



# Mathematical Model and Algorithm for Determination of Minimum Antenna Mast Height for Terrestrial Line of Sight Microwave Link with Zero Path Inclination

Wali Samuel<sup>1</sup>, Steve Worgu<sup>2</sup>, Elsie Chidinma Anderson<sup>2</sup>

<sup>1</sup>Department of Electrical/Electronic and Computer Engineering, University of Uyo, Uyo, Nigeria

<sup>2</sup>Department of Electrical/ Computer Engineering, Port Harcourt Polytechnic, Rumuola, Port Harcourt, Nigeria

## Email address:

samwalliuy@yahoo.com (W. Samuel), stevewa438@yahoo.com (S. Worgu)

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**Abstract:** In this paper, mathematical model and algorithm for determination of minimum antenna mast height for terrestrial line of sight microwave link with zero path inclination is presented. The mathematical expressions developed are used for computing relevant link parameters while the algorithm gives the procedure for using the mathematical expressions for computing the minimum antenna mast heights. Sample 40km 10GHz Ku-band microwave link is used to demonstrate the application of the model and algorithm; in this case, the transmitter is located at longitude 7.711747 and latitude 5.178536 and the receiver is located at longitude 8.039903 and latitude 5.055223. The link is required to make a minimum of 100% clearance with respect to the Fresnel zone 1. The results show that the transmitter and receiver antenna are at the same line of sight height of 158.7 m whereas the transmitter antenna mast height is 68.8 m while the receiver antenna mast height is 109.7m. Also, the maximum obstruction height of 128.58m occurred at a distance of 14306.98m from the transmitter with percentage clearance of 100% in respect of Fresnel zone 1. The result shows that the model can be used to ensure that the specified percentage clearance is achieved through the application of the models presented in this paper.

**Keywords:** Antenna Mast Height, Path Inclination, Microwave Link, Line Of Sight Communication, Percentage Clearance

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## 1. Introduction

Microwave signals travel in a straight line [1, 2]. Consequently, they are used for Line Of Site (LOS) communication. Researches however, have found that microwave signal do not travel in a pencil tip straight line. Rather, the signal spread from the transmitter and terminate at the receiver forming an ellipsoidal shape that is described by Fresnel and modeled as Fresnel zones [3, 4, 5, 6]. Also, secondary radioclimatic parameter called effective earth radius factor (k-factor) affect the signal path [8-10]. At different values of k-factor the signal can bend away from the earth, towards the earth or it can travel in a straight line with respect to the earth surface. Particularly, the effective earth radius factor is dependent on the three primary radioclimatic parameters, namely, temperature, pressure and relative humidity. The effect of the effective earth radius factor is modeled using the concept of earth bulge [11].

Importantly, in LOS communication system, the transmitter antenna and the receiver antenna are installed at a height that will ensure that no obstacle obstruct the LOS between the (straight line joining) transmitter and the receiver. In order to ensure that the required LOS clearance is achieved, during the design of the LOS communication link the Fresnel Zone clearance and the earth bulge are taken into consideration along with the elevation profile of the terrain. Also, the path inclination, which is the angle subtended by the LOS with the horizontal is considered in the determination of the antenna mast height. In the case where the path inclination is zero, it means that the transmitter and the receiver antenna heights are equal. However, that does not mean that the transmitter mast and the receiver antenna mast heights are equal.

In any case, there are several approaches that can be used to determine the minimum transmitter mast and the receiver

antenna mast heights when the path inclination is zero. In this paper, one of such methods is presented based on a given Fresnel geometry for LOS link. The method assumes that the required LOS percentage clearance with respect to Fresnel zone 1 is specified along with the expected obstruction height. Also, the path elevation profile is required as input data for the model. The requisite mathematical expressions and algorithm for the computation of the minimum transmitter mast and the receiver antenna mast heights are presented.

## 2. Theoretical Background

### 2.1. Fresnel Geometry for the Line of Sight Link

The Fresnel geometry for the line of sight link is given in figure 1. In the model, it is assumed that the transmitter has the lower antenna height, especially, when the two antenna heights are not equal. However, if the transmitter antenna height is higher than the receiver antenna height, then the analysis still holds by swapping the receiver subscripts with that of the transmitter.

