International Journal of Theoretical and Applied Mathematics
2021; 7(3): 53-56
http://www.sciencepublishinggroup.com/j/ijtam
doi: 10.11648/j.ijtam.20210703.12
ISSN: 2575-5072 (Print); ISSN: 2575-5080 (Online)

Sensitivity Analysis of Linear Programming in Decision Making Model

Shek Ahmed[1], Jakia Sultana[2], Tanzila Yeasmin Nilu[2], Shamima Islam[2]

[1]Department of Mathematics, University of Barishal, Barishal, Bangladesh

[2]Department of Computer Science and Engineering, Green University of Bangladesh, Dhaka, Bangladesh

Email address:
shekmathdu@gmail.com (S. Ahmed)

To cite this article:
Shek Ahmed, Jakia Sultana, Tanzila Yeasmin Nilu, Shamima Islam. Sensitivity Analysis of Linear Programming in Decision Making Model. *International Journal of Theoretical and Applied Mathematics*. Vol. 7, No. 3, 2021, pp. 53-56. doi: 10.11648/j.ijtam.20210703.12

Received: April 23, 2021; **Accepted**: May 11, 2021; **Published**: May 31, 2021

Abstract: The term Sensitivity Analysis (SA), sometimes called the post optimality analysis, refers to an analysis of the effect on the optimal solution of changes in the parameters of problem on the current optimal solution. Simplex method is an iterative procedure which gives the optimal solution to a Linear Programming Problem (LPP) in a finite number of steps or gives an indication that there is an unbounded solution whereas SA serves as an integral part of solving LPP and is normally carried out after getting optimal solution. In this research work, Sensitivity Analysis is used to understand the effect of a set of independent variables on some dependent variable under certain specific conditions. In order to determine the possible effect of independent parameters, we considered the changes in the input data of the optimal solution. This notion is actually based on the idea of Sensitivity Analysis. And it is found that all the possible alternative decision making converges in the neighborhood of the optimal solution. To avoid numerical complexity, we use LINDO software to show the changes in the input data and optimal solution.

Keywords: Linear Programming Problem (LPP), Sensitivity Analysis (SA), Simplex Method (SM), Shadow Price, Basic and Non-basic Variable

1. Introduction

Linear Programming, sometimes known as linear optimization, is the problem of maximizing or minimizing a linear function over a convex polyhedron specified by linear and non-negativity constraints [2, 3]. The earliest LP was first developed by Leonid Kantorovich in 1939. To plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy LP was used during World War II. It was kept secret until 1951. After war, many industries found its use in their daily planning. The founders of the subject are George B. Dantzig, who published the Simplex Method (SM) in 1951. The success of simplex method is described by Dantzig's assignment of 70 people to 70 jobs. The computing power required to scan all the permutations to select the best assignment is vast and impossible. He observed that it takes only a moment to find the optimum solution using the Simplex Method (SM), which is effectively noticing that a solution exists in the corners of the polygon described by the equations formed from the given constraints. Linear Programming (LP) is a tool for solving optimization problems. The development of LP has been ranked among the most important scientific advances of the mid-20th century. Its impact since just 1950 has been extraordinary. Today it is a standard tool that has saved many thousands or millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world and its use in other sectors of society has been spreading rapidly. A major proportion of all scientific computation on computers is devoted to the use of LP [9, 12].

For discussing the variation of different parameters in Linear Programming Problem sensitivity analysis plays a vital role. It gives a direct test of optimality of a particular problem due to the change of a component of cost or requirement vector. It works as a decisive step in the model building and result communication process. Using sensitivity analysis, we can obtain necessary insights on the model behavior, on its structure and on its response to changes in the model inputs. For different purpose of our life we can use sensitivity analysis.

2. Literature Review

The literature on Sensitivity Analysis (SA) is vast and diverse. In late 1980's and early 1990's several researchers and scientists were involved in the fields of operations research employed on the Linear Programming (LP) Sensitivity Analysis (SA) and some noteworthy advances were formed in LP. And SA. The research in the field of SA was extensively carried out by many O. R. specialists. Many researchers like Murty (1983), Luenberger (1984), Eschenbach and Mc Keague (1989), Bazaraa and Jarvis (1990), Hamby (1994), Murty (1995), Clemson (1995), Bradley et al. (1977), Gass S. (1997). Gal and Greenberg (1997) worked on SA parameter but excluded the simultaneous changes in the LP parameter. Arsham (1992) studied the SA for the parameters of structured problems. Khan et al. (2011) studied the profit in products by using LP techniques and SA [10, 11].

