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Topological Structure of Fuzzy PU-new Ideal in PU-algebra

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Abstract: In this manuscript, we consider the fuzzification of the notion PU-new ideal of PU-algebra and define the fuzzy topological terms such that fuzzy topology, τ-open fuzzy set, fuzzy neighborhood, fuzzy interior, sequence of fuzzy sets, fuzzy neighborhood system, fuzzy continuity of a function with respect to PU-new ideal of PU-algebra. We explore the new theorems and related properties of above mention notions with respect to PU-new ideal on PU-algebras. Such that for $(Ʒ, \tau)$ to be a TSFP on Ʒ and the set $\dot{F}$ is a fuzzy in Ʒ and $N_{\dot{F}}$ be a fuzzy neighborhood system of $\dot{F}$ then the finite intersection of elements of '$N_{\dot{F}}$' is also an element of '$N_{\dot{F}}$' also any fuzzy set of Ʒ which contains an element of "$N_{\dot{F}}$" is also an element of "$N_{\dot{F}}$". Furthermore we prove the conditions with respect to fuzzy neighborhood, convergence of a sequence of fuzzy sets, fuzzy interior set of a fuzzy set under which a fuzzy set $\dot{F}$ is τ-open. We show that how the function Ψ from $(Ʒ_1,\tau)$ to $(Ʒ_2,\omega)$ is fuzzy continuous. We prove that if Ψ is a fuzzy continuous function then for every fuzzy set $\dot{F}$ in $Ʒ_1$, inverse of each neighborhood of $\Psi(\dot{F})$ is a neighborhood of a fuzzy set $\dot{F}$.

Keywords: PU-algebra, Fuzzy PU-new Ideal, Fuzzy Topology, Topology of Fuzzy PU-new Ideal, Fuzzy Continuity

1. Introduction

The concept of topological space is one of the important concepts that deal with sets theory and this notion is discussed in different fields [1-13]. Imai and Iseki [14] introduced two classes of abstract algebras, which are the BCK − algebra and the BCI − algebra [15]. Next, a wide class of abstract algebras (BCH −algebras) was introduced by Hu [16] in 1983 and Li [17] in 1985. Further, they showed that the classes of BCK − algebras were proper subclasses of the Khalil 98 classes of BCI − algebras and the classes of BCI − algebras were proper subclasses of the classes of BCH-algebras. Next, some new classes of algebras were given [18-22]. Also, many applications on set theory of groups and topological spaces are shown [23-28]. A Fuzzy set, which is a class of objects with a continuum of grades of membership, was a concept proposed by Zadeh [29] in 1965. After that many concepts were introduced on Fuzzy sets like fuzzy topology by Chang [30] in 1968 and in recent years, some interesting studies and applications on fuzzy sets have been discussed by Shuker [31-35] and others. Jun [36] combined the structure of Fuzzy topological spaces with that of a Fuzzy BCK-algebra to formulate the theory of Fuzzy topological BCK-algebras. In 1999, the concept of d − algebra, which is another generalization of BCK-algebras, is introduced by Neggers and Kim [37]. Also, the notion of d −ideal in d −algebra is discussed by Jun, Neggers and Kim [38]. After that, they introduced the notions of fuzzy d −sub algebra, fuzzy d −ideal, fuzzy B − algebras, fuzzy BCI − algebras and the relations among them are shown [39-41]. Keawrahun and Leerawat [42] determined another class of abstract algebras known as SU-Algebra. In 2018, Muralikrishna and Vijayan [43] combined SU-algebras and fuzzy topology. During 2015 Mostafa [44] introduced a new structured algebra known as PU-algebra while fuzzy PU-new ideal of PU-algebra have been discussed by Mostafa [45].

In this manuscript, we consider the fuzzification of the notion PU-new ideal of PU-algebra and define the fuzzy topological terms such that fuzzy topology, τ-open fuzzy set, fuzzy neighborhood, fuzzy interior, sequence of fuzzy sets, fuzzy neighborhood system, fuzzy continuity of a function with respect to PU-new ideal of PU-algebra. We explore the new theorems and related properties of above mention notions with respect to PU-new ideal on PU-algebras. Furthermore we prove that under what conditions a fuzzy set

$\dot{F}$ is τ-open. We show that how the function Ψ from $(Ẕ_1,\tau)$ to $(Ẕ_2,\omega)$ is fuzzy continuous. we prove that if Ψ is a fuzzy continuous function then for every fuzzy set $\dot{F}$ in $Ẕ_1$, inverse of each neighborhood of $\Psi(\dot{F})$ is a neighborhood of $\dot{F}$.

