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Unsteady MHD Free Convective Three Phase Flow Through Porous Medium Sandwiched Between Viscous Fluids

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Abstract: In this paper, the unsteady MHD free convective flow through porous medium sandwiched between electrically conducting viscous-incompressible fluids in a horizontal channel with heat and mass transfer, with the assumptions that the upper and lower channel are non-porous (clear regions) and the middle channel as porous respectively have been studied. The governing equations of the flow were transformed to ordinary differential equation by a regular perturbation method and the expression for the velocity, temperature, and concentration for the flow were obtained. It is observed that the fluid velocity decreases with an increase in Prandtl number, Radiation parameter, Hartmann number and Schmidt number. Some of these governing parameters had little effect on the velocity profile while others had significant effect on this velocity profile. The same was seen on the temperature profile and concentration profile. Finally the governing parameters had effects on the flow and this study does aid in the practical usage of such flow or when confronted with such a flow.

Keywords: MHD, Free Convective, Porous Medium, Unsteady Flow, Heat and Mass Transfer

1. Introduction

The flow of viscous fluids through porous meduim is very much prevalent in nature: therefore such studies have been attracting the considerable attention of engineers and scientist all over the world. Convection in porous media finds applications in oil extraction, thermal energy storage and flow through filtering devices.

The hydro-magnetic convection with heat and mass transfer in porous medium has been studied due to its importance in the design of under-ground water energy storage system,Soil-sciences in design of MHD generators and accelaretors in geophysics, nuclear power reactors etc. Magneto-hydrodynamics is currently undergoing a greate period of enlargement and differentialtion of sbject matter. The magneto-dynamic has wide applications in the field of nuclear engineering heat transfer,mechanical engineering, chemical engineering, aerodynamic, solar collector, heat exchangers, liquid metals, elecrolyte and ionized gases.

Looking at the various wide applications of such flows numerous scholar have paid their attaintionin it. Joseph et al [5] studied the unsteady MHD free convective two immiscible fluids flow in a horizontal channel with heat and mass transfer. They assumed that the upper channel and lower channel are porous and non – porous respectively. The governing equations were transformed to ordinary differential equations by a regular pertubation method. Sulochana and Sandeep [14] considered flow and heat behavior of MHD dusty nanofluid past a porous stretching/shrinking cylinder at different temperatures. Hall current effects on convective heat and mass transfer flow of viscous fluids in a vertical wavy channel was considered by

Veerasuneela et al [11]. Parabhakararao [4] studied unsteady free convection (MHD) flow of an incompressible electrically conducting viscos fluid through porous medium between two vertical plates. Choudhury and Kumar Das [10] studied visco-elastic MHD free convective flow through porous media in the present of radiation and chemical reaction with heat and mass transfer. Effect of variable heat source/sink on chemically reacting 3D slip flow caused by a slandering stretching sheet was studied by Babu and Sandeep [13]. Dada et al [8] investigated the unsteady radiative and MHD free convective two immiscible fluid flows through a horizontal channel. The upper channel is assumed to be porous while the lower is non-porous. The partial differential equations governing the fluid flow are transformed to ordinary differential form by a regular perturbation method and the analytical solutions for each fluid flow are obtained and these solutions matched at the interface. The results are presented for various values of the fluid parameters such as, Grashof number, radiative parameter, and frequency parameter, Prandtl number, viscosity and conductivity ratio. It is found among others results that as the thermal radiation from the wall temperature decreases, the temperature profiles and thermal boundary layers increases.

According to Mateen, [6], problems relating to the petroleum industry, plasma physics, Magneto-fluid dynamic involve multi-fluid flow situation. However he found among other results that as the Schmidt number increase, the concentration decreases which causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Transient magnetohydrodynamic flow of two immiscible fluids through a horizontal channel was carried out by Mateen [7]. Sugunama et al [12] presented run up flow of Rivilin – Ericksen fluid through a porous medium in a channel.

Kumar et al [3] presented unsteady MHD and heat transfer of two viscous immiscible fluids through a porous medium in a horizontal channel. The unsteady flow and heat transfer of porous media sandwiched between viscous fluids was studied by Umavathi et al [2]

The aim of this paper is to extend the paper investigated by Kumar et al [1] titled "unsteady MHD free convective flow through porous medium sandwiched between viscous fluids." In our paper we considered the presence of foreign bodies in the fluid flow and added the concentration equation in order to have mass transfer.

2. Formulation of Problem

Consider the free-convective flow of a viscous incompressible electrically conducting fluid through a porous medium squeezed in between viscous fluids bounded by two infinite horizontal parallel porous plates under the action of uniform magnetic field applied normal to the direction of flow. The walls of the plates are extending in x' and z' directions and the y' direction is taken as normal to the plates.

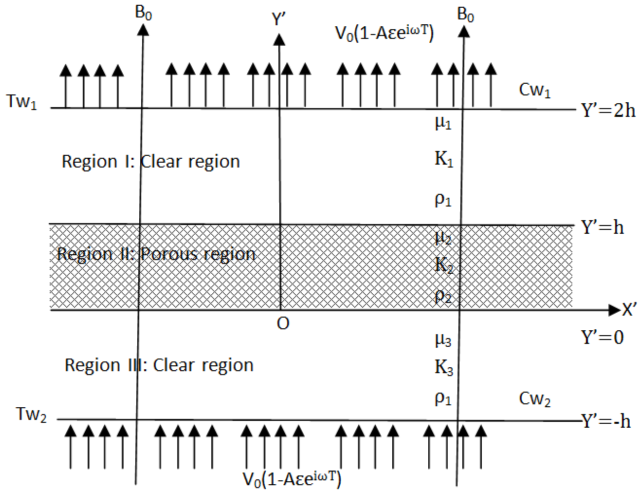

Fig. 1. *Geometrical Configuration.*

The rate of injection of fluid through lower plate is equal to rate of suction of fluid through upper plate. Buoyancy force is due to combined presence of fluid density gradient and body force is proportional to density. The lower plate is lying in the lane y' = -h and kept at temperature Tw_2 whereas the upper plate held at temperature Tw_1 has been placed in the plane y' = 2h with $Tw_2 >> Tw_1$. The temperature of the lower plate exceeds that of upper plate and the density decreases in the direction of gravitational force i.e. if the temperature difference exceeds from the critical value conditions are unstable and the buoyancy force are able to overcome the retarding influence of viscous forces, as a result the free convective take place. The viscous fluid flowing through region -h≤y'≤0 (region III clear region) and -h≤y'≤2h (region I clear region) having densities ρ_1 and ρ_3 dynamic viscosities μ_1 and μ_3 specific heat at constant pressure C_{p1} and C_{p3} thermal conductivity k_1 and k_3 region 0≤y'≤h (region II porous region) having density ρ_2 dynamic viscosity μ_2 and specific heat at constant pressure C_{p2} and thermal conductivity K_2. The flow is such that the effect of induced magnetic field is neglected. The magnetic Reynolds number is assumed very small, further the magnetic field is not strong enough to cause joule heating (electrical dissipation). The flow in the channel is assumed to be fully developed unsteady and laminar and the fluid properties are constant. As the bounding surface is infinite in length along x-axis therefore, all the variables are function of y' and t' only i.e. ∂P'/∂x'=0

Hence the governing equation (dynamic fluid and max-well equation) of flow inside the channel for three different region on taking.

REGION-I: Clear Region

$$\frac{\partial V_1'}{\partial y'} = 0 \tag{1}$$

$$\rho_1\left(\frac{\partial U_1'}{\partial t'} + V_1'\frac{\partial U_1'}{\partial y'}\right) = \mu_1\frac{\partial^2 U_1'}{\partial y'^2} - \frac{\partial P'}{\partial x'} - \sigma B_0^2 U_1' + \rho_1 g \beta'_{f_1}\left(T_1' - T_{w_1}'\right) + \rho_1 g \beta_{c_1}'\left(C_1' - C_{w_1}'\right) \tag{2}$$

$$\rho_1 C_p\left(\frac{\partial T_1'}{\partial t'} + V_1'\frac{\partial T_1'}{\partial y'}\right) = k_1\frac{\partial^2 T_1'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C_1'}{\partial t'} + V_1'\frac{\partial C_1'}{\partial y'} = D_1\frac{\partial^2 C_1'}{\partial y'^2} - k_{c'}\left(C_1' - C_{w_1}'\right) \tag{4}$$

REGION-II: Porous Region

$$\frac{\partial V_2'}{\partial y'} = 0 \tag{5}$$

$$\rho_2\left(\frac{\partial U_2'}{\partial t'} + V_2'\frac{\partial U_2'}{\partial y'}\right) = \mu_2\frac{\partial^2 U_2'}{\partial y'^2} - \frac{\partial P'}{\partial x'} - \sigma B_0^2 U_2' - \frac{\mu_2}{K'}U_2' + \rho_2 g\beta_{f_2}\left(T_2' - T_{w_2}'\right) + \rho_2 g\beta_{c_2}'\left(C_2' - C_{w_2}'\right) \tag{6}$$

$$\rho_2 C_p\left(\frac{\partial T_2'}{\partial t'} + V_2'\frac{\partial T_2'}{\partial y'}\right) = k_2\frac{\partial^2 T_2'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{7}$$

$$\frac{\partial C_2'}{\partial t'} + V_2'\frac{\partial C_2'}{\partial y'} = D_2\frac{\partial^2 C_2'}{\partial y'^2} - k_{c'}\left(C_2' - C_{w_1}'\right) \tag{8}$$

REGION-III: Clear Region

$$\frac{\partial V_3'}{\partial y'} = 0 \tag{9}$$

$$\rho_3\left(\frac{\partial U_3'}{\partial t'} + V_3'\frac{\partial U_3'}{\partial y'}\right) = \mu_3\frac{\partial^2 U_3'}{\partial y'^2} - \frac{\partial P'}{\partial x'} - \sigma B_0^2 U_2' - \frac{\mu_2}{K'}U_3' + \rho_3 g\beta_{f_3}\left(T_3' - T_{w_3}'\right) + \rho_3 g\beta_{c_3}'\left(C_3' - C_{w_3}'\right) \tag{10}$$

$$\rho_3 C_p\left(\frac{\partial T_3'}{\partial y'} + V_3'\frac{\partial T_3'}{\partial y'}\right) = k_3\frac{\partial^2 T_3'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{11}$$

$$\frac{\partial C_2'}{\partial t'} + V_2'\frac{\partial C_2'}{\partial y'} = D_3\frac{\partial^2 C_3'}{\partial y'^2} - k_{c'}\left(C_3' - C_{w_3}'\right) \tag{12}$$

Assuming that the boundary and interface conditions on velocity are no slip, given that at the boundary and interface, the fluid particles are at rest, prompting the **x'-** component of the velocity to varnish at the wall.

Therefore, the boundary and interface conditions on the velocity for both fluids are:

$$U_1'(2h) = 0, U_1'(h) = U_2'(h), U_2'(0) = U_3'(0), U_3'(-h) = 0, \mu, \frac{\partial u_1'}{\partial y} = \mu_2\frac{\partial u_2'}{\partial y}, at\ y' = h,$$

$$\mu_2\frac{\partial u_2'}{\partial y} = \mu_3\frac{\partial u_3'}{\partial y}, at\ y' = 0 \tag{13}$$

The boundary and interface conditions on the temperature field for both fluids are:

$$\left.\begin{array}{c} T_1'(2h) = T_{w_1}', T_1'(h) = T_2'(h),\ T_2'(0) = T_3'(0),\ T_3'(-h) = T_{W_2}', \\ k_1\frac{\partial T_1'}{\partial y} = k_2\frac{\partial T_2'}{\partial y},\ at\ Y' = h, k_2\frac{\partial T_2'}{\partial y'} = k_3\frac{\partial T_3'}{\partial y'}\ at\ y' = 0 \end{array}\right\} \tag{14}$$

The boundary and interface conditions on the concentration field for both fluids are:

$$\left.\begin{array}{c} C_1'(2h) = C_{w_1}',\ C_1'(h) = C_2'(h), \\ C_2'(0) = C_3'(0), \\ C_3'(-h) = C_{w_2}', D_1\frac{\partial C_1'}{\partial y} = D_2\frac{\partial C_2'}{\partial y}\ \ at\ y' = h \\ D_2\frac{\partial C_2'}{\partial y'} = D_3\frac{\partial C_3'}{\partial y'}\ at\ y' = 0 \end{array}\right\} \tag{15}$$

The continuity equations (1), (5) and (9) implies that $V_1'V_2'$ and V_3' are independent of y', they can be at most a function of time alone. Hence we can write

$$V' = V_0(1 + \varepsilon A e^{i\omega t}) \tag{16}$$

Assuming that $V_1' = V_2' = V_3' = V'$.