However, the prevailing works concerning our research scope is limited. Most of the previous work on sensitivity analysis were focused on long methodologies and a significant amount of time consumed to arrive at a solution of sensitivity analysis. Yang (1990) introduced two kinds of SA. First one, defines region the properties of sensitivity while the second one is the positive sensitivity analysis [1, 4]. This method has been presented in many papers and textbooks up to now (Dantzig (1963, 1978); Gal, (1979)).

3. Linear Programming Problem (LPP)

The general problem of linear programming is to optimize a linear function subject to linear equality and inequality constraints. In other words, we need to determine the values of $x_1, \ldots, x_n$ that solve the program [7, 8]:

(LP) Maximize (Minimize) $z = \sum_{i=1}^{n} c_i x_i$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \{\leq, =, \geq\} b_i \ (i = 1, \ldots, m) \quad (1)$$

Where one and only one of the signs $\leq, =, \geq$ holds for each constraint in (2.1), and the sign may vary from one constraint to another. Here, $c_j, b_i, and\ a_{ij}$ are known real numbers.

$$T = \{(x_1, \ldots, x_n)^T : (x_1, \ldots, x_n)^T \in R^n$$
$$and\ (2.1) hold\ at\ (x_1, \ldots, x_n)^T \quad (2)$$

The set T is also called the constraint set or feasible set, or feasible region of (LP).

4. Sensitivity Analysis (SA)

The investigation that deals with changes in the optimal solution due to changes in the parameters is called Sensitivity Analysis (SA) or post-optimality analysis. It plays an important rule to illustrate the nature of Linear Programming Problem (LPP) [3-6]. SA gives a direct test of Sensitivity of a particular problem due to the variation of a component of cost or requirement vector etc. Mainly Sensitivity Analysis (SA) is in general a post-optimality test which normally carried out after optimal solution is obtained. The optimal solution of the linear programming problem depends upon the parameters $(c_j, a_{ij}\ and\ b_i)$ of the problem. It the optimal value of the objective function is relative sensitive to changes in certain parameters, special care should be taken in estimating these parameters and selecting a solution which does well for most of their likely values. Then, it is quite important to know the range of the cost for which optimal solution remains optimal. It governs the variety of a component of the cost or requirement vector when all other component remains unchanged, so that optimal solution remains unaffected. Such type of change is called discrete change [13-15].

The changes in the linear programming problem which are usually studied by sensitivity analysis include:

i. Coefficients (c_j) of the objective function.

ii. Change in the right-hand side (b_i) constants

5. Proposed Model

A recognized Company manufactures, assembles, and rebuilds material handling equipment used in warehouses and distribution centers. One product, called a Lift trainer, is assembled from four components: a frame, a motor, two supports, and a metal strap. Companies production schedule calls for 5000 Lift trainers to be made next month. Walton purchases the motors from an outside supplier, but the frames, supports, and straps may be either manufactured by the company or purchased from an outside supplier. Manufacturing and purchase costs per unit are shown.

Table 1. *Component cost.*

Component	Manufacturing Cost	Purchase Cost
Frame	Tk 38.00	Tk 51.00
Support	Tk 11.50	Tk 15.00
Strap	Tk 6.50	Tk 7.50

Three departments are involved in the production of these components. The time (in minutes per unit) required to process each component in each department and the available capacity (in hours) for the three departments are as follows:

Table 2. *Required time and capacity of three department.*

	Department		
Component	Cutting	Milling	Shaping
Frame	3.5	2.2	3.1
Support	1.3	1.7	2.6
Strap	0.8	-	1.7
Capacity (hours)	350	420	680

The company wants to determine the total number of components which is to be manufactured and purchased. Also wants to fix the payments for an additional hour of time in the shaping department.

