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Combined Heat and Mass Transfer Steady Flow of Viscous Fluid over a Vertical Plate with Large Suction

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Abstract: Combined heat and mass transfer of a viscous fluid along a semi-infinite vertical plate with large suction is studied analytically. Perturbation technique is used as main tool for the analytical approach. Viscous fluid behavior of heat and mass transfer over a vertical plate with large suction has been considered and its similarity equations have been obtained. Similarity equations of the corresponding momentum, temperature and concentration equations are derived by employing the usual similarity technique. The dimensionless similarity equations for momentum, temperature and concentration equation solved analytically by perturbation technique. The obtained numerical values of fluid velocity, temperature and concentration are drawn in figures. The results are discussed in detailed with the help of graphs to observe the effects of various parameters on the flow variables. Lastly, the important findings are concluded here.

Keywords: Mass Transfer, Heat Transfer, Eckert Number, Schmidt Number, Large Suction

1. Introduction

The heat transfer flows play a decisive role in many engineering applications as distillation, condensation, evaporation, rectification and absorption of a fluid as well as in fluids condensing or boiling at a solid surface. The heat transfer processes are of great interest in power engineering, metallurgy, astrophysics and geophysics. A natural convective heat transfer flow of fluid was first studied by *Finston* (1956) [1]. *Sparrow* and *Gregg* (1958) computed a similar solution for laminar free convection from a non-isothermal vertical plate [2]. A Finite difference solution of transient free convective flow over an isothermal plate was obtained by *Soundalgekar* and *Ganesan* (1981) [3]. A numerical study on the natural convective cooling problem of a vertical plate was completed by *Camargo et al*. (1996) [4].

The processes of mass transfer are of great interest in the production of materials in order to obtain the desired properties of a substance. Separation processes in chemical engineering such as the drying of solid materials, distillation, extraction and absorption are all affected by the process of mass transfer. Chemical reactions including combustion processes are often decisively determined by the mass transfer. *Callahan* and *Marner* (1976) studied a free convective

unsteady flow with mass transfer past a semi-infinite plate [5]. An investigation on free convective unsteady flow with mass transfer past an infinite vertical porous plate with constant suction was completed by *Soundalgekar* and *Wavre* (1977) [6]. Transient free convection flow on a semi-infinite vertical plate with mass transfer was observed by *Soundalgekar* and *Ganesan* (1980) [7].

The combined heat and mass transfer flows play a special role in power engineering, metallurgy, condensation, evaporation and rectification of a fluid. In the separation processes as drying of solid materials, distillation, extraction and absorption; the combined heat and mass transfer occur due to buoyancy forces caused by temperature difference and concentration difference. *Pera* and *Gebhart* (1971) was the first author to study the natural convective heat and mass transfer problem [8]. *Singh et al.* (2003 investigated the combined heat and mass transfer hydromagnetic flow of a viscous incompressible fluid along an infinite vertical porous plate [9]. *Chamkha* and *Khaled* (2001) investigated the problem of coupled heat and mass transfer from an inclined plate in the presence of absorption [10]. Hence the main aim of this research project was to study a combined heat and mass transfer steady flow of a viscous fluid along a semi-infinite vertical plate with large suction.

2. Mathematical Model of Flow

A heat and mass transfer steady flow of a viscous fluid along a semi-infinite vertical plate was considered. The flow was assumed to be in the $x-$ direction, which was chosen along the plate in upward direction and $y-$axis was normal to it. Initially, it was considered that the plate as well as the fluid particle was at rest at the same temperature $T(=T_\infty)$ and the same species concentration level $C(=C_\infty)$ at all points, where C_∞ and T_∞ are fluid concentration and temperature species of uniform flow respectively. It was also assumed that the fluid particles outside the boundary layer moved with a uniform velocity U_0. The suitable physical configuration with co-ordinate systems are shown in Fig 2.1.
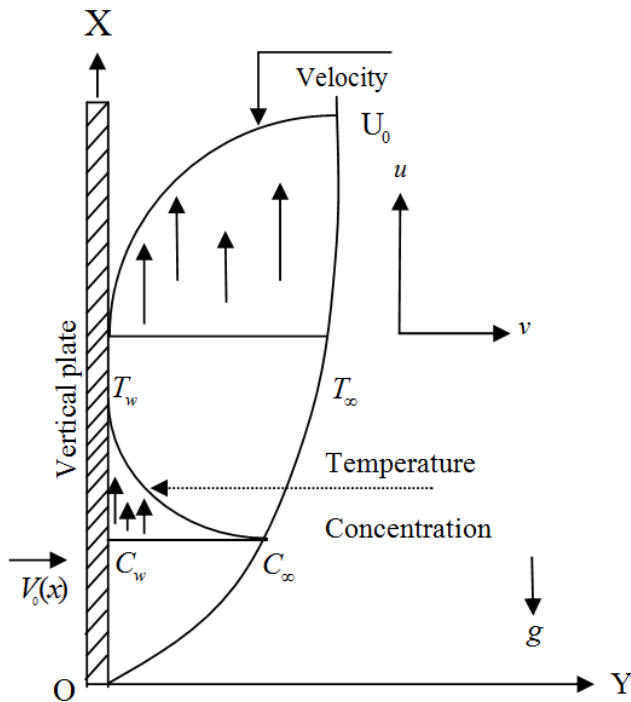

