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Research Article

Mathematical Modeling of Reaction Diffusion Equation for Facilitated Emigration of Planar Electrode in a Non-Linear Process at Electroactive Polymer Film

Uma Andiappan, Swaminathan Rajagopal*

PG & Research Department of Mathematics, Vidhyaa Giri College of Arts and Science (Affiliated to Alagappa University), Puduvayal, India

Abstract

In this article, we propose a simple and effective methods to resolve the reaction diffusion equation for facilitated emigration of planar electrode in a steady state non-linear process that arises in the context of the electroactive polymer film. The mathematical modeling presented here suggest a substrate and an immobilized catalyst form a complex. By applying the two effective analytical approach namely Homotopy Analysis Method and Exp-Function Method, an approximate analytical expression for the substrate concentration for planar electrode is established. Moreover, the analytical approach of the current for the experimental outcomes is established. The efficiency of the methods is demonstrated by contrasting the numerical simulation with the Analytical findings. The derived analytical outcomes are compared with numerical data which is obtained by using Matlab software and it is transpires that they correspond adequately. Also the comparison of computational outcomes with dimensionless concentration of planar electrode substrate in its analytical representation established in table. In these table results depicts for different amount of reaction and diffusion parameters our new result agree rather well with the numerical findings. The error percentage of our results employing Homotopy Analysis Method and Exp-Function Method with numerical results presented. The solution is also graphically presented. It provides a satisfactory agreement for all parameter setting under comparison.

Keywordsk

Mathematical Modeling, Nonlinear Differential Equation, Reaction Diffusion Equation, Electroactive Polymer Film, Homotopy Analysis Method, Exp-Function Method

1. Introduction

Over the last two decades, interest has risen in innovative materials that alter their structure and mechanical behavior in response to outside non-mechanical stimulation. Electroactive polymers are prominent representatives of intelligent materials because they respond to an electro field's excitation by deforming in bulk and altering their material behavior. Electrode surfaces modified with electroactive polymer films are frequently employed in electrocatalysis and chemical sensor technologies. In the past 20 years, several simplified mathematical models that explain electrocatalysis in electroactive

*Corresponding author: swaminathanmath@gmail.com (Swaminathan Rajagopal)

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polymer film have been developed [1-5]. Nonlinear reaction-diffusion equations are frequently created and solved during the inquiry, leading to the establishment of empirical estimation for the current amperometric response. Due to the simulation of the reaction-diffusion process in a finite diffusion space, the analysis is exceedingly tricky, where Michaeli's Menten Equation gives the chemical reaction term. When formulating the differential equation, including the chemical reaction component alongside the Fick diffusion term frequently generates a nonlinear form that is difficult to solve using conventional analytical techniques. This technique often occurs when a complicated rate expression represents the interaction between the substrate and the active sensing catalysis. As a result, approximations of solutions must be found. On polymer-modified electrodes, significant progress has been achieved in simulating current response at a stable state arising from mediated electrocatalysis.

Many valuable summaries of recent developments in this field was published by numerous scientists [6-16]. Dharmalingam et al [17]. used Akbari Ganji Method to found the analytical expression of electroactive polymer film. Rajendran et al. [18] obtained the analytical solution of substrate concentration using Taylor Series Method. The analytical study to find the concentration of various electrodes geometries employing hyperbolic function method was established by Rekha et al. [19] In this work, we present approximate analytical formulation for the substrate concentration in the case of planar electrode at electroactive polymer film by applying Homotopy Analysis Method and Exp-Function Method. The issue is then numerically stimulated and then the solution is also graphically presented by using Matlab Software.

2. Mathematical Modeling

The detailed discussion of the model has already been explained [20]. So, we only briefly describe the model used here. The mechanism by which the substrate and the sensing element interact, along with the substrate's diffusion in the Nernst diffusion phase and the polymer matrix, are all considered in the model.

We presume the catalyst and substrate will react using Michaeli's Menten kinetics, as shown in Ref. [20]

$$A + B \overset{\mathcal{K}_M}{\longrightarrow} [AB] \rightarrow [QB'] \overset{k_C}{\rightarrow} Q + B' \tag{1}$$

$$B' \overset{k'_E}{\rightarrow} B$$

Where B and $\mathcal{C}'$ – The catalytically active form of immobilized catalyst, A-Substrate, Q -Product, $[AB]$-enzyme substrate complex, $[QB']$ – product enzyme complex, $\mathcal{K}_M$-Michaeli's Menten constant, k_B – Catalytic constant.