2. Preliminaries

In this section we take into account some of the basic definitions which play an essential role in the proof of theorems.

Definition (1): Let $Ẕ \neq \{\}$ then the mapping T: $Ẕ \rightarrow [0, 1]$ is called a fuzzy set of Ẕ.

Definition (2): Suppose that T_1 and T_2 are any two fuzzy sets of Ẕ then $T_1 U T_2$ is also a fuzzy set of Ẕ which is defined as

$$(T_1 U\, T_2)(z) = \max \{(T_1)(z), (T_2)(z)\}, \text{ for any } z \in Ẕ.$$

Definition (3): Suppose that T_1 and T_2 are any two fuzzy sets of Ẕ then $T_1 \cap T_2$ is also a fuzzy set of Ẕ which is defined as

$$(T_1 \cap T_2)(z) = \min \{(T_1)(z), (T_2)(z)\}, \text{ for any } z \in Ẕ.$$

Definition (4): Suppose that T1 and T2 are any two fuzzy sets of a set Ẕ and if T1 $\subset$ T2 then (T1) (z) $\leq$ (T2) (z), $\forall$ z$\in$ Ẕ.

Definition (5): Suppose that T is any fuzzy set of a set Ẕ then the complement T` of T is defined by T`(z) = 1− T (z), $\forall$ z $\in$ Ẕ.

Definition (6): Suppose that τ is a family of fuzzy sets in a set Ẕ then τ is said to be a fuzzy topology if it satisfies the following conditions

(τ_{i}): $\bar{0}, \bar{1} \in \tau$ where $\bar{0} = T(z) = 0, \forall z \in Ẕ$ and $\bar{1} = T(z) = 1, \forall z \in Ẕ$.

(τ_{ii}): If $T_1, T_2 \in \tau$ then $T_1 \cap T_2 \in \tau$.

(τ_{iii}): If $T_i \in \tau$ for all $i \in I$ then $\bigcup_{i \in I} T_i \in \tau$, where I represents the indexing set.

Definition (7): Let Ẕ is a non-vacuous set and τ be a family of fuzzy sets in Ẕ then the pair (Ẕ,τ) is termed as a fuzzy topological space while the elements of τ are termed as τ-open fuzzy sets in Ẕ.

Definition (8): Suppose that (Ẕ,τ) is a fuzzy topological space and let the fuzzy set T1 be in Ẕ. Then a fuzzy set T2 $\in$ τ is defined to be a neighborhood of T1 if there exists a τ-open fuzzy set δ such that T1 $\subset \delta \subset$ T2 i.e. (T1) (z) $\leq$ δ(z) $\leq$ (T2) (z), $\forall$ z $\in$ Ẕ.

Definition (9): Let T1 and T2 are any two fuzzy sets in (Ẕ,τ) and let T1$\supset$ T2 then T2 is said to be an interior of T1 if T1 is a neighborhood T2.

Definition (10): If T_1 is any fuzzy set in $(Ẕ,\tau)$ then all of its fuzzy interior sets union is also its interior which is represented by T_1°.

Definition (11): A PU-algebra (Ẕ, $*$, 0) is a class of the type (2, 0) algebras satisfying (PU-i) and (PU-ii) conditions for all a, b, c $\in$ Ẕ, where

(PU-i) . $0 * a = 0$

(PU-ii) . $(a * c) * (b * c) = b * a$

Example (1): Suppose that Ẕ $=\{0, a, b, c\}$ in which binary operation * is defined

Table 1. Tabular arrangement of the values of the set Ẕ satisfying the axioms of PU-algebra.

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Shows (Ẕ $=\{0, a, b, c\}$, $*$) is a PU-Algebra.

Definition (12): Let (Ẕ,$*$,0) is a PU-algebra and let $\dot{F}$ be a fuzzy subset in Ẕ then $\dot{F}$ is called a fuzzy PU-Subalgebra of Ẕ if $\dot{F}(z_1 * z_2) \geq \min\{\dot{F}(z_1), \dot{F}(z_2)\}, \forall z_1, z_2 \in Ẕ$.

