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Research Article

Deriving Mathematical Equations for Measuring the Holes of the Nay (Arabic Flute)

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Abstract

Developing and discovering things requires the process of collecting relevant data and information, which is then analyzed to draw conclusions that enable us to move from the contemplation stage to the innovation stage. It is evident that many musical instruments have undergone evolution over the decades, and this development continues to the present day. Before the commencement of the development and innovation phase, it is preceded by a stage of data and information collection and analysis, laying the foundation for a scientific basis relying on academic methods. The researcher believes that presenting an innovation or development process in a specific instrument should be followed by a mathematical analysis of the sound holes. The research aims to analyze the dimensions of the Nay instrument mathematically and derive mathematical equations for measuring its holes. The significance of the research lies in analyzing the holes of all sound degrees of the Nay instrument and deriving mathematical equations to measure the dimensions of the instrument, contributing to the overall development and elevation of research in Arab musical instruments, especially the Nay. The results were as follows: The ratio between the holes is equal, and equations were determined to calculate the positions of the Nay's holes.

Keywords

Nay, Nay Holes, Mathematical Equations for Nay, Nay Acoustics, Length of Nay Holes, Nay Frequency

1. Introduction

The evolution and discovery of things require the collection of data and information related to them. Subsequently, this data is analyzed, leading to conclusions that enable us to move from the contemplation phase to the innovation phase. Undoubtedly, many musical instruments have been developed over the past decades, and development continues. Before reaching the stage of development and innovation, there is a phase of collecting data and information, followed by analysis, to establish a scientific foundation based on academic methods that enable researchers to answer research questions or test the validity of hypotheses.

Therefore, the researcher believes that the process of innovation or development in musical instruments must be preceded by an analysis of hole positions mathematically. This involves deriving mathematical equations to measure the positions on the Nay (Arabic Flute). Such equations enable researchers to develop the instrument and advance research in Arabic musical instruments in general, and the Nay in particular.

The set of used Nays: The Nay is a musical instrument with a specific musical scale. Therefore, the musician carries a set of Nays with different musical scales, determined by the

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lengths of these instruments. The musician can replace them to achieve the desired musical scale. The set carried by the musician includes the main Nay, auxiliary Nays, and additional Nays, given the difference in musical scales.

1. Main Nay: It is the primary instrument in the set and is named after the specific musical scale it represents. The musician can interchange Nays of varying lengths to achieve different musical scales.
2. Auxiliary Nays: This group assists the main Nay and shares the same sound layer but differs in the musical scale produced. Each auxiliary Nay is named after the second degree of the Nay. The auxiliary Nays include:
   1) Al-Husayni (La)
   2) Al-Nawa (Sol)
   3) Al-Jiharkah (Fa)
   4) Al-Buslik (Mi Natural)
   5) Al-Rast (Do)
   6) Al-Ajam (Si Bemol)
3. Additional Nays: This group consists of Nays from the previous set but does not adhere to the international sound layer (known as the piano layer). The purpose of these Nays, in the case of working with singers who do not adhere to a specific layer, is to provide flexibility. Sound Layers of the Nay:
   1) Large Sound Layer (Basic):
      a) Al-Duka (Re)
      b) Al-Nawa (Sol)
      c) Al-Husayni (La)
      d) Al-Buslik (Mi)
   2) Small Sound Layer:
      a) Al-Rast (Do)
      b) Al-Jiharkah (Fa)
      c) Al-Ajam (Si Bemol)
   3) Medium Sound Layer:
      a) Dukah and Half tone (Re#)
      b) Nawa and Half tone (Sawt#)
      c) Husayni and Half tone (La#)

Completing this set are Al-Kurdan Nay (Do1) as an answer to Al-Rast, Al-Mahir Nay (Re1) as an answer to Al-Dukah, and Mahur Nay (Si Natural). These three Nays are not classified into the three sound layers and are rarely used. [1]

Sound Spaces Produced by the Nay: Blowing into the Nay is classified into four types, each producing six natural tones:

a) The Resolution Preceding Resolution Blow: This gentle blow with slightly open lips produces a thick sound layer.
b) The Resolution Blow: A weak blow with slightly parted lips, producing a thick sound layer, similar to the previous tones.
c) The Middle Blow: A moderate blow with slightly closed lips, producing tones between resolution and answer. It is a moderately sharp sound layer and corresponds to the previous tones.
d) The Answer Blow: A strong blow with tightly closed lips, producing a very sharp sound layer. This sound is considered an answer to the tones produced by the resolution blow. [2]

The Nay was traditionally used in various religious, festive, and wartime occasions. In ancient times, there were two types of flutes, the long flute and the short flute, but the short flute was more commonly used during that period. The number of its holes ranged from two to six. [7]

The Nay is considered a crucial instrument in Arabic music ensembles, often relied upon by many music directors for solo performances. Additionally, it serves as an accompanying instrument for singers during vocal performances. Players of the Nay are required to deliver improvisational variations and innovative phrases, whether at the beginning of a musical piece or in instrumental or vocal sections throughout various works.