The diffusion coefficients of a substrate in the Nernst diffusion layer and in the film are denoted by $\mathfrak{D}_A$ and $\mathfrak{D}'_A$ respectively. The coefficient of partition and the substrate's partitioning through the polymer film are assume to be accurate. The rate of the reactions takes on the Michaeli's Menten from. Using the non-dimensional parameters, the limiting equation for reaction diffusion is expressed as,

$$\frac{d\mathcal{P}^2(\zeta)}{d\zeta^2} + \frac{a}{\mu}\frac{d\mathcal{P}(\zeta)}{d\zeta} - \frac{\lambda\mathcal{P}(\zeta)}{1+\delta\mathcal{P}(\zeta)} = 0 \tag{2}$$

Subject to the boundary conditions

$$\frac{d\mathcal{P}(\zeta)}{d\zeta} = 0 \text{ when } \zeta = 0$$

$$\mathcal{P}(\zeta) = 1, \text{ when } \zeta = 1$$

The normalized current reaction can be represented as

$$\Omega = \frac{\xi}{n\mathfrak{D}_A\mathcal{K}_M}$$

The dimensionless parameters are,

$$\mathcal{P} = \frac{A}{\mathcal{K}A^{\infty}};\ \zeta = \frac{\mu}{\xi};\ \delta = \frac{\mathcal{K}A^{\infty}}{\mathcal{K}_M};\ \lambda = \frac{k\xi^2}{\mathfrak{D}_A} \tag{3}$$

Where P – non-dimensional concentration of substrate, ζ – dimensionless distance, λ- Non-dimensional diffusion- parameter, δ-dimensionless saturation, a – number of electrodes involved in a charge transfer, μ -distance from electrode, ξ -uniform thickness film, A^{∞}- Bulk substrate concentration. In case of a planar, cylindrical and spherical electrode, the value of a is zero, one and two.

The planar electrode equation can be formulated as follows according to steady state scenarios,

$$\frac{d\mathcal{P}^2(\zeta)}{d\zeta^2} - \frac{\lambda\mathcal{P}(\zeta)}{1+\delta\mathcal{P}(\zeta)} = 0 \tag{4}$$

with the boundary conditions

$$\frac{d\mathcal{P}(0)}{d\zeta} = 0 \text{ and } \mathcal{P}(1) = 1 \tag{5}$$

The non-dimensional current written as

$$\Omega = \tau\left(\frac{d\mathcal{P}(\zeta)}{d\zeta}\right)_{\zeta=1} \tag{6}$$

3. Analytical Formulation of the Concentration

Two various methods namely Homotopy Analysis Method and Exp-Function Method are used to derive the analytical expression for the concentration of substrate of planar electrode.

3.1. Homotopy Analysis Method

Nonlinear equations are solved by numerous methods

Homotopy perturbation Method [21, 22], Akbari Ganji Method [23, 24], Variational Iteration Method etc., One efficient algebraic technique, Homotopy Analysis Method, can approximate semi analytic solutions to reaction diffusion nonlinear equations. It was first developed by Liao Shijun in 1992. It is an easy technique to verify and implement the convergence of a several solutions using the convergent control parameter. More Application of Homotopy Analysis Method can be phrased in numerous articles [25-30].

By Homotopy Analysis Method, we first construct the zeroth order deformation equation by taking H(ζ) = 1. We build the homotopy as follows

$$(1-p)\ \frac{d\mathcal{P}^2(\zeta)}{d\zeta^2} = ph[\frac{d\mathcal{P}^2(\zeta)}{d\zeta^2} - \lambda\left(\delta+\frac{1}{P}\right)^{-1}] \tag{7}$$

This equation (7) has an approximate solution

$$\frac{d\mathcal{P}(\zeta)}{d\zeta} = \frac{d\mathcal{P}_0(\zeta)}{d\zeta} + p\frac{d\mathcal{P}_1(\zeta)}{d\zeta} + p^2\frac{d\mathcal{P}_2(\zeta)}{d\zeta} + \ldots \tag{8}$$