Fig. 2.1. Physical Configuration of Flow.

The following is the system of coupled governing non-linear partial differential equations.

The continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)$$
$$+g\beta^*(C-C_\infty)+v\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

The Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho\,c_p}\frac{\partial^2 T}{\partial y^2}$$
$$+\frac{v}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The Concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

and the boundary conditions are,

$$u=0,\ v=V_0(x),\ T=T_w\quad C=C_w\ \text{at y}=0$$
$$u=U_0,\ v=0,\qquad T\to T_\infty\quad C\to C_\infty\ \text{at y}\to\infty$$

where, x,y be the Cartesian coordinate; u,v are velocity components, g is the local acceleration due to gravity, β is the is the thermal expansion coefficient, β^* is the is the concentric expansion coefficient, T_w denotes the wall temperature, C_w is the species concentration at the wall, v is the kinematic viscosity, ρ is density, k is thermal conductivity, C_p is specific heat at constant pressure, D_m is coefficient of mass diffusivity, m is mass per unit area.

3. Mathematical Formulation

Since the researchers goal was to attain similarly solutions of the problem the researchers introduced the following variables,

$$\left.\begin{array}{c} \eta=y\sqrt{\dfrac{U_0}{2\,v\,x}} \\[2mm] \psi=\sqrt{2\,v\,U_0\,x}\ f(\eta) \\[2mm] u=\dfrac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\dfrac{\partial\psi}{\partial x} \end{array}\right\} \tag{5}$$

Where,

$$u=U_0 f'(\eta) \tag{6}$$

$$u\frac{\partial u}{\partial x}=-\frac{U_0^{\,2}}{2x}\eta f'(\eta)f''(\eta) \tag{7}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{U_0^{\,2}}{2\,v\,x}f'''(\eta) \tag{8}$$

$$\therefore v=\sqrt{\frac{v\,U_0}{2x}}\left[\eta f'(\eta)-f(\eta)\right] \tag{9}$$

Above values we get these;

$$f''' + ff'' + G_r\theta + G_m\varnothing=0 \tag{10}$$

$$\theta^{"} + P_r f \theta^{'} + P_r E_c f^{"2} = 0 \qquad (11)$$

$$\varnothing^{"} + S_C f(\eta) \varnothing^{'}(\eta) = 0 \qquad (12)$$

where, $G_r = \dfrac{2x}{U_0^{\;2}} g\beta(T_w - T_\infty)$ is the Grashof Number and

$G_m = \dfrac{2x}{U_0^{\;2}} g\beta(C_w - C_\infty)$ is the Modified Grashof

Number. $P_r = \dfrac{\mu c_p}{k} = \dfrac{\rho \upsilon c_p}{k}$ is the Prandtl Number and

$E_C = \dfrac{U_0^{\;2}}{c_p(T_w - T_\infty)}$ is the Eckert Number.

  With the boundary conditions

$$f = f_w, \; f' = 0, \; \theta = 1, \; \varnothing = 1 \quad \text{at} \quad \eta = 0$$

$$f' = 1, \quad \theta = 0, \; \varnothing = 0 \qquad \text{at} \quad \eta \to \infty$$

Where, $f_w = -V(x)\sqrt{\dfrac{2x}{U_0 \nu}}$ the transpiration parameter

and prime is denotes derivatives with respect to η . Here $f_w > 0$ indicates the suction and $f_w < 0$ the injection.

