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A State Estimation H_∞ Issue for Continuous-Time Impulsive Genetic Regulatory Networks with Random Delays Via Sampled-Data Approach

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Abstract: This paper addresses the state-estimation H_∞ problem for continuous-time impulsive genetic regulatory networks (GRNs) with random delays, using a sampled-data approach. Genetic regulatory networks are fundamental in controlling gene expression and protein synthesis, governed by regulatory interactions between transcription factors and mRNA (Messenger Ribonucleic Acid) binding sites. To estimate mRNA and protein concentrations, sampled measurements replace continuous measurements in this framework. We propose a new model that leverages impulsive control strategies to regulate mRNA and protein dynamics under conditions with random delays. The primary contribution of this study is the derivation of sufficient conditions that guaranteeing that impulsive genetic regulatory networks is globally asymptotically stable is derived. By introducing a discontinuous Lyapunov- Krasovskii functional, sufficient stability analysis has been rooted in terms of LMIs: Linear Matrix Inequalities. By applying Wirtinger inequality technique, conservation of the impulsive GRNs system is globally asymptotically stable in the mean- square sense have been diminished greatly. Eventually, a numerical example is given to the feasibility and advantages of the developed results.

Keywords: Genetic Regulatory Networks (GRNs), Robust State Estimation, Random Delays, Impulsive Equation

1. Introduction and System Formulation

Gene regulatory networks play an important role in the molecular mechanism underlying biological process. The mutual interaction between a set of genes or proteins or small molecules to control the ratio of transcription is referred to as "Genetic regulatory networks" briefly called GRNs.Transcription and Translation are the main process of gene expression, which is composed by proteins. Until now, various models have been developed to describe the GRNs. Some of the theorems are Bayesian model, Boolean network model, continuous network model, the differential equations model.

Many of the results on the stability analysis for GRNs with time- delay and discrete time varying systems have been published, see [2, 4, 6, 8, 9, 16]. In [11–13, 15, 17, 20], the authors discussed the random delays and stochastic delays in modelling GRNs and in [5, 10, 14, 21, 22] the researchers deeply studied in time-delay at network areas. However, an important issue is discretization of continuous time signals in the process of using computers to conduct the addressed problem is called sampling.

Furthermore the sampled data approach have been presented to handle the analysis and synthesis problems of GRNs in many important literature, see [1, 7, 19]. Moreover many systems are identified by changes at definite instants due to rapid perturbations, which results to impulsive effects [3, 18].

Motivated by the above deliberations, the main contribution of this paper can be summarised as follows:

1. To examine the approximation concerns for the continuous - time impulsive GRNs with random delays and extrinsic distribution into the problem and modelled

the robust H_∞ state estimator for a class of continuous-time impulsive GRNs.
2. By applying the Lyapunov stability theory, we found some sufficient conditions are satisfying the terms of linear matrix inequality(LMI).
3. Finally, a numerical example is given to view the capability of our results.

2. Problem Formulation

In this paper, we deliberate the Consequential Genetic regulatory networks with time- varying delays, external disturbances and impulses

$$\begin{aligned} \dot{m}_i(t) &= -a_i m_i(t) + g(p_1(t-\zeta_1(t)),...,p_n(t-\zeta_n(t))) + e_{mi}u(t) \\ m(t_k) &= Lm(t_k^-), k \in Z^+, t = t_k, \\ \dot{p}_i(t) &= -c_i p_i(t) + d_i m_i(t-\xi(t)) + e_{p_i}u(t), \\ p(t_k) &= Jp(t_k{}^-), k \in Z^+, t = t_k \end{aligned} \tag{1}$$

where $m_i(t) \in \mathbb{R}(i = 1, 2, 3...n)$ and $p_i(t) \in \mathbb{R}(i = 1, 2, ...n)$ are the concentrations of mRNA and protein of the i^{th} gene at time t, respectively. a_i and c_i represents the degradation rates of mRNA and protein. d_i denotes the translation rate. $\zeta_1(t) \in [\zeta_{m_1}, \zeta_{M_1}]$ and $\zeta_2(t) \in [\zeta_{m_2}, \zeta_{M_2}]$ are random delays, the regulation function $g_i(\cdot)$ with this form

$g_i(p_1(t), p_2(t), ..., p_n(t)) = \sum_{j=1}^{n} g_{ij}(P_j(t))$ is called sum logic [18], the function $g_{ij}(P_j(t))$ is a monotonic function and satisfies the Hill form [19].

$$g_{ij}(P_j(t)) = \begin{cases} \gamma_{ij}\frac{(P_j(t)/\beta)^H}{1+(P_j(t)/\beta)^H}, \text{if transcription factor j is an activator of gene i,} \\ \alpha_{ij}\frac{1}{1+(P_j(t)/\beta)^H}, \text{if transcription factor j is an repressor of gene i.} \end{cases}$$

Where H is the Hill Co-efficient, β is a positive scalar, γ_{ij} is a bounded constant representing the dimensionless transcriptional rate of transcription factor j to gene i. $v(t)$ is the external disturbance belonging to $L_2([0, \infty), \mathbf{R})$, while e_{m_i} and e_{p_i} are used to express the intensities of the external disturbances of the mRNA and the protein respectively.
Impulsive GRNs(1) can be rewritten as

$$\begin{aligned} \dot{m}_i(t) &= -a_i m_i(t) + \sum_{j=1}^{n} V_i j b_j(P_j(t-\zeta(t)) + I_i + e_{mi}u(t)) \\ m(t_k) &= Lm(t_k^-), k \in Z^+, t = t_k, \\ \dot{p}_i(t) &= -c_i p_i(t) + d_i m_i(t-\xi(t)) + e_{p_i}u(t), \\ p(t_k) &= Jp(t_k^-), k \in Z^+, t = t_k, i = 1, 2, ...n \end{aligned} \tag{2}$$

Where $b_j(x) = \frac{(x/\beta)^H}{(1+(x/\beta)^H)}$,// $I_i = \sum_{j \in U_i} \gamma_{ij}$ and U_i is the set of all the transcription factors j which is a repressor of gene i. The matrix $V = (V_{ij})$ is the coupling matrix of GRNS, which is defined as follows:

$$V_{ij} = \begin{cases} \gamma_{ij}, if\ \ transcription\ \ factor\ \ j\ \ is\ \ an\ \ activator\ \ of\ \ gene\ \ i, \\ 0, if\ \ there\ \ is\ \ no\ \ link\ \ from\ \ node\ \ j\ \ to\ \ node\ \ i, \\ -\gamma_{ij}, if\ \ transcription\ \ factor\ \ j\ \ is\ \ a\ \ repressor\ \ of\ \ gene\ \ i. \end{cases}$$