Substituting (8) in (7)

$$(1-p)\left[\frac{d\mathcal{P}_0{}^2(\zeta)}{d\zeta^2} + p\frac{d\mathcal{P}_1{}^2(\zeta)}{d\zeta^2} + p^2\frac{d\mathcal{P}_2{}^2(\zeta)}{d\zeta^2} + \ldots\right] = ph[\frac{d\mathcal{P}_0{}^2(\zeta)}{d\zeta^2} + p\frac{d\mathcal{P}_1{}^2(\zeta)}{d\zeta^2} + p^2\frac{d\mathcal{P}_2{}^2(\zeta)}{d\zeta^2} + \ldots) - \lambda\left(\delta+\frac{1}{\mathcal{P}_0+p\mathcal{P}_1+p^2\mathcal{P}_2+\ldots}\right)^{-1}] \tag{9}$$

Comparing the coefficients of various powers of p in (9), we get

$$p^0 : \frac{d\mathcal{P}_0{}^2(\zeta)}{d\zeta^2} = 0 \tag{10}$$

$$p^1 : \frac{d\mathcal{P}_1{}^2(\zeta)}{d\zeta^2} = \frac{d\mathcal{P}_0{}^2(\zeta)}{d\zeta^2}(h+1) - h[\lambda(\left(\delta+\frac{1}{\mathcal{P}_0}\right)^{-1}] \tag{11}$$

Solving equation (10)

$$\frac{d\mathcal{P}_0(\zeta)}{d\zeta} = \alpha_1\zeta + \alpha_2 \tag{12}$$

Where α_1 and α_2 are constants.
Using the boundary conditions (5), we get

$$\alpha_1 = 0\ and\ \alpha_2 = 1 \tag{13}$$

Substitute (13) in (12), we get $\mathcal{P}_0(\zeta) = 1$
Solving equation (11), we get

$$\frac{d\mathcal{P}_1(\zeta)}{d\zeta} = -h\lambda[\frac{\zeta}{(1+\delta)}] + c_1\zeta \tag{14}$$

$$\mathcal{P}_1(\zeta) = -h\lambda[\frac{\zeta^2}{2(1+\delta)}] + c_1\zeta + c_2 \tag{15}$$

where c_1 and c_2 are constants.

Under the boundary conditions (5), we get

$$c_1 = 0\ and\ c_2 = 1 + \frac{h\lambda}{2(1+\delta)} \tag{16}$$

Equation (15) becomes

$$\mathcal{P}_1(\zeta) = \frac{h\lambda}{2(1+\delta)}(1-\zeta^2) \tag{17}$$

According to Homotopy Analysis Method, the analytical expression of the concentration of the substrate as

$$\mathcal{P}(\zeta) = \mathcal{P}_0 + \mathcal{P}_1 = 1 + \frac{h\lambda}{2(1+\delta)}(1-\zeta^2) \tag{18}$$

The analytical expression of steady state current solved by using Homotopy Analysis Method as,

$$\Omega = \tau(\frac{-h\lambda}{(1+\delta)}) \tag{19}$$

3.2. Exp-Function Method

Exp-Function Method is more effective and simpler than other methods and a numerous solution can be found in the same time [31, 32]. We are able to discover more general answers in the Exp- Function which is more universal than the sinh and tanh-function, so we can find more general solutions in Exp-Function method.

According to Exp-Function Method, the trial solution of (4) is in the form as,

$$\mathcal{P}(\zeta) = C_1e^{\omega\zeta} + C_2e^{-\omega\zeta} \tag{20}$$

Where C_1, C_2 are constants.

By the use of Boundary conditions (5), we get the values of constants

$$P'(0) = \omega C_1e^{\omega(0)} - \omega C_2e^{-\omega(0)}, \text{ when t = 0}$$

$$\text{Then we get } C_1 = C_2 \tag{21}$$

$$\mathcal{P}(1) = C_1e^{\omega} + C_2e^{-\omega}, \text{ when t = 1}$$

$$\text{Therefore } C_2 = \frac{1}{e^{\omega}+e^{-\omega}} \tag{22}$$

Replacing the constants in (20),

$$\mathcal{P}(\zeta) = \frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}, \tag{23}$$

where ω is the constant coefficient.

$$\text{Consider } (t): \frac{d\mathcal{P}^2(\zeta)}{d\zeta^2} - \frac{\lambda\mathcal{P}(\zeta)}{1+\delta\mathcal{P}(\zeta)} = 0 \tag{24}$$