ε is a very small positive quantity such that $\varepsilon A \ll 1$. Here, it is assumed that the transpiration velocity V' varies periodically with time about a non-zero constant mean velocity, V_0.

By using the following dimensionless quantities:

$$U_i = \frac{U_i'}{u}, y = \frac{y'}{h}, t = \frac{t'\vartheta_1}{h^2}, V = \frac{h}{\vartheta_1}V_1' = \frac{V}{V_0}, P = \frac{-h^2}{\mu_1 u}\left(\frac{\partial P'}{\partial x'}\right), \theta_i = \frac{T_i' - T_{w_1}'}{T_{w_2}' - T_{w_1}'}, Pr = \frac{\mu_1 C_p}{k_1}, \alpha_1 = \frac{\mu_2}{\mu_1}, \beta_1 = \frac{k_2}{k_1}, \beta_2 = \frac{K_3}{K_1},$$

$$\tau_1 = \frac{\rho_2}{\rho_1}, \gamma_1 = \frac{D_2}{D_1}, m_1 = \frac{\beta_{f_2}}{\beta_{f_1}}, m_2 = \frac{\beta_{f_3}}{\beta_{f_1}}\eta_1 = \frac{\beta_{c_2}^*}{\beta_{c_1}^*}, K^2 = \frac{h^2}{K'}, Sc = \frac{\vartheta_1}{D_1}, C_i = \frac{C_i' - C_{w_1}'}{C_{w_2}' - C_{w_1}'}, M^2 = \frac{\sigma h^2 B_0^2}{\mu_1}, F = \frac{4I'h_1^2}{k_1}, \frac{\partial q_r}{\partial y} = 4\left(T_i' - T_{w_1}'\right)I',$$

$$Gr = \frac{\rho_1 g h^2 \beta_{f_1}(T_{w_2}' - T_{w_1}')}{\mu_1 u}, \xi_1 = \frac{1}{\tau_1} = \frac{\rho_1}{\rho_2}, \gamma_2 = \frac{D_3}{D_1}\tau_2 = \frac{\rho_3}{\rho_1},$$

$$Gc = \frac{\rho_1 g h^2 \beta_{c_1}^*(C_{w_2}' - C_{w_1}')}{\mu_1 u}.k^2 = \frac{h^2}{k'}\eta_2 = \frac{\beta_{c_3}'}{\beta_{c_1}'}, \xi_2 = \frac{1}{\tau_2} = \frac{\rho_3}{\rho_1}$$

Equations (2), (3), (4), (6), (7), (8), (10), (11) and (12) becomes.

REGION-I

$$\frac{\partial U_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial U_1}{\partial y} = \frac{\partial^2 U_1}{\partial y^2} + P - M^2 U_1 + Gr\theta_1 + GcC_1 \tag{17}$$

$$\frac{\partial \theta_1}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial \theta_1}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta_1}{\partial y^2} - \frac{F\theta_1}{Pr} \tag{18}$$

$$\frac{\partial C_1}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial C_1}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C_1}{\partial y^2} - K_c C_1 \tag{19}$$

REGION-II

$$\frac{\partial U_2}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial U_2}{\partial y} = \alpha_1 \xi_1 \frac{\partial^2 U_2}{\partial y^2} + \xi_1 P - \xi_1 M^2 U_2 - \alpha_1 \xi_1 K^2 U_2 + Grm_1\theta_2 + Gc\eta_1 C_2 \tag{20}$$

$$\frac{\partial \theta_2}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial \theta_2}{\partial y} = \frac{\beta_1 \xi_1}{Pr}\frac{\partial^2 \theta_2}{\partial y^2} - \frac{F\xi_1 \theta_2}{Pr} \tag{21}$$

$$\frac{\partial C_2}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial C_2}{\partial y} = \frac{\gamma_1}{Sc}\frac{\partial^2 C_2}{\partial y^2} - K_C' C_2 \tag{22}$$

REGION-III

$$\frac{\partial U_3}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial U_3}{\partial y} = \alpha_2 \xi_2 \frac{\partial^2 U_3}{\partial y^2} + \xi_2 P - \xi_2 M^2 U_3 - \alpha_1 \xi_1 K^2 U_3 + Grm_2\theta_3 + Gc\eta_2 C_3 \tag{23}$$

$$\frac{\partial \theta_3}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial \theta_3}{\partial y} = \frac{\beta_2 \xi_2}{Pr}\frac{\partial^2 \theta_3}{\partial y^2} - \frac{F\xi_2 \theta_3}{Pr} \tag{24}$$

$$\frac{\partial C_3}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial C_3}{\partial y} = \frac{\gamma_2}{Sc}\frac{\partial^2 C_3}{\partial y^2} - K_C' C_2 \tag{25}$$

The boundary and interface, conditions in dimensionless form are given as follows

$$\left.\begin{array}{c} U_1(2) = 0,\ U_1(1) = U_2(1)U_2(0) = U_3(0),\ U_3(-1) = 0 \\ \frac{\partial U_1}{\partial y} = \alpha_1 \frac{\partial u_2}{\partial y}\ at\ Y' = 1 \\ \frac{\partial U_2}{\partial y} = \frac{\alpha_2}{\alpha_1}\frac{\partial U_3}{\partial y}\ at\ y = 0 \end{array}\right\} \tag{26}$$

$$\left.\begin{array}{c} \theta_1(2) = 0,\ \theta_1(1) = \theta_2(1) \\ \theta_2(0) = \theta_3(0), \\ \theta_3(-1) = 1,\ \frac{\partial \theta_1}{\partial y} = \beta_1 \frac{\partial \theta_2}{\partial y},\ at\ y = 1 \\ \frac{\partial \theta_2}{\partial y} = \frac{\beta_2}{\beta_1}\frac{\partial \theta_3}{\partial y}\ at\ y = 0 \end{array}\right\} \tag{27}$$

$$\left.\begin{array}{c} C_1(2) = 0,\ C_1(1) = C_2(1) \\ C_2(0) = C_3(0) \\ C_3(-1) = 1,\ \frac{\partial C_1}{\partial y} = \alpha_1 \frac{\partial C_2}{\partial y},\ at\ y = 1, \\ \frac{\partial C_2}{\partial y} = \frac{\alpha_2}{\alpha_1}\frac{\partial C_3}{\partial y}\ at\ y = 0 \end{array}\right\} \tag{28}$$

3. Method of Solution/Solution of the Problem

In order to solve the governing equations (17) to (25) under the boundary and interface conditions (26) to (28), we expand $U_1(y,t), \theta_1(y,t), C_1(y,t), U_2(y,t), \theta_2(y,t), C_2(y,t), C_3(y,t)$ as a power series in the perturbative parameter ε. Here, we assumed small amplitude of oscillation ($\varepsilon A \ll 1$), thus,

$$U_1(y,t) = U_{10}(y) + \varepsilon e^{i\omega t} U_{11}(y)$$

$$\theta_1(y,t) = \theta_{10}(y) + \varepsilon e^{i\omega t} \theta_{11}(y)$$

$$C_1(y,t) = C_{10}(y) + \varepsilon e^{i\omega t} C_{11}(y)$$

$$U_2(y,t) = U_{20}(y) + \varepsilon e^{i\omega t} U_{21}(y)$$

$$\theta_2(y,t) = \theta_{20}(y) + \varepsilon e^{i\omega t} \theta_{21}(y)$$

$$C_2(y,t) = C_{20}(y) + \varepsilon e^{i\omega t} C_{21}(y)$$

$$U_3(y,t) = U_{30}(y) + \varepsilon e^{i\omega t} U_{31}(y)$$

$$\theta_3(y,t) = \theta_{30}(y) + \varepsilon e^{i\omega t} \theta_{31}(y)$$

$$C_3(y,t) = C_{30}(y) + \varepsilon e^{i\omega t} C_{31}(y)$$

By substituting the above set of equations into equations (17) to (28), equating the coefficients of the like powers of ε and neglecting the terms containing $\boldsymbol{\varepsilon^2}$, we obtain the following set of ordinary differential equations in periodic and non-periodic terms,

REGION-I

Non-Periodic Terms:

$$\frac{\partial^2 U_{10}}{\partial y^2} - \frac{\partial U_{10}}{\partial y} - M^2 U_{10} = -P - Gr\theta_{10} - GcC_{10} \tag{29}$$

$$\frac{\partial^2 \theta_{10}}{\partial y^2} - Pr\frac{\partial \theta_{10}}{\partial y} - F\theta_{10} = 0 \tag{30}$$

$$\frac{\partial^2 C_{10}}{\partial y^2} - Sc\frac{\partial C_{10}}{\partial y} - S_C K_C C_{10} = 0 \tag{31}$$

Periodic terms:

$$\frac{\partial^2 U_{11}}{\partial y^2} - \frac{\partial U_{11}}{\partial y} - (M^2 + i\omega)U_{11} = \frac{\partial U_{10}}{\partial y} - Gr\theta_{11} - GcC_{11} \tag{32}$$

$$\frac{\partial^2 \theta_{11}}{\partial y^2} - Pr\frac{\partial \theta_{11}}{\partial y} - (F + i\omega Pr)\theta_{11} = Pr\frac{\partial \theta_{10}}{\partial y} \tag{33}$$

$$\frac{\partial^2 C_{11}}{\partial y^2} - Sc\frac{\partial C_{11}}{\partial y} - (S_C K_C + i\omega Sc) = Sc\frac{\partial C_{10}}{\partial y} \tag{34}$$

REGION-II

Non-Periodic terms:

$$\frac{\partial^2 U_{20}}{\partial y^2} - \frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y} - \left(\frac{\xi_1 M^2 + \alpha_1\xi_1 K^2}{\alpha_1\xi_1}\right)U_{20} = -\frac{P}{\alpha_1} - \frac{Grm_1}{\alpha_1\xi_1}\theta_{20} - \frac{Gc\eta_1}{\alpha_1\xi_1}C_{20} \tag{35}$$

$$\frac{\partial^2 \theta_{20}}{\partial y^2} - \frac{Pr}{\beta_1\xi_1}\frac{\partial \theta_{20}}{\partial y} - \frac{F}{\beta_1}\theta_{20} = 0 \tag{36}$$

$$\frac{\partial^2 C_{20}}{\partial y^2} - \frac{Sc}{\gamma_1}\frac{\partial C_{20}}{\partial y} - \frac{K_C S_C}{\gamma_1}C_{20} = 0 \tag{37}$$

Periodic Terms:

$$\frac{\partial^2 U_{21}}{\partial y^2} - \frac{1}{\alpha_1\xi_1}\frac{\partial U_{21}}{\partial y} - \left(\frac{\xi_1 M^2 + \alpha_1\xi_1 K^2 + i\omega}{\alpha_1\xi_1}\right)U_{21} = \frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y} - \frac{Grm_1}{\alpha_1\xi_1}\theta_{21} - \frac{Gc\eta_1}{\alpha_1\xi_1}C_{21} \tag{38}$$

$$\frac{\partial^2 \theta_{21}}{\partial y^2} - \frac{Pr}{\beta_1\xi_1}\frac{\partial \theta_{21}}{\partial y} - \left(\frac{F\xi_1 + i\omega Pr}{\beta_1\xi_1}\right)\theta_{21} = \frac{Pr}{\beta_1\xi_1}\frac{\partial \theta_{20}}{\partial y} \tag{39}$$

$$\frac{\partial^2 C_{21}}{\partial y^2} - \frac{Sc}{\gamma_1}\frac{\partial C_{21}}{\partial y} - \left(\frac{K_C S_C + i\omega S_C}{\gamma_1}\right)C_{21} = \frac{Sc}{\gamma_1}\frac{\partial C_{20}}{\partial y} \tag{40}$$