5.1. Formulation of Our Proposed Model

The following linear programming problem is formulated for making decision:

We define the decision variables as follows:

FM=Number of frames manufactured

FP=Number of Frames Purchased

SM=Number of supports manufactured

SP=Number of Supports purchased

TM=Number of straps manufactured

TP=Number of straps purchased

The objective function is to minimize the total cost, including manufacturing costs and purchase costs. Using the cost per unit data, we write the objective function as

Min 38 FM + 5I FP + 11.5 SM + 15 SP + 6.5 TM + 7.5 TP

At least 5000 lift trainers to be made next month. So at least 5000 Frames, Straps and 10000 Supports be manufactured or purchased to make 5000 lift masters.

The first three constraints are

$FM + FP \geq 5000$ (Frames)

$SM + SP \geq 10000$ (Supports)

$TM + TP \geq 5000$ (Straps)

The manufacturing times for the components are expressed in minutes. So, we state the total manufacturing capacity constraints in minutes, with the 350 hours of cutting time capacity becomes $60 \times 350=21000$ minutes.

Using the manufacturing time for cutting, milling and shaping department from the table, we have

$3.5\ FM + 1.3SM + 0.8\ TM \leq 21{,}000$

$2.2\ FM + 1.7\ SM \leq 25{,}200$

$3.1\ FM + 2.6\ SM + 1.7\ TM \leq 40{,}800$

The complete formation of companies makes or buy problem with all decision variable greater or Less than zero is

Min 38 FM + 51 FP + 11.5 SM + 15 SP + 6.5 TM + 7.5 TP

Subject to

$3.5\ FM + 1.3SM + 0.8\ TM \leq 21000$

$2.2\ FM + 1.7\ SM \leq 25200$

$3.1\ FM + 2.6\ SM + 1.7\ TM \leq 40{,}800$

$FM + FP \geq 5000$

$SM + SP \geq 10000$

$TM + TP \geq 5000$

$FM, FP, SM, SP, TM, TP \geq 0$

5.2. Computer Based Solution

Table 3. *Global optimal solution.*

Objective value	368076.9
Infeasibilities	0.000000
Total solver iterations	6
Model Class	LP
Total variables	6
Nonlinear variables	0
Integer variables	0
Total constraints	7
Nonlinear constraints	0
Total non-zeros	20
Nonlinear non-zeros	0

Table 4. *Optimum value with reduced cost of decision variable.*

Variable	Value	Reduced Cost
FM	5000.0	0.000000
FP	0.000000	3.576923
SM	2692.308	0.000000
SP	7307.692	0.000000
TM	0.000000	1.153846
TP	5000.0	0.000000

Table 5. *Slack or surplus value with dual price.*

Row	Slack or Surplus	Dual Price
1	368076.9	-1.000000
2	0.000000	2.692308
3	9623.077	0.000000
4	18300.00	0.000000
5	0.000000	-47.42308
6	0.000000	-15.000
7	0.000000	-7.500000

Ranges in which the basis is unchanged.

Table 6. *Objective coefficient ranges.*

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
FM	38.00000	3.576923	47.42308
FP	51.00000	INFINITY	3.576923
SM	11.50000	1.875000	1.328571
SP	15.00000	1.328571	1.875000
TM	6.500000	INFINITY	1.153846
TP	7.500000	1.153846	7.500000

Table 7. *Constraint ranges.*

Row	Current RHS	Allowable Increase	Allowable Decrease
2	21000.00	7358.824	3500.000
3	25200.00	INFINITY	9623.077
4	40800.00	INFINITY	18300.00
5	5000.000	1000.000	2714.286
6	10000.000	INFINITY	7307.692
7	5000.000	INFINITY	5000.000

5.3. Result Summary

Table 8. *Result.*

	Manufacture	Purchase
Frames	5000	0
Supports	2692	7308
Straps	0	5000

Total Cost=Tk368,076.91.

Subtract values of slack variables from minutes available to determine minutes used. Divide by 60 to determine hours of production time used.

Table 9. *Elapsed time duration.*

Constraint	
1	Cutting: Slack=0; 350 hours used
2	Milling:(25200 - 9623) / 60=259.62hours
3	Shaping:(40800 - 18300) / 60=375 hours

Additional hours are not needed, there are already more hours available than are being used.

5.4. Analyzing Result by Sensitivity Analysis

The computer solution of these 6 variables, 6 Constraints liner problem is given below:

The optimal solution indicates that all 5000 frames, 2692 supports should be manufactured and the remaining 7308 supports and 5000 straps should be purchased.