Therefore, a Nay player is indispensable in musical ensembles due to the significant role and unique contributions it brings alongside other instruments. [8]

At present, the craftsmanship of crafting the Nay instrument relies heavily on the maker's expertise in selecting the reed and determining the locations for the holes on its body. These holes are strategically placed to produce the fundamental and derived musical scale tones. The precision of Nay instrument craftsmanship directly influences its cost, making it essential to consider maintaining accurate proportions in the hole placements. Additionally, the emitted sounds should adhere to correct musical dimensions. The front of the Nay typically features six holes and one hole at the back, designated for the left-hand thumb. For the right hand, four holes are allocated, three of which are positioned in front, above the holes for the right hand, and one hole for the index finger, situated above all the holes. [9]

In the twentieth century, numerous developments and modifications were introduced to the Ney instrument. Many musicians and instrument enthusiasts attempted to enhance this instrument to keep pace with musical advancements and address the inherent challenges. Consequently, Ney players faced difficulties in rendering non-fixed tones across the Ney's range. Various attempts were made by players and instrument craftsmen to overcome these challenges, with modifications initiated as early as the beginning of the twentieth century and persisting until today. These modifications were led by a group of players dedicated to improving the Ney's performance and adapting it to contemporary musical requirements. [10]

Ismail Al-Badri* has created the "Badr Nay," designed to overcome the difficulty of performing in the lower register of the scale. This was challenging for musicians due to the need for long flutes that made it difficult for the player to control the grip beyond the finger holes. After several attempts, Ismail Al-Badri achieved success by introducing bends in the length of the air column, making the instrument shorter while retaining its original length of approximately 90 cm. Ismail Al-Badri presented this instrument to the committee for musical instruments, and after its success, he named it the "Badr

Nay." Later, he added the keys to it. [11]

Then, Rizk Suleiman* innovated the "Modern Arabic Ney." His idea involved incorporating elements of the flute's development into the Ney by adding keys and metal valves. This innovation aimed to utilize a single Ney (called "Doka") and eliminate the need for auxiliary Ney instruments. This was achieved by dividing the air column and determining the positions of additional tones on it. Suleiman added a metal key for each missing tone, allowing musicians to play a scale of twenty-four quarter-tones. This enabled Ney players to replace a set of Ney instruments with a single Ney. The instrument was named the "Modern Arabic Ney," and its adoption was not hindered by its cost or the difficulty of playing it. [12]

Then came the complaint about the lack of suitable keys for the nay instrument, as the keys used were those of the ancient Western flute, which craftsmen used to install in the nay. Many craftsmen consider revisiting the idea of Gerges Saad and Abdelhamid Mish'al, by opening two additional holes on the sides of the nay and using the small finger (pinky) of both hands to cover them, similar to what is found in the recorder, which has an additional hole used in playing two notes (Re Bemol, La Bemol) naturally without any effort. [13]

It is worth noting that the nay instrument has evolved in its manufacturing and performance. Auxiliary nays have emerged, allowing the musician to depict maqamat (musical modes) and melodies. Additional side holes, as well as keys, have been introduced, increasing the number of tones and consequently expanding the range of maqamat that a single nay can produce, ensuring that the role of the nay remains significant. The art of playing musical instruments has undergone significant development, enabling musicians to demonstrate high technical skills, such as melodic leaps, rapid scales, chromatic scales, and arpeggios. Additionally, musicians have embraced collaboration with other instruments, reflecting the continuous evolution of musical skills in general. [14]

The nay instrument is considered a crucial element in Arab music ensembles, and undoubtedly, attention to the instrument, whether in playing, composing, or studying, is of utmost importance due to its prominent role among various musical instruments. As we mentioned, the nay holds a special significance in the performance of Arab music throughout the centuries. It is used both as a solo instrument to express this music in various situations in Egyptian music. It follows established maqamat (musical modes) and rhythms, sometimes conveying melancholy and other times expressing joy. [15]

2. The Relationship Between Music and Mathematics

The connection between music and mathematics is intricate and profound, as these two arts are intertwined through various mathematical elements that manifest deep mathematical interactions. This relationship is evident in several aspects:

1) Frequencies and Oscillations:

In physics and mathematics, frequencies and oscillations are integral to wave studies. In music, tones and sounds are composed of recurring oscillations and specific frequencies. Musical harmonies rely on mathematical ratios between these frequencies to achieve consonance and harmonic balance. [5]

2) Rhythm and Coherence:

Rhythm in music represents a systematic organization of time, which can be examined mathematically. Melody and rhythm form a regular and coherent pattern following mathematical laws in the distribution of duration and timing. [6]

3) Harmony and Scale:

In music, harmony relies on mathematical relationships between sounds and melodies. Scales and harmonies form mathematical models that determine the sequence and progression of pitches and tones.