Definition (13): Let (Ẕ,$*$,0) is a PU-algebra and let $\dot{F}$ be a fuzzy subset in Ẕ then $\dot{F}$ is called a fuzzy PU new-ideal of Ẕ if $\dot{F}((z_1*(z_2*z_3))*z_3) \geq \min\{\dot{F}(z_1), \dot{F}(z_2)\}, \forall z_1, z_2, z_3 \in Ẕ$.

3. Topological Structure of Fuzzy PU-new Ideal (TSFP)

In this portion we describe the topological structure of fuzzy PU-new ideal (TSFP) on PU-Algebra (Ẕ,$*$,0). For simplicity we use the notations $\dot{F}_1$, $\dot{F}_2$, $\dot{F}_3$ etc. for representing the elements of "τ".

Definition (14): Suppose that (Ẕ,$*$,0) is a PU-Algebra then the pair $(Ẕ,\tau)$ is said to be a topological structure of a fuzzy PU-new ideal (TSFP) on a PU-algebra (Ẕ,$*$,0) if the family τ of fuzzy PU-new ideals in Ẕ holds the following axioms.

(τ_1): $\bar{0}, \bar{1} \in \tau$

(τ_2): If any $\dot{F}_1, \dot{F}_2 \in \tau \Longrightarrow \dot{F}_1 \cap \dot{F}_2 \in \tau$.

(τ_3): If $\dot{F}_i \in \tau$ for all $i \in I \Longrightarrow \bigcup_{i \in I} \dot{F}_i \in \tau$ where I is an indexing set.

Definition (15): If $(Ẕ,\tau)$ is a topological structure of a fuzzy PU-new ideal (TSFP) than any member of τ is called τ-open fuzzy set in the PU-Algebra Ẕ.

Definition (16): Let us consider the PU-Algebra which has been defined in [example 2.12] therefore for that algebra we define the fuzzy PU-new ideals $\dot{F}_i$, i = 1, 2, 3, 4, 5, 6 on Ẕ as follows

$$\dot{F}_1(z) = \begin{cases} 0.7; & 0 \\ 0.4; & a \\ 0.4; & b \\ 0.5; & c \end{cases} \quad \dot{F}_2(z) = \begin{cases} 0.6; & 0 \\ 0.3; & a \\ 0.3; & b \\ 0.4; & c \end{cases}$$

$$\dot{F}_3(z) = \begin{cases} 0.8; & 0 \\ 0.5; & a \\ 0.5; & b \\ 0.6; & c \end{cases} \quad \dot{F}_4(z) = \begin{cases} 0.5; & 0 \\ 0.2; & a \\ 0.2; & b \\ 0.3; & c \end{cases}$$

$$\dot{F}_5(z) = \begin{cases} 0.9; & 0 \\ 0.5; & a \\ 0.5; & b \\ 0.7; & c \end{cases} \quad \dot{F}_6(z) = \begin{cases} 0.7; & 0 \\ 0.4; & a \\ 0.4; & b \\ 0.6; & c \end{cases}$$

Then $(Ẕ, \tau = \{\bar{0}, \bar{1}, \dot{F}_1, \dot{F}_2, \dot{F}_3, \dot{F}_4, \dot{F}_5, \dot{F}_6\})$ is a TSFP on a PU-algebra

Definition(17): Let $(Ẕ,\tau)$ is a topological structure of a fuzzy PU-new ideal (TSFP) on a PU-Algebra Ẕ then the fuzzy set $\dot{F}_1$ in $(Ϛ,\tau)$ is defined to be a fuzzy neighborhood of $\dot{F}_2$ if there exists a τ-open set Θ so that

$$\dot{F}_2 \subset \Theta \subset \dot{F}_1 \text{ or } \dot{F}_2(z) \leq \Theta(z) \leq \dot{F}_1(z),\ \forall\, z \in Ϛ$$