REGION-III

Non-Periodic terms:

$$\frac{\partial^2 U_{30}}{\partial y^2} - \frac{1}{\alpha_2\xi_2}\frac{\partial U_{30}}{\partial y} - \frac{M^2}{\alpha_2}U_{30} = -\frac{P}{\alpha_2} - \frac{Grm_1}{\alpha_2\xi_2}\theta_{30} - \frac{Gc\eta_2}{\alpha_2\xi_2}C_{30} \tag{41}$$

$$\frac{\partial^2 \theta_{30}}{\partial y^2} - \frac{Pr}{\beta_2\xi_2}\frac{\partial \theta_{30}}{\partial y} - \frac{F}{\beta_2}\theta_{30} = 0 \tag{42}$$

$$\frac{\partial^2 C_{30}}{\partial y^2} - \frac{Sc}{\gamma_2}\frac{\partial C_{30}}{\partial y} - \frac{K_C S_C}{\gamma_2}C_{30} = 0 \tag{43}$$

Periodic Terms:

$$\frac{\partial^2 U_{31}}{\partial y^2} - \frac{1}{\alpha_2\xi_2}\frac{\partial U_{31}}{\partial y} - \left(\frac{\xi_2 M^2 + i\omega}{\alpha_2\xi_2}\right)U_{31} = \frac{1}{\alpha_2\xi_2}\frac{\partial U_{30}}{\partial y} - \frac{Grm_1}{\alpha_1\xi_1}\theta_{31} - \frac{Gc\eta_2}{\alpha_2\xi_2}C_{31} \tag{44}$$

$$\frac{\partial^2 \theta_{31}}{\partial y^2} - \frac{Pr}{\beta_2\xi_2}\frac{\partial \theta_{31}}{\partial y} - \left(\frac{F\xi_2 + i\omega Pr}{\beta_2\xi_2}\right)\theta_{31} = \frac{Pr}{\beta_2\xi_2}\frac{\partial \theta_{30}}{\partial y} \tag{45}$$

$$\frac{\partial^2 C_{31}}{\partial y^2} - \frac{Sc}{\gamma_2}\frac{\partial C_{31}}{\partial y} - \left(\frac{K_C S_C + i\omega S_C}{\gamma_2}\right)C_{21} = \frac{Sc}{\gamma_2}\frac{\partial C_{30}}{\partial y} \tag{46}$$

The equations (22) to (33) are ordinary linear coupled differential equations with constant coefficients. The corresponding boundary and interface conditions then become:

Non-Periodic terms:

$$\left.\begin{array}{c} U_{10}(2)=0, U_{10}(1)=U_{20}(1), \\ U_{20}(0)=U_{30}(0),\ U_{30}(-1)=0 \\ \frac{\partial U_{10}}{\partial y}=\alpha_1\frac{\partial u_{20}}{\partial y},\ at\ Y=1,\ \frac{\partial U_{20}}{\partial y}=\frac{\alpha_2}{\alpha_1}\frac{\partial U_{30}}{\partial y},\ at\ Y=0 \end{array}\right\} \quad (47)$$

$$\left.\begin{array}{c} \theta_{10}(2)=0,\ U_{10}(1)=U_{20}(1) \\ \theta_{20}(0)=\theta_{30}(0),\ \theta_{30}(-1)=0 \\ \frac{\partial\theta_{10}}{\partial y}=\beta_1\frac{\partial\theta_{20}}{\partial y},\ at\ Y=1 \\ \frac{\partial\theta_{20}}{\partial y}=\frac{\beta_2}{\beta_1}\frac{\partial\theta_{30}}{\partial y}\ at\ y=0 \end{array}\right\} \quad (48)$$

$$\left.\begin{array}{c} C_{10}(2)=0,\ U_{10}(1)=U_{20}(1), \\ C_{20}(0)=U_{30}(0),\ U_{30}(-1)=1 \\ \frac{\partial C_{10}}{\partial y}=\gamma_1\frac{\partial C_{20}}{\partial y},\ at\ y=1 \\ \frac{\partial C_{20}}{\partial y}=\frac{\gamma_2}{\gamma_1}\frac{\partial C_{30}}{\partial y}\ at\ y=0 \end{array}\right\} \quad (49)$$

Periodic Terms:

$$\left.\begin{array}{c} U_{11}(2)=0,\ U_{11}(1)=U_{21}(1) \\ U_{21}(0)=U_{31}(0),\ U_{31}(-1)=0 \\ \frac{\partial U_{11}}{\partial y}=\alpha_1\frac{\partial U_{21}}{\partial y}, at\ y=1 \\ \frac{\partial U_{21}}{\partial y}=\frac{\alpha_2}{\alpha_1}\frac{\partial U_{31}}{\partial y}\ at\ y=0 \end{array}\right\} \quad (50)$$

$$\left.\begin{array}{c} \theta_{11}(2)=0,\ \theta_{11}(1)=\theta_{21}(1) \\ \theta_{21}(0)=\theta_{31}(0),\ \theta_{31}(-1)=0 \\ \frac{\partial\theta_{11}}{\partial y}=\beta_1\frac{\partial\theta_{21}}{\partial y}, at\ y=1 \\ \frac{\partial\theta_{11}}{\partial y}=\frac{\beta_2}{\beta_1}\frac{\partial\theta_{21}}{\partial y}\ at\ y=0 \end{array}\right\} \quad (51)$$

$$\left.\begin{array}{c} C_{11}(2)=0,\ C_{11}(1)=C_{21}(1) \\ C_{21}(0)=C_{31}(0),\ C_{31}(-1)=1 \\ \frac{\partial C_{11}}{\partial y}=\gamma_1\frac{\partial C_{21}}{\partial y}, at\ y=1 \\ \frac{\partial C_{11}}{\partial y}=\frac{\gamma_2}{\gamma_1}\frac{\partial C_{21}}{\partial y}\ at\ y=0 \end{array}\right\} \quad (52)$$

The analytical solutions of the differential equations (29) to (46) are readily obtainable under the boundary conditions (47) to (52). They are:

$$U_{10}(y)=C_5e^{m_5y}+C_6e^{m_6y}+K_1+K_2e^{m_1y}+K_3e^{m_2y}+K_4e^{m_3y}+K_5e^{m_4y} \quad (53)$$

$$U_{20}(y)=C_{17}e^{m_{17}y}+C_{18}e^{m_{18}y}+K_{20}+K_{21}e^{m_{13}y}+K_{22}e^{m_{14}y}+K_{23}e^{m_{15}y}+K_{24}e^{m_{16}y} \quad (54)$$

$$U_{30}(y)=C_{29}e^{m_{29}y}+C_{30}e^{m_{30}y}+K_{39}+K_{40}e^{m_{25}y}+K_{41}e^{m_{26}y}+K_{42}e^{m_{27}y}+K_{43}e^{m_{28}y} \quad (55)$$

$$\theta_{10}(y)=C_1e^{m_1y}+C_2e^{m_2y} \quad (56)$$

$$\theta_{20}(y)=C_{13}e^{m_{13}y}+C_{14}e^{m_{14}y} \quad (57)$$

$$\theta_{30}(y)=C_{25}e^{m_{25}y}+C_{26}e^{m_{26}y} \quad (58)$$

$$C_{10}(y)=C_3e^{m_3y}+C_4e^{m_4y} \quad (59)$$

$$C_{20}(y)=C_{15}e^{m_{15}y}+C_{16}e^{m_{16}y}\ C_{30}(y)=C_{27}e^{m_{27}y}+C_{28}e^{m_{28}y} \quad (60)$$

$$U_{11}(y)=C_{11}e^{m_{11}y}+C_{12}e^{m_{12}y}+K_{10}e^{m_1y}+K_{11}e^{m_2y}+K_{12}e^{m_3y}+K_{13}e^{m_4y}+K_{14}e^{m_5y}+K_{15}e^{m_6y}+K_{16}e^{m_7y}+K_{17}e^{m_8y}+K_{18}e^{m_9y}+K_{19}e^{m_{10}y} \quad (61)$$

$$U_{21}(y)=C_{23}e^{m_{23}y}+C_{24}e^{m_{24}y}+K_{29}e^{m_{13}y}+K_{30}e^{m_{14}y}+K_{31}e^{m_{15}y}+K_{32}e^{m_{16}y}+K_{33}e^{m_{17}y}+K_{34}e^{m_{18}y}+K_{35}e^{m_{19}y}+K_{36}e^{m_{20}y}+K_{37}e^{m_{21}y}+K_{38}e^{m_{22}y} \quad (62)$$

$$U_{31}(y)=C_{35}e^{m_{35}y}+C_{36}e^{m_{36}y}+K_{48}e^{m_{25}y}+K_{49}e^{m_{26}y}+K_{50}e^{m_{27}y}+K_{51}e^{m_{28}y}+K_{52}e^{m_{29}y}+K_{53}e^{m_{30}y}+K_{54}e^{m_{31}y}+K_{55}e^{m_{32}y}+K_{56}e^{m_{33}y}+K_{57}e^{m_{34}y} \quad (63)$$

$$\theta_{11}(y)=C_7e^{m_7y}+C_8e^{m_8y}+K_6e^{m_1y}+K_7e^{m_2y} \quad (64)$$

$$\theta_{21}(y)=C_{19}e^{m_{19}y}+C_{20}e^{m_{20}y}+K_{25}e^{m_{13}y}+K_{26}e^{m_{14}y} \quad (65)$$

$$\theta_{31}(y)=C_{31}e^{m_{31}y}+C_{32}e^{m_{32}y}+K_{44}e^{m_{25}y}+K_{45}e^{m_{26}y} \quad (66)$$

$$C_{11}(y)=C_9e^{m_9y}+C_{10}e^{m_{10}y}+K_8e^{m_3y}+K_9e^{m_4y} \quad (67)$$

$$C_{21}(y)=C_{21}e^{m_{21}y}+C_{22}e^{m_{22}y}+K_{27}e^{m_{15}y}+K_{28}e^{m_{16}y} \quad (68)$$

$$C_{31}(y)=C_{33}e^{m_{33}y}+C_{34}e^{m_{34}y}+K_{46}e^{m_{27}y}+K_{47}e^{m_{28}y} \quad (69)$$

Where

$$V_1 = F + i\omega Pr, V_2 = S_cK_c + i\omega Sc, V_3 = M^2 + i\omega, V_4 = \frac{Pr}{\beta_1\xi_1}, V_5 = \frac{F}{\beta_1}, V_6 = \frac{Sc}{\gamma_1}, V_7 = \frac{S_cK_c}{\gamma_1}, V_8 = \frac{1}{\alpha_1\xi_1}, V_9 = \frac{M^2+\alpha_1K^2}{\alpha_1}, V_{10} = \frac{P_r}{\beta_1\xi_1}, V_{11} = \frac{\xi_1F+i\omega P_r}{\alpha_1\xi_1}, V_{12} = \frac{K_cS_c+i\omega S_c}{\alpha_1}, V_{13} = \frac{\xi_1M^2+i\omega+K^2\alpha_1}{\alpha_1\xi_1}, V_{14} = \frac{P_r}{\beta_2\xi_2}, V_{15} = \frac{F}{\beta_2}, V_{16} = \frac{S_c}{\alpha_2}, V_{17} = \frac{S_cK_c}{\alpha_2}, V_{18} = \frac{1}{\alpha_2\xi_2}, V_{19} = \frac{M^2}{\alpha_2}, V_{20} = \frac{\xi_2F+i\omega P_r}{\beta_2\xi_2}, V_{21} = \frac{K_cS_c+i\omega S_c}{\alpha_2}, V_{22} = \frac{M^2\xi_2+i\omega}{\alpha_2\xi_2},$$

$$m_1 = \frac{Pr+\sqrt{Pr^2+4F}}{2}, m_2 = \frac{Pr+\sqrt{(Pr)^2+4F}}{2}, m_3 = \frac{P_r-\sqrt{(P_r)^2-4F}}{2}, m_4 = \frac{S_c-\sqrt{(S_c)^2+4S_cK_c}}{2}, m_5 = \frac{S_c+\sqrt{(S_c)^2+4S_cK_c}}{2},$$

$$m_6 = \frac{1-\sqrt{1+4M^2}}{2}, m_7 = \frac{1+\sqrt{1+4M^2}}{2}, m_8 = \frac{Pr-\sqrt{Pr^2+4V_1}}{2}, m_9 = \frac{Sc+\sqrt{Sc^2+4V_2}}{2},$$