The Nay consists of six front holes and one back hole. As illustrated in Diagram

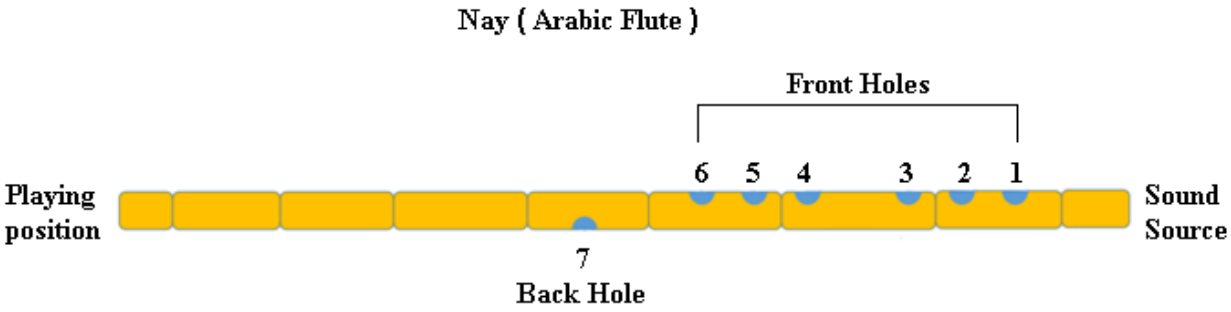

Figure 1. *Nay (Arabic Flute).*

Significance of the Study: The significance lies in analyzing the dimensions of all the pitch holes of the Nay instrument and deriving mathematical equations to measure the holes. This aims to provide a deep understanding of the relationship between the geometry of the holes and the produced sound. Through this analysis, it contributes to the development and

enhancement of Nay design, enabling musicians to achieve a wider tonal range and greater musical accuracy. Furthermore, this research can contribute to advancing the scientific understanding of the fundamentals of sound and musical instruments, opening the door to new possibilities for innovating and developing advanced and efficient musical instruments.

2.1. Research Questions

a) Are the frequencies of the pitch grades equal for the Nay?
b) How can the holes of the Nay be measured?
c) What is the ratio of the holes on the Nay?

Research Method: This research adopts the deductive approach.

Research Sample: The Nay (Arabic flute).

2.2. Instruments of Study

a) Nay pitch grades.
b) Software for measuring tone frequencies.

3. Results

By answering the research questions, the researcher inferred the following data.

The researcher wrote the musical notes for the "Al-Duka Nay" (Re) as follows:

Table 1. *Nay Al-Duka (Re).*

Holes	-	First	Second	Third	Fourth	Fifth	Sixth	Seventh (The Back)
Melodies	C	D	E ♭	E ♮	F	F#	G	B ♭
	Do	Re	Mi Bemol	Mi Half Bemol	Fa	F sharp	Sol	Si Bemol
Frequency	261.6	293.7	311.1	321.3	349.2	370	392	466.2

Table 1 illustrates the tones of the Nay instrument along with the frequency of each tone.

Then measure the frequency ratio for each pitch by dividing the frequency of the C note by the hole frequency as follows:

$$\frac{(DO)\,frequency}{hole\ frequency}$$

Ratio of the D note: 261.6/293.7 = 9/8
Ratio of the E Bemol note 261.6/311.1 = 6/5
Ratio of the E Half Diminished note: 261.6/321.3 = 5/4
Ratio of the F note 261.6/349.2 = 4/3
Ratio of the F sharp note: 261.6/370 = 7/5
Ratio of the G note: 261.6/392 = 2/3
Ratio of the back hole note: 261.6/466.2 = 5/9

And in the same manner, the frequencies of the notes for different pitches of the Nay instrument were written, and the ratios for these sound pitches were measured and then recorded in the following tables:

Table 2. *Al-Rast Nay (Do).*

Holes	-	First	Second	Third	Fourth	Fifth	Sixth	Seventh (The Back)
Melodies	B ♭ 3	C 4	D ♭ 4	D ♮ 4	E ♭ 4	E 4	F 4	A ♭ 4
	Si	Do	Re	Re quarter Bemol	Mi	Mi sharp	F	La Bemol
Frequency	233.1	261.6	277.2	291.4	311.1	329.6	349.2	415.3
Ratio	1	8/9	5/6	4/5	¾	5/7	2/3	5/9