To find the value of m by the way of getting $f'(t=1)=0$

Using (23) in (24), we obtain,

$$f(\zeta):\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)''-\frac{\lambda\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)}{1+\delta\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)}=0 \quad (25)$$

$$f(\zeta):\ \omega\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)'-\frac{\lambda\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)}{\frac{(e^{\omega}+e^{-\omega})+\delta(e^{\omega\zeta}+e^{-\omega\zeta})}{(e^{\omega}+e^{-\omega})}}=0$$

$$f(\zeta):\ \omega^2\left(\frac{e^{\omega\zeta}+e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)-\frac{\lambda(e^{\omega\zeta}+e^{-\omega\zeta})}{(e^{\omega}+e^{-\omega})+\delta(e^{\omega\zeta}+e^{-\omega\zeta})}=0 \quad (26)$$

Differentiate (26) with respect to t,

$$f'(\zeta):\ \omega^3\left(\frac{e^{\omega\zeta}-e^{-\omega\zeta}}{e^{\omega}+e^{-\omega}}\right)-\lambda\left(\frac{\omega(e^{\omega\zeta}-e^{-\omega\zeta})\left(e^{\omega}+e^{-\omega}+\delta(e^{\omega\zeta}+e^{-\omega\zeta})\right)+\delta\omega(e^{\omega\zeta}-e^{-\omega\zeta})(e^{\omega\zeta}+e^{-\omega\zeta})}{\left((e^{\omega}+e^{-\omega})+\delta(e^{\omega\zeta}+e^{-\omega\zeta})\right)^2}\right)=0 \quad (27)$$

Substituting ζ =1in (27),

$$f'(\zeta=1):\ \omega^3\left(\frac{e^{\omega}-e^{-\omega}}{e^{\omega}+e^{-\omega}}\right)-\lambda\left(\frac{\omega(e^{\omega}-e^{-\omega})\left(e^{\omega}+e^{-\omega}+\delta(e^{\omega}+e^{-\omega})\right)+\delta\omega(e^{\omega}-e^{-\omega})(e^{\omega}+e^{-\omega})}{\left((e^{\omega}+e^{-\omega})+\delta(e^{\omega}+e^{-\omega})\right)^2}\right)=0$$

Then, $f'(\zeta=1)=\frac{\omega(e^{2\omega}-1)\left(-\omega^2+\left(\frac{\lambda}{1+\delta}\right)-\left(\frac{\delta\lambda}{(1+\delta)^2}\right)\right)}{e^{2\omega}+1}$ (28)

After Simplifying (28), we obtain

$$\omega=\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)} \quad (29)$$

By Exp-Function Method, the approximate analytical formulation of the concentration of substrate as

$$\mathcal{P}(\zeta)=\frac{e^{\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)\zeta}+e^{-\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)\zeta}}{e^{\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}+e^{-\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}} \quad (30)$$

The normalized current obtained by using EFM can be expressed as,

$$\Omega=\tau\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\left(\frac{e^{\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}-e^{-\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}}{e^{\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}+e^{-\left(\sqrt{\lambda\left(\frac{1}{1+\delta}-\frac{\delta}{(1+\delta)^2}\right)}\right)}}\right) \quad (31)$$

4. Numerical Simulation

The efficiency of the methods is demonstrated by contrasting the numerical simulation of the equation of (4) with the Analytical findings (18) and (30). Table 1 and Table 2 compares the computational outcomes with dimensionless concentration of planar electrode $\mathcal{P}(\zeta)$ substrate in its analytical representation. These tables show that, for different amount of δ and λ, our new result agree rather well with the numerical findings. It provides a satisfactory agreement for all parameter setting under comparison. Our results employing Homotopy Analysis Method and Exp-Function Method with numerical results have maximum relative error rate of 0.0045% and 0.0078% respectively.

Table 1. *Deviation between Numerical result (4) and Analytical results (18) and (30) of the substrate concentration in the case of planar electrode for various values of saturation parameter δ and for fixed diffusion parameter λ = 0.1 and h = -1.*

	$\lambda=0.1,\delta=1,h=-1$					$\lambda=0.1,\delta=0.01,h=-1$				
T	**Num. Value**	**HAM of (18)**	**Error% of HAM**	**EFM of (3)**	**Error of EFM**	**Num. Value**	**HAM of (18)**	**Error of HAM**	**EFM of (30)**	**Error of EFM**
0.0	0.9753	0.9750	0.0003	0.9876	0.0123	0.9524	0.9504	0.0020	0.9529	.0.0005
0.1	0.9755	0.9752	0.0003	0.9877	0.0122	0.9528	0.9509	0.0019	0.9533	0.0005
0.2	0.9762	0.9760	0.0002	0.9881	0.0119	0.9542	0.9524	0.0018	0.9547	0.0005
0.3	0.9775	0.9772	0.0003	0.9887	0.0112	0.9565	0.9549	0.0016	0.9571	0.0006
0.4	0.9791	0.9790	0.0001	0.9896	0.0105	0.9598	0.9584	0.0014	0.9603	0.0005
0.5	0.9816	0.9812	0.0004	0.9907	0.0091	0.964	0.9628	0.0012	0.9646	0.0006
0.6	0.984	0.9840	0.0004	0.9920	0.008	0.9692	0.9683	0.0009	0.9697	0.0005
0.7	0.9876	0.9872	0.0004	0.993	0.0057	0.9754	0.9747	0.0007	0.9758	0.0004
0.8	0.991	0.9910	0.000	0.9955	0.0045	0.9826	0.9821	0.0005	0.9829	0.0003