4. Mathematical Analysis

$$\left.\begin{aligned}
\xi &= \eta f_w, \\
f(\eta) &= f_w \, F(\xi), \\
\theta(\eta) &= f_w^2 \, H(\xi) \\
\varnothing(\eta) &= f_w^2 \, G(\xi)
\end{aligned}\right\} \qquad (13)$$

  By using the conversion in the equation (10) and (11) we have,

$$\begin{aligned}
&f_w^4 \, F^{'''} + f_w F f_w^3 F^{''} + G_r f_w^2 H \\
&+ G_m f_w^2 G = 0
\end{aligned} \qquad (14)$$

$$\begin{aligned}
&f_w^4 H^{''} + P_r f_w F \cdot f_w^3 H^{'} \\
&+ P_r E_C f_w^6 F^{''2} = 0
\end{aligned} \qquad (15)$$

$$f_w^4 G^{"} + S_C f_w F f_w^3 G^{'} = 0 \qquad (16)$$

  Therefore the equation (13) with boundary conditions as given below

$$\left.\begin{aligned}
&F^{'''} + F\,F^{''} + \varepsilon\,G_r H + \varepsilon\,G_m G = 0 \\
&H^{''} + P_r\,F\,H^{'} + P_r E_C \frac{1}{\varepsilon}\,F^{''2} = 0 \\
&G^{"} + S_C\,FG^{'} = 0
\end{aligned}\right\} \qquad (17)$$

$$F = 1, \; F' = 0, \; H = \varepsilon, G = 0 \quad \text{at} \quad \xi = 0$$

$$F' = \varepsilon, \quad H = 0, G = 0 \qquad \text{at} \quad \xi \to \infty$$

where, $\varepsilon = \dfrac{1}{f_w^2}$

  Now for the large solution $f_w > 1$ .If f_w is very large then ε will be very small. Therefore following Bestman (1990) and Singh & Dikshit (1988), F, H and G can be expended in terms of the small perturbation quantity ε as,

$$F(\xi) = 1 + \varepsilon\,F_1(\xi) + \varepsilon^2 F_2(\xi) + \qquad (18)$$

$$H(\xi) = \varepsilon\,H_1(\xi) + \varepsilon^2 H_2(\xi) + \qquad (19)$$

$$G(\xi) = \varepsilon\,G_1(\xi) + \varepsilon^2 G_2(\xi) + \qquad (20)$$

Taking the order $O(\varepsilon)$ the system (17) becomes,

$$\left.\begin{aligned}
&F_1^{'''} + F_1^{''} = 0 \\
&H_1^{''} + P_r\,H_1^{'} + P_r\,E_C\,F_1^{''2} = 0 \\
&G_1^{"} + S_C\,G_1^{'} = 0
\end{aligned}\right\} \qquad (21)$$

With boundary condition

$$\left.\begin{aligned}
&F_1 = 0, \; F_1' = 0, \; H_1 = 1, G_1 = 1 \quad \text{at} \quad \xi = 0 \\
&F_1' = 1, \; H_1 = 0, G_1 = 0 \qquad \text{at} \quad \xi \to \infty
\end{aligned}\right\}$$

Again for order $O(\varepsilon^2)$ the system (17) becomes,

$$\left.\begin{aligned}
&F_2^{'''} + F_2^{''} + F_1 F_1^{''} + G_r H_1 + G_m G_1 = 0 \\
&H_2^{''} + P_r\left(F_1 H_1' + H_2' + 2E_C F_1' F_2''\right) = 0 \\
&G_2^{"} + S_C\left(F_1 G_1' + G_2'\right) = 0
\end{aligned}\right\} \qquad (22)$$

with boundary condition

$$\left.\begin{aligned}
&F_2 = 0, \; F_2' = 0, \; H_2 = 0, G_2 = 0 \quad \text{at} \quad \xi = 0 \\
&F_2' = 0, \; H_2 = 0, G_2 = 0 \qquad \text{at} \quad \xi \to \infty
\end{aligned}\right\}$$

5. Solution

  The first order solution becomes;

$$F_1 = -1 + \xi + e^{-\xi}, \; H_1 = e^{-P_r \xi} + A_{11}\,e^{-2\xi}$$

$$\text{and} \; G_1 = e^{-S_C \xi} \, .$$

and the second order solution becomes

$$F_2 = -3\xi e^{-\xi} - \xi e^{-\xi} - \frac{1}{2}\xi^2 e^{-\xi}$$
$$+ A_{21}e^{-P_r\,\xi} + A_{22}e^{-2\xi} + A_{23}e^{-S_c\,\xi}$$

$$H_2 = A_{27}e^{-S_c\,\xi} + A_{28}e^{-3\xi} + \left(B_{12} + B_{13}\xi + B_{14}\xi^2\right) e^{-2\xi}$$
$$+ \left(A_{30}\,\xi - B_{11}\xi^2\right)e^{-P_r\,\xi} + A_{31} e^{-\xi\,(P_r+1)}$$