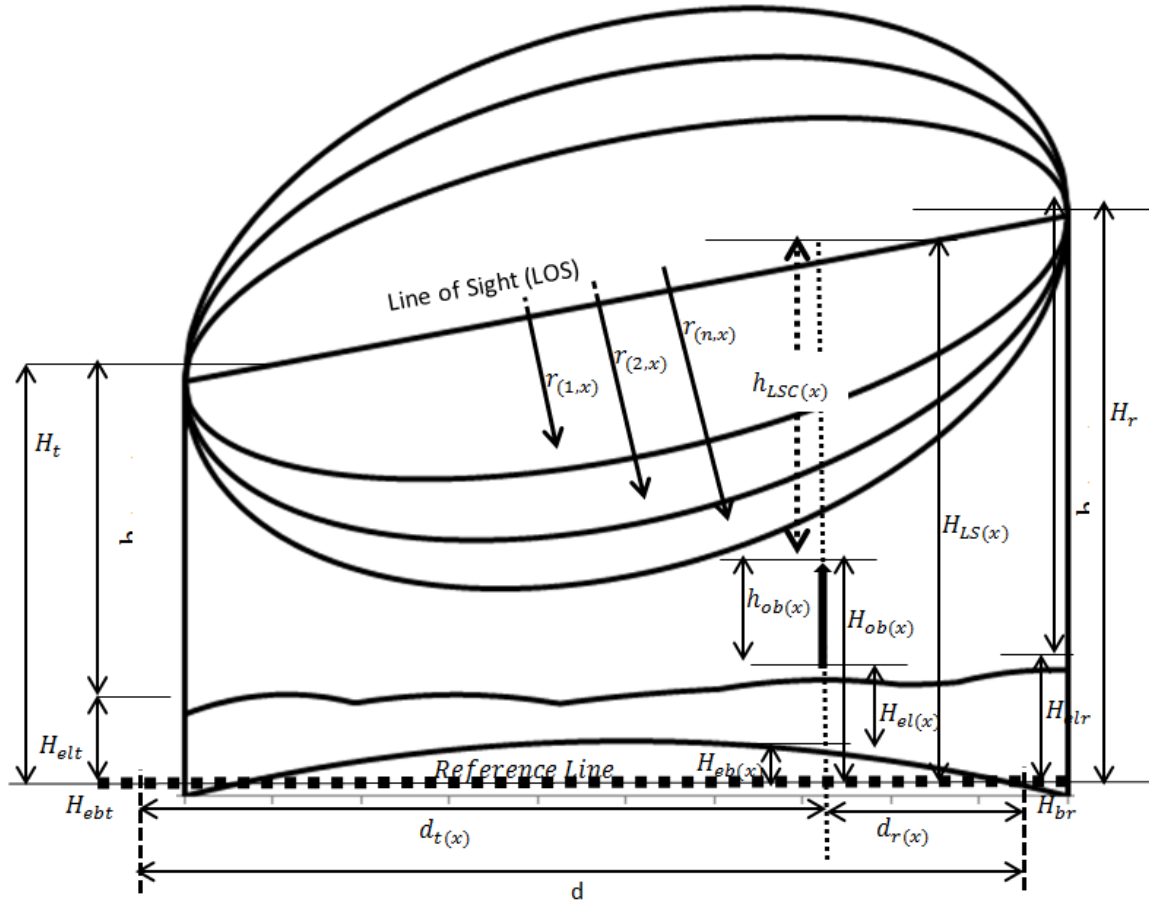


Figure 1. Fresnel Geometry For The Line Of Sight Link.

### 2.2. The Elevation Profile

The model make use of the link elevation profile which is the set of data on the elevation at various points between the transmitter and the receiver. The terrain elevation profile provides a number of elevation points and their distance from the transmitter and the receiver. Each elevation point consist of the elevation height and the distance of the point from the transmitter or distance from the receiver. Let  $n_e$  be the number of elevation points taken from the transmitter location to the receiver location. Also, let  $H_{el}(x)$  be the elevation at point  $x$ , where  $x = 1, 2, 3, \dots, n_e$ ; let  $d_{t(x)}$  be the distance of location  $x$  from the transmitter; let  $d_{r(x)}$  be the distance of location  $x$  from the receiver, where  $x = 1, 2, 3, \dots, n_e$  and let  $d$  be the distance (in meters) between the transmitter and the receiver. Then,

$$d = d_{r(x)} + d_{t(x)} \quad (1)$$

$$d_{r(x)} = d - d_{t(x)} \quad (2)$$

Since the transmitter is taken as the reference point for the measurements, then, the transmitter is located at  $x = 0$  and the receiver is located at  $x = n_e$ . Therefore,

$$d_t = d_{t(0)} = 0 \quad (3)$$

$$d_r = d_{t(n_e)} = d \quad (4)$$

$H_{elt}$  is the elevation at the transmitter location where  $x = 0$ , hence,  $H_{elt} = H_{el(0)}$

