda_i v_i^2} T_i^{*5/4}, \frac{\partial \mathrm{C}_i^*}{\partial \mathrm{Y}} = 0 \quad (13)$$

$$-\tan\varphi \leq X \leq 0;\ 0 \leq Y \leq 0{,}06$$

$$\mathrm{U}_i^* = \mathrm{V}_i^* = 0 \quad \frac{\partial T_i^*}{\partial X} = 0, \frac{\partial C_i^*}{\partial X} = 0 \quad (14)$$

$$l/H \leq X \leq \tan\varphi;\ 0 \leq Y \leq 0{,}06$$

$$\mathrm{U}_i^* = \mathrm{V}_i^* = 0 \quad \frac{\partial T_i^*}{\partial X} = O, \frac{\partial C_i^*}{\partial X} = 0 \quad (15)$$

5. Numerical Methodology

Equations (6-10) are discretized using the finite difference method (FDM). these numerical equations describe the discrete properties of the fluid at each node of the studied domain. the algebraic equations that emerge are expressed in tridiagonal matrices and solved step by step through Thomas algorithm for diffusing the energy and equations, Gauss for the equation of motion.

6. Discussions of Results

We present the numerical results of our calculations obtained with the Comsol Multiphysics 4.2 software, in the form of temperature curves and velocity profiles. The curves in Figures 2-9 indicate the thermal and fluid behavior of the air confined in the vicinity of the four walls of our trapezoidal cavity model. These curves are obtained for values of $Ra = 10^3 - 10^5$.

6.1. Evolution of the Air Temperature as a Function of the Rayleigh Number in the Vicinity of the Different Walls of the Trapezoidal Cavity

We study the temperature evolution for different values of the Rayleigh number (Ra) along the different walls of the trapezoidal cavity.

We note that the curves in figure 2 show that the temperature of the upper wall (small base) reaches its maximum when the contact between the air and the slanted wall was under pressure of high temperature fluxes Q=100 W/m^2 is achieved. At this stage, the heat exchange by conduction-convection becomes important. Then, the temperature gradually decreases until reaching a minimum value in the part in contact with the adiabatic wall. In this part the temperature difference between the air and the wall begins to drop, which generates a weak flux. These results show that the temperature of the upper horizontal wall remains high for Rahigh Rayleigh numbers $Ra = 10^3 - 10^5$.

The curves in figure 3 show that the temperature of the lower wall (large base) reaches its maximum when the contact between the air and the slanted wall was under pressure of high temperature fluxes Q=100 W/m^2 is achieved. We note that the heat exchange by conduction-convection is important in the area close to the slanted wall was under pressure of high temperature fluxes Q=100 W/m^2. Then it gradually decreases until reaching a minimum value in the part in contact with the adiabatic wall. In this part the temperature difference between the air and the wall begins to drop, which generates a weak flux. These results show that the temperature of the lower horizontal wall remains low for high Rayleigh numbers Ra$Ra = 10^3 - 10^5$.