$$G_2 = (C_{11} + C_{12}\xi + C_{13}\xi^2)e^{-S_c\,\xi} + C_{11}e^{-\xi}$$

6. Results and Discussions

The analytical solutions were obtained by using the perturbation technique. In order to analyze the physical situation of the model, the researchers computed the numerical values of the flow variables for different values of suction parameter (f_w), Grashof number (G_r), Modified Grashof Number (G_m), Schmidt number (S_c), Eckert Number (E_c) and the Prandtl Number (P_r). The fluid velocity, temperature and concentration versus the non-dimensional coordinate variable η are displayed in Figs. 6.1 - 6.15. It is noted in Fig. 6.1 that the velocity remains unchanged with the increasing of Eckert Number (E_c). Fig.6.2 represents the velocity increase with the increase of the value of the suction parameter (f_w). From Fig. 6.3 it can be seen that the velocity decreases with the increase of the value of the Grashof number (G_r). Fig. 6.4 shows that the velocity remains unchanged with the increase of the value of the Prandtl Number (P_r). In Fig. 6.5, the velocity decreases with the increase of the value of the Schmidt number (S_c). In Fig. 6.6, the fluid temperature decreases with the increase of the value of the Eckert Number (E_c). Fig. 6.7 shows that the temperature decreases with the increase of the value of the suction parameter (f_w). It is found in Fig. 6.8 that the temperature remains unchanged with the increase of the value of the Grashof number (G_r). Fig. 6.9 shows that the temperature decreases with the increase of the value of the Prandtl Number (P_r). Fig. 6.10 shows that the temperature increases with the increase of the value of the Schmidt number (S_c). Fig. 6.11 shows that the concentration remains unchanged with the increase of the value of the Eckert Number (E_c). It is shown in Fig. 6.12 that the concentration decreases with the increase of the value of the suction parameter (f_w). Fig. 6.13 shows that the concentration remains unchanged with the increase of the value of the Grashof number (G_r). It is observed from Fig. 6.14 that the concentration remains unchanged with the increase of the value of the Prandtl Number (P_r). Fig. 6.15 shows that the concentration decreases with the increase of the value of the
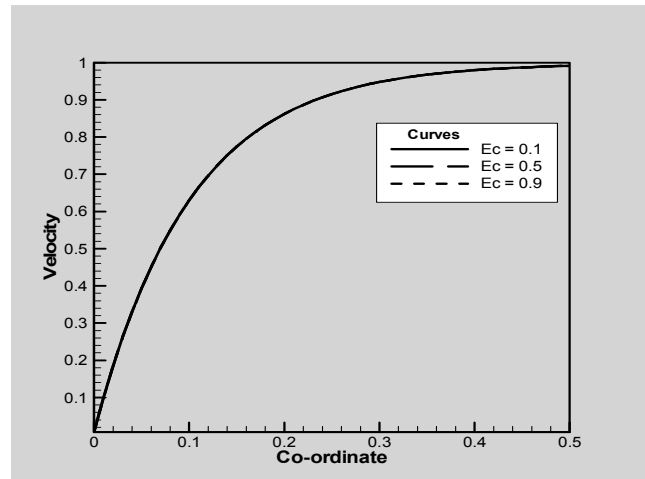
Schmidt number (S_c).

Fig. 6.1. *Velocity profiles for different values of* E_C *with* $G_m = 0.2$, $G_r = 0.2$, $P_r = 0.5$, $f_w = 10.0$ *and* $S_C = 0.3$.
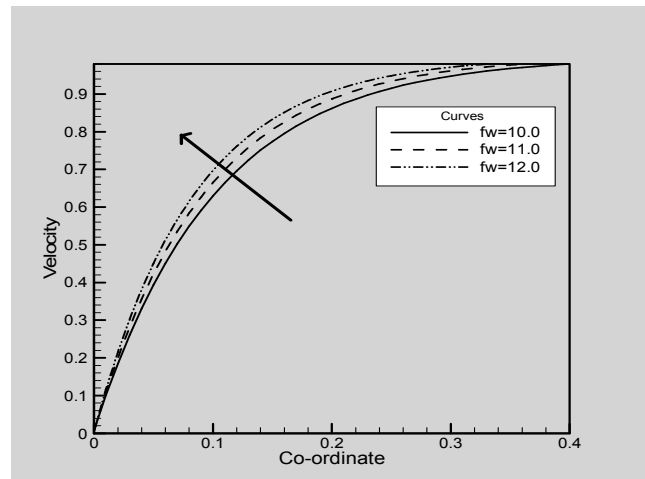