Rewriting GRNs(2) in compact form, we have

$$\begin{aligned} \dot{m}(t) &= -Am(t) + Vb(p(t)-\zeta(t)) + I + E_m u(t) \\ m(t_k) &= Lm(t_k^-), k \in Z^+, t = t_k \\ \dot{p}(t) &= -Cp(t) + Dm(t-\xi_t)) + E_p u(t) \\ p(t_k) &= Jp(t_k^-), k \in Z^+, t = t_k \end{aligned} \tag{3}$$

Where

$$\begin{aligned}
m(t) &= Col\{m_1(t), m_2(t), ..., m_n(t))\} \\
p(t) &= Col\{p_1(t), p_2(t), ..., p_n(t))\} \\
A &= diag\{a_1, a_2, ...a_n\} \\
I &= Col\{I_1, I_2, ...I_n\} \\
C &= diag\{c_1, c_2, ...c_n\} \\
D &= diag\{d_1, d_2, ...d_n\} \\
E_m &= Col\{e_{m_1}, e_{m_2}, ...e_{m_n}\} \\
E_p &= Col\{e_{p_1}, e_{p_2}, ...e_{p_n}\} \\
b(p(t)) &= Col\{b_1(p_1(t)), b_2(p_2(t)), ...b_n(p_n(t))\}
\end{aligned}$$

Suppose that (m^*, p^*) is an equilibrium points of GRNS(3) in the disturbance free case.

We shift the equilibrium (m^*, p^*) to the origin by letting $x(t) = m(t) - m^*, y(t) = p(t) - p^*$.

Then we can obtain the following equations:

$$\begin{aligned}
\dot{x}(t) &= -Ax(t) + Vf(y(t - \zeta(t))) + E_m u(t) \\
x(t_k) &= Lx(t_k^-), k \in Z^+, t = t_k \\
\dot{y}(t) &= -Cy(t) + Dx(t - \xi(t))) + E_p u(t) \\
y(t_k) &= Jy(t_k^-), k \in Z^+, t = t_k
\end{aligned} \tag{4}$$

Where

$$\begin{aligned}
x(t) &= Col\{x_1(t), x_2(t), ..., x_n(t)\} \\
y(t) &= Col\{y_1(t), y_2(t), ..., y_n(t)\} \\
f(y(t)) &= b(y(t) + p^*) - b(p^*).
\end{aligned}$$

Note that $g_j(\cdot)$ is a bounded and monotonically increasing function, hence we have

$$0 \leq \frac{f_i(y_i(t))}{y_i(t)} \leq K_i, \forall y_i(t) \neq 0, i = 1, 2, ...n \tag{5}$$

The parameter uncertainties are inevitable in GRNS, due to model errors and the change of environment.

Now, we consider the following uncertain IGRNS;

$$\begin{aligned}
\dot{x}(t) &= -(A + \Delta A(t))x(t) + Vf(y(t) - \zeta(t)) + I + E_m u(t) \\
x(t_k) &= Lx(t_k^-), k \in Z^+, t = t_k \\
\dot{y}(t) &= -(C + \Delta C(t))y(t) + Dx(t - \xi(t)) + E_p u(t) \\
y(t_k) &= Jy(t_k^-), k \in Z^+, t = t_k
\end{aligned} \tag{6}$$

the parameter uncertainties $\Delta A(t)$ and $\Delta C(t)$ satisfy:

$[\Delta A(t), \Delta C(t)] = K_1 M(t)[N_A, N_C]$.

Where K_1, N_A and N_C are some given constants matrices with approximate dimensions, $M(t)$ is the uncertain matrix with Lebesgue measurable elements satisfying $M^T(t)M(t) \leq I, \forall t \geq 0$.

Assumption 1 [11]: Considering the probability distribution of the time- delays $\zeta(t)$ and $\xi(t)$, for some given scalars ζ_0 and ξ_0, two sets of functions are defined as

$$\xi_1(t) = \begin{cases} \xi(t), & for\ t \in \Psi_1, \\ 0, & for\ t \in \Psi_2 \end{cases}$$

$$\xi_2(t) = \begin{cases} 0, & for\ t \in \Psi_1, \\ \xi(t), & for\ t \in \Psi_2 \end{cases}$$

$$\zeta_1(t) = \begin{cases} 0, & for\ t \in \Psi_3, \\ \zeta, & for\ t \in \Psi_4 \end{cases}$$

$$\zeta_1(t) = \begin{cases} \zeta(t), & for\ t \in \Psi_3, \\ 0, & for\ t \in \Psi_4 \end{cases}$$

Where $\Psi_1 = \{t : \xi(t) \in [\xi_m, \xi_0)\}$,
$\Psi_2 = \{t : \xi(t) \in [\xi_0, \xi_M)\}$,
$\Psi_3 = \{t : \zeta(t) \in [\zeta_m, \zeta_0)\}$,
$\Psi_4 = \{t : \zeta(t) \in [\zeta_0, \zeta_M)\}$,

From the definitions of the Ψ_1, Ψ_2, Ψ_3 and Ψ_4, it can be seen that $t \in \Psi_1$ means that the event $\xi(t) \in [\xi_m, \xi_0)$ occurs $t \in \Psi_2$ means that the event $\xi(t) \in [\xi_0, \xi_M)$ occurs $t \in \Psi_3$ means that the event $\zeta(t) \in [\zeta_m, \zeta_0)$ occurs and $t \in \Psi_4$ means that the event occurs $\zeta(t) \in [\zeta_0, \zeta_M)$ occurs. Then the stochastic variables $\alpha(t)$ and $\beta(t)$ can be defined as:

$$\alpha(t) = \begin{cases} 1, t \in \Psi_1 \\ 0, t \in \Psi_2, \end{cases}$$

$$\beta(t) = \begin{cases} 1, t \in \Psi_3 \\ 0, t \in \Psi_4. \end{cases}$$

Assume that $\alpha(t)$ and $\beta(t)$ are Bernoulli distributed sequences with

Prob $\{\alpha(t) = 1\} = E\{\alpha(t)\} = \alpha_0$
Prob $\{\alpha(t) = 0\} = 1 - E\{\alpha(t)\} = 1 - \alpha_0$
Prob $\{\beta(t) = 1\} = E\{\beta(t)\} = \beta_0$
Prob $\{\beta(t) = 0\} = 1 - E\{\beta(t)\} = 1 - \beta_0$,

where $0 \le \alpha_0 \le 1$ and $0 \le \beta_0 \le 1$ are constants, $E\{\alpha(t)\}$ $E\{\beta(t)\}$ are the expectations of $\alpha(t)$ and $\beta(t)$, respectively.