$$m_{10} = \frac{Sc-\sqrt{Sc^2+4V_2}}{2}, m_{11} = \frac{1+\sqrt{1+4V_3}}{2}, m_{12} = \frac{1-\sqrt{1+4V_3}}{2}, m_{13} = \frac{V_4+\sqrt{V_4^2+4V_5}}{2},$$

$$m_{14} = \frac{V_4-\sqrt{V_4^2+4V_5}}{2}, m_{15} = \frac{V_6-\sqrt{V_6^2+4V_7}}{2}, m_{16} = \frac{V_6+\sqrt{V_6^2+4V_7}}{2}, m_{17} = \frac{V_8+\sqrt{V_8^2+4V_9}}{2}, m_{18} = \frac{V_7-\sqrt{V_8^2+4V_9}}{2},$$

$$m_{19} = \frac{V_{10}+\sqrt{V_{10}^2+4V_{11}}}{2}, m_{20} = \frac{V_{10}-\sqrt{V_{10}^2+4V_{11}}}{2}, m_{21} = \frac{V_{12}+\sqrt{V_{12}^2+4V_{13}}}{2}, m_{22} = \frac{V_{12}-\sqrt{V_{12}^2+4V_{13}}}{2},$$

$$, m_{23} = \frac{V_8+\sqrt{V_8^2+4V_{14}}}{2}, m_{24} = \frac{V_8-\sqrt{V_8^2+4V_{14}}}{2},$$

$$m_{25} = \frac{V_{15}+\sqrt{V_{15}^2+4V_{16}}}{2}, m_{26} = \frac{V_{15}-\sqrt{V_{15}^2+4V_{16}}}{2},$$

$$m_{27} = \frac{V_{17}-\sqrt{V_{17}^2+4V_{18}}}{2},$$

$$m_{28} = \frac{V_{17}+\sqrt{V_{17}^2+4V_{18}}}{2}, m_{29} = \frac{V_{19}+\sqrt{V_{19}^2+4V_{20}}}{2}, m_{30} = \frac{V_{19}-\sqrt{V_{19}^2+4V_{20}}}{2}, m_{31} = \frac{V_{15}+\sqrt{V_{15}^2+4V_{21}}}{2}, m_{32} = \frac{V_{15}-\sqrt{V_{15}^2+4V_{21}}}{2},$$

$$m_{33} = \frac{V_{17}+\sqrt{V_{17}^2+4V_{22}}}{2}, m_{34} = \frac{V_{17}-\sqrt{V_{17}^2+4V_{22}}}{2}, m_{35} = \frac{V_{19}+\sqrt{V_{19}^2+4V_{23}}}{2}, m_{36} = \frac{V_{19}-\sqrt{V_{19}^2+4V_{23}}}{2},$$

$$r_1 = e^{2(m_1-m_2)}, r_2 = e^{m_{26}}$$

$$r_3 = e^{(m_{26}-m_{25})}, r_4 = m_2e^{-m_2}, r_5 = m_1 - m_2e^{(m_1-m_2)},$$

$$r_6 = \beta_1(m_{14}e^{(m_{14}-m_{13})} - m_{13}),$$

$$r_7 = e^{-m_4}, r_8 = 1 - e^{(m_3-m_4)}, r_9 = e^{(m_{16}-m_{15})} - 1,$$

$$r_{10} = m_4e^{-m_4}, r_{11} = m_3 - m_4e^{(m_3-m_4)}, r_{12} = \gamma_1(m_{16}e^{(m_{16}-m_{15})} - m_{15}),$$

$$r_{13} = 1 - e^{(m_5-m_6)}, r_{14} = e^{(m_{18}-m_{17})} - 1, r_{15} = m_5 - m_6e^{(m_5-m_6)},$$

$$r_{16} = \alpha_1(m_{18}e^{(m_{18}-m_{17})} - m_{17}), r_{17} = 1 - e^{(m_7-m_8)}, r_{18} = e^{(m_{20}-m_{19})} - 1,$$

$$r_{19} = m_7 - m_8e^{(m_7-m_8)}, r_{20} = \beta_1(m_{20}e^{(m_{20}-m_{19})} - m_{19}), r_{21} = 1 - e^{(m_9-m_{10})},$$

$$r_{22} = e^{(m_{22}-m_{21})} - 1, r_{23} = m_9 - m_{10}e^{(m_9-m_{10})}, r_{24} = \gamma_1(m_{22}e^{(m_{22}-m_{21})} - m_{21}),$$

$$r_{25} = 1 - e^{(m_{11}-m_{12})}, r_{26} = e^{(m_{24}-m_{23})} - 1, r_{27} = m_{11} - m_{12}e^{(m_{11}-m_{12})},$$

$$r_{28} = \alpha_1(m_{24}e^{(m_{24}-m_{23})} - m_{23}), C_1 = \frac{r_1r_6-r_3r_4}{r_3r_5-r_2r_6}, C_2 = C_1r_1, C_4 = C_3r_{22}, C_{16} = \frac{r_{36}-C_{15}r_{34}}{r_{35}},$$

$$C_{26} = r_2 + C_{25}r_3, C_{28} = r_{23} + C_{27}r_{24}, C_{25} = \frac{r_2-C_{13}-C_{14}}{r_4}, C_{27} = \frac{r_{23}-C_{15}-C_{16}}{r_{25}}, C_{14} = \frac{r_{15}-C_{13}r_{13}}{r_{14}}, C_{13} = \frac{C_1r_{16}-r_{18}}{r_{17}}, C_{15} = \frac{C_3r_{37}-r_{39}}{r_{38}},$$

$$C_1 = \frac{r_{17}r_{21}-r_{18}r_{20}}{r_{19}r_{17}-r_{16}r_{20}}, C_3 = \frac{r_{38}r_{42}-r_{39}r_{41}}{r_{38}r_{40}-r_{37}r_{41}}, C_5 = \frac{Q_7r_{54}-Q_6r_{56}}{r_{54}r_{55}-r_{53}r_{56}}, C_{29} = \frac{Q_1-C_{17}-C_{18}}{r_{43}}$$

$$C_6 = -A_1\, e^{-2m_6} - C_5 e^{2(m_5-m_6)}, C_{30} = -A_4\, e^{m_{30}} - C_{29} e^{(m_{30}-m_{29})} C_7 = \frac{Q_{14} r_{68} - Q_{13} r_{70}}{r_{68} r_{69} - r_{67} r_{70}},$$

$$C_8 = A_5\, e^{-m_8} - C_7 e^{(m_7-m_8)}, \quad C_9 = \frac{Q_6 - Q_5 r_{24}}{r_{22} r_{23} - r_{21} r_{24}}, C_{31} = \frac{C_{19} + C_{20} - Q_9}{r_{58}}$$

$$C_{10} = -A_{13}\, e^{-2m_{10}} - C_9 e^{2(m_9-m_{10})}, \quad C_{17} = \frac{C_5 r_{53} - Q_6}{r_{54}},$$

$$C_{34} = -A_{16} e^{m_{34}} - C_{33} e^{(m_{34}-m_{33})}, \quad C_{19} = \frac{Q_{13} - C_7 r_{67}}{r_{68}},$$

$$C_{12} = -A_{19} e^{-2m_{12}} - C_{11} e^{2(m_{11}-m_{12})}, \quad C_{21} = \frac{Q_5 - C_9 r_{21}}{r_{22}},$$

$$C_{22} = -A_{10} e^{m_{22}} - C_{21} e^{(m_{22}-m_{21})},$$

$$K_1 = \frac{P}{M^2}, K_2 = -\frac{GrC_1}{m_1^2 - m_1 - M^2}, K_3 = -\frac{GrC_2}{m_2^2 - m_2 - M^2},$$

$$K_4 = -\frac{GcC_3}{m_3^2 - m_3 - M^2}, K_5 = -\frac{GcC_4}{m_4^2 - m_4 - M^2}, K_6 = \frac{PrC_1 m_1}{m_1^2 - Prm_1 - V_1}, K_7 = \frac{PrC_2 m_2}{m_2^2 - Prm_2 - V_1},$$

$$K_8 = \frac{ScC_3 m_3}{m_3^2 - Scm_3 - V_2}, K_9 = \frac{ScC_4 m_4}{m_4^2 - Scm_4 - V_2},$$

$$K_{10} = \frac{K_2 m_1 - GrK_6}{m_1^2 - m_1 - V_3}, K_{11} = \frac{K_3 m_2 - GrK_7}{m_2^2 - m_2 - V_3}, K_{12} = \frac{K_4 m_3 - GcK_8}{m_3^2 - m_3 - V_3}$$

$$K_{13} = \frac{K_5 m_4 - GcK_9}{m_4^2 - m_4 - V_3}$$

$$K_{14} = \frac{C_5 m_5}{m_5^2 - m_5 - V_3}, K_{15} = \frac{C_6 m_6}{m_6^2 - m_6 - V_3},$$

$$K_{16} = -\frac{GrC_7}{m_7^2 - m_7 - V_3}, K_{17} = -\frac{GrC_8}{m_8^2 - m_8 - V_3},$$

$$K_{18} = -\frac{GcC_9}{m_9^2 - m_9 - V_3}, K_{19} = -\frac{GcC_{10}}{m_{10}^2 - m_{10} - V_3},$$

$$K_{20} = \frac{P}{V_9 \alpha_1}, K_{21} = -\frac{G_r \varphi_1 V_8 C_{13}}{m_{13}^2 - V_8 m_{13} - V_9}, K_{22} = -\frac{V_8 Gr \varphi_1 C_{14}}{m_{14}^2 - V_8 m_{14} - V_9}, K_{23} = -\frac{V_8 Gc \eta_1 C_{15}}{m_{15}^2 - V_8 m_{15} - V_9}, K_{24} = -\frac{V_8 Gc \eta_1 C_{16}}{m_{16}^2 - V_8 m_{16} - V_9},$$

$$K_{25} = \frac{V_{10} C_{13} m_{13}}{m_{13}^2 - V_{10} m_{13} - V_{11}}, K_{26} = \frac{V_{10} C_{14} m_{14}}{m_{14}^2 - V_{10} m_{14} - V_{11}},$$

$$K_{27} = \frac{V_{12} C_{15} m_{15}}{m_{15}^2 - V_{12} m_{15} - V_{13}}, K_{28} = \frac{V_{12} C_{16} m_{16}}{m_{16}^2 - V_{12} m_{16} - V_{13}},$$

$$K_{29} = \frac{V_8 (K_{21} m_{13} - GrK_{25} \varphi_1)}{m_{13}^2 - V_8 m_{13} - V_{14}}, K_{30} = \frac{V_8 (K_{22} m_{14} - G_r \varphi_1 K_{26})}{m_{14}^2 - V_8 m_{14} - V_{14}}, K_{31} = \frac{V_8 (K_{23} m_{15} - GcK_{27} \eta_1)}{m_{15}^2 - V_8 m_{15} - V_{14}}$$

$$K_{32} = \frac{V_8 (K_{24} m_{16} - GcK_{28} \eta_1)}{m_{16}^2 - V_8 m_{16} - V_{14}},$$

$$K_{33} = \frac{V_8 C_{17} m_{17}}{m_{17}^2 - V_8 m_{17} - V_{14}},$$

$$K_{34} = \frac{V_8 C_{18} m_{18}}{m_{18}^2 - V_8 m_{18} - V_{14}}, K_{35} = -\frac{V_8 Gr \varphi_1 C_{19}}{m_{19}^2 - V_8 m_{19} - V_{14}},$$

$$K_{36} = -\frac{V_8 Gr \varphi_1 C_{20}}{m_{20}^2 - V_8 m_{20} - V_{14}},$$

$$K_{37} = -\frac{V_8 Gc \eta_1 C_{21}}{m_{21}^2 - V_8 m_{21} - V_{14}}, K_{38} = -\frac{V_8 Gc \eta_1 C_{22}}{m_{22}^2 - V_8 m_{22} - V_{14}},$$

$$k_{39} = \frac{p}{v_{20} \alpha_2},$$

$$K_{40} = -\frac{V_{19} Gr \varphi_2 C_{25}}{m_{25}^2 - V_{19} m_{25} - V_{20}},$$