Fig. 6.2. *Velocity profiles for different values of* f_w *with* $G_m = 0.2$, $G_r = 0.2$, $P_r = 0.5$, $E_C = 0.1$ *and* $S_C = 0.3$.
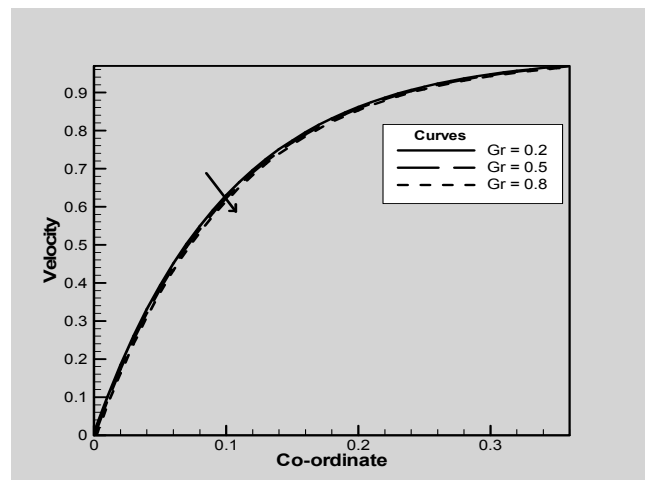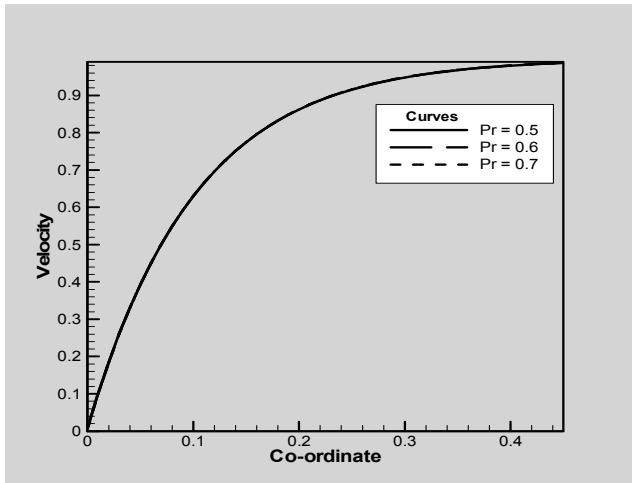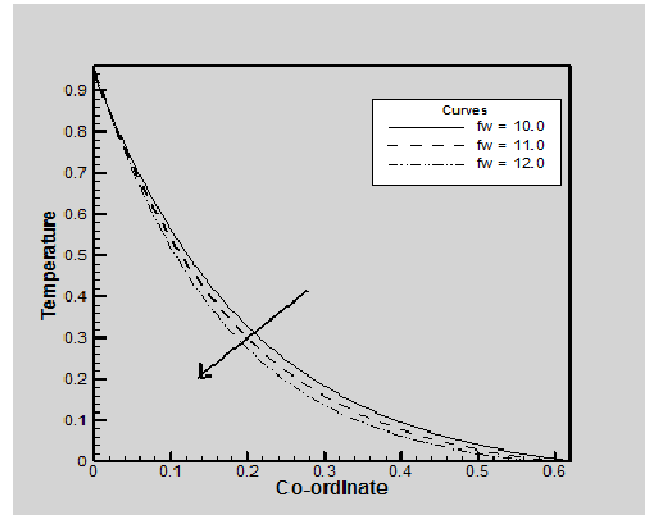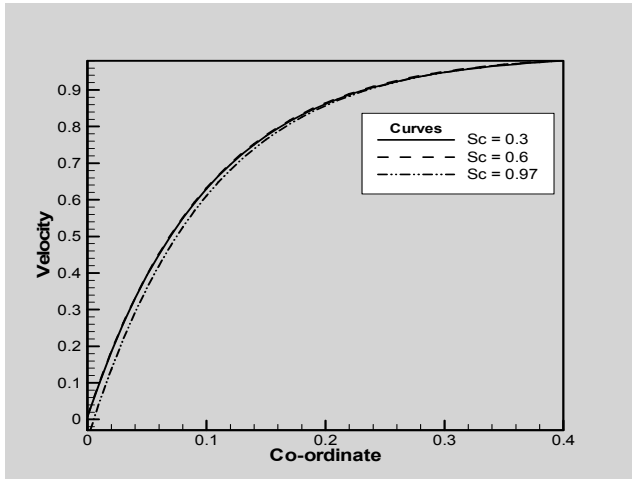