Definition (18): Consider the PU-Algebra which has been defined in [example 2.12] and the TSFP $(Ϛ,\tau)$ which has defined in example 3.3 from these examples we have noted that $\dot{F}_5$ is a fuzzy neighborhood of a fuzzy set $\dot{F}_2$ such that

$$\dot{F}_2(z) \leq \dot{F}_3(z) \leq \dot{F}_5(z)$$

Definition (19): Let $\dot{F}_1$ and $\dot{F}_2$ are any two fuzzy sets in a TSFP $(Ϛ,\tau)$ on a PU-algebra Ϛ and suppose that $\dot{F}_1 \supset \dot{F}_2$ then $\dot{F}_2$ is called a fuzzy interior of $\dot{F}_1 \Leftrightarrow \dot{F}_1$ is a fuzzy neighborhood of $\dot{F}_2$.

Remarks (1): If $\dot{F}_1$ is any fuzzy set in $(Ϛ,\tau)$ then all of its fuzzy interior sets union is also its interior which is designated by $\dot{F}_1^{\circ}$.

Example (2): Consider the PU-Algebra which has been defined in example 2.12 and the TSFP $(Ϛ,\tau)$ which has been defined in example 3.3. From these examples we have noted that $\dot{F}_5$ is a fuzzy neighborhood of fuzzy sets $\dot{F}_1$, $\dot{F}_2$, $\dot{F}_4$, $\dot{F}_6$ and

$$\dot{F}_2(z) \leq \dot{F}_1(z) \leq \dot{F}_5(z),\ \forall\, z \in Ϛ$$

$$\dot{F}_1(z) \leq \dot{F}_3(z) \leq \dot{F}_5(z),\ \forall\, z \in Ϛ$$

$$\dot{F}_4(z) \leq \dot{F}_1(z) \leq \dot{F}_5(z),\ \forall\, z \in Ϛ$$

$$\dot{F}_6(z) \leq \dot{F}_3(z) \leq \dot{F}_5(z),\ \forall\, z \in Ϛ$$

Which shows that $\dot{F}_1$, $\dot{F}_2$, $\dot{F}_4$, $\dot{F}_6$ are the fuzzy interiors of $\dot{F}_5$ and

$\dot{F}^{\circ} = \cup\{\dot{F}_1, \dot{F}_2, \dot{F}_4, \dot{F}_6\} = \max\{\dot{F}_1(z), \dot{F}_2(z), \dot{F}_4(z), \dot{F}_6(z)\} = \dot{F}_6(z)$, which implies that $\dot{F}^{\circ}$ is also an interior of $\dot{F}_5$.

Definition (20): Let $(Ϛ,\tau)$ is a TSFP on Ϛ and let a set $\{\dot{F}_n : n \geq 1 \wedge n \in N\}$ is a sequence of fuzzy sets in a PU-algebra Ϛ such a set of sequences is said to be contained in $\dot{F}$ Where $\dot{F}$ is a fuzzy set $\Leftrightarrow$ there exists $i \in N$ so that $n \geq i \Longrightarrow \dot{F}_n \subset \dot{F}$.

Definition (21): Let $(Ϛ,\tau)$ is a TSFP on Ϛ then a sequence $\{\dot{F}_n : n \geq 1 \wedge n \in N\}$ of fuzzy PU-new ideals in Ϛ converges to a fuzzy PU-new ideal $\dot{F}$ in Ϛ $\Leftrightarrow$ every fuzzy neighborhood of $\dot{F}$ contains such a sequence.

Definition (22): Consider the PU-Algebra which has been defined in [example 2.12]. For that algebra we determine the fuzzy PU-new ideals $\dot{F}i, i = 1,2,3,4,5,6,7,8$ on Ϛ as follows.

$$\dot{F}_1(z) = \begin{cases} 0.7; & 0 \\ 0.4; & a \\ 0.4; & b \\ 0.5; & c \end{cases} \quad \dot{F}_2(z) = \begin{cases} 0.6; & 0 \\ 0.3; & a \\ 0.3; & b \\ 0.4; & c \end{cases}$$

$$\dot{F}_3(z) = \begin{cases} 0.8; & 0 \\ 0.5; & a \\ 0.5; & b \\ 0.6; & c \end{cases} \quad \dot{F}_4(z) = \begin{cases} 0.5; & 0 \\ 0.2; & a \\ 0.2; & b \\ 0.3; & c \end{cases}$$