$H_{elr}$  is the elevation at the receiver location where  $x = n_e$ , hence,  $H_{elr} = H_{el(n_e)}$

### 2.3. The Earth Bulge

Earth bulge is the height an obstruction is raised higher in elevation (into the path) owing to earth curvature. Earth bulge is given as [12];

$$H_{eb(x)} = \frac{(d_t(x))(d_r(x))}{12.75 * K} \quad (5)$$

Where  $H_{eb(x)}$  is the height (in meters) of the earth bulge at location  $x$  between the transmitter and the receiver;  $H_{ebt}$  is the height (in meters) of the earth bulge at the transmitter mast location;  $H_{ebr}$  is the height (in meters) of the earth bulge at the receiver mast location;  $d_t(x)$  and  $d_r(x)$  are as defined earlier. For Line of Sight (LOS) point-to-point links design K-factor of 4/3 is often used. At the transmitter,  $d_t(x) = 0$ , hence,

$$H_{ebt} = H_{eb(x)} = \frac{(0)(d_r(x))}{12.75 * K} = 0 \quad (6)$$

Similarly, at the receiver,  $d_r(x) = 0$ , hence,

$$H_{ebr} = H_{eb(x)} = \frac{(d_t(x))(0)}{12.75 * K} = 0 \quad (7)$$

In essence, at the transmitter and at the receiver, the earth bulge is zero.

### 2.4. Radius of the Fresnel Zones

In many cases obstructions do exist in the signal path. For acceptable clear line of sight, at least 60% clearance is required in the first Fresnel zone. So, obstructions in the signal path must maintain a clearance height that is at least 60% of the radius of the first Fresnel zone. Theoretically, there are infinite number of Fresnel zones in any LOS link. Let  $\lambda$  be the wavelength of the radio wave; let  $c$  be the speed of the radio wave (where  $c = 3 \times 10^8 \text{ m/s}$  and let  $f$  be the frequency of the radio wave in Hz, then, the radius of the  $n$ th Fresnel zone ( $r_{(n,x)}$ ) at location  $x$  is given as [13-15];

$$r_{(n,x)} = \sqrt{\frac{n(\lambda(d_t(x))(d_r(x)))}{(d_t(x) + d_r(x))}}; \text{ for } n=1,2,3,\dots \text{ and } d_t(x) \gg r_{(n,x)} \text{ and } d_r(x) \gg r_{(n,x)} \quad (8)$$

$\lambda$  in metres is given as;

$$\lambda = \frac{c}{f} \quad (9)$$

Let  $P_{C(n)}$  be the percentage clearance allowed for the Fresnel zone  $n$ , given in % where  $P_{C(n)}$  is positive if the obstacle tip is below the line of sight and  $P_{C(n)}$  is negative if the obstacle tip is above the line of sight. It must be noted that  $P_{C(n)} = 0\%$  when the tip of the obstruction is on the LOS;  $P_{C(n)} = 100\%$  when the tip of the obstruction is below the LOS at a clearance height equal to  $r_{C(n,x)}$ ;  $P_{C(n)} = -100\%$  when tip of the obstruction is above the LOS at a clearance height equal to  $r_{C(n,x)}$  above the LOS. Let  $h_{cl(n,x)}$  be the clearance height required at point  $x$  for clear line of sight with respect to the Fresnel zone  $n$ . Then,

$$h_{cl(n,x)} = \left( \frac{P_{C(n)}}{100} \right) r_{(n,x)} = \left( \frac{P_{C(n)}}{100} \right) \sqrt{\frac{n(\lambda(d_t(x))(d_r(x)))}{(d_t(x) + d_r(x))}} \quad (10)$$