Fig. 6.3. *Velocity profiles for different values of* G_r *with* $G_m = 0.2$, $E_C = 0.1$, $P_r = 0.5$, $f_w = 10.0$ *and* $S_C = 0.3$.
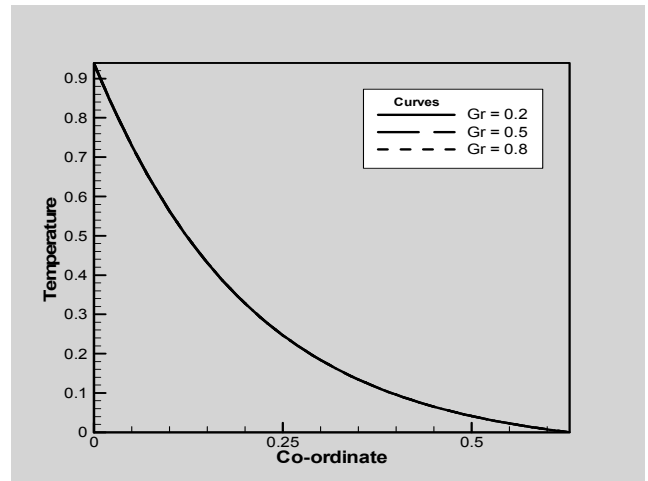
Fig. 6.4. *Velocity profiles for different values of* P_r *with* $G_m = 0.2$, $E_C = 0.1$, $G_r = 0.2$, $f_w = 10.0$ *and* $S_C = 0.3$.
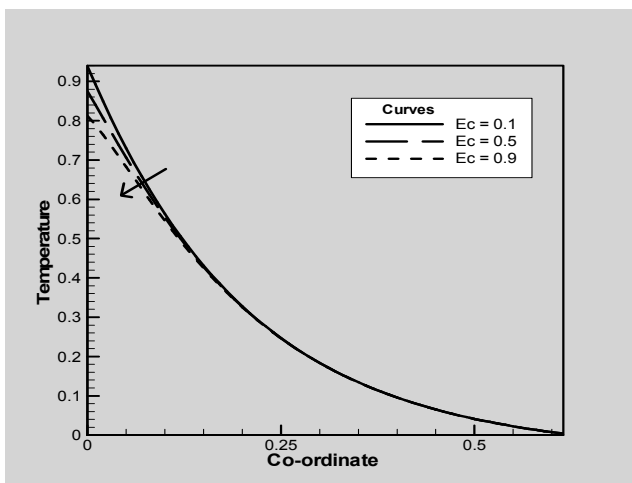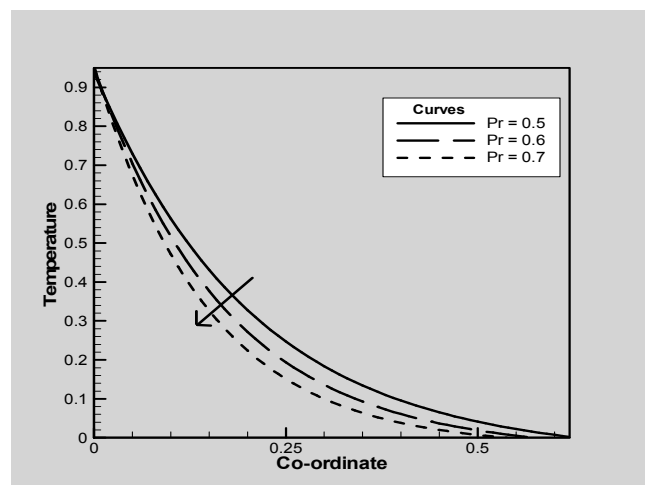

Fig. 6.5. *Velocity profiles for different values of* S_C *with* $G_m = 0.2$, $E_C = 0.1$, $G_r = 0.2$, $f_w = 10.0$ *and* $P_r = 0.5$.

Fig. 6.6. *Temperature profiles for different values of* E_C *with* $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $f_w = 10.0$ *and* $P_r = 0.5$.