Remark 1:
From the Assumption 1 it is easy to see that,
$E\{\alpha(t)\} = \alpha_0, E\{(\alpha(t) - \alpha_0)^2\} = \alpha_0(1 - \alpha_0)$
$E\{\beta(t)\} = \beta_0, E\{(\beta(t) - \beta_0)^2\} = \beta_0(1 - \beta_0)$

By Assumption 1, the IGRNS (6), can be rewritten as

$$\begin{aligned} \dot{x}(t) &= -(A + \Delta A(t))x(t) + \alpha(t)Vf(y(t - \zeta_1(t))) + (1 - \alpha(t))Vf(y(t - \zeta_2(t))) + E_m u(t) \\ x(t_k) &= Lx(t_k^-), k \in Z^+, t = t_k \\ \dot{y}(t) &= -(C + \Delta C(t))y(t) + \beta(t)Dx(t - \xi_1(t))) + (1 - \beta(t))Dx(t - \xi_2(t)) + E_p u(t) \\ y(t_k) &= Jy(t_k{}^-), k \in Z^+, t = t_k \end{aligned} \tag{7}$$

which is equivalent to

$$\begin{aligned} \dot{x}(t) &= -(A + \Delta A(t))x(t) + \alpha_0 Vf(y(t) - \zeta_1(t))) + (1 - \alpha_0)Vf(y(t - \zeta_2(t))) \\ &\quad +(\alpha(t) - \alpha_0)V[f(y(t - \zeta_1(t))) - f(y(t - \zeta_2(t)))] + E_m u(t) \\ x(t_k) &= Lx(t_k{}^-), k \in Z^+, t = t_k \\ \dot{y}(t) &= -(C + \Delta C(t))y(t) + \beta_0 Dx(t - \xi_1(t)) + (1 - \beta_0)Dx(t - \xi_2(t)) \\ &\quad +(\beta(t) - \beta_0)D[x(t - \xi_1(t)) - x(t - \xi_2(t))] + E_p u(t) \\ y(t_k) &= Jy(t_k{}^-), k \in Z^+, t = t_k \end{aligned} \tag{8}$$

the initial condition of the GRNS (8) are given by

$$\begin{aligned} x_i(t) &= \Psi_i(t) \in C([-\xi, 0], R), \\ y_i(t) &= \Psi_i(t) \in C([-\zeta, 0], R), \end{aligned}$$

where $C([-\xi, 0], \mathbb{R})$ and $C([-\zeta, 0], \mathbb{R})$ denote the set of all continuous functions from $[-\xi, 0]$ and $[-\zeta, 0]$ to $\mathbb{R}$.

Now we define the measurement outputs of the network as follows.

$$Z_x(t) = Qx(t), Z_y(t) = Py(t) \tag{9}$$

where $Z_x(t)$ and $Z_y(t) \in \mathbb{R}^m$ are the measurement outputs of the GRNS(8), Q and P are known constant matrices with appropriate dimensions.

Similar to [13], the measurement outputs have already been sampled before transmitted to the estimator side.

That is

$$\tilde{Z}_x(t) = Z_x(t_k), \tilde{Z}_y(t) = Z_y(t_k), t_k \le t < t_{k+1} \tag{10}$$

Where $\tilde{Z}_x(t)$ and $\tilde{Z}_y(t) \in \mathbb{R}^m$ are the actual inputs of the estimator side, t_k is the updating instant of the zero order hold, $0 = t_0 < t_1 < ... < t_k...$, satisfying $lim_{t\to\infty} t_k = \infty$.

Let the sampling period be $h = t_{k+1} - t_k$. Using the available sampling outputs $\tilde{Z}_x(t)$ and $\tilde{Z}_y(t)$, we can construct the following state estimator:

$$\begin{aligned} \dot{\tilde{x}}(t) &= -A\tilde{x}(t) + R_1[\tilde{Z}_x(t) - Q\tilde{x}(t_k)], \tilde{x}(0) = 0 \\ \dot{\tilde{y}}(t) &= -C\tilde{y}(t) + R_2[\tilde{Z}_y(t) - P\tilde{y}(t_k)], \tilde{y}(0) = 0 \end{aligned} \tag{11}$$

Where $\tilde{x}(t)$ and $\tilde{y}(t)$ are the estimations of $x(t)$ and $y(t)$, R_1 and R_2 are the estimator gain matrices to be determined later.

Defining the error vectors by $e_x(t) = x(t) - \hat{x}(t)$ and $e_y(t) = y(t) - \hat{y}(t)$,

We can get the following error dynamical system from (8) and (11):

$$\begin{aligned} \dot{e}_x(t) &= -Ae_x(t) - R_1 Qe_x(t_k) - \Delta A(t)x(t) + E_m u(t) + \alpha_0 Vf(y(t - \zeta_1(t))) + (1 - \alpha_0) \\ &\quad \times Vf((y(t - \zeta_2(t))) + (\alpha(t) - \alpha_0)V[f(y(t - \zeta_1(t))) - f(y(t - \zeta_2(t)))] \\ \dot{e}_y(t) &= -Ce_y(t) - R_2 Pe_y(t_k) - \Delta C(t)y(t) + E_p u(t) + \beta_0 Dx(t - \xi_1(t))) + (1 - \beta_0) \\ &\quad \times Dx(t - \xi_2(t))) + (\beta(t) - \beta_0)D[x(t - \xi_1(t)) - x(t - \xi_2(t))], t_k \le t < t_{k+1} \end{aligned} \tag{12}$$

Subsequently, we define a function $d(t) = t - t_k$, then the error system (12) can be written as:

$$\begin{aligned} \dot{e_x}(t) &= -Ae_x(t) - R_1 Qe_x(t - d(t)) - \Delta A(t)x(t) + E_m u(t) + \alpha_0 Vf(y(t - \zeta_1(t))) + (1 - \alpha_0) \\ &\quad \times Vf((y(t - \zeta_2(t))) + (\alpha(t) - \alpha_0)V[f(y(t - \zeta_1(t))) - f(y(t - \zeta_2(t)))] \\ \dot{e_y}(t) &= -Ce_y(t) - R_2 Pe_y(t - d(t)) - \Delta C(t)y(t) + E_p u(t) + \beta_0 Dx(t - \xi_1(t))) + (1 - \beta_0) \\ &\quad \times Dx(t - \xi_2(t))) + (\beta(t) - \beta_0)D[x(t - \xi_1(t)) - x(t - \xi_2(t))], t_k \le t < t_{k+1} \end{aligned} \tag{13}$$

where $0 \leq d(t) < h$.

