

# Ring Theory Application to Musical Tones Compositions

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**Abstract:** This paper critically analyzed the idea of using musical tones and mathematical techniques to composed a beautiful, listenable musical sound. observed the tone behavior and connection between mathematical concepts (such as rings) and musical notes. The twelve, (12), musical notes were used for the composition, which formed a ring with addition, +, and multiplication, \*, operations, such that additive semigroup and multiplicative group forms a commutative group and semigroup respectively. This was done as a specific technique for the composition of music in which, with the aid of mathematical matrix developed, an equal importance is given to all twelve musical tones in the chromatic scale. The ring theorem is a branch of mathematics that is applicable to music composition, contrary to what people might think, as shown by a practical demonstration of mathematical techniques applied to the twelve musical tones that produced a good sound when played on the keyboard, one of the fundamental musical instruments used for translation of keys to sound. There were some claims that had supporting evidence as a prove that, mathematics is one of the tools for music composition. The research is a prove that, ring theory which is an aspect of mathematics that can be used to formulate music using the twelve musical notes, the behaviors and sound produced attested to the fact that mathematics is musical friendly.

**Keywords:** Abstract Algebra, Ring Theory, Music, Musical Notes, Mathematics, Musical Notes Composition, Group Theory

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## 1. Introduction

The enormous application of mathematics to sciences and other disciplines can't be overlooked. There are questions people ask, is mathematics applicable? How can it be applied to a real life situation? These and more questions. This research shows that mathematics is not just a classroom work as insinuated by many but applicable everywhere either obvious or not. [6, 12]. In 2018, Samuel, H. T. et. al and Bobis et al. applied group theory to the arrangement of musical notes. [2, 8] Music and Mathematics, a lot of researchers have shown that, there is a great connection between the two disciplines for more than two thousand years ago. Ring theory is an aspect of mathematics that deals with algebraic structures housed by abstract algebra in mathematics. The common algebraic structures in abstract algebra are: rings, fields, and vectors space with some specific operations and axioms. Rings theory is very essential in the field of science such as Physics, Chemistry, and

Material Sciences and other areas like Music. etc.

Experts in music uses mathematical techniques to develop, express and transfer their musical ideas to good sounds. This research will lay emphasize on application of rings theory to how the twelve musical notes can be formulated mathematically without bias. A demonstration of the mathematical music twelve tones arrangement shall be carried out as proof. Readers can check [1, 2].

Abstract algebra which is an area in mathematics that studies the algebraic structures such as group, rings, fields and vectors space are all endowed with additional operations and axioms. Ring theory is a special structure which study algebraic structures in which addition and multiplication are defined and have similar properties to those operations defined for the integers. On the other hand, it is a study of ring structures, special classes of rings are; group rings, division rings, universal enveloping algebras as well as an array of properties that proved to be of interest both within the theory itself and for it applications. [3, 4]

In 2021, Silva A., et al opined that, the effectiveness of the teaching approach to checking the relationship between mathematics and music associated with performance of mathematics learning and whether this effectiveness depends on the musical knowledge of students to identify didactically relevant patterns. It is observed that music is the referent of real life and as the activities unfold, it becomes a mathematical object, and which allows students to experience different epistemic levels when artefacts are used with the status of epistemic materials. His results showed that whether, or not, the students have critical musical knowledge, the “doing math with music” approach is more effective than conventional approaches for the same mathematical subject. His study based on the importance of distinguishing the concepts of “quality of epistemic activity” and “epistemic level”. He effectively established that there is a relationship between them which, however, is not direct. It also shows that it is advantageous from the point of view of mathematical learning to start from lower epistemic levels (closer to real life) and to have a progression of the epistemic level at which the epistemic activity takes place. However, the progressive tendency of the epistemic level with which the mathematical object is dealt with may be concomitant with a certain variability of the epistemic level and that, under certain conditions, this variability may favor the attainment of higher levels of abstraction. [11, 15]

## 2. Music Theory

Researchers have tremendously worked on music theory in on different dimensions been a big field due to its nature as connected to mathematics. Samuel Hwere Tsok, (2018), opined that music is more or less a fine arts course which

major in the combination of musical notes to produce sound in other to express ones thought or feelings. The basic notes in music are:  $C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B$  [2, 7, 13].

Mathematics generally look abstract to people and that makes it difficult to comprehend some important concept. This research shows the behavior of the musical notes as a direct effect of ring theory with better understanding that there is a connection between mathematics and music. Bijana, J. et al, (2018), asserted that ‘mathematics helps in creation of music and in listening to music’, [5, 9].