Fig. 6.7. *Temperature profiles for different values of* f_w *with* $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $E_C = 0.1$ *and* $P_r = 0.5$.
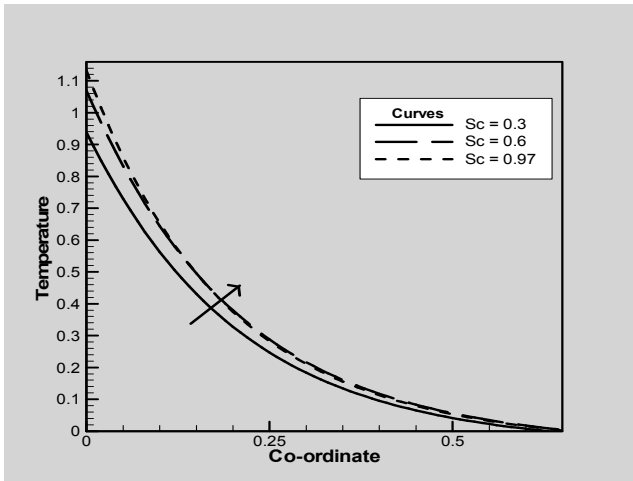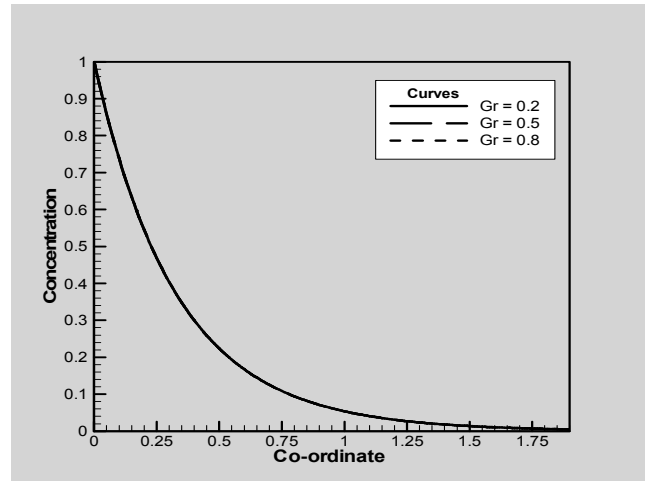

Fig. 6.8. *Temperature profiles for different values of* G_r *with* $G_m = 0.2$, $S_C = 0.3$, $E_C = 0.1$, $f_w = 10.0$ *and* $P_r = 0.5$.
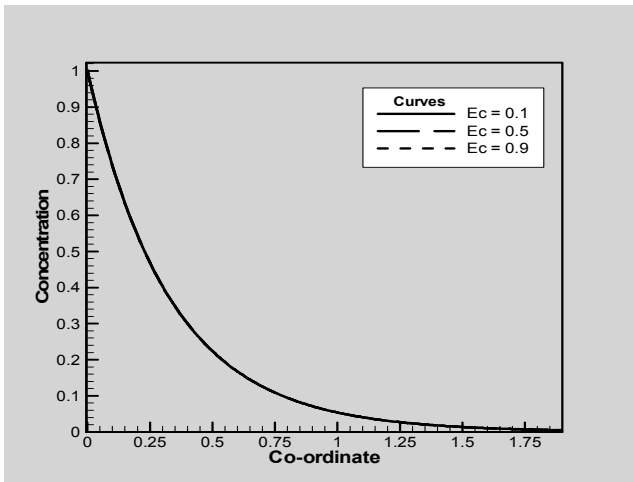

Fig. 6.9. *Temperature profiles for different values of* P_r *with* $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $f_w = 10.0$ *and* $E_C = 0.1$.
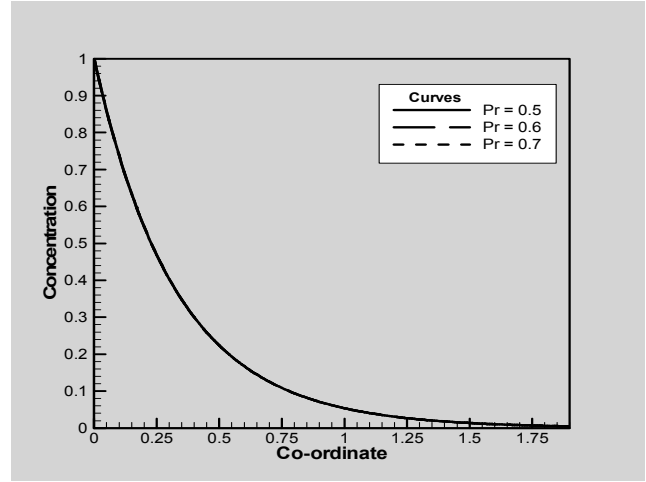
Fig. 6.10. Temperature profiles for different values of S_C with $G_m = 0.2$, $E_C = 0.1$, $G_r = 0.2$, $f_w = 10.0$ and $P_r = 0.5$.