$\lambda = 0.1, \delta = 1, h = -1$						$\lambda = 0.1, \delta = 0.01, h = -1$				
T	**Num. Value**	**HAM of (18)**	**Error% of HAM**	**EFM of (3)**	**Error of EFM**	**Num. Value**	**HAM of (18)**	**Error of HAM**	**EFM of (30)**	**Error of EFM**
0.9	0.9957	0.995	0.0007	0.997	0.0013	0.9908	0.9905	0.0003	0.9909	0.0001
1.0	1.0000	1.0000	0.000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
Average Error%			0.0003		0.0078			0.0001		0.0004

Table 2. *Comparison of concentration of substrate in the case of planar electrode between Numerical result (4) and obtained new Analytical results (18) and (30) for various values of diffusion parameter λ and for fixed saturation parameter δ =0.01 and h = -1.*

$\lambda = 0.01, \delta = 0.01, h = -1$						$\lambda = 0.2, \delta = 0.01, h = -1$				
T	**Num. Value**	**HAM of (18)**	**Error% of HAM**	**EFM of (30)**	**Error of EFM**	**Num. Value**	**HAM of (18)**	**Error of HAM**	**EFM of (30)**	**Error of EFM**
0.0	0.9951	0.9950	0.0001	0.9951	0.0001	0.9085	0.9009	0.0076	0.9093	0.0008
0.1	0.9951	0.9950	0.0001	0.9951	0.0001	0.9094	0.9019	0.0075	0.9102	0.0008
0.2	0.9953	0.9952	0.0001	0.9953	0.0001	0.9118	0.9049	0.0069	0.9129	0.0011
0.3	0.9955	0.9954	0.0001	0.9955	0.0001	0.9168	0.9099	0.0069	0.9174	0.0006
0.4	0.9958	0.9958	0.0000	0.9958	0.0000	0.9225	0.9168	0.0057	0.9236	0.0011
0.5	0.9963	0.9962	0.0001	0.9963	0.0001	0.9306	0.9257	0.0049	0.9317	0.0011
0.6	0.9968	0.9968	0.0000	0.9968	0.0000	0.9406	0.9366	0.004	0.9416	0.001
0.7	0.9975	0.9974	0.0001	0.9975	0.0001	0.9526	0.9495	0.0031	0.9534	0.0008
0.8	0.9982	0.9982	0.0000	0.9982	0.0000	0.9664	0.9643	0.0021	0.9670	0.0006
0.9	0.9991	0.999	0.0001	0.999	0.0001	0.9822	0.9811	0.0011	0.9825	0.0003
1.0	1.0000	1.0000	0.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
Average Error%			$6.36e^{-5}$		$6.36e^{-5}$			0.0045		0.0007

5. Results and Discussion

Equation (18) and (30) are the newly developed approximative analytical expressions for the concentration of substrate in Homotopy Analysis method and Exp-Function Method respectively. Also, the reaction of the normalized current is presented in Eqn. (19) and (31). The prediction was observed that the analytical and numerical results agreed well. The diffusion parameter λ and the saturation δ both affect the materials concentration. The layer thickness ξ, the diffusion coefficient of the substrate $\mathcal{D}_A$ within the polymer film and the parameter of reaction diffusion λ which establishes the relationship between the rate of substrate diffusion over the film to the amount of the chemical reaction that occurs in the layer.

Figure 1 represents that the dimensionless substrate concentration for the various values of δ and the constant value of $\lambda = 0.1$ and h = -1. According to Figure 1, it evident that the concentration of substrate $\mathcal{P}(\zeta)$ increases whenever the saturation factor δ increases for the fixed amount of diffusion parameter λ. For $\delta \geq 5$, the concentration of material becomes constant. For every large amount of non-dimensional distance, the analytical outcomes which are derived by using Homotopy analysis method and Exp-Function method are coincide.