Bobis, J. and Still, K., 2023, proved that, mathematics and music are compactable and worked on the integration of mathematics and music into primary school educational curriculum and opined that, experienced teachers are needed to carry out the integration which means, teachers my need to reconceptualise what it is that makes an activity ‘musical’ and ‘mathematical’. This will build the confidence in the student and creating the likeness in mathematics as its relate to music. [10, 14]

## 3. Ring Theory

Definition 1.1

A ring is a set,  $\mathbb{R}$ , together with two operations (+) and (.) satisfying the following properties:

- (a)  $\mathbb{R}$  is an abelian group under addition i. e.  $\mathbb{R}$  is closed under addition, there is an additive identity called 0, every element  $a \in \mathbb{R}$  has an additive inverse  $-a \in \mathbb{R}$ , and addition is associative and commutative.
- (b)  $\mathbb{R}$  is closed under multiplication and multiplication is associative. i. e.

$$\forall a, b \in \mathbb{R}, a \cdot b \in \mathbb{R}$$

Table 1. Table Showing Commutative Group  $\langle \mathbb{Z}_{12}, + \rangle$ .

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10

$$\forall a, b, c \in \mathbb{R}, a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

Table 2. Table Showing Musical Translation of Commutative Group  $\langle \mathbb{Z}_{12}, + \rangle$ .

+	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
C	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
C#	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
D	D	D#	E	F	F#	G	G#	A	A#	B	C	C#
D#	D#	E	F	F#	G	G#	A	A#	B	C	C#	D
E	E	F	F#	G	G#	A	A#	B	C	C#	D	D#
F	F	F#	G	G#	A	A#	B	C	C#	D	D#	E
F#	F#	G	G#	A	A#	B	C	C#	D	D#	E	F
G	G	G#	A	A#	B	C	C#	D	D#	E	F	F#

+	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
G#	G#	A	A#	B	C	C#	D	D#	E	F	F#	G
A	A	A#	B	C	C#	D	D#	E	F	F#	G	G#
A#	A#	B	C	C#	D	D#	E	F	F#	G	G#	A
B	B	C	C#	D	D#	E	F	F#	G	G#	A	A#

Table 2 clearly show that addition under  $\langle \mathbb{Z}_{12}, + \rangle$  is closed. Set with these elements together with a binary operation of addition is called a group known as abelian i. e. commutative group. [4]

*Table 3. Table Showing Commutative Group  $\langle \mathbb{Z}_{12}, . \rangle$ .*

.	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	0	2	4	6	8	10
3	0	3	6	9	0	3	6	9	0	3	6	9
4	0	4	8	0	4	8	0	4	8	0	4	8
5	0	5	10	3	8	1	6	11	4	9	2	7
6	0	6	0	6	0	6	0	6	0	6	0	6
7	0	7	2	9	4	11	6	1	8	3	10	5
8	0	8	4	0	8	4	0	8	4	0	8	4
9	0	9	6	3	0	9	6	3	0	9	6	3
10	0	10	8	6	4	2	0	10	8	6	4	2
11	0	11	10	9	8	7	6	5	4	3	2	1

*Table 4. Table Showing Musical Translation of Semi-Group  $\langle \mathbb{Z}_{12}, . \rangle$ .*

.	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
C	C	C	C	C	C	C	C	C	C	C	C	C
C#	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
D	C	D	E	F#	G#	A#	C	D	E	F#	G#	A#
D#	C	D#	F#	A	C	D#	F#	A	C	D#	F#	A
E	C	E	G#	C	E	G#	C	E	G#	C	E	G#
F	C	F	A#	D#	G#	C#	F#	B	E	A	D	G
F#	C	F#	C	F#	C	F#	C	F#	C	F#	C	F#
G	C	G	D	A	E	B	F#	C#	G#	D#	A#	F
G#	C	G#	E	C	G#	E	C	G#	E	C	G#	E
A	C	A	F#	D#	C	A	F#	D#	C	A	F#	D#
A#	C	A#	G#	F#	E	D	C	A#	G#	F#	E	D
B	C	B	A#	A	G#	G	F#	F	E	D#	D	C#

Table 4 shows that the addition in this set is a closed operation, so the set with these elements and the operation addition present a group and that is abelian (commutative) group. [4]

### 4. Conclusion

The research has extensively show that ring theory which is an aspect in mathematics that can be used to formulate music using the twelve musical notes, the behaviors and sound produced attested to the fact that mathematics is applicable to music from all indication and all group axioms in relation to ring theory are satisfied. It is suggested that the understanding of abstract part of mathematics will be a strong tool for musicians in other to have a well logical structured musical composition that will bring about healing of minds and souls to their audience. It has been established that musical tone behavior in relation with mathematics (ring theory) is possible and practicable. The twelve, (12), musical notes composition form a ring, with addition and multiplication operations, such that additive semigroup and multiplicative group forms a commutative group and semigroup respectively as a specific technique for the

composition of music in which by the help of mathematical matrix developed, an equal importance is given to all the twelve musical tones equal opportunity in the chromatic scale. A practical demonstration of the mathematical techniques as it applied to the twelve musical tones was carried out with a good sound as outcome when played on keyboard, which shows that abstract algebra as an area of mathematics housing the ring theorem is applicable as against human’s thought.

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