By defining

$$\dot{\bar{x}}(t) = [x^T(t) e_x^{T(t)}]^T$$

$$\dot{\bar{y}}(t) = [y^T(t) e_y^{T(t)}]^T$$

Combining (8) and (13) we get the following augmented system

$$\begin{aligned}
\dot{\bar{x}}(t) &= -\bar{A}\bar{x}(t) - \bar{Q}\bar{x}(t-d(t)) + \alpha_0 \bar{V} f(R\bar{y}(t-\zeta_1(t))) + (1-\alpha_0)\bar{V} f(R\bar{y}(t-\zeta_2(t))) \\
&\quad + \bar{E}_m u(t) + (\alpha(t)-\alpha_0)\bar{V}[f(R\bar{y}(t-\zeta_1(t))) - f(R\bar{y}(t-\zeta_2(t)))] \\
x(t_k) &= Lx(t_k{}^-), k \in Z^+, t = t_k \\
\dot{\bar{y}}(t) &= -\bar{C}\bar{y}(t) - \bar{P}\bar{y}(t-d(t)) + \beta_0 \bar{D}R\bar{x}(t-\xi_1(t)) + (1-\beta_0)\bar{D}R\bar{x}(t-\xi_2(t))) \\
&\quad + \bar{E}_p u(t) + (\beta(t)-\beta_0)\bar{D}[R\bar{x}(t-\xi_1(t)) - x(t-\xi_2(t))] \\
y(t_k) &= Jy(t_k{}^-), k \in Z^+, t = t_k
\end{aligned} \tag{14}$$

where

$$\bar{A} = \begin{pmatrix} A+\Delta A(t) & 0 \\ \Delta A(t) & A \end{pmatrix}, \quad \bar{Q} = \begin{pmatrix} 0 & 0 \\ 0 & R_1 Q \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} C+\Delta C(t) & 0 \\ \Delta C(t) & C \end{pmatrix}, \quad \bar{V} = \begin{pmatrix} V \\ V \end{pmatrix}$$

$$\bar{P} = \begin{pmatrix} 0 & 0 \\ 0 & R_2 P \end{pmatrix}, \quad \bar{D} = \begin{pmatrix} D \\ D \end{pmatrix}, \quad R = \begin{pmatrix} I & 0 \end{pmatrix}, \quad \bar{E_m} = \begin{pmatrix} E_m \\ E_m \end{pmatrix}, \quad \bar{E_p} = \begin{pmatrix} E_p \\ E_p \end{pmatrix}$$

Let us introduce the following definitions and lemmas.

Definition 2.1. For all non- zero $v(t) \in L_2[0, +\infty)$, the error system (13) is said to satisfy a prescribed H_∞ disturbance attenuation level γ, if it satiesfies the following inequality under the zero- initial condition:

$$E\{||e||_2^2\} \leq \gamma^2 ||v_2^2|| \tag{15}$$

where $e(t) = [e_x^T(t) e_y^T(t)]^T$.

Definition 2.2. For a given functional $V : C([\zeta, 0]; R^n) \times \mathbb{R}_+ \to \mathbb{R}$, its infinitesimal operator L is defined as

$LV(x_t, t) = \lim_\Delta \to 0^- \frac{1}{\Delta}[E(V(x_{t+\Delta}, t+\Delta)/(x_t) - V(x_t, t)]$.

Lemma 2.1. (Extended Wirtinger inequality) Let $Z(t) \in V[c, d)$ and $Z(c) = 0$. Then , for any $n \times n$ matrix $H > 0$, the following inequality holds:

$\int_c^d Z^T(s) H Z(s) ds \leq \frac{4(d-c)^2}{\pi^2} \int_c^d \dot{Z}^T(s) R \dot{Z}(s) ds.$

Lemma 2.2. (Lower Bounds Theorem) Let $r_1, r_2, ..., r_N : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset G of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies:

$$\min_{\alpha_i | \alpha_i > 0, \sum \alpha_i = 1} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$g_{i,j} : \mathbb{R}^m \to \mathbb{R}, g_{i,j}(t) = g_{i,j}(t), \begin{pmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{pmatrix} \geq 0.$$

Lemma 2.3. Let E, F and G be real matrices of appropriate dimensions with $G^T G \leq I$. Then for any scalar $\epsilon > 0$,

$EGF + (EGF)^T \leq \epsilon E E^T + \epsilon^{-1} F F^T$.

Main Results: In this section, to obtain the sufficient condition under which the disturbance level $v(t) = 0$ is globally asymptotically stable in the mean - square sense and satisfies the H_∞ performance constraints (15) under the zero-initial condition for all non-zero $v(t)$.

Theorem 2.1. Let the estimator gain matrices R_1 and R_2 be given. For given positive scalars $\varsigma_m, \varsigma_M, \varsigma_0, \xi_m, \xi_M$ and ξ_0, the error system with $v(t) = 0$ is globally asymptotically stable in the mean- square sense, if there exist matrices $S_1 > 0, S_2 > 0, T_j > 0, U_j > 0 (j = 1, 2, ..., 6), V_1 > 0, V_2 > 0$, diagonal matrices $W_1 > 0, W_2 > 0, W_3 > 0$, and any appropriate dimensional matrices $Z_1, Z_2, \widetilde{U_j} (j = 1, 2, ..., 6)$, such that the following LMIs hold:

$$\Phi < 0 \tag{16}$$

$$\begin{pmatrix} U_i & \widetilde{U_i} \\ * & U_i \end{pmatrix} \geq 0 (i = 1, 2, ..., 6) \tag{17}$$