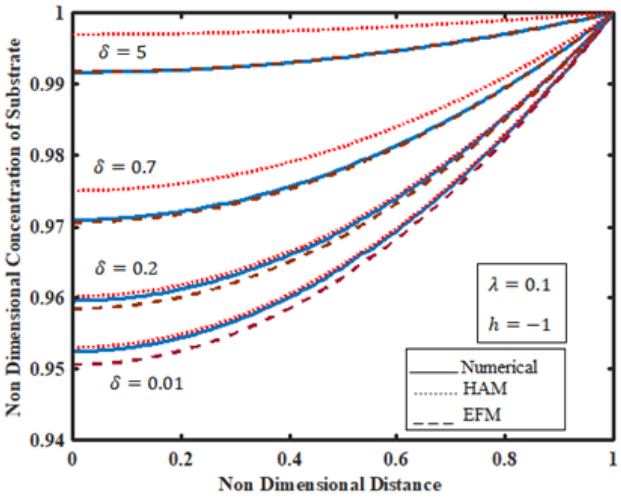

Figure 1. *Comparison of Analytical and Numerical outcomes for different values of* δ *with fixed amount of* λ*=0.1.*

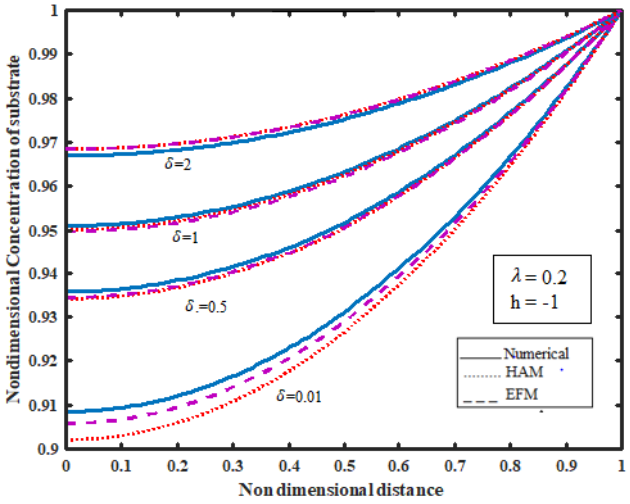

Figure 2. *Comparison of Analytical and Numerical results for fixed* λ*=0.2 with various amount of* δ*.*

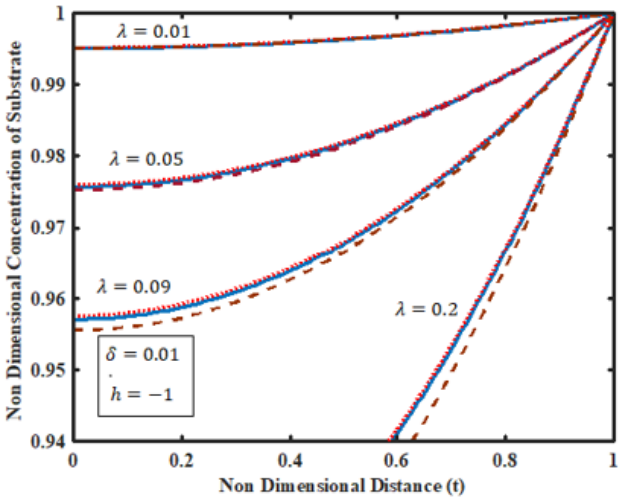

Figure 3. *Comparison of Analytical and Numerical results for fixed* δ*=0.01 with various amount of* λ*.*

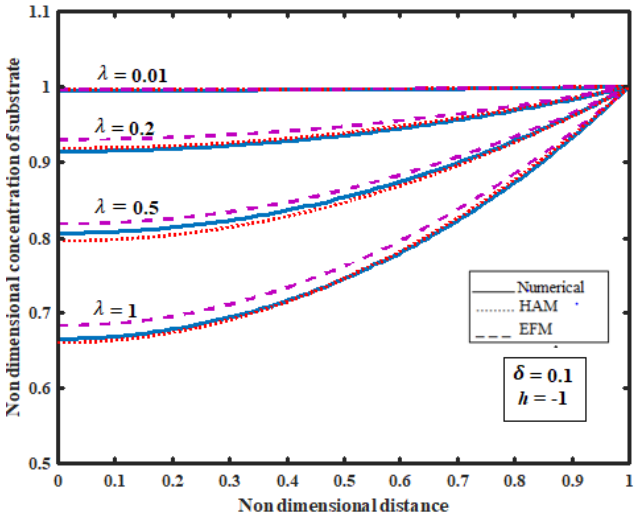

Figure 4. *Plot of analytical finding contrast with numerical results for various values of* λ *with* $\delta = 0.1$*.*