$$\dot{F}_5(z) = \begin{cases} 0.9; & 0 \\ 0.5; & a \\ 0.5; & b \\ 0.7; & c \end{cases} \quad \dot{F}_6(z) = \begin{cases} 0.7; & 0 \\ 0.4; & a \\ 0.4; & b \\ 0.6; & c \end{cases}$$

$$\dot{F}_7(z) = \begin{cases} 0.4; & 0 \\ 0; & a \\ 0; & b \\ 0.1; & c \end{cases} \quad \dot{F}_8(z) = \begin{cases} 0.8; & 0 \\ 0.2; & a \\ 0.2; & b \\ 0.3; & c \end{cases}$$

Then $(Ϛ, \tau = \{\bar{0}, \bar{1}, \dot{F}_1, \dot{F}_2, \dot{F}_3, \dot{F}_4, \dot{F}_5, \dot{F}_6, \dot{F}_7, \dot{F}_8\})$ is a TSFP on a PU-algebra Ϛ and take

$$\dot{F} = \dot{F}_4(z) = \begin{cases} 0.5; & 0 \\ 0.2; & a \\ 0.2; & b \\ 0.3; & c \end{cases}$$

Consider the sequence $\dot{F}_n = \{\dot{F}_1, \dot{F}_2, \dot{F}_6, \dot{F}_7\}$ while $\dot{F}_4(z) \leq \dot{F}_8(z) \leq \dot{F}_3(z)$ and $\dot{F}_4(z) \leq \dot{F}_8(z) \leq \dot{F}_5(z)$ thus $\dot{F}_3(z)$ and $\dot{F}_5(z)$ are the neighborhoods of $\dot{F}$ and $\dot{F}_n$ is contained in $\dot{F}_3$ and $\dot{F}_5$. Ultimately the sequence $\dot{F}_n$ converges to $\dot{F}$.

Theorem (1): Let $(Ϛ,\boldsymbol{\tau})$ is a TSFP on Ϛ. A fuzzy set $\dot{F}$ is $\boldsymbol{\tau}$-open iff for every fuzzy set G that $\dot{F}$ contains, $\dot{F}$ is a fuzzy neighborhood of G.

Proof: Let the fuzzy set $\dot{F}$ is τ-open. Let G be any fuzzy set contained in $\dot{F}$ so that $G \subset \dot{F}$. Since $\dot{F}$ is open therefore we have $G \subset \dot{F} \subset \dot{F}$ which implies that $\dot{F}$ is a fuzzy neighborhood of G.

Conversely, for every fuzzy set G that $\dot{F}$ contains, $\dot{F}$ is a fuzzy neighborhood of G. it is obvious that $\dot{F} \subset \dot{F}$ by our supposition $\dot{F}$ is a fuzzy neighborhood of $\dot{F}$ therefore there exists τ-open set Θ so that $\dot{F} \subset \Theta \subset \dot{F}$. Which implies $\dot{F} = \Theta \Longrightarrow \dot{F}$ is τ-open.

Definition (23): Let $(Ϛ,\tau)$ is a TSFP on a PU-Algebra. Let the set $\dot{F}$ be a fuzzy in Ϛ then the set that contains all fuzzy neighborhoods of $\dot{F}$ is called fuzzy neighborhood system of $\dot{F}$ which we denote by " $N_{\dot{F}}$ ".

Theorem (2): Let $(Ϛ,\tau)$ is a TSFP on Ϛ and the set $\dot{F}$ is supposed to a fuzzy in Ϛ and $N_{\dot{F}}$ be a fuzzy neighborhood system of $\dot{F}$ then

(i) The finite intersection of elements of '$N_{\dot{F}}$' is also an element of '$N_{\dot{F}}$'.

(ii) Any fuzzy set of Ϛ which contains an element of "$N_{\dot{F}}$" is also an element of "$N_{\dot{F}}$".