Where $\Phi = [\Phi_{i,j}]_{21\times 21}$ is symmetric, with

$$\begin{aligned}
\Phi_{1,1} &= -S_1\bar{A} - \bar{A}^T S_1 + T_1 + T_2 + T_3 + T_4 - U_3 - \frac{\pi^2}{4}V_1,\\
\Phi_{1,2} &= -S_1\bar{Q} - \widetilde{U}_3^T + U_3 + \frac{\pi^2}{4}V_1,\\
\Phi_{1,3} &= \widetilde{U}_3^T,\ \ \Phi_{1,9} = -\bar{A}^T Z_1^T,\ \ \Phi_{1,20} = \alpha_0 S_1\bar{Q},\\
\Phi_{1,21} &= (1-\alpha_0)S_1\bar{Q},\\
\Phi_{2,2} &= -2U_3 + \widetilde{U}_3 + \widetilde{U}_3^T - \frac{\pi^2}{4}V_1,\\
\Phi_{2,3} &= -\widetilde{U}_3^T + U_3,\\
\Phi_{2,9} &= -\bar{Q}^T Z_1^T,\ \ \Phi_{3,3} = -T_4 - U_3,\\
\Phi_{4,4} &= -U_1 - T_1,\ \ \Phi_{4,5} = -\widetilde{U}_1^T + U_1,\\
\Phi_{4,6} &= \widetilde{U}_1^T,\ \ \Phi_{5,5} = -2U_1 + \widetilde{U}_1 + \widetilde{U}_1^T,\\
\Phi_{5,6} &= -\widetilde{U}_1^T + U_1,\ \ \Phi_{5,10} = \beta_0(\bar{D}R)^T S_2,\\
\Phi_{5,18} &= \beta_0(\bar{D}R)^T {Z_2}^T,\ \ \Phi_{6,6} = -U_1 - U_2 - T_2,\\
\Phi_{6,7} &= -\widetilde{U}_2^T + U_2,\ \ \Phi_{6,8} = \widetilde{U}_2^T,\ \ \Phi_{7,7} = -2U_2 + \widetilde{U}_2 + \widetilde{U}_2^T,\\
\Phi_{7,8} &= -\widetilde{U}_2^T + U_2,\ \ \Phi_{7,10} = (1-\beta_0)(\bar{D}R)^T S_2,\\
\Phi_{7,18} &= (1-\beta_0)(\bar{D}R)^T Z_2^T,\ \ \Phi_{8,8} = -U_2 - T_3,\\
\Phi_{9,9} &= -Z_1 - Z_1^T + \xi_1^2 U_1 + \xi^2 U_2 + g^2 U_3 + g^2 V_1,\\
\Phi_{9,10} &= \alpha_0 Z_1\bar{Q},\\
\Phi_{9,11} &= (1-\alpha_0)Z_1\bar{Q},\\
\Phi_{10,10} &= -S_2\bar{C} - \bar{C}^T S_2 + T_5 + T_6 + T_7 + T_8 - U_6 - \frac{\pi^2}{4}V_2,\\
\Phi_{10,11} &= -S_2\bar{P} - \widetilde{U}_6^T + U_6 + \frac{\pi^2}{4}V_2,\\
\Phi_{10,12} &= \widetilde{U}_6^T,\ \ \Phi_{10,18} = -\bar{C}^T Z_2^T,\\
\Phi_{10,19} &= R^T K W_1,\ \ \Phi_{11,11} = -2U_6 + \widetilde{U}_6 + \widetilde{U}_6^T - \frac{\pi^2}{4}V_2,\\
\Phi_{11,12} &= -\widetilde{U}_6^T + U_6,\ \ \Phi_{11,18} = -\bar{P}^T + Z_2^T,\ \ \Phi_{12,12} = -T_8 - U_6,\\
\Phi_{13,13} &= -U_4 - T_5,\ \ \Phi_{13,14} = -\widetilde{U}_4^T + U_4,\\
\Phi_{13,15} &= \widetilde{U}_4^T,\ \ \Phi_{14,14} = -2U_4 + \widetilde{U}_4 + \widetilde{U}_4^T,\\
\Phi_{14,15} &= \widetilde{U}_4^T + U_4,\ \ \Phi_{14,20} = (R)^T K W_2,\\
\Phi_{15,15} &= -U_4 - U_5 - U_6,\ \ \Phi_{15,16} = \widetilde{U}_5^T + U_5,\\
\Phi_{15,17} &= \widetilde{U}_5^T,\ \ \Phi_{16,16} = -2U_5 + \widetilde{U}_5 + \widetilde{U}_5^T,\\
\Phi_{16,17} &= -\widetilde{U}_5^T + U_5,\ \ \Phi_{16,21} = R^T K W_3,\\
\Phi_{17,17} &= -U_5 - T_7,\\
\Phi_{18,18} &= -Z_2 - Z_2^T - {\xi_1}^2 U_4 + {\xi_2}^2 U_5 + g^2 U_6 + g^2 V_2,\\
\Phi_{19,19} &= -2W,\ \ \Phi_{20,20} = -2W_2,\ \ \Phi_{21,21} = -2W_3,\\
\xi_1 &= \xi_0 - \xi_m,\ \ \xi_2 = \xi_M - \xi_0,\ \ \varsigma_1 = \varsigma_0 - \varsigma_m,\ \ \varsigma_2 = \varsigma_M - \varsigma_0,
\end{aligned}$$

and the rest terms of Φ are zero.

Proof: We choose the following discontinuous Lyapunov functional to prove our results

$$V(t, \bar{x}_t, \bar{y}_t) = \sum_{i=1}^{4} V_i(t, \bar{x}_t, \bar{y}_t) \tag{18}$$

Where

$$\begin{aligned}
V_1(t,\bar{x}_t,\bar{y}_t) &= \bar{x}^T(t)S_1x(t)+\bar{y}^T(t)S_2y(t)\\
V_2(t,\bar{x}_t,\bar{y}_t) &= \int_{t-\xi_m}^{t}\bar{x}^T(s)T_1\bar{x}(s)ds+\int_{t-\xi_0}^{t}\bar{x}^T(s)T_2\bar{x}(s)ds+\int_{t-\xi_M}^{t}\bar{x}^T(s)T_3\bar{x}(s)ds+\int_{t-g}^{t}\bar{x}^T(s)T_4\bar{x}(s)ds\\
&\quad+\int_{t-\zeta_m}^{t}\bar{y}^T(s)T_5\bar{y}(s)ds+\int_{t-\zeta_0}^{t}\bar{y}^T(s)T_6\bar{y}(s)ds+\int_{t-\zeta_M}^{t}\bar{y}^T(s)T_7\bar{y}(s)ds+\int_{t-g}^{t}\bar{y}^T(s)T_8\bar{y}(s)ds,\\
V_3(t,\bar{x}_t,\bar{y}_t) &= \xi_1\int_{-\xi_0}^{-\xi_m}\int_{t+\theta}^{t}\dot{\bar{x}}^T(s)U_1\dot{\bar{x}}(s)dsd\theta+\xi_2\int_{-\xi_M}^{-\xi_0}\int_{t+\theta}^{t}\dot{\bar{x}}^T(s)U_2\dot{\bar{x}}(s)dsd\theta+g\int_{-g}^{0}\int_{t+\theta}^{t}\dot{\bar{x}}^T(s)U_3\dot{\bar{x}}(s)dsd\theta\\
&\quad+\xi_1\int_{-\xi_0}^{-\xi_m}\int_{t+\theta}^{t}\dot{\bar{y}}^T(s)U_4\dot{\bar{y}}(s)dsd\theta+\xi_2\int_{-\xi_M}^{-\xi_0}\int_{t+\theta}^{t}\dot{\bar{y}}^T(s)U_5\dot{\bar{y}}(s)dsd\theta+g\int_{-g}^{0}\int_{t+\theta}^{t}\dot{\bar{y}}^T(s)U_6\dot{\bar{y}}(s)dsd\theta,\\
V_4(t,\bar{x}_t,\bar{y}_t) &= g^2\int_{t_k}^{t}\dot{\bar{x}}^T(s)V_1\dot{\bar{x}}(s)ds+g^2\int_{t_k}^{t}\dot{\bar{y}}^T(s)V_2\dot{\bar{y}}(s)ds-\frac{\pi^4}{4}\int_{t_k}^{t}(\bar{x}(s)-\bar{x}(t_k))^TV_1(\bar{x}(s)-\bar{x}(t_k))ds\\
&\quad-\frac{\pi^4}{4}\int_{t_k}^{t}(\bar{y}(s)-\bar{y}(t_k))^TV_2(\bar{y}(s)-\bar{y}(t_k))ds
\end{aligned}$$

According to Lemma 1, it is easy to see that $V_4(t)\geq 0$.
Furthermore, $V_4(t)$ vanishes at $t=t_k$. So we have $\lim\limits_{t\to t_k^-}V(t)\geq V(t_k)$.