$$K_{41} = -\frac{V_{19}Gr\varphi_2C_{26}}{m_{26}^2-V_{19}m_{26}-V_{20}},$$

$$K_{42} = -\frac{V_{19}Gc\eta_2C_{27}}{m_{27}^2-V_{19}m_{27}-V_{20}},$$

$$K_{43} = -\frac{V_{19}Gc\eta_2C_{28}}{m_{28}^2-V_{19}m_{28}-V_{20}},$$

$$K_{44} = \frac{V_{15}m_{25}C_{25}}{m_{25}^2-V_{15}m_{25}-V_{21}}, K_{45} = \frac{V_{15}m_{26}C_{26}}{m_{26}^2-V_{15}m_{26}-V_{21}},$$

$$K_{46} = \frac{V_{17}m_{27}C_{27}}{m_{27}^2-V_{17}m_{27}-V_{22}}, K_{47} = \frac{V_{17}m_{28}C_{28}}{m_{28}^2-V_{17}m_{28}-V_{22}},$$

$$K_{48} = \frac{V_{19}(K_{40}m_{25}-GrK_{44}\varphi_2)}{m_{25}^2-V_{19}m_{25}-V_{23}}, K_{49} = \frac{V_{19}(K_{41}m_{26}-G_r\varphi_2K_{45})}{m_{26}^2-V_{19}m_{19}-V_{23}},$$

$$K_{50} = \frac{V_{19}(K_{42}m_{27}-GcK_{45}\eta_2)}{m_{27}^2-V_{19}m_{27}-V_{23}}, K_{51} = \frac{V_8(K_{24}m_{16}-GcK_{28}\eta_1)}{m_{16}^2-V_8m_{16}-V_{14}}$$

$$K_{52} = \frac{V_{19}m_{29}C_{29}}{m_{29}^2-V_{19}m_{29}-V_{23}}, K_{53} = \frac{V_{19}m_{30}C_{30}}{m_{30}^2-V_{19}m_{30}-V_{23}},$$

$$K_{54} = -\frac{V_{19}Gr\varphi_2C_{31}}{m_{31}^2-V_{19}m_{31}-V_{23}},$$

$$K_{55} = -\frac{V_{19}\varphi_2C_{32}G_r}{m_{32}^2-V_{19}m_{32}-V_{23}}, K_{56} = -\frac{V_{19}G_c\eta_2C_{33}}{m_{33}^2-V_{19}m_{33}-V_{23}}, K_{57} = -\frac{\eta_2V_{19}G_cC_{34}}{m_{34}^2-V_{19}m_{34}-V_{23}}$$

$$A_1 = K_1 + K_2e^{2m_1} + K_3e^{2m_2} + K_4e^{2m_3} + K_5e^{2m_4},$$

$$A_2 = K_1 + K_2e^{m_1} + K_3e^{m_2} + K_4e^{m_3} + K_5e^{m_4}$$

$$A_3 = K_{20} + K_{21} + K_{22} + K_{23} + K_{24} - (K_{39} + K_{40} + K_{41} + K_{42} + K_{43}),$$

$$A_4 = K_{39} + K_{40}e^{-m_{25}} + K_{41}e^{-m_{26}} + K_{42}e^{-m_{27}} + K_{43}e^{-m_{28}}$$

$$A_5 = K_2e^{m_1}m_1 + K_3m_2e^{m_2} + K_4m_3e^{m_3} + K_5m_4e^{m_4} - \alpha_1(K_{21}m_{13}e^{m_{13}} + K_{22}m_{14}e^{m_{14}} + K_{23}m_{15}e^{m_{15}} + K_{24}m_{16}e^{m_{16}}$$

$$A_6 = \alpha_1(K_{21}m_{13} + K_{22}m_{14} + K_{23}m_{15} + K_{24}m_{16}) - \alpha_2(K_{40}m_{25} + K_{41}m_{26} + K_{42}m_{27} + K_{43}m_{28}, A_7 = K_6e^{2m_1} + K_7e^{2m_2}$$

$$A_8 = K_6e^{m_1} + K_7e^{m_2}) - (K_{25}e^{m_{13}} + K_{26}e^{m_{14}}), A_9 = K_{25} + K_{26} - (K_{44} + K_{45})$$

$$A_{10} = K_{44}e^{-m_{25}} + K_{45}e^{-m_{26}} - 1, A_{11} = K_{27} + K_{28} - (K_8 + K_9), A_{13} = K_8e^{2m_3} + K_9e^{2m_4}$$

$$A_{12} = \gamma_1(K_{27}m_{15} + K_{28}m_{16}) - (K_8m_3 + K_9m_4), A_{14} = K_8e^{m_3} + K_9e^{m_4} - (K_{27}e^{m_{15}} + K_{28}e^{m_{16}})$$

$$A_{15} = K_{27} + K_{28} - (K_{46} + K_{47})$$

$$A_{16} = K_{46}e^{-m_{27}} + K_{47}e^{-m_{28}}, A_{18} = \gamma_1(K_{27}m_{15} + K_{28}m_{16}) - \gamma_2(K_{46}m_{17} + K_{47}m_{28})\ Q_1 = A_3 + A_1e^{-m_6} - A_2e^{m_{18}}$$

$$A_{17} = K_8e^{m_3}m_3 + K_9m_4e^{m_4} - \gamma_1(K_{27}m_{15}e^{m_{15}} + K_{28}m_{16}e^{m_{16}})$$

$$Q_2 = A_4r_{14} + A_1r_{14}m_6e^{-m_6} - \alpha_1A_2r_{14}m_{18}e^{m_{18}}, Q_3 = A_7 - A_5e^{-m_8} - A_6e^{m_{20}}$$

$$Q_4 = A_8r_{18} - A_5r_{18}m_8e^{-m_8} - \beta_1A_6r_{18}m_{20}e^{m_{20}}, Q_5 = A_{11} - A_9e^{-m_{10}} - A_{10}e^{m_{22}}$$

$$Q_6 = A_{12}r_{22} - A_9r_{22}m_{10}e^{-m_{10}} - \gamma_1A_{10}r_{22}m_{22}e^{m_{22}}$$

$$Q_7 = A_{15} + A_{13}e^{-m_{12}} - A_{14}e^{m_{24}}, Q_8 = A_{16}r_{26} + A_{13}r_{26}m_{12}e^{-m_{12}} - \alpha_1A_{14}r_{26}m_{24}e^{m_{24}}$$

4. The Coefficient of Skin Friction, Nusselt Number and Sherwood Number Are Given as

Coefficient of skin friction

$$C_f(U) = \left[\frac{\partial U_{10}}{\partial y}\right]_{y=2} + \varepsilon e^{i\omega t}\left[\frac{\partial U_{11}}{\partial y}\right]_{y=2}$$

$$C_f(L) = [\frac{\partial U_{20}}{\partial y}]_{y=-1} + \varepsilon e^{i\omega t}[\frac{\partial U_{21}}{\partial y}]_{y=-1}$$

Nusselt number

$$\mathrm{Nu(U)} = \left[\frac{\partial\theta_{10}}{\partial y}\right]_{y=2} + \varepsilon e^{i\omega t}\left[\frac{\partial\theta_{11}}{\partial y}\right]_{y=2}$$

$$\mathrm{Nu(L)} = \left[\frac{\partial\theta_{20}}{\partial y}\right]_{y=-1} + \varepsilon e^{i\omega t}\left[\frac{\partial\theta_{21}}{\partial y}\right]_{y=-1}$$

Sherwood number

$$\mathrm{Sh(U)} = \left[\frac{\partial C_{10}}{\partial y}\right]_{y=2} + \varepsilon e^{i\omega t}\left[\frac{\partial C_{11}}{\partial y}\right]_{y=2}$$

$$\mathrm{Sh(L)} = \left[\frac{\partial C_{20}}{\partial y}\right]_{y=-1} + \varepsilon e^{i\omega t}\left[\frac{\partial C_{21}}{\partial y}\right]_{y=-1}$$

5. Discussion of Results

An attempt has been made to study the velocity, temperature and concentration fields in an unsteady MHD free convective flow through a porous medium sandwiched between viscous fluids with heat and mass transfer in a horizontal channel. In order to get the physical insight into the problem, the velocity, temperature and concentration have been discussed by assigning the various numerical values to M (magnetic parameter), Kc (chemical parameter), Pr (prandtl number), Sc (Schmidt number) etc.

Figure 2 presents the effect of prandtl number Pr on the velocity profile. It is observe that as the momentum diffusivity (kinematic viscosity) gradually dominate the thermal diffusivity, the velocity of the flow decreases with slight alteration in the porous region-II, while the variation in the velocity has not much significant in the clear region-I and III.

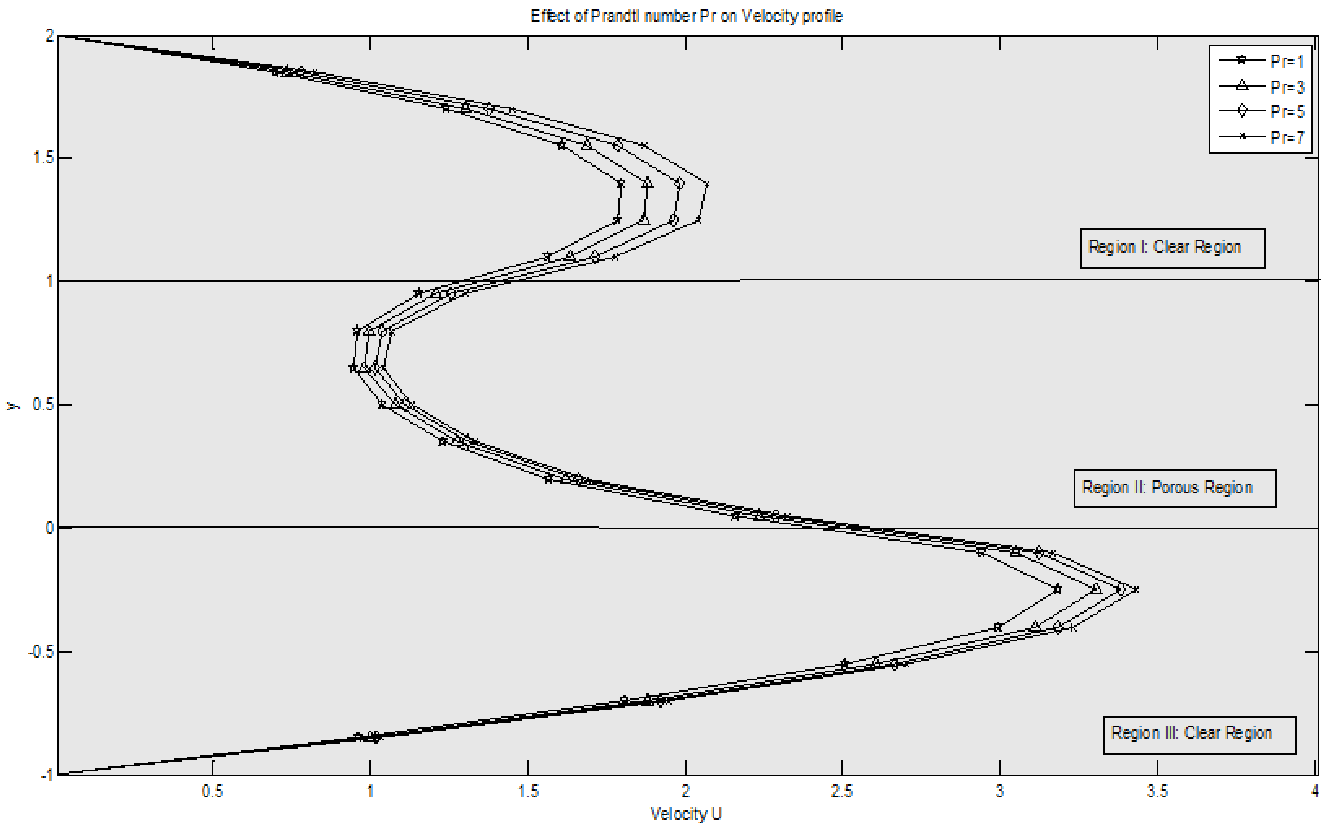

Figure 2. *Velocity distribution versus y for different values of prandlt number Pr when K=1, P=1,* $\beta_2 = \eta_2 = \alpha_2 = m_2 = 1 = m_1 = \eta_1 . \omega = 10, \omega t = \pi/6, G_r = 5, \varepsilon = 0.025$ *and A=1.*

Figure 3 depicts the effect of Radiation parameter F on velocity profile. The radiation parameter has no significant effect on the velocity field as it can be seen, although an increase in the Radiation parameter has slight decrease on the velocity in region-II with slight increase in region-III.