### 2.5. Obstruction Location and Height

In the model, the reference line for measuring the earth bulge is the chord (line segment) that joins the sea level at the transmitter and the sea level at the receiver. In the model of figure 1 the Reference Line is represented by thick dotted horizontal line at the base of figure 1. Let  $h_{ob(x)}$  be the height of the obstruction at point  $x$ , where  $h_{ob(x)}$  is measured from the ground level (where the ground level is at the top of the elevation point at point  $x$ ) and it does not include the elevation and earth bulge at point  $x$ . The elevation point is measured from the sea level. Let  $H_{ob(x)}$  be the overall height of the obstruction at point  $x$ , where  $H_{ob(x)}$  is measured from the Reference Line.  $H_{ob(x)}$  includes the elevation at point  $x$  and also include the earth bulge at point  $x$ , Then;

$$H_{ob(x)} = h_{ob(x)} + H_{eb(x)} + H_{el(x)} \quad (11)$$

### 2.6. Determination of Minimum LOS Height at Location $x$

For the specified percentage clearance  $P_{C(n)}$ , the minimum LOS height at location  $x$  is given as  $H_{LSR(x)}$ , where;

$$H_{LSR(x)} \geq H_{ob(x)} + h_{cl(n,x)} \quad (12)$$

$$H_{LSR(x)} \geq H_{eb(x)} + H_{el(x)} + h_{ob(x)} + \left\{ \left( \frac{P_{C(n)}}{100} \right) \sqrt{\frac{n(\lambda(d_t(x))(d_r(x)))}{(d_t(x) + d_r(x))}} \right\} \quad (13)$$

Let  $X_{LSMAX}$  be the distance from the transmitter to the point at which the tip of the obstruction attains its maximum height,  $H_{LSMAX}$  and let  $H_{LSMAX}$  be the maximum height attained by the tip of the obstruction at location  $X_{LSMAX}$  and distance  $d_{LSMAX}$  from the transmitter. Then,

$$H_{LSMAX} = \text{Maximum} \left( H_{eb(x)} + H_{el(x)} + h_{ob(x)} + \left\{ \left( \frac{P_{C(n)}}{100} \right) \sqrt{\frac{n(\lambda(d_t(x))(d_r(x)))}{(d_t(x) + d_r(x))}} \right\} \right) \quad (14)$$

for  $x = X_{LSMAX}$

### 2.7. Determination of the Minimum Antenna Height When the Path Inclination Is Zero, ( $H_t = H_r$ )

Let  $H_t$  be the overall height (in meters) of the transmitter antenna, including the elevation measured from the sea level and the earth bulge at the transmitter. Also, let  $H_r$  be the overall height (in meters) of the receiver antenna, including the elevation measured from the sea level and the earth bulge at the receiver. Let path inclination be denoted as  $\varepsilon_p$  where;

$$\varepsilon_p = \frac{|H_t - H_r|}{d} \quad (15)$$

When the path inclination is zero, then,

$$H_r = H_t \quad (16)$$

The transmitter and receiver antenna mast heights are

$$H_t \geq H_{LSMAX} \text{ for all } x \geq 0 \quad (17)$$

$$H_t \geq \text{Maximum} \left( H_{eb(x)} + H_{el(x)} + h_{ob(x)} + \left\{ \left( \frac{P_{C(n)}}{100} \right) \sqrt{\frac{n(d_t(x))(d_r(x))}{(d_t(x) + d_r(x))}} \right\} \right) \quad (18)$$

Let  $H_t = H_{LSR(x)} = H_{LSMAX}$  and let  $h_{LSC(x)}$  be the actual LOS clearance height (in meters) from the tip of the obstruction at location  $x$  to the line of sight is given as;

$$h_{LSC(x)} = H_{ob(x)} - H_{LSMAX} \quad (19)$$

Note:  $h_{LSC(x)}$  is negative if  $H_{ob(x)} < H_{LSMAX}$ , that is, if the obstruction tip is below the line of sight. In this case,  $P_{C(n)} > 0$ .

$h_{LSC(x)} = 0$  if  $H_{ob(x)} = H_{LSMAX}$ , that is, if the obstruction tip is just on the line of sight. In this case,  $P_{C(n)} = 0$ .

$h_{LSC(x)}$  is positive if  $h_{LSC(x)} > H_{LSMAX}$  that is, if the obstruction tip is above the line of sight. In this case,  $P_{C(n)} < 0$

Let  $h_{t(mast)}$  be the height (in meters) of the transmitter antenna mast measured from the ground and let  $h_{r(mast)}$  be the height (in meters) of the receiver antenna mast measured from the ground

Then,

$$H_t = h_{t(mast)} + H_{elt} + H_{ebt} \quad (20)$$

$$h_{t(mast)} = H_t - (H_{elt} + H_{ebt}) \quad (21)$$

$$h_{t(mast)} = H_{LSMAX} - (H_{elt} + H_{ebt}) \quad (22)$$

But  $H_{ebt} = 0$ , then;

$$h_{t(mast)} = H_{LSMAX} - H_{elt} \quad (23)$$

selected so that the resultant overall heights of the antennas, namely,  $H_t$  and  $H_r$  will ensure clear line of sight between the transmitter and the receiver antennas.