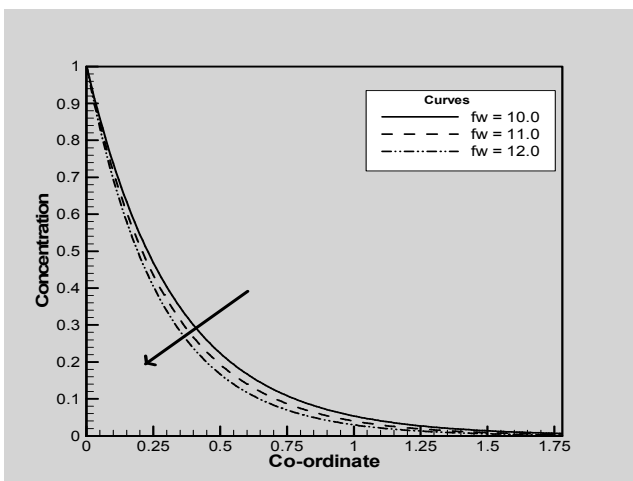

Fig. 6.13. Concentration profiles for different values of G_r with $G_m = 0.2$, $S_C = 0.3$, $f_w = 10.0$, $E_C = 0.1$ and $P_r = 0.5$.
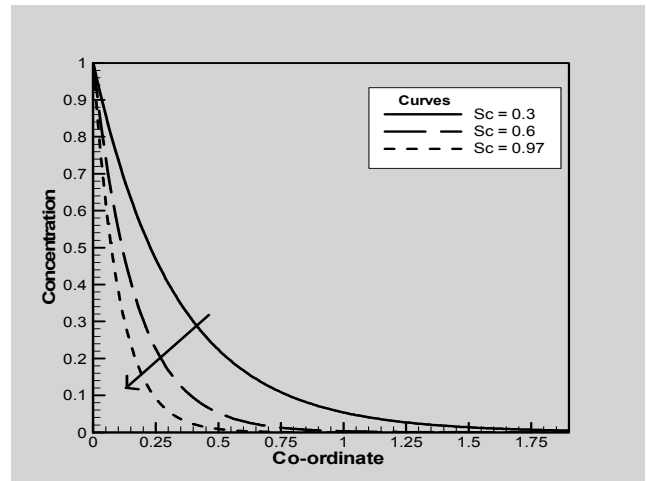

Fig. 6.11. Concentration profiles for different values of E_C with $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $f_w = 10.0$ and $P_r = 0.5$.

Fig. 6.14. Concentration profiles for different values of P_r with $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $E_C = 0.1$ and $f_w = 10.0$.

Fig. 6.12. Concentration profiles for different values of f_w with $G_m = 0.2$, $S_C = 0.3$, $G_r = 0.2$, $E_C = 0.1$ and $P_r = 0.5$.

Fig. 6.15. Concentration profiles for different values of S_C with $G_m = 0.2$, $f_w = 10.0$ $G_r = 0.2$, $E_C = 0.1$ and $P_r = 0.5$.

7. Conclusions

Viscous fluid behaviors of heat and mass transfer over a vertical plate with large suction have been observed in this work. The resulting governing dimensionless coupled non-linear partial differential equations have been solved analytically by perturbation technique. The results are shown for different values of important parameters as the Eckert number, suction parameter, Prandtl number, Grashof number and Schimdt number. Some of the important findings obtained from the graphical representation of the results are listed below:

1. The velocity profiles increases with the increase of suction parameter f_w while the velocity profile decreases with the increase of grashof number G_r.

2. The temperature profiles increases with the increase of suction parameter f_w while the temperature profiles decrease with the increase of Prandtl number P_r.

3. The concentration profiles decreases with the increase of Grashof Number (G_r) and Schimdt number S_C.

Acknowledgement

Nomenclature

S_c	Schmidt number
G_r	Grashof number
G_m	Modified Grashof number
C	Concentration
D_c	Mass Diffusivity
u	Velocity component in the x-direction
v	Velocity component in the y-direction
w	Velocity component in the z-direction
∇	Differential operator
P_r	Prandtl number
k	Thermal Conductivity
f_w	Suction parameter
β	Co-efficient of thermal expansion
β^*	Concentric expansion coefficient
δ	Boundary layer thickness
μ	Co-efficient of viscosity
υ	Co-efficient of kinematic viscosity
ρ	Density of the fluid

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