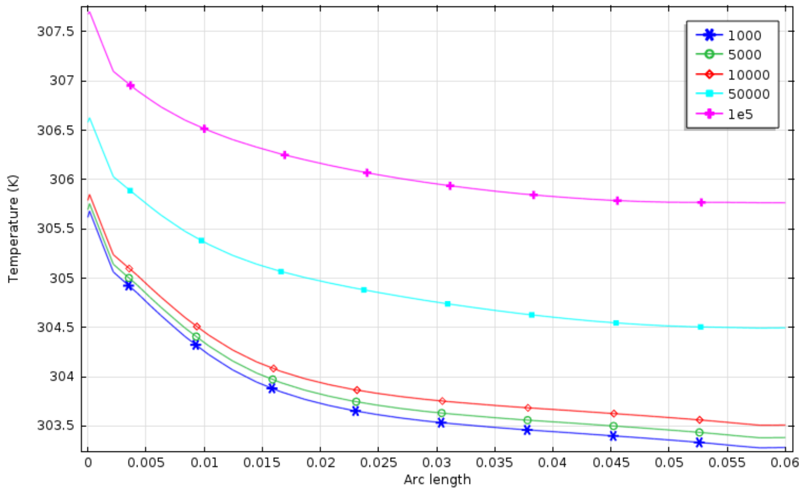

Figure 2. *Temperature variation along the upper horizontal wall.*

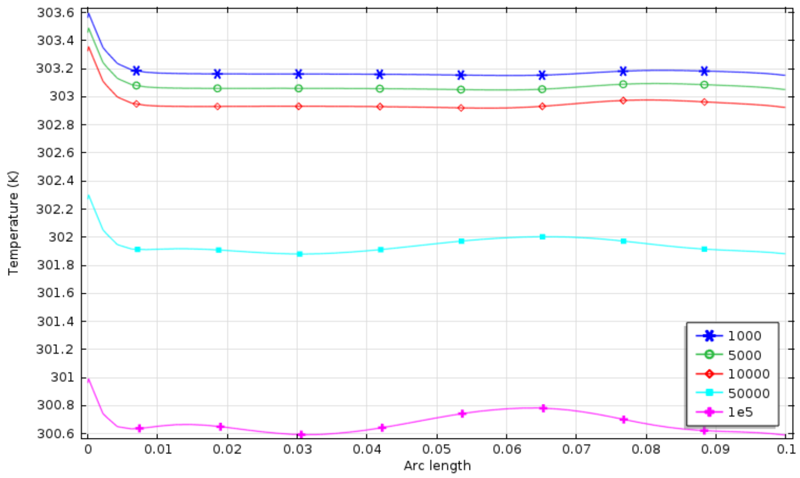

Figure 3. *Temperature variation along the lower horizontal wall (large base).*

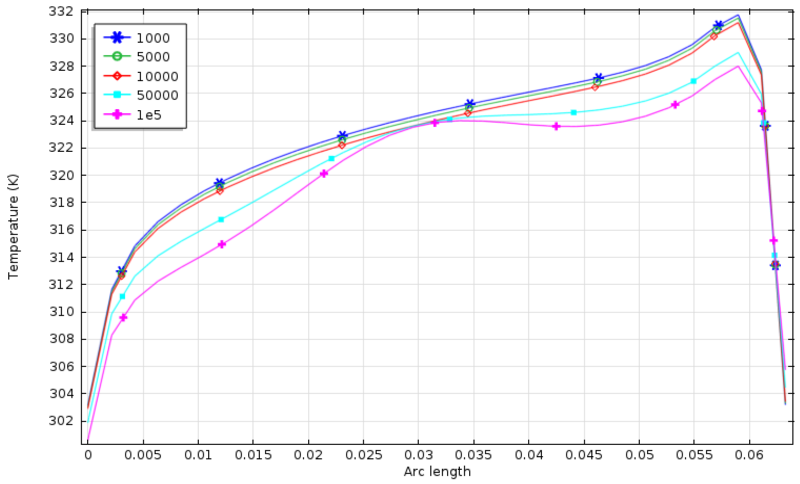

Figure 4. *Temperature variation along the wall subjected to the flow of heat* $(Q = 100W/m^2)$.

The curves in Figure 4 show that the temperature of the wall exposed to low temperature change in the lower part of the trapezoidal cavity reaches its maximum in the upper part. We note that the heat exchange by conduction-convection is important in the upper part of the cavity. These results show that the temperature of the lower horizontal wall remains low for Rahigh Rayleigh numbers. $Ra = 10^3 - 10^5$