Also,

$$H_r = h_{r(mast)} + H_{elr} + H_{ebr} \quad (24)$$

$$h_{r(mast)} = H_r - (H_{elr} + H_{ebr}) \quad (25)$$

$$h_{r(mast)} = H_{LSMAX} - (H_{elr} + H_{ebr}) \quad (26)$$

But  $H_{ebr} = 0$ , then;

$$h_{r(mast)} = H_{LSMAX} - H_{elr} \quad (27)$$

### 3. Results and Discussion

The path profile used is obtained using online Geocontext-Profiler - an online topographic software available at <http://www.geocontext.org/publ/2010/04/profiler/en/>. The path profile data are taken for a transmitter located at longitude 7.711747 and latitude 5.178536 and the receiver located at longitude 8.039903 and latitude 5.055223. The elevation profile in Table 1 is plotted in figure 2. In Table 1 and figure 2, there is a total of 512 elevation data points and the link is 40,000 km long. The link's maximum elevation of 98.3 meters occurred at 11,415.1 meters from the transmitter and the link's minimum elevation of 10.2 meters occurred at a distance of 33,788.8 meters from the transmitter.

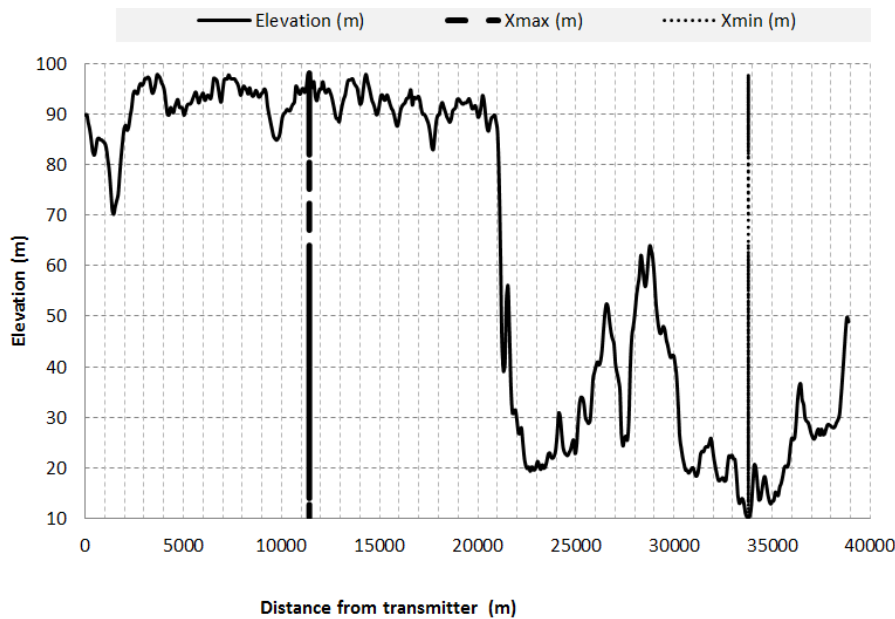


Figure 2. The Elevation profile for the Link.

**Table 1.** The Elevation Profile Of The Link.

Elevation Data point	Distance (m)	Elevation (m)	Elevation Data point	Distance (m)	Elevation (m)
1	0	89.8	264	20014.6	89.4
12	837.1	84.9	276	20927.8	88.6
24	1750.3	78	288	21841	30.8
36	2663.5	94.1	300	22754.2	20.2
48	3576.7	96	312	23667.4	23
60	4490	90.3	324	24580.6	22.5
72	5403.2	92.3	336	25493.8	29.8
84	6316.4	93.6	348	26407	47.2
96	7229.6	97	360	27320.2	26.8
108	8142.8	95.2	372	28233.5	58
120	9056	94.4	384	29146.7	49.9
132	9969.2	87.6	396	30059.9	40.5
144	10882.4	94.2	408	30973.1	20
151	11415.1	98.3	420	31886.3	25.9
156	11795.6	92.7	432	32799.5	22.4
168	12708.9	90.5	444	33712.7	10.3
180	13622.1	97	445	33788.8	10.2
192	14535.3	94	456	34625.9	18.3
204	15448.5	93	468	35539.1	18.3
216	16361.7	93	480	36452.4	36.7
228	17274.9	90	492	37365.6	27.7
240	18188.1	92.3	504	38278.8	29
252	19101.3	92.6	512	38887.6	48.9
Maximum Elevation Data point	Maximum Elevation (m)	Maximum Elevation Distance From The Transmitter, Xmax (m)	Minimum Elevation Data point	minimum Elevation (m)	Minimum Elevation Distance From The Transmitter, Xmin (m)
151	98.3	11415.1	445	10.2	33788.8