Differentiating equation (18) and taking expectation on it, we get

$$E\{LV(t,\bar{x}_t,\bar{y}_t)\} = \sum_{i=1}^{4}E\{V_i(t,\bar{x_t}),\bar{y}_t\} \tag{19}$$

$$E\{LV_1(t,\bar{x}_t,\bar{y}_t)\} = 2\bar{x}^T(t)S_1\dot{\bar{x}}(t)+2\bar{y}^T(t)S_2\dot{\bar{y}}(t) \tag{20}$$

$$\begin{aligned}
E\{LV_2(t,\bar{x}_t,\bar{y}_t)\} &= \bar{x}_t^T(T_1+T_2+T_3+T_4)\bar{x}_t+\bar{y}_t^T(T_1+T_2+T_3+T_4)\bar{y}_t\\
&\quad-\bar{x}_t^T(t-\xi_m)T_1\bar{x}(t-\xi_m)-\bar{x}_t^T(t-\xi_0)T_2\bar{x}(t-\xi_0)\\
&\quad-\bar{x}_t^T(t-\xi_M)T_3\bar{x}(t-\xi_M)-\bar{x}_t^T(t-g)T_4\bar{x}(t-g)\\
&\quad-\bar{y}_t^T(t-\zeta_m)T_5\bar{y}(t-\zeta_m)-\bar{y}_t^T(t-\zeta_0)T_6\bar{y}(t-\zeta_0)\\
&\quad-\bar{y}_t^T(t-\zeta_M)T_7\bar{y}(t-\zeta_M)-\bar{y}_t^T(t-g)T_8\bar{y}(t-g)
\end{aligned} \tag{21}$$

$$\begin{aligned}
E\{LV_3(t,\bar{x}_t,\bar{y}_t)\} &= \dot{\bar{x}}^T(t)(\xi_1^2U_1+\xi_2^2U_2+g^2U_3)\dot{\bar{x}}(t)+\dot{\bar{y}}^T(t)(\zeta_1^2U_4+\zeta_2^2U_5+g^2U_6)\dot{\bar{y}}(t)\\
&\quad-\xi_1\int_{t-\xi_0}^{t-\xi_m}\dot{\bar{x}}^T(s)U_1\dot{\bar{x}}(s)ds-\xi_2\int_{t-\xi_M}^{t-\xi_0}\dot{\bar{x}}^T(s)U_2\dot{\bar{x}}(s)ds-g\int_{t-g}^{t}\dot{\bar{x}}^T(s)U_3\dot{\bar{x}}(s)ds\\
&\quad-\zeta_1\int_{t-\zeta_0}^{t-\zeta_m}\dot{\bar{y}}^T(s)U_4\dot{\bar{y}}(s)ds-\zeta_2\int_{t-\zeta_M}^{t-\zeta_0}\dot{\bar{y}}^T(s)U_5\dot{\bar{y}}(s)ds-g\int_{t-g}^{t}\dot{\bar{y}}^T(s)U_6\dot{\bar{y}}(s)ds
\end{aligned} \tag{22}$$

$$\begin{aligned}
E\{LV_4(t,\bar{x}_t,\bar{y}_t)\} &= g^2\dot{\bar{x}}^T(t)V_1\dot{\bar{x}}(t)+g^2\dot{\bar{y}}^T(t)V_2\dot{\bar{y}}(t)\\
&\quad-\frac{\pi^4}{4}\begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-dt)\end{bmatrix}^T\begin{bmatrix}V_1 & -V_1\\ * & V_1\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-dt)\end{bmatrix}\\
&\quad-\frac{\pi^4}{4}\begin{bmatrix}\bar{y}(t)\\ \bar{y}(t-dt)\end{bmatrix}^T\begin{bmatrix}V_2 & -V_2\\ * & V_2\end{bmatrix}\begin{bmatrix}\bar{y}(t)\\ \bar{y}(t-dt)\end{bmatrix}
\end{aligned} \tag{23}$$

It follows that Lemma $*$ that we can obtain

$$\begin{aligned} -\xi_1 \int_{t-\xi_0}^{t-\xi_m} \dot{\bar{x}}^T(s) U_1 \dot{\bar{x}}(s) ds \quad &\leq \quad - \begin{bmatrix} \bar{x}(t-\xi_1(t)) - \bar{x}(t-\xi_0) \\ \bar{x}(t-\xi_m) - \bar{x}(t-\xi_1(t)) \end{bmatrix}^T \begin{bmatrix} U_1 & \tilde{U}_1 \\ * & U_1 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{x}(t-\xi_1(t)) - \bar{x}(t-\xi_0) \\ \bar{x}(t-\xi_m) - \bar{x}(t-\xi_1(t)) \end{bmatrix} \end{aligned} \tag{24}$$

Similar to (24), we have

$$\begin{aligned} -\xi_2 \int_{t-\xi_M}^{t-\xi_0} \dot{\bar{x}}^T(s) U_2 \dot{\bar{x}}(s) ds \quad &\leq \quad - \begin{bmatrix} \bar{x}(t-\xi_2(t)) - \bar{x}(t-\xi_M) \\ \bar{x}(t-\xi_0) - \bar{x}(t-\xi_2(t)) \end{bmatrix}^T \begin{bmatrix} U_2 & \tilde{U}_2 \\ * & U_2 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{x}(t-\xi_2(t)) - \bar{x}(t-\xi_M) \\ \bar{x}(t-\xi_0) - \bar{x}(t-\xi_2(t)) \end{bmatrix} - g \int_{t-g}^{t} \dot{\bar{x}}^T(s) U_3 \dot{\bar{x}}(s) ds \end{aligned} \tag{25}$$

$$\begin{aligned} &\leq \quad - \begin{bmatrix} \bar{x}(t-d(t)) - \bar{x}(t-g) \\ \bar{x}(t) - \bar{x}(t-d(t)) \end{bmatrix}^T \begin{bmatrix} U_3 & \tilde{U}_3 \\ * & U_3 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{x}(t-d(t)) - \bar{x}(t-g) \\ \bar{x}(t) - \bar{x}(t-d(t)) \end{bmatrix} - \zeta_1 \int_{t-\zeta_0}^{t-\zeta_m} \dot{\bar{y}}^T(s) U_4 \dot{\bar{y}}(s) ds \end{aligned} \tag{26}$$

$$\begin{aligned} &\leq \quad - \begin{bmatrix} \bar{y}(t-\zeta_1(t)) - \bar{y}(t-\zeta_0) \\ \bar{y}(t-\zeta_m) - \bar{y}(t-\zeta_1(t)) \end{bmatrix}^T \begin{bmatrix} U_4 & \tilde{U}_4 \\ * & U_4 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{y}(t-\zeta_1(t)) - \bar{y}(t-\zeta_0) \\ \bar{y}(t-\zeta_m) - \bar{y}(t-\zeta_1(t)) \end{bmatrix} - \zeta_2 \int_{t-\zeta_M}^{t-\zeta_0} \dot{\bar{y}}^T(s) U_5 \dot{\bar{y}}(s) ds \end{aligned} \tag{27}$$