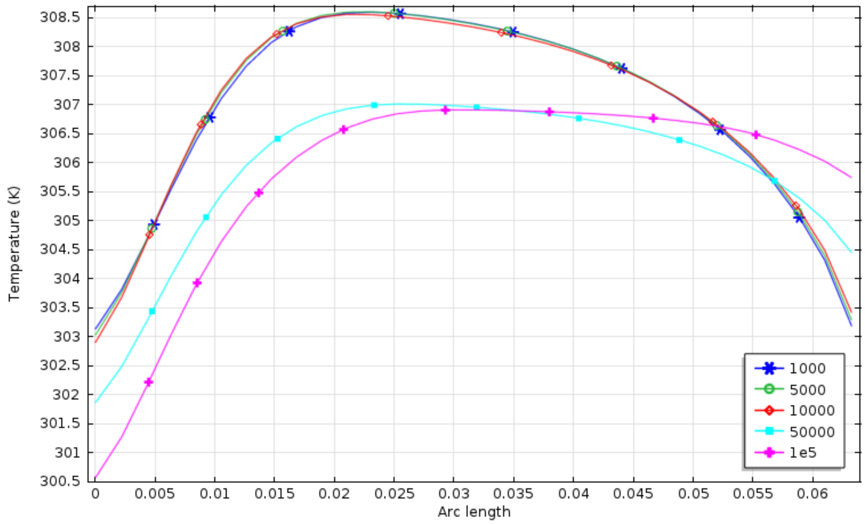

Figure 5. *Temperature variation along the adiabatic inclined wall.*

The curves in figure 5 show that the adiabatic wall temperature which is low in the parts close to the horizontal walls of the trapezoidal cavity reaches its maximum in the middle of the wall. We note that the heat exchange by conduction-convection remains important in the central part of the cavity. These results show that the adiabatic wall temperature is low for Rahigh Rayleigh numbers $Ra = 10^3 - 10^5$

6.2. Evolution of the Air Speed as a Function of the Rayleigh Number in the Vicinity of the Different Walls of the Trapezoidal Cavity

The curves in Figures 6-9 show the evolution of air speeds in the vicinity of the different walls.

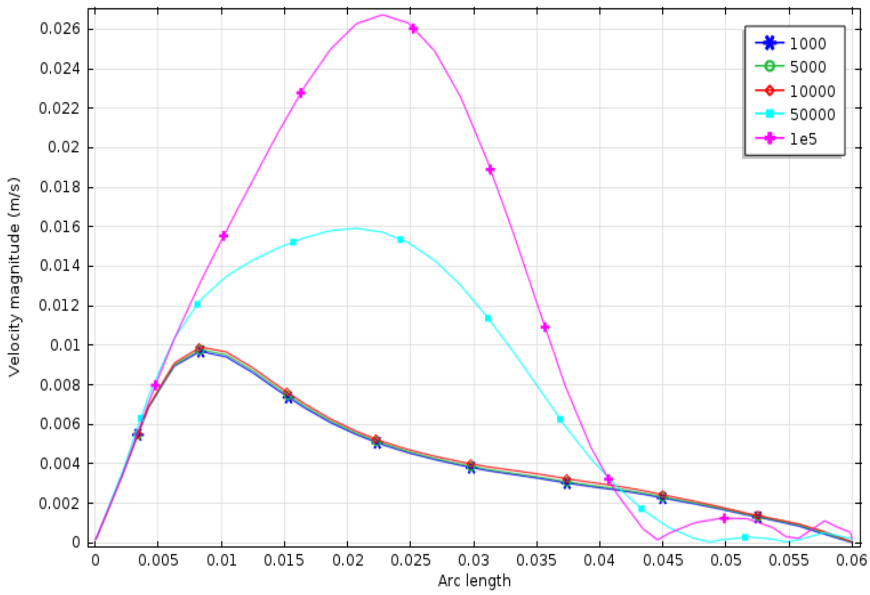

Figure 6. *Evolution of air speed near the upper wall (small base).*

The curves in Figure 6 show that for values of Ra=10^3-10^4 the speed in the vicinity of the large base increases from 0 m/s to reach its maximum at 0.002 m/s in the part close to the wall subjected to the flow. In the following, we note that the air speed decreases and tends towards 0 m/s in the part close to the adiabatic wall. These results show that the exchanges by conduction-convection between the air and the large base are important in the part close to the wall subjected to the heat flux Q=100 W/m^2.

When Ratends to 10^5, the velocity in the vicinity of the large base increases faster from 0 m/s to reach its maximum at 0.027 m/s towards the middle of the small base. Then, this velocity decreases and tends to 0 m/s. This result shows that for high Rayleigh numbers, the conduction-convection exchanges between the air and the large base are important in the central part of the trapezoidal cavity. Also, the velocity remains low for Rayleigh values between 10^3 and 10^4.