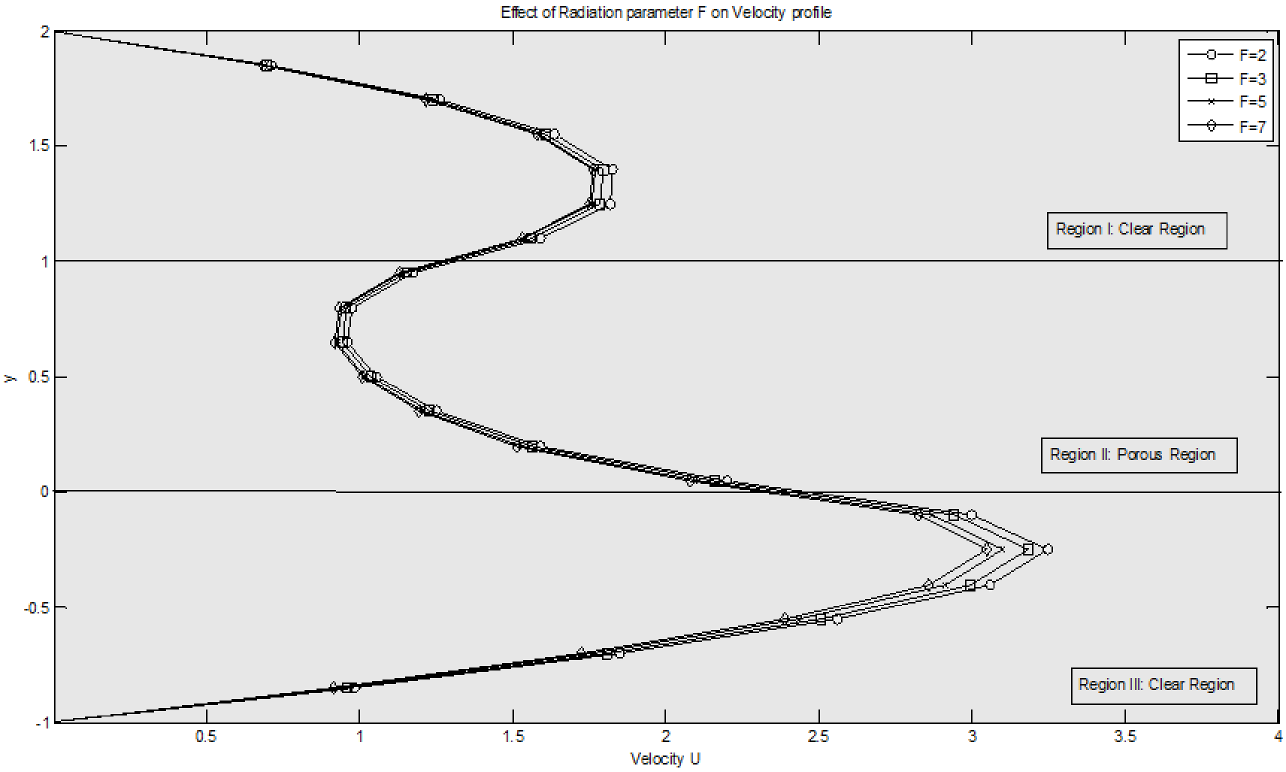

Figure 3. *Velocity distribution versus y for different values of thermal radiation parameter when K=1, P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6,G_r=5,\varepsilon=0.025$ *and A=1.*

Figure 4 illustrates the effect of Schmidt number Sc on velocity filed. It is observed that gradual domination of viscous diffusion rate over the molecular (mass) diffusion rate contribute to a decrease in the velocity field in region-I and II, and rapid increase of velocity in region-III.

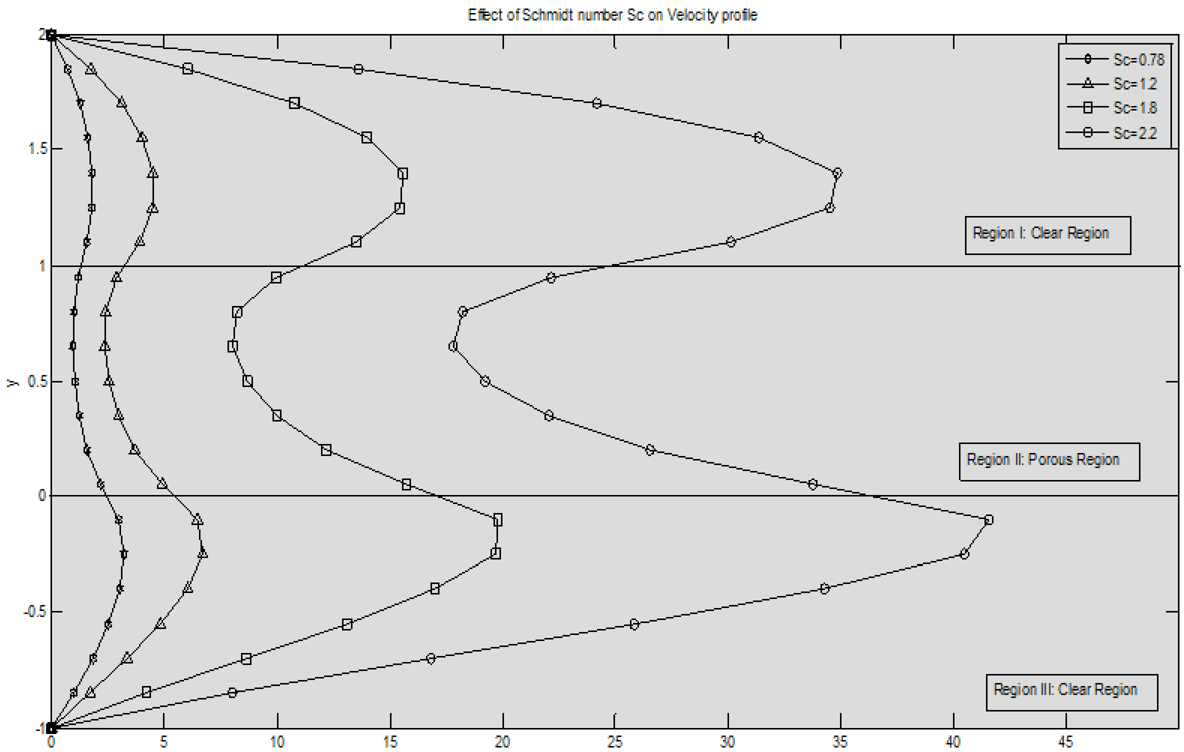

Figure 4. *Velocity distribution versus y for different values of Schmidt Sc number when K=1, P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6,G_r=5,\varepsilon=0.025$ *and A=1.*

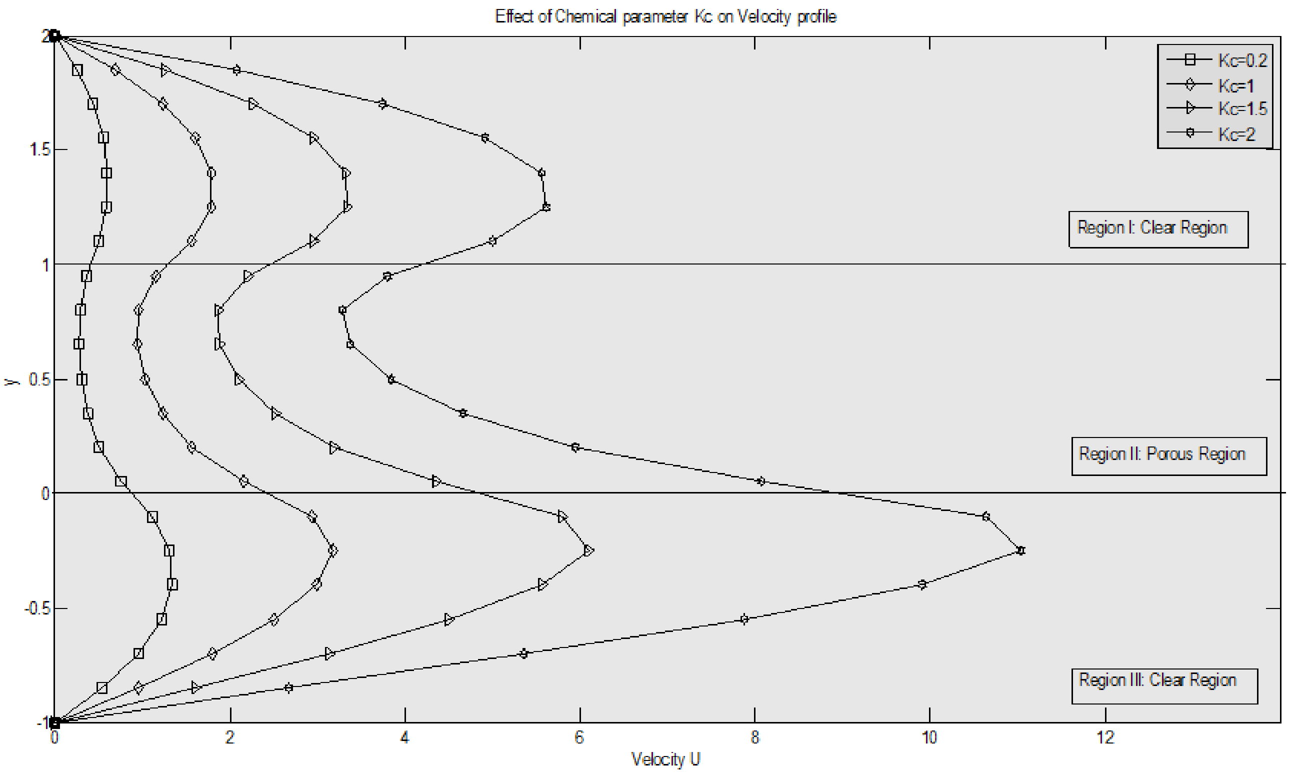

Figure 5. *Velocity distribution versus y for different values chemical parameter Kc when K=1 P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5,\varepsilon=0.025$ *and A=1.*

Figure 6 highlights the effect of Hartmann number on the velocity profile. It is shown that as the electromagnetic force increasingly dominates over the viscous force, the velocity decreases randomly throughout the channels as the Hartmann number increases, this decrease is as the result of the presence of magnetic field applied transverse to the flow which would suppress turbulence thereby coursing decrease in the velocity.

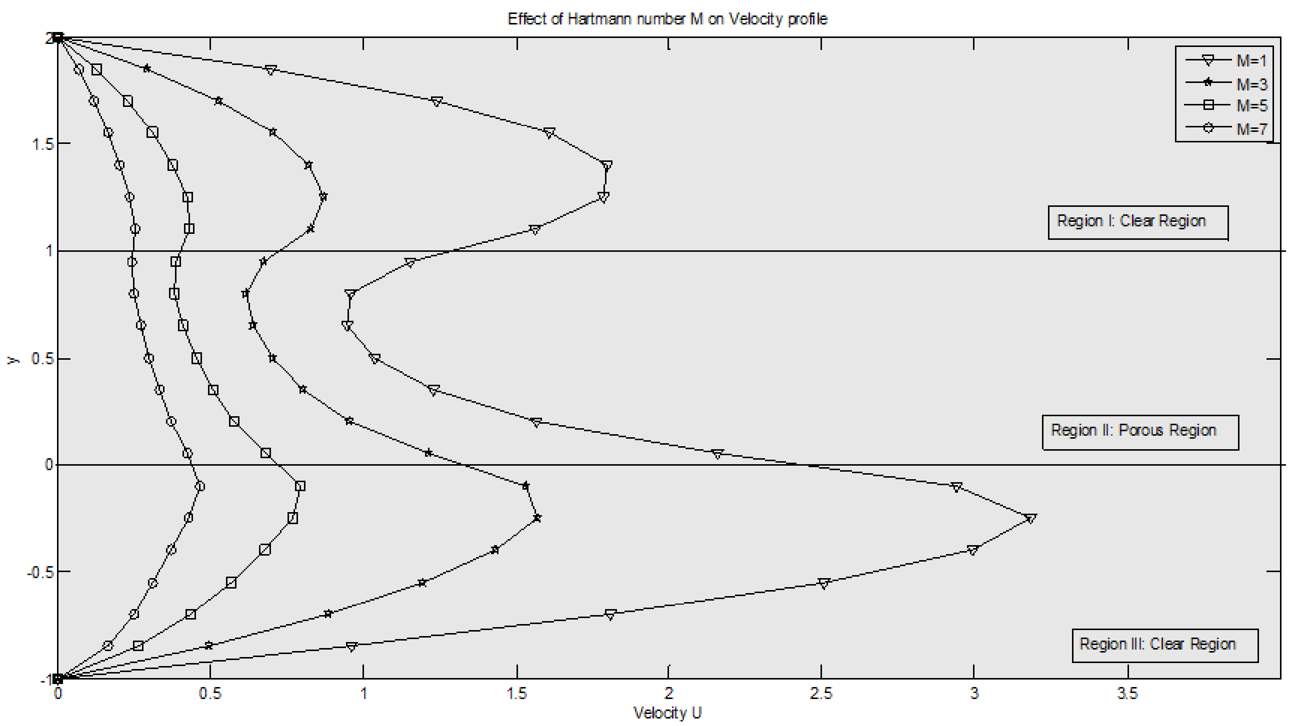

Figure 6. *Velocity distribution versus y for difference values of Magnetic parameter M when Pr=1. K=1, P=1.* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5,\varepsilon=0.025$ *and A=1.*

Figure 7 depicts the effect of Schmidt number on the magnitude of the fluid concentration. It is noticed from the figure that fluid concentration increases with an increase of Schmidt number, and attain its maximums at the interface of region III, also the magnitude of promotion is large with curve shape at the interface of region I.