$$\begin{aligned} &\leq \quad - \begin{bmatrix} \bar{y}(t-\zeta_2(t)) - \bar{y}(t-\zeta_M) \\ \bar{y}(t-\zeta_0) - \bar{y}(t-\zeta_2(t)) \end{bmatrix}^T \begin{bmatrix} U_5 & \tilde{U}_5 \\ * & U_5 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{y}(t-\zeta_2(t)) - \bar{y}(t-\zeta_M) \\ \bar{y}(t-\zeta_0) - \bar{y}(t-\zeta_2(t)) \end{bmatrix} - g \int_{t-g}^{t} \dot{\bar{y}}^T(s) U_6 \dot{\bar{y}}(s) ds \end{aligned} \tag{28}$$

$$\begin{aligned} &\leq \quad - \begin{bmatrix} \bar{y}(t-d(t)) - \bar{y}(t-g) \\ \bar{y}(t) - \bar{y}(t-d(t)) \end{bmatrix}^T \begin{bmatrix} U_6 & \tilde{U}_6 \\ * & U_6 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{y}(t-d(t)) - \bar{y}(t-g) \\ \bar{y}(t) - \bar{y}(t-d(t)) \end{bmatrix} \end{aligned} \tag{29}$$

Besides from (5), we obtain

$$\begin{aligned} f_i(y_i(t)) \left[f_i(y_i(t)) - k_i y_i(t) \right] \quad &\leq \quad 0 \\ f_i(y_i(t-\zeta_1(t))) \left[f_i(y_i(t-\zeta_1(t))) - k_i y_i(t-\zeta_1(t)) \right] \quad &\leq \quad 0, \\ f_i(y_i(t-\zeta_2(t))) \left[f_i(y_i(t-\zeta_2(t))) - k_i y_i(t-\zeta_2(t)) \right] \quad &\leq \quad 0, \end{aligned}$$

where $i = 1, 2, ...n$ Hence for the diagonal matrices $W_1 > 0, W_2 > 0$ and $W_3 > 0$, the following inequalities hold.

$$2(R\bar{y}(t))^T K W_1 f(R\bar{y}(t)) - 2f^T(R\bar{y}(t) W_1 f(R\bar{y}(t)) \quad \geq \quad 0 \tag{30}$$

$$2(R\bar{y}(t-\zeta_1(t)))^T K W_2 f(R\bar{y}(t-\zeta_1(t))) - 2f^T(R\bar{y}(t-\zeta_1(t)) W_2 f(R\bar{y}(t-\zeta_1(t)) \quad \geq \quad 0 \tag{31}$$

$$2(R\bar{y}(t-\zeta_2(t)))^T K W_3 f(R\bar{y}(t-\zeta_2(t))) - 2f^T(R\bar{y}(t-\zeta_2(t)) W_3 f(R\bar{y}(t-\zeta_2(t)) \quad \geq \quad 0 \tag{32}$$

Where $K = diag\{k_1, k_2, ...k_n\}$.

It is also known that the following equations are always true

$$\begin{aligned} E\{2\dot{\bar{x}}^T(t)Z_1[-\dot{\bar{x}}(t)-\bar{A}\bar{x}(t)-\bar{Q}\bar{x}(t-d(t))+\alpha_0\bar{V}f(R\bar{y}(t-\zeta_1(t))) \\ +(1-\alpha_0)\bar{V}f(R\bar{(y)}(t-\zeta_2(t)))+(\alpha(t)-\alpha_0)\bar{V}f(R\bar{y}(t-\zeta_1(t)))-f(R\bar{y}(t-\zeta_2(t)))]\} &= 0 \quad (33) \\ E\{2\dot{\bar{y}}^T(t)Z_2[-\dot{\bar{y}}(t)-\bar{C}\bar{y}(t)-\bar{P}\bar{y}(t-dt)+\beta_0\bar{D}R\bar{x}(t-\xi_1(t)) \\ +(1-\beta_0)\bar{D}R\bar{x}(t-\xi_2(t))+(\beta(t)-\beta_0)\bar{D}[R\bar{x}(t-\xi_1(t))-Rx(t-\xi_2(t))]\} &= 0 \quad (34) \end{aligned}$$

By combining (18)-(34) and setting

$$\begin{aligned} \Psi(t) = Col\{&\bar{x}(t),\bar{x}(t-d(t)),\bar{x}(t-g),\bar{x}(t-\xi_m),\bar{x}(t-\xi_1(t)),\bar{x}(t-\xi_0),\bar{x}(t-\xi_2(t)),\bar{x}(t-\xi_M), \\ &\dot{\bar{x}}(t),\bar{y}(t),\bar{y}(t-d(t)),\bar{y}(t-g),\bar{y}(t-\zeta_m),\bar{y}(t-\zeta_1(t)),\bar{y}(t-\zeta_0),\bar{y}(t-\zeta_2(t)),\bar{y}(t-\zeta_M), \\ &\dot{\bar{y}}(t),f(R\bar{y}(t)),f(R\bar{y}(t-\zeta_1(t))),f(R\bar{y}(t)-\zeta_2(t))\}, \end{aligned}$$

We obtain,

$$E\{V(t,\bar{x}_t,\bar{y}_t)\} \leq \Psi^T(t)\Phi\Psi(t), \quad (35)$$

Where Φ is defined in (16).
Obviously, the inequalities (16) and (35) imply $E\{V(t,\bar{x}_t,\bar{y}_t)\}<0$.
Then, from the Lyapunov stability theorem , it is easy to find that the error system with $v(t)=0$ is globally asymptotically stable in the mean square sense.
On the other hand from the conditions of equation (18) and theorem (1), we note that,

$$\begin{aligned} V_1(t_k,x(t_k),j)-V_1(t_k^-,\bar{x}(t_k^-),i) &= x_m^T(t_k)L_jx_m(t_k)-x_m^T(t_k^-)L_ix_m(t_k^-) \quad (36) \\ &= x_m^T(t_k^-)D_{ik}^TL_jD_{ik}x_m(t_k^-)-x_m^T(t_k^-)L_ix_m(t_k^-) \\ &= x_m^T(t_k^-)(D_{ik}^TL_jD_{ik}-L_i)x_m(t_k^-) \\ &\leq 0 \\ V_1(t_k,x_m(t_k),j) &\leq V_1(t_k^-,x_m(t_k^-),i), k\in Z_+. \quad (37) \end{aligned}$$

This proves that the system (14) with impulsive effect is asymptotically stable in the mean square.
Hence the proof.