Proof (i): Let $(Ϛ,\tau)$ be a TSFP on Ϛ. Let the set $\dot{F}$ is a fuzzy in Ϛ and $N_{\dot{F}}$ be a fuzzy neighborhood system of $\dot{F}$. Let G, H $\in N_{\dot{F}}$ then G and H are the neighborhoods of $\dot{F}$ therefore there exist open fuzzy sets G_o and H_o of G and H respectively so that

$$\dot{F} \subset G_o \subset G \tag{1}$$

$$\dot{F} \subset H_o \subset H \tag{2}$$

from (1) and (2) we have

$$\dot{F} \subset G_o \cap H_o \subset G \cap H \tag{3}$$

Equation (3) reveals that G∩H is a fuzzy neighborhood of $\dot{F} \Longrightarrow$ intersection of two elements of $N_{\dot{F}}$ is also an element of

$N_{\dot{F}}$ and it conclusively results that the finite intersection of elements of $N_{\dot{F}}$ is also an element of $N_{\dot{F}}$.

Proof (ii): Let G is a fuzzy set containing an element of $N_{\dot{F}}$ and let that element is 'J'. This means that G contains a neighborhood J of $\dot{F}$ that is $J \subset G$ where $J \in N_{\dot{F}}$. Since J is a fuzzy neighborhood of $\dot{F}$ therefore there must exists τ-open set ' Θ ' such that $\dot{F} \subset \Theta \subset J \subset G \Rightarrow \dot{F} \subset \Theta \subset G \Rightarrow G$ is a neighborhood of $\dot{F}$. Hence $G \in N_{\dot{F}}$.

Theorem (3): Let (Z,τ) be a TSFP on Z. Let the fuzzy set $\dot{F}$ be in Z then

(i) $\dot{F}^o$ is τ-open as well as the largest τ-open fuzzy set contained in $\dot{F}$.

(ii) $\dot{F}$ is τ-open $\Leftrightarrow \dot{F}^o = \dot{F}$.

Proof (i): Let (Z,τ) be a TSFP on Z. Suppose that fuzzy set $\dot{F}$ is in Z. By fuzzy interior set definition, the fuzzy interior set of $\dot{F}$ is again a fuzzy interior which we denote by $\dot{F}^o$. Therefore there exists a τ- open set ' Θ ' such that

$$\dot{F}^o \subset \Theta \subset \dot{F} \Rightarrow \dot{F}^o \subset \Theta \quad (4)$$

While the set Θ is a fuzzy interior of $\dot{F}$ therefore we have

$$\Theta \subset \dot{F}^o \quad (5)$$

from (3) and (4) we get $\dot{F}^o = \Theta \Rightarrow \dot{F}^o$ is τ-open as well as the largest τ-open fuzzy set that $\dot{F}$ contains.

Proof (ii): Let the fuzzy set $\dot{F}$ is τ-open then

$$\dot{F} \subset \dot{F}^o \quad (6)$$

from part (i) of this theorem $\dot{F}^o$ is contained in $\dot{F}$ which implies that

$$\dot{F}^o \subset \dot{F} \quad (7)$$

Equations (5) and (6) we get $\dot{F}^o = \dot{F}$.

Conversely, suppose that $\dot{F}^o = \dot{F}$ so then by remark 3.7, union of all fuzzy interior sets of $\dot{F}$ is $\dot{F}^o$ which is by itself a fuzzy interior of $\dot{F}$. Thus $\dot{F}$ is a fuzzy neighborhood of $\dot{F}^o$. Hence fuzzy set $\dot{F}$ is open.

Theorem (4): Let (Z,τ) be a TSFP on Z. If every fuzzy set in (Z,τ) has a countable fuzzy neighborhood system then the fuzzy set $\dot{F}$ is τ-open iff each sequence of fuzzy sets $\{\dot{F}_n: n \geq 1 \wedge n \in N\}$ converging to fuzzy set G that $\dot{F}$ contains is eventually contained in $\dot{F}$.

Proof: Let $\dot{F}$ be an τ-open fuzzy set. Let each given sequence $\{\dot{F}_n: n \geq 1 \wedge n \in N\}$ of fuzzy sets converges to G. Since $\dot{F}$ is τ-open containing G which implies that $\dot{F}$ is a neighborhood of $G \Rightarrow \dot{F}$ contains $\{\dot{F}_n: n \geq 1 \wedge n \in N\}$.