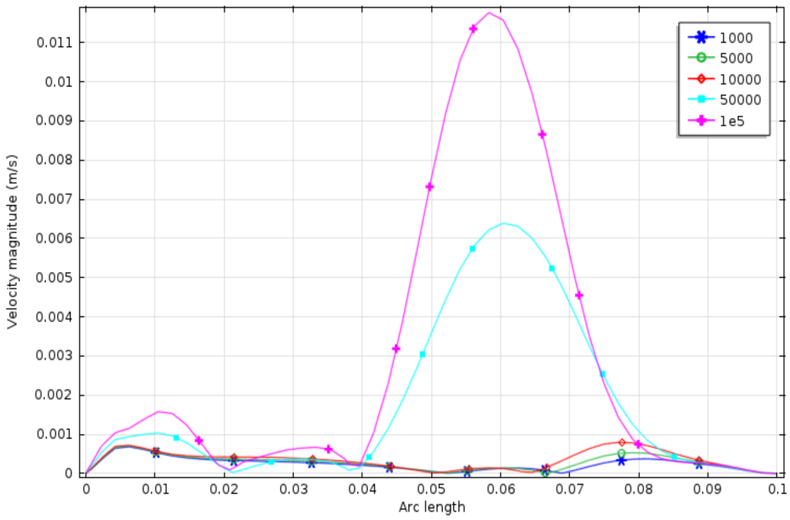

Figure 7. *Evolution of the speed of the confined air near the wall lower horizontal.*

The curves in Figure 7 show that for values of Ra=10^3-10^4 the air speed varies slightly along the lower horizontal wall (large base). When Ra=5.10^4-10^5 the speed is low in the parts close to the inclined walls but high towards the middle of the lower horizontal wall (large base).

We find that the convective exchanges between the air and the lower horizontal wall (large base) remain significant in the middle of the large base of the cavity. These results also show that the velocity remains high in the vicinity of the lower horizontal wall (large base) for high Rayleigh numbers Ra.

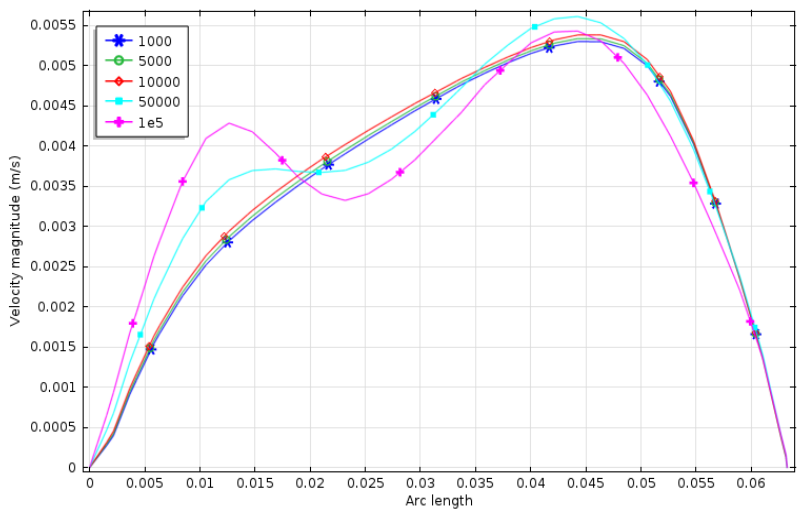

Figure 8. *Evolution of the speed of the confined air in the vicinity of the wall inclined and exposed to movement* $(Q = 100W/m^2)$.

We note that the curves in figure 8 show that the air velocity remains high near the inclined wall exposed to high temperature flux in the upper part of the trapezoidal cavity. When the Rayleigh number Ra varies from 10^3 to 10^4, the velocity goes from 0 m/s in the lower part of the cavity to reach its maximum 0.0055 m/s in the upper part. Then this velocity decreases to 0.0016 m/s in the vicinity of the upper horizontal wall (small base).

For Ra=10^5, we find that the velocity increases from 0 m/s to 0.00425 m/s in the lower part near the lower horizontal wall (large base). Then the velocity decreases slightly to 0.0034 m/s, then it increases again to 0.0556 m/s in the upper part near the small base. Finally, the velocity decreases to 0.0018 m/s in the vicinity of the upper horizontal wall (small base).

These results show that the convective exchange between the air and the inclined wall subjected to the movement of heat is significant in the superior part of the trapezoid-shaped cavity.