**Table 2.** The Link Clearance Parameters.

Elevation Data Point Number	Distance (m)	Elevation (m)	Earth Bulge (m)	Obstruction height(m)	Radius Of Fresnel Zone 1 (m)	Line Of Sight Clearance Height (m)	Percentage Clearance Of Fresnel Zone 1 (%)
1	0	89.8	0	99.8	0	-58.8185	0
24	1750.32	81.8	3.8	91.8	12.93	-66.8461	-517
48	3576.74	103.5	7.4	113.5	18.02	-45.2013	-251
72	5403.17	103	10.6	113	21.57	-45.6927	-212
96	7229.59	110.5	13.5	120.5	24.26	-38.1905	-157
120	9056.01	110.3	15.9	120.3	26.36	-38.4033	-146
144	10882.44	112.1	17.9	122.1	27.99	-36.533	-130
168	12708.86	110.1	19.6	120.1	29.25	-38.5907	-132
189	14306.98	118.6	20.7	128.6	30.07	-30.0722	-100
192	14535.28	114.8	20.8	124.8	30.17	-33.8383	-112
216	16361.71	114.7	21.7	124.7	30.79	-33.9353	-110
256	19405.74	114.5	22.2	124.5	31.18	-34.1746	-110
264	20014.55	111.6	22.2	121.6	31.17	-37.0346	-119
288	21840.97	52.7	21.9	62.7	30.94	-95.9249	-310
312	23667.4	44.2	21.2	54.2	30.44	-104.497	-343
408	30973.09	34.4	14.4	44.4	25.11	-114.234	-455
432	32799.51	34.1	11.7	44.1	22.66	-114.522	-505
456	34625.93	27	8.7	37	19.48	-121.665	-625
480	36452.36	41.9	5.2	51.9	15.11	-106.78	-707
504	38278.78	30.3	1.4	40.3	7.74	-118.322	-1528
512	40000.00	48.9	0	58.9	0	-99.7385	-

**Table 3.** The Antenna Mast Heights and Other Link Clearance Parameters.

Link Parameter	Value
Height of Transmitter Antenna Mast, (m)	68.8
Height of Receiver Antenna Mast, (m)	109.7
Height of Line Of Sight (m) = Transmitter Antenna Height (m) = Receiver Antenna Height (m)	158.7
Maximum Height Of The Tip Of Obstruction (m)	128.58
Distance of Maximum Obstruction Height, (m)	14306.98
Minimum Line Of Sight Clearance Height,	-30.07
Distance of Minimum Line Of Sight Clearance Height From The Transmitter, (m)	14306.98
Radius Of The First Fresnel Zone At The Minimum Clearance Height (m)	30.07
Percentage Clearance of First Fresnel Zone At The Minimum Clearance Height (%)	-100
Specified Percentage Clearance (%)	-100

In Table 2 the maximum obstruction height of 128.58 m occurred at a distance of 14306.98m from the transmitter. The Percentage Clearance of 100% is achieved at the maximum obstruction point. This is equal to the 100% clearance given in the link design specification. In Table 3, the transmitter and receiver antenna are at the same line of sight height of 158.7 m. However, the transmitter antenna mast height is 68.8 m while the receiver antenna mast height is 109.7m. Maximum earth bulge of 22.2 occurred at the middle of the like at a distance of 19405.74 m from the transmitter.

## 4. Conclusion

Mathematical model and algorithm for determination of minimum antenna mast height for terrestrial line of sight microwave link with zero path inclination is presented. The mathematical expressions developed are used for computing relevant link parameters while the algorithm gives the procedure for using the mathematical expressions for computing the minimum antenna mast heights. Sample Ku-band microwave link is used to demonstrate the application of the model and algorithm. The result shows that the model can be used to ensure that the specified percentage clearance is achieved through the application of the models presented in this paper.

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