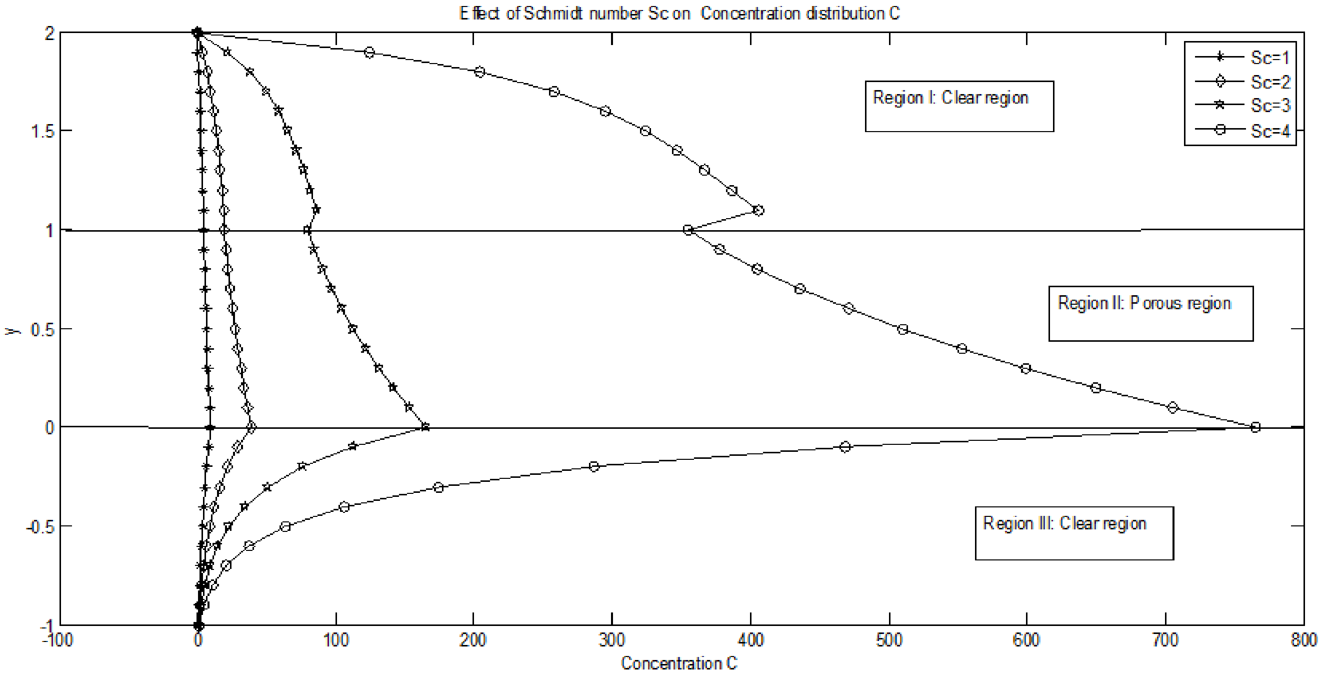

Figure 7. *Concentration distribution versus y for different values of Schmidt number Sc when K=1 P=1,* $\beta_2 = \eta_2 = \alpha_2 = m_2 = 1 = m_1 = \eta_1 . \omega = 10, \omega t = \pi/6, G_r = 5, \varepsilon = 0.025$ *and A=1.*

Figure 8 depicts the effect of chemical parameter Kc on concentration distribution. It is observe that concentration of the fluid increase with increase in the chemical parameter, it is also examine that the concentration increase rapidly at each increase of the chemical parameter.

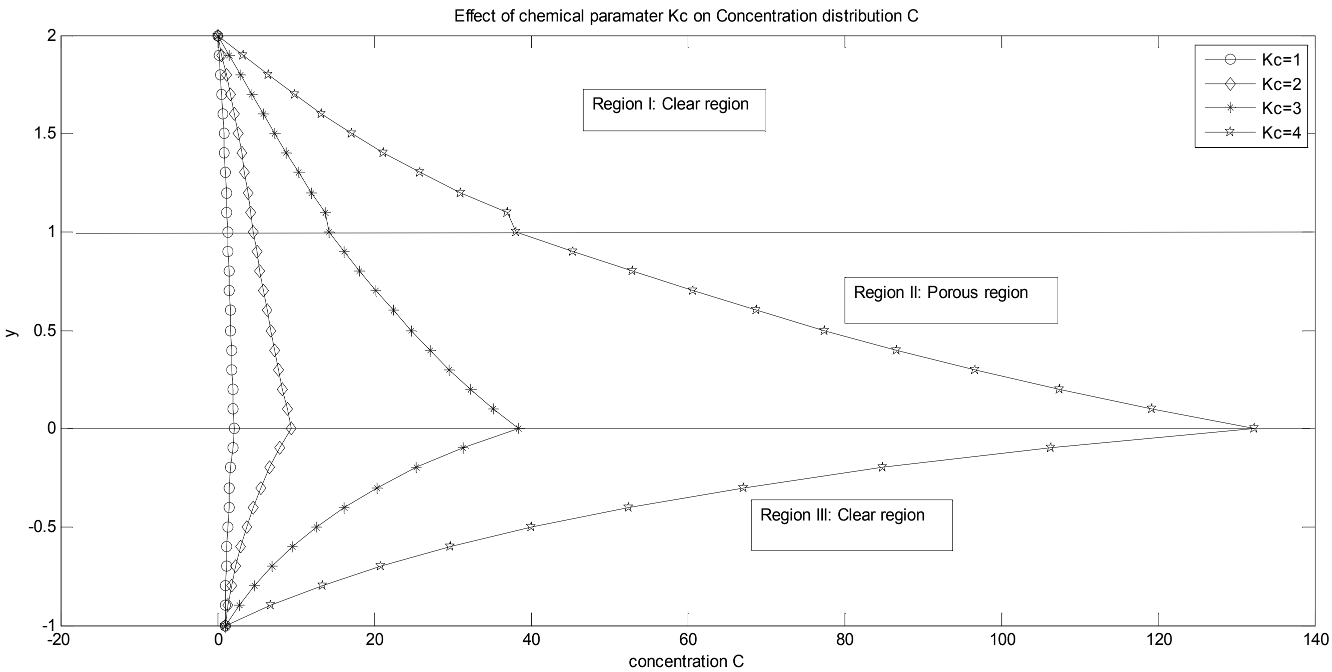

Figure 8. *Concentration distribution versus y for different values of Chemical parameter Kc when K=1 P=1,* $\beta_2 = \eta_2 = \alpha_2 = m_2 = 1 = m_1 = \eta_1 . \omega = 10, \omega t = \pi/6, G_r = 5, \varepsilon = 0.025$ *and A=1.*

Figure 9 and 10 highlight the effect of thermal conductivity ratio β_1 and β_2 on the velocity profile. It is noticed in figure 9 that an increment in the thermal conductivity ratio β_1 props up the fluid velocity in region I, while opposite behavior is observed in both region II and III. While in figure 10 was apprise that increment in the thermal conductivity ratio β_2 support the fluid velocity with uniform increase in region III and rapid decrease in region I also attain it maximum value at the interface of the region II and III. Therefor thermal conductivity has the tendency to accelerate the fluid flow.

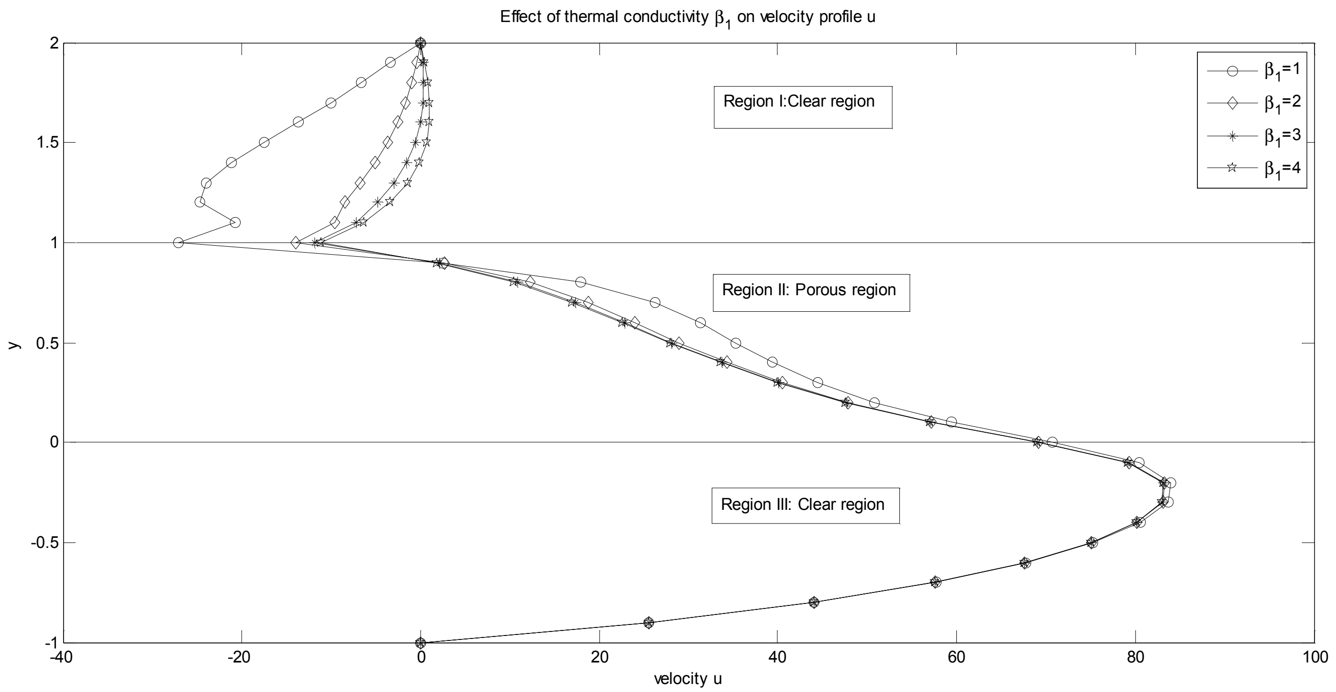

Figure 9. *Velocity distribution versus y for different values of thermal conductivity* β_1 *when K=1 P=1,* $\beta_2 = \eta_2 = \alpha_2 = m_2 = 1 = m_1 = \eta_1 . \omega = 10, \omega t = \pi/6, G_r = 5, \varepsilon = 0.025$ *and A=1.*