Figure 2 shows that the comparison of analytical and numerical outcomes of the concentration for fixed $\lambda = 0.2$ with the various saturation parameter. It depicts that the substrate evolves into less saturated in the concentration although the diffusion parameter remains elevated. The concentration on the electrode surface have been considered to be directly proportional to the saturation parameter δ.

Figure 3 demonstrate that the non-dimensional concentration of substrate for the different values of λ and the fixed value of $\delta = 0.01$ and h = -1. Figure 3 leads to the conclusion that the material concentration rises when the diffusion parameter λ falls for the fixed factor δ. When $\lambda \leq 0.01$, the concentration of substrate is uniform. That means, the inclined curve immerged as straight line. The parameter has an impact which is inversely correlated to the diffusion parameter.

Figure 4 illustrates that the plot of new analytical findings contrast with numerical simulation for different values of diffusion parameter with $\delta = 0.1$. According to Figure 4, the increasing amount of diffusion parameter decreases the concentration of substrate on the planar electrode. The substrate concentration $\mathcal{P}(\zeta)$ reaches attains its most extreme value and decreases for $\zeta \leq 1$ due to very large amount of non-dimensional distance at ζ=1.

Figure 5 depicts that the consequences of diffusion parameter on the current according to demonstrating the steady state current compared with the substrate concentration in the polymer modified electrode surface for an ensemble of various saturation parameter values. From the findings, the large amount of diffusion parameter increases the concentration of molar flux.

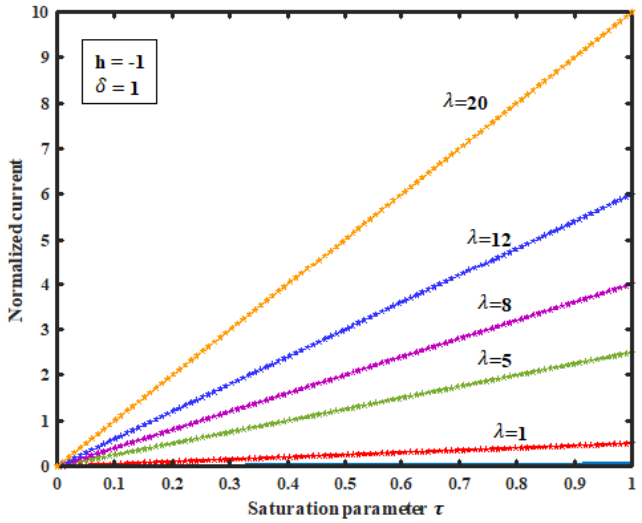

Figure 5. *Influence of diffusion parameter λ on normalized current for fixed $\delta = 1$.*

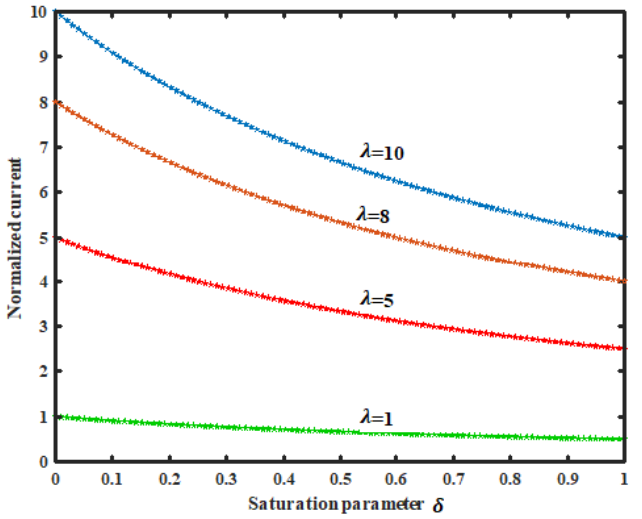

Figure 7. *Influence of diffusion parameter λ on normalized current versus saturation parameter δ.*