Table 3. *Al-Duka Nay (Re).*

Holes	**-**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh (The Back)**
Melodies	C 4	D 4	E♭4	E♮4	F 4	F# 5	G 4	B♭4
	Do	Re	Mi Bemol	Mi quarter Bemol	Fa	F sharp	Sol	Si Bemol
Frequency	261.6	293.7	311.1	321.3	349.2	370	392	466.2
Ratio	1	8/9	5/6	4/5	3/4	5/7	2/3	5/9

Table 4. *Al-Buslik Nay (Mi).*

Holes	-	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh (The Back)**
Melodies	D 4	E 4	F 4	F♯4	G 4	G# 4	A 4	C 5
	Re	Mi	Fa	Fa quarter sharp	Fa	Sol sharp	la	Do Bemol
Frequency	293.7	329.6	349.2	359.8	392	415.3	440	523.3
Ratio	1	8/9	5/6	4/5	3/4	5/7	2/3	5/9

Table 5. *Al-Jiharkah Nay (Fa).*

Holes	**-**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh (The Back)**
Melodies	E♭4	F4	G♭4	G♮ 4	A♭4	A 4	B♭4	D♭5
	Mi	Fa	Sol Bemol	Sol quarter Bemol	La Bemol	La	Si Bemol	Re Bemol
Frequency	311.1	349.2	370	389	415.3	440	466.2	554.4
Ratio	1	8/9	5/6	4/5	3/4	5/7	2/3	5/9

Table 6. *Al-Nawa Nay (Sol).*

Holes	**-**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh (The Back)**
Melodies	F 4	G 4	A♭4	A♮4	B♭4	B 4	C 5	E♭5
	Fa	Sol	La Bemol	La quarter Bemol	Si Bemol	Si	Do	Re Bemol
Frequency	349.2	392	415.3	436.5	466.2	493.9	523.3	622.3
Ratio	1	8/9	5/6	4/5	3/4	5/7	2/3	5/9

Table 7. *Al-Husayni Nay (La).*

Holes	**-**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh (The Back)**
Melodies	G 4	A 4	B♭4	B♮ 4	C 5	C# 5	D 5	F 5
	Sol	La	Si Bemol	Si quarter Bemol	Do	Do sharp	Re	Fa
Frequency	392	440	466.2	490	523.3	554.4	587.3	698.5

Holes	-	First	Second	Third	Fourth	Fifth	Sixth	Seventh (The Back)
Ratio	1	8/9	5/6	4/5	3/4	5/7	2/3	5/9

Through the data in the previous tables, we conclude that the ratios between the holes are equal to each other. Thus, we can establish a relationship to measure the dimensions of the holes as a percentage (an equation derived by the researcher), and it is as follows:

And the ratio of the holes can be derived from the following table:

Table 8. *Hole ratio.*

Holes	First	Second	Third	Fourth	Fifth	Sixth	Seventh (The Back)
The hole ratio	85.5%	81.3%	77.1%	69.4%	65.2%	61%	47.5%

Table 8 illustrates the ratio of the finger holes on the Nay, as evident from the table.

And here comes the last step, which is how to calculate the holes of the Nay instrument during:

1) Determine the length of the desired musical pitch.
2) Substitute it into the following equation for each hole (an equation derived by the researcher).

$$\frac{length\ Nay * hole\ ratio}{100}$$

3) Calculate the diameter of the Nay through the following equation: (an equation derived by the researcher).

$$\frac{length\ Nay}{34}$$

4. Discussion

Clarifying the relationship between mathematics and music can be intriguing. This research follows a descriptive and content analysis approach. The previous study and the current study agreed on the interest in connecting mathematics with music.

However, the previous study differed from the current study in several aspects. One of them is the difference in methodology, where the current study focused on elucidating the relationship between the Nay instrument and mathematics specifically. In contrast, the previous study concentrated on explaining the general relationship between mathematics and music. This highlights the unique focus of the current study on the specific connection between the Nay instrument and mathematical concepts, adding additional value to understanding the correlation between these two aspects. [4]

Analysis and Extraction of Characteristics of Arabic Music for the Nay Instrument, Computerized Notation. This research follows the methodology of descriptive content analysis.

The previous study aligned with the current study in the interest in analyzing the Nay. However, the previous study differed from the current study in the methodology and in the computerized analysis of the Nay instrument's characteristics. The current study focuses on mathematically analyzing the dimensions of the Nay instrument and deriving mathematical equations for measuring its dimensions. [3]

5. Conclusion

Through a study analyzing the distances and pitch degrees of the Nay, the researcher concluded a universal ratio that can be applied to all flute instruments. This provides an opportunity for instrument makers to enhance Nay development based on these determined distances and ratios.

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Conflicts of Interest

The authors declare no conflicts of interest.

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