The total cost associated with the optimal make or buy plan is TK 368076.91.

The dual price for the manufacturing capacity cutting department (row2) is 2.692308. This price indicates that the hour of manufacturing capacity is worth TK 2.692308 per minute or (2.692308×60)=TK 161.53848 per hour.

The range of feasibility of cutting department (row2) Shows that this conclusion is valid until the amount of time increase to 7358.882 min/ 122.647 hours and decreases to 3500.00 or 58.33 hours.

Sensitivity analysis also indicates that a change in prices charged by the outside supplies can affect the optimal solution.

For instance, the range of optimality for objective function coefficient for FP lower bound is (51-3.576923)=47.423077 to no upper bound. If the purchase price for frames remains at 3.576923 or more the number of frames purchased (FP) will remain to zero. How if the purchase price drops below 3.576923, Company should begin to purchase rather than manufacture the frame component. Similar sensitivity analysis conclusions about the purchase price range can be drown for another component.

6. Conclusion

In this research, Sensitivity Analysis (SA) is applied to make the decision in identifying the key variables that influence in the cost and benefits of the project of a company. SA indicates that a change in prices charged by the outside supplies can affect the optimal solution. It is also observed that the changes in the positive right-hand side constant shows the maximum possible optimal solution for decision making problem and the shadow prices of various constraints help in determining how much one can afford to pay for increased in the constrained resources.

References

[1] Adler I, Monteiro R. D. C. (1992), A geometric view of parametric linear programming, Algorithmica Vol: 161-176.

[2] M. A. I. Bhuiyan and Shek Ahmed. 2017. A new Computer Oriented Technique for solving Linear Programming Problem Using Bender's Decomposition Method. Barishal University Journal Part 1. Vol 4 (2): 413-426.

[3] Tanzila Yeasmin Nilu, Shek Ahmed and M. A. I. Bhuiyan 2017. A Study of Sensitivity Analysis in Linear Programming problem and its Implementation in Real Life. Green University Bangladesh Journal of Science and Engineering, Vol 4 (1): 85-92.

[4] Winston, W. L., 1994. Linear Programming: Applications and Algorithm, Dunbury Press, Bellmont, California, U.S.A.

[5] Hashnayne Ahmed and Shek Ahmed 2019. A Comparative Study on Harvesting Plan Predicting insurance with Two-Stage Stochastic Analysis. International Journal on data Science and Technology. 5 (4): 73-82.

[6] Dahiya K, Verma V (2005), Sensitivity analysis in linear programming with bounded variables, ASOR Bulletin 24 (3), 2-19.

[7] Tanzila Yeasmin Nilu, Shek Ahmed and Hashnayne Ahmed 2020. Analysis of Diet Choice towards a Proper Nutrition Plan by Linear Programming. Science Journal of Applied Mathematics and Statistics. 2020; 8 (5): 59-66.

[8] Yang B. H. (1990), "A study on sensitivity analysis for a non-extreme optimal solution in linear programming", Ph. D. Thesis, Seoul National University, Republic of Korea.

[9] Tanzila Yeasmin Nilu, Hashnayne Ahmed and Shek Ahmed 2019. Mathematical Modeling for Launch Vessel Operators at Internal Waterways Transportation in Bangladesh, Green University Bangladesh Journal of Science and Engineering, Vol 6 (1): 46-53.

[10] Jansen B, JJ de Jong, Ross C, Terlaky T (1997), Sensitivity analysis in linear programming: just be careful, European Journal of Operational Research 101, 15-28.

[11] Monteiro R. D. C., Mehrotra S (1996), "A general parametric approach and its implications to sensitivity analysis in interior point methods, Mathematical Programming" 92, 65-82.

[12] Murty K. G. (1976), Linear and combinatorial programming, John Wiley & Sons INC., New York London Sydney Toronto.

[13] Park C, Kim W, Lee S, Park S (2004), Positive sensitivity analysis in linear programming, Asia-Pacific Journal of Operational Research 21 (1), 53-68.

[14] Roos. CT., Terlaky T, Vial (1997), Theory and algorithms for linear optimization, New York, John Wiley & Sons.

[15] Taha, H. A., Operations research: An introduction, 8th Ed. Pearson Princeton hall.