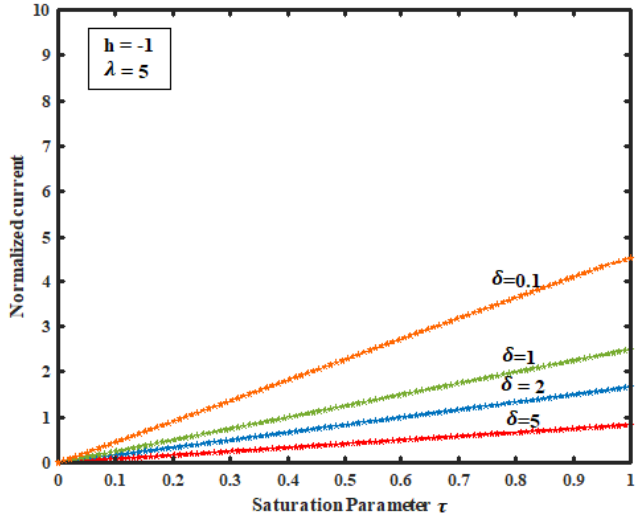

Figure 6. *Influence of saturation parameter δ on normalized current for fixed $\lambda = 5$.*

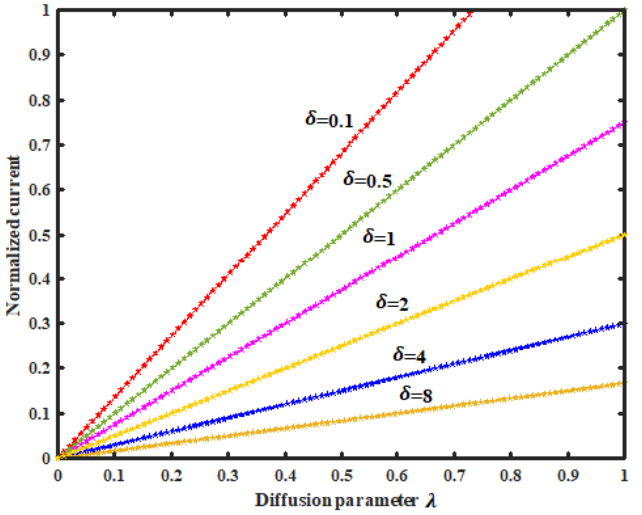

Figure 8. *Influence of saturation parameter δ on normalized current versus diffusion parameter λ.*

Figure 6 represent the plot of normalized current versus saturation parameter τ for various values of δ with $\lambda = 5$ From this figure, the concentration level decreases for all large values of saturation parameter. That is, the impact of δ inversely proportional to the normalized current for fixed amount of diffusion parameter.

Figure 7 demonstrate that the concentration of current with respect to the saturation parameter δ for numerous amounts of diffusion parameter λ and for constant $\tau = 1$. According to Figure 7, it evident that the impact of greater amount of diffusion parameter on normalized current increases the concentration level. At $\lambda \leq 1$, the concentration of current attains its steady state.

Figure 8. established that the relation between the normalized current and the saturation parameter δ depends on the diffusion parameter λ. If the amount of saturation parameter δ rises, then the concentration level decreases and reaches the steady state. That means, the concentration of current inversely proportional to the saturation parameter δ.

6. Conclusion

A mathematical model of the facilitated diffusion of a planar electrode in a nonlinear process at an electroactive polymer film is discussed in this work. The approximate analytical formulation of concentrations is achieved using Homotopy Analysis Method and Exp-Function Method. The substrate concentration of planar electrode in the electroactive

film has been examined employing our analytical results for a range of parameter values. These two analytical techniques are also examined with Matlab solution. Homotopy Analysis Method prevails over the Exp-Function Method in terms of efficiency and leads to a highly satisfactory result. Non-steady state scenarios are also possible to resolved with these methods as well. Furthermore, the outcome of this research offers the prospect of developing the approach in order to obtain an approximation of substrate concentrations and normalized flux for various electrode geometries.

Abbreviations

HAM	Homotopy Analysis Method
EFM	Exp-Function Method

Author Contributions

Uma Andiappan: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing

Swaminathan Rajagopal: Conceptualization, Formal Analysis, Project administration, Resources, Supervision, Validation, Visualization

Conflicts of Interest

The authors declare no conflicts of interest.

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Research Fields

Uma Andiappan: Mathematical Modeling, Non Linear Differential Equations, Boundary Value Problems, Analytical Methods, Reaction Diffusion Process in Chemical Sciences

Swaminathan Rajagopal: Mathematical Modeling, Differential Equations, Analytical Methods, Numerical Simulations, Initial and Boundary value problems