Conversely, For every $G \subset \dot{F}$ we suppose that $H_1, H_2, \ldots, H_n, \ldots$ is a neighborhood system of G. Let $G_n = \bigcup_1^n \{H_i\}$. Then for the sequence $G1, G2, \ldots, Gn, \ldots$ contained in every fuzzy neighborhood of G there exists $i \in N$ so that for $n > i$, $G_n \subset \dot{F}$. Thus G_n are the fuzzy neighborhoods of G. Hence $\dot{F}$ is τ-open.

Definition (24): Let Ψ is a mapping from Z_1 to Z_2 and ν is a fuzzy set in Z_2 then Ψ^{-1} is defined as

$$\nu_{\Psi^{-1}}(p) = \nu(\Psi(p)), \forall p \in Z_1$$

Suppose that μ is a fuzzy set in Z_2 then image of such a set is given by

$$\mu(q) = \begin{cases} \sup \mu(p)_{p \in \Psi^{-1}(q)}, & if\ \Psi^{-1}(q) \neq \emptyset \\ 0, & otherwise \end{cases}$$

Definition (25): Let (Z_1,τ) and (Z_2,ω) are the TSFP on PU-algebras Z_1 and Z_2 respectively. Then the function Ψ from (Z_1,τ) to (Z_2,ω) is said to be fuzzy continuous function if every ω-open fuzzy set has its inverse as τ-open fuzzy set.

Theorem (5): Let (Z_1,τ) and (Z_2,ω) are the TSFP on PU-algebras Z_1 and Z_2 respectively then the function Ψ from (Z_1,τ) to (Z_2,ω) is fuzzy continuous function iff any closed fuzzy set has its inverse as a closed set.

Proof: Let Ψ is a fuzzy continuous function then by definition 3.18 inverse of any ω-open fuzzy set is τ-open fuzzy set. Suppose that the fuzzy set ω` is closed in Z_2 then

$\mu_{\Psi^{-1}(\omega`)}(p) = \mu`_{\omega}(\Psi(p)) = \mu_{\omega`}(\Psi(p)) = 1 - \mu_{\omega}(\Psi(p)) = 1 - \mu_{\Psi^{-1}(\omega)}(p) = \mu`_{\Psi^{-1}(\omega`)}(p) \Rightarrow \Psi^{-1}(\omega`) = \{\Psi^{-1}(\omega)\}`, \forall p \in Z_1$

Since Ψ is a fuzzy continuous therefore any closed fuzzy set has its inverse as a closed set.

Conversely, suppose that open fuzzy set in Z_2 is ω. Then

$$\mu_{\Psi^{-1}(\omega)}(p) = \mu_{\omega}(\Psi(p)), \forall p \in Z_1.$$

We know that any closed fuzzy set has its inverse as a closed set therefore each open fuzzy set has its inverse as an open set $\Rightarrow$ Ψ is a fuzzy continuous.

Theorem (6): Let (Z_1,τ) and (Z_2,ω) are the TSFP on PU-algebras Z_1 and Z_2 respectively and Ψ is a function from (Z_1, τ) to (Z_2, ω) then for every $\dot{F}$ in Z_1 where $\dot{F}$ is a fuzzy set the inverse of any neighborhood of $\Psi(\dot{F})$ is a neighborhood of $\dot{F}$ iff for every fuzzy set $\dot{F}$ in Z_1 and every neighborhood N of $\Psi(\dot{F})$ there is a neighborhood M of $\dot{F}$ such that $\Psi(M) \subset N$.

Proof: Let $\tilde{\dot{F}}$ is a set that contains all fuzzy sets of Z_1. Let $\widetilde{N}$ and $\widetilde{M}$ are the families of neighborhoods of fuzzy sets and their images. Suppose that $\dot{F} \in \tilde{\dot{F}}$. Let $N \in \widetilde{N}$, $M \in \widetilde{M}$ are the neighborhoods of $\Psi(\dot{F})$ and $\dot{F}$ respectively. Since the inverse of each neighborhood of $\Psi(\dot{F})$ is a neighborhood of $\dot{F}$. Therefore;

$$\Psi(M) = \Psi(\Psi^{-1}(N)) \quad (8)$$

If $\Psi^{-1}(q) \neq \emptyset$

Then

$\mu_{\Psi(\Psi^{-1}(N))}(q) = \sup \mu_{\Psi^{-1}(N)}(r)_{r \in \Psi^{-1}(q)} = \sup\{\mu_N(\Psi(r))\}_{r \in \Psi^{-1}(q)} = \mu_N(q)$, for all $q \in Z_2$. If $\Psi^{-1}(q) \neq \emptyset$, then $\mu_{\Psi(\Psi^{-1}(N))}(q) = 0$.