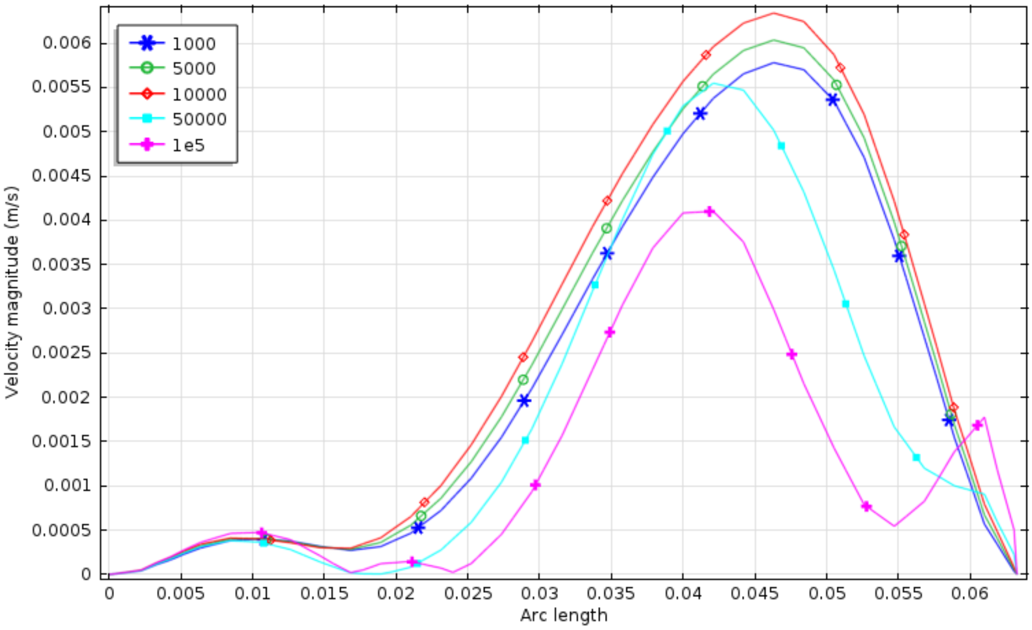

Figure 9. *Evolution of air speed near the inclined wall adiabatic.*

The curves in Figure 9. show that the air velocity near the adiabatic inclined wall is high in the upper part of the trapezoidal cavity. We see that as the Rayleigh number Ra increases, the air velocity decreases near the adiabatic inclined wall.

These results show that the convective exchange between the air and the adiabatic inclined wall is important in the upper part of the trapezoidal cavity.

7. Conclusion

We have numerically studied the evolutions of temperatures and internal velocities along the different walls by natural convection in our restricted trapezoid-shaped cell model. At the final stage of our simulation calculations, we scrutinized the impact of Rayleigh number Ra on the evolution of temperatures and velocities of the confined air along the different walls of the trapezoidal cavity for a constant heat flux Q=100 W/m ^{2} subjected to an inclined wall. The results obtained show that:

1. the temperature of the interior air in the vicinity of the upper wall increases with the intensification in the Rayleigh number $Ra = 10^3 - 10^5$. Also, the convective exchanges of the confined air and the different walls become preponderant with the increase in the Rayleigh number.
2. the speed of the confined air remains high along the horizontal walls for number $Ra = 10^5$. It reaches a maximum value of 0.027 m/s and 0.012 m/s respectively in the vicinity of the upper and lower horizontal walls.
3. the convective exchange between the air and the inclined walls remains significant in the upper part of the trapezoidal cavity.

Abbreviations

Gr	Grashof Number
Nu	Nusselt Number

Ra	Rayleigh Number
u	Velocity Component in x Direction, m/s
v	Velocity Component in the y Direction, m/s
x, y	Cartesian Coordinates, m
X, Y	Dimensionless Cartesian Coordinates, m
C	Concentration of Air in the Cavity
C_p	specific heat volume at stable pressure
T	Temperature, K
P	Pressure, Pa
Pr	Prandtl Number
Sc	Schmidt Number
t	Time, s
D	Mass Diffusion Coefficient, $m^2.s^{-1}$
g	Coefficient of Acceleration of Gravity, ms^{-2}
l	Length of the Small Base, m
H	Height of the Trapezoid, m
L	Length of the Large Base, m
λ	Thermal Conductivity ($Wm^{-2}.K$)
τ	Dimensionless Time
φ	Angle
ρ	Density, $kg.m^3$
β	Coefficient of Expansion of Air in the Cavity
υ	Kinematic Viscosity, $m^2.s^{-1}$
a	Ambient Environment
i	Interior of the Cavity
0	Initial
T	Thermal
C	Mass
*	Dimensionless Value

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Author Contributions

Windé Nongué Daniel Koumbem: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Bouwèreou Bignan-Kagomna: Funding acquisition, Methodology, Supervision, Validation, Visualization