3. A Numerical Example

In this section, a numerical example is given to demonstrate the effectiveness of the proposed method for estimating concentrations of the GRNs.
Example 1. Considering delayed GRNs (8), the parameters are given as follows.

$$A=\begin{pmatrix}0.4 & 0\\ 0 & 0.1\end{pmatrix},\quad V=\begin{pmatrix}-0.6 & 0.2\\ 0.2 & -0.7\end{pmatrix},\quad C=\begin{pmatrix}0.1 & 0\\ 0 & 0.3\end{pmatrix},\quad D=\begin{pmatrix}0.2 & 0\\ 0 & 0.7\end{pmatrix}$$

$$E_m=\begin{pmatrix}0.46 & 0\\ 0 & 0.5\end{pmatrix},\quad E_p=\begin{pmatrix}0.5 & 0\\ 0 & 0.9\end{pmatrix},\quad E_1=\begin{pmatrix}0.1 & -0.1\\ 0.1 & 0.3\end{pmatrix}$$

Let $f(y)=\frac{y^2}{1+y^2}$ is taken as the regulatory function. It can be easily checked that the derivative of $f(y)$ is less than 0.65. Assume that time-delays $\xi(t)\in[0.3,0.6]$ and $\zeta(t)\in[0.4,0.1]$ are not continuously differentiable, suppose $\xi_0=0.5,\zeta_0=0.2,\alpha_0=0.4$ and $\beta_0=0.2$. By Theorem 2.6, we can obtain the following feasible parameters. Due to space consideration, we only provide a part of the feasible solutions here.

$$Q=\begin{pmatrix}0.0231 & -0.0306\\ -0.0306 & 0.1218\end{pmatrix},\quad P=\begin{pmatrix}0.0463 & -0.1003\\ -0.1003 & 0.0243\end{pmatrix}$$

Let $h=0.9$ and $\gamma=1$. Then, we can obtain that $\xi_m=0.1,\xi_M=0.6,\zeta_m=0.2$ and $\zeta_M=0.4$.

$$R_1=\begin{pmatrix}0.5440 & 0.0003\\ 0.0003 & 1.7873\end{pmatrix},\quad R_2=\begin{pmatrix}0.0050 & 0.0000\\ 0.0000 & 0.0081\end{pmatrix}.$$

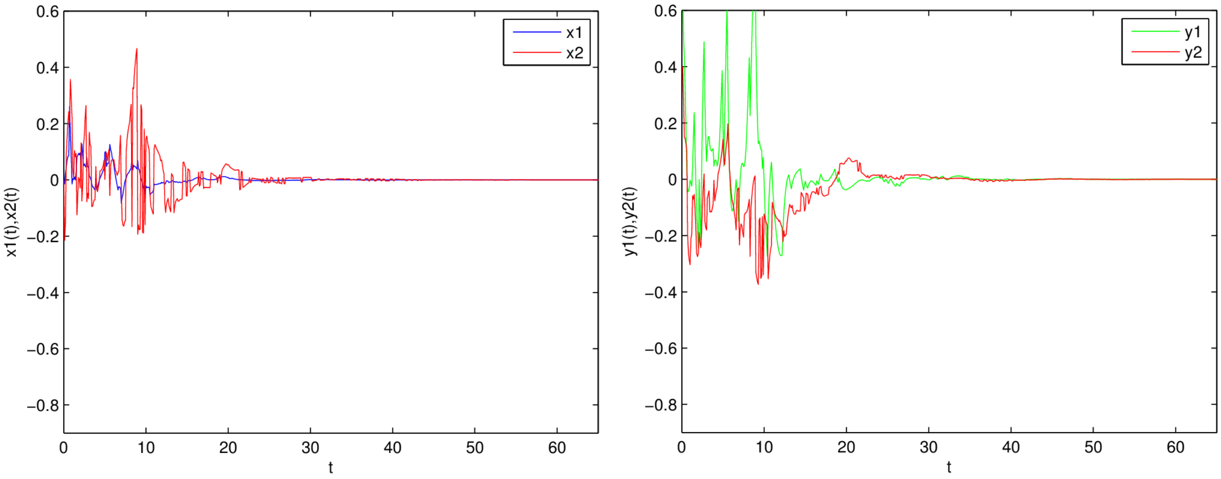

Figure 1. *mRNA and Protein concentrations with impulsive effects.*

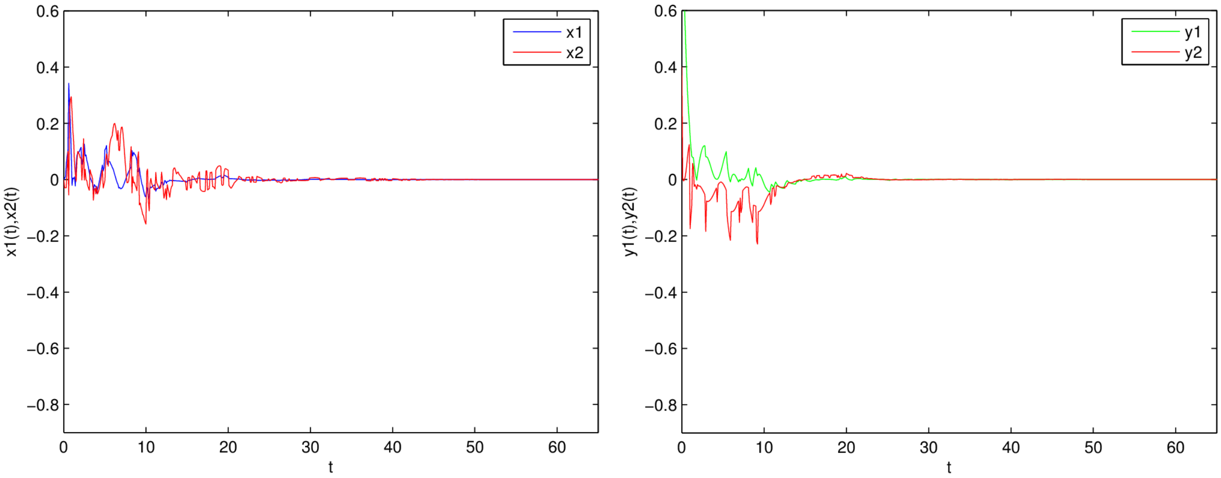

Figure 2. *mRNA and Protein concentrations without impulsive effects.*

From Theorem 2.6, one can conclude that the continuous-time GRNs (7) with impulsive effects are globally asymptotically stable. The concentrations of mRNAs and proteins with impulsive effects are illustrated in Figure 1 with the initial conditions $x(0) = [0.01 \;\; 0.02]^T$, $y(0) = [0.1 \;\; 0.2]^T$ and the concentrations of mRNAs and proteins without impulsive effects are illustrated in Figure 2 with the initial conditions $x(0) = [0.01 \;\; 0.1]^T$ and $y(0) = [0.1 \;\; 0.3]^T$.

4. Conclusion

In this paper, we have investigated the robust sampled- data H_∞ state estimation problem for continuous- time impulsive genetic regulatory networks subject to random delays. By constructing a discontinuous Lyapunov-Krasovskii functional, sufficient stability analysis has been rooted in terms of LMIs. By applying Wirtinger inequality technique, conservation of the impulsive GRNs system is globally asymptotically stable in the mean- square sense have been diminished greatly. Eventually, a numerical example is given to the feasibility and advantages of the developed results.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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