Hence $\mu_{\Psi(\Psi^{-1}(N))}(q) \leq \mu_N(q)$ for all $q \in Z_2$ and $\Psi(\Psi^{-1}(N)) \subset N$,

Equation (7) implies $\Psi(M) \subset N$.

Conversely, Let N is a neighborhood of $\Psi(\dot{F})$. Since there is a neighborhood M of $\dot{F}$ such that $\Psi(M) \subset N$.

Hence;

$$\Psi^{-1}(\Psi(M)) \subset \Psi^{-1}(N) \quad (9)$$

Then for any $p \in Z_1$ we have

$$\mu_{\Psi^{-1}(\Psi(M))}(p) = \mu_{\Psi(M))}(\Psi(p)) = \sup\{\mu_M(r)\}_{r \in \Psi^{-1}(\Psi(p))} \geq \mu_M(p).$$

Thus $M \subset \Psi^{-1}(\Psi(M))$ and equation (2) implies $M \subset \Psi^{-1}(\Psi(M)) \subset \Psi^{-1}(N)$.

Hence $\Psi^{-1}(N)$ is a neighborhood of M.

Theorem (7): Let (Z_1,τ) and (Z_2,ω) are the TSFP on PU-algebras Z_1 and Z_2 respectively and Ψ is a function from (Z_1,τ) to (Z_2,ω). If Ψ is a fuzzy continuous function then for every fuzzy set $\dot{F}$ in Z_1, inverse of each neighborhood of $\Psi(\dot{F})$ is a neighborhood of $\dot{F}$.

Proof.: Assume that $\tilde{\dot{F}}$ contains all fuzzy sets of Z_1. Let $\widetilde{N}$ is a family of neighborhoods on $\tilde{\dot{F}}$. Let $\dot{F} \in \tilde{\dot{F}}$ and $N \in \widetilde{N}$ then the set $\dot{F}$ being a fuzzy in Z_1 and N being a neighborhood of $\Psi(\dot{F})$. There exists an open neighborhood M of $\Psi(\dot{F})$ so that $\Psi(\dot{F}) \subset M \subset N$

And;

$$\Psi^{-1}(\Psi(\dot{F})) \subset \Psi^{-1}(M) \subset \Psi^{-1}(N) \qquad (10)$$

Since Ψ is a fuzzy continuous therefore $\Psi^{-1}(M)$ is open. Moreover $\mu_{\Psi^{-1}(\Psi(\dot{F}))}(p) = \mu_{\Psi(\dot{F})}(\Psi(p)) = \sup\{\mu_{\dot{F}}(r)\}_{r \in \Psi^{-1}(\Psi(p))} \geq \mu_{\dot{F}}(p)$ for all $p \in Z_1$. Therefore, $\dot{F} \subset \Psi^{-1}(\Psi(\dot{F}))$. Thus equation (1) implies $\dot{F} \subset \Psi^{-1}(\Psi(\dot{F})) \subset \Psi^{-1}(M) \subset \Psi^{-1}(N)$ and $\subset \Psi^{-1}(M) \subset \Psi^{-1}(N)$. Hence $\Psi^{-1}(N)$ is a neighborhood of $\dot{F}$.

4. Conclusion

In this manuscript various important results evaluated by inserting the concept of fuzzy topology in fuzzy PU-new ideal. It can also be expanded to interval valued fuzzy and intuitionistic fuzzy substructures on PU-algebras by combining with particular framework of fuzzy topology.: It has no doubt that the research along this line can be kept up, and indeed, some results in this manuscript have already made up a foundation for further exploration concerning the further progression of PU-algebras. These definitions and main results can be similarly extended to some other algebraic systems such as BCH-algebras, Hilbert algebras, BF-algebras, J-algebras, WS-algebras, CI-algebras, BCL-algebras, BP-algebras and BO-algebras, Z- algebras and so forth. The main purpose of this research is that the future work is to further investigate the fuzzy topology in PU-algebras, which may have a lot of applications in different branches of theoretical physics and computer sciences.

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