Noufou Bagaya: Funding acquisition, Methodology, Supervision, Validation, Visualization

Issaka Ouédraogo: Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Windé Nongué Daniel Koumbem, Issaka Ouédraogo, Wend Dolean Arsène Ilboudo, Péléga Florent Kieno; Numerical Study of the Thermal Performance of Three Roof Models in Hot and Dry Climates, Modeling and Numerical Simulation of Material Science, 2021, 11, 35-46, https://orcid.org/10.4236/mnsms.2021.112003

[2] Ouedraogo, I. (2009) Modeling and Optimization of a Bioclimatic Roof for the Passive Air Conditioning of a Typical Habitat of Burkina Faso. University of Ouagadougou, Ouagadougo.

[3] I. Ouédraogo, A. Ouédraogo, K. Palm, B. Zeghmati, MODELING OF A BIOCLIMATIC ROOF USING NATURAL VENTILATION, International Scientific Journal for Alternative Energy and Ecology № 6(62) 2008.

[4] Richard. JDD, and Brager. SG (2002). Thermal comfort in naturally ventilated buildings. Energy and buildings. Revision to ASHRAE Standard 55; 34(6): 549-561. https://orcid.org/10.1016/S0378-7788(02)00005-1

[5] Issaka Ouédraogo, Windé Nongué Daniel Koumbem, Noufou Bagaya and Alioune Ouédraogo; THE THERMAL COMFORT INVESTIGATION OF TWO TRADITIONAL HABITAT MODELS IN THE SUB-SAHELIAN AREA, International Journal of Advanced Research. 10(10), 1096-1104, 2022, https://orcid.org/10.21474/IJAR01/15585

[6] Ouedraogo Issaka, Simonis Priscilla, Ouedraogo Alioune and Zeghmati Belkacem (2016). Study of Air Temperatures within the Enclosure of aModel of Traditional Habitat Bilobate andRectangular. British Journal of Applied Science & Technology. 16(3): 1-7.

[7] Arouna Kaboré, Zoma Vincent, Palm Kalifa and Bathiebo Dieudonné Joseph Two-Dimensional Modeling of Heat Transfers in a Ventilated Test Cell Built with Various Local Materials, Physical Science International Journal 25(8): 14-31, 2021; https://orcid.org/10.9734/PSIJ/2021/v25i830275

[8] Windé Nongué Daniel Koumbem, Issaka Ouédraogo, Noufou Bagaya and Pelega Florent Kieno; Thermal Behavior of the Natural Convection of Air Confined in a Trapezoidal Cavity, Current Journal of Applied Science and Technology 40(12): 69-80, 2021; https://orcid.org/10.9734/CJAST/2021/v40i1231382

[9] Khalil LASFER 1, Mounir BOUZAIANE 2, Taieb LILI3, NUMERICAL STUDY OF TURBULENT NATURAL CONVECTION IN A TRAPEZOIDAL CAVITY, 13th International Thermal Conference, Albi, France from August 28 to 30, 2007.

[10] R. Zarrit, MS Boumaza, S. Kherrour and B. Dadda, Natural convection in an inclined rectangular cavity of different aspect ratios, Renewable Energy Review Vol. 19 No. 1(2016) 97 – 109.

[11] Djatout A, Douha M, Hami O, Rahmani L, Mebarki B. Study of natural convection in laminar regime in a square cavity inclined at an angle α, Journal of Scientific Research. 2010; 1. 12.

[12] Ahmed Kadhim Hussein, Finite volume simulation of natural convection in trapezoidal Cavity filled with various fluids and heated from Top Wall, Universal Journal of Fluid Mechanics. 2013; 1: 24-36.

[13] Aparna B, Seetharamu KN. FEM Analysis of Natural Convection Flows in Porous Trapezoidal Enclosure, International Journal of Innovative Research in Advanced Engineering (IJIRAE). 2017; 04(4).

[14] Fakour M, Ganji DD, Khalili A, Bakhshi A. Study of heat transfer in nanofluid MHD flow in a channel with Permeable walls, begellhouse, Heat Transfer Research. 2017; 48(3): 221–238.

[15] Pensiri Sompong and Supot Witayangkurn, Natural Convection in Trapezoidal Enclosure with wavy top surface, Journal of Applied Mathematics. 2013; 840632: 7.