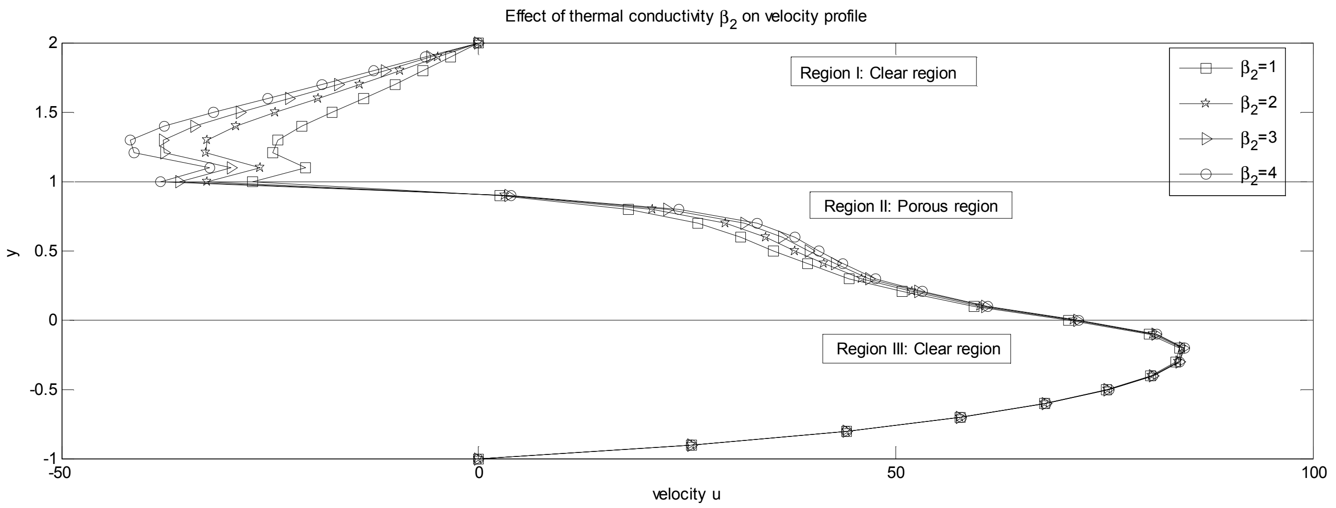

Figure 10. *Velocity distribution versus y for different values of thermal conductivity* β_2 *when* $K = 1, P = 1$, $\beta_2 = \eta_2 = \alpha_2 = m_2 = 1 = m_1 = \eta_1 . \omega = 10, \omega t = \pi/6, G_r = 5, \varepsilon = 0.025$ *and A=1.*

Figure 11 and 12 depict the effect of thermal conductivity ratio β_1 and β_2 on the magnitude of the fluid temperature. It is observed that in figure 11 that the fluid temperature increase in region I as the thermal conductivity ratio generate heat while the opponent behavior was noticed closed to the interface of region II and III. While in figure 12 shows that increase in thermal conductivity β_2 prop's up the magnitude of the fluid temperature throughout the regions.

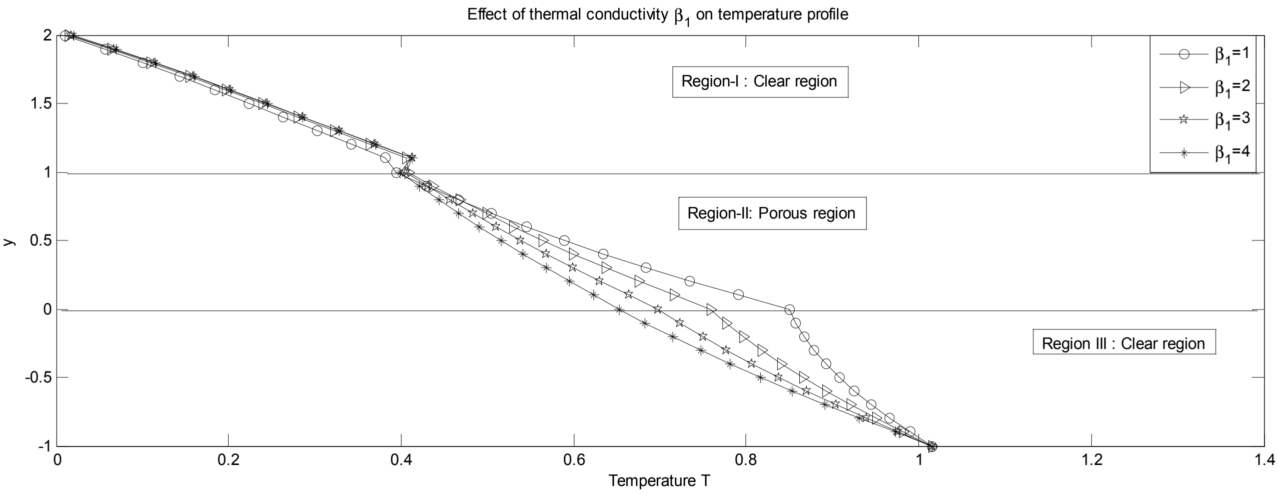

Figure 11. *Temperature distribution versus y for different values of thermal conductivity* β_1 *when K=1, P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5, \varepsilon=0.025$ *and A=1.*

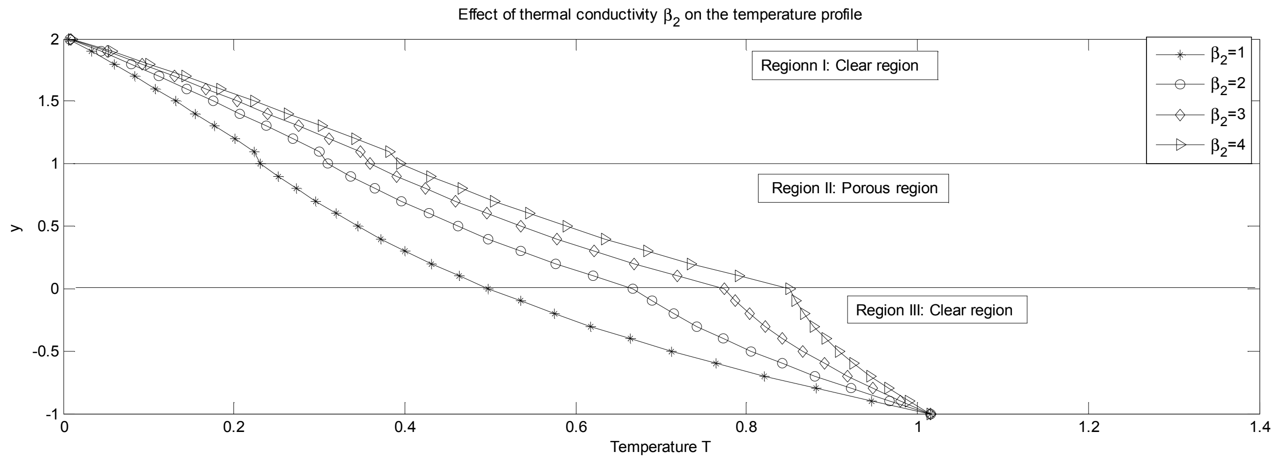

Figure 12. *Temperature distribution versus y for different values of thermal conductivity* β_2 *when K=1, P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5, \varepsilon=0.025$ *and A=1.*

Figure 13 present the effect of radiation parameter on the magnitude of temperature profile. It is observed that an increase in the radiation parameter lead to uniform decrease throughout the channel.

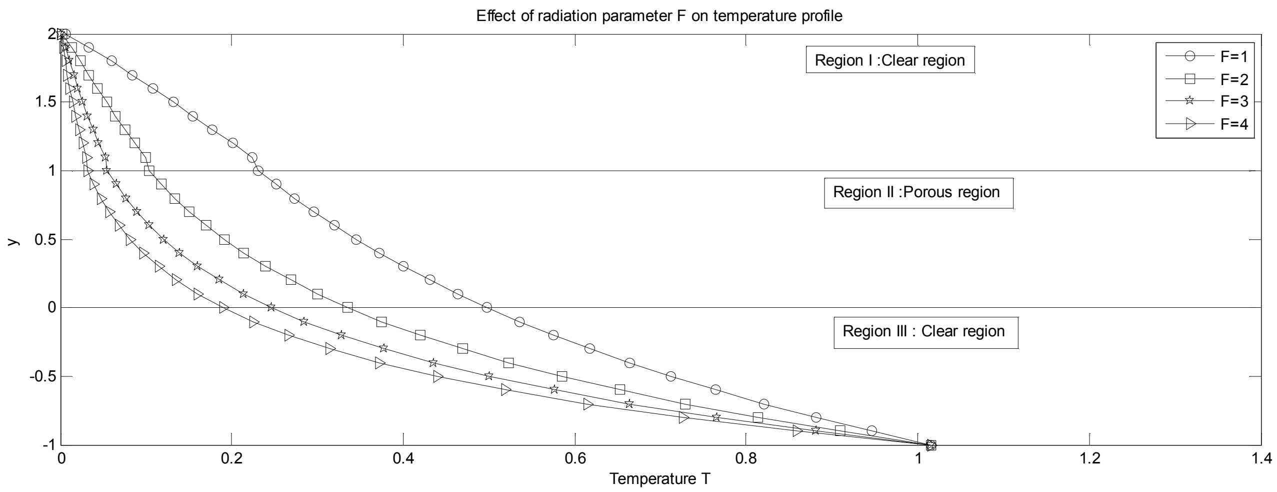

Figure 13. *Temperature distribution versus y for different values of Radiation parameter F when K=1, P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5, \varepsilon=0.025$ *and A=1.*

Figure 14 depict the effect of Prandtl number Pr on the magnitude of the fluid temperature. It is noticed that as the Prandtl number is increasing the temperature too is increasing in all the regions.

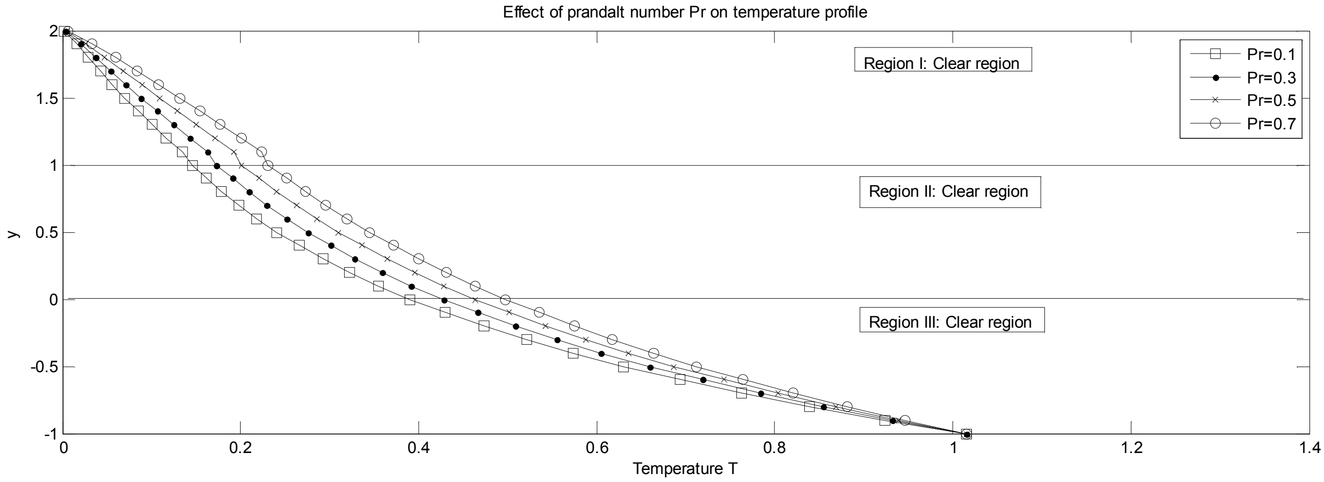

Figure 14. *Temperature distribution versus y for different values of Prandlt number Pr when K=1 P=1,* $\beta_2=\eta_2=\alpha_2=m_2=1=m_1=\eta_1.\omega=10,\omega t=\pi/6, G_r=5,\varepsilon=0.025$ *and A=1.*

Table 1: Shows that the coefficient of skin-friction at the upper plate and lower plate in case of unsteady flow is less than that of steady flow. Further in case of unsteady flow, the coefficient of skin-friction at the upper plate increase with an increase of the chemical parameter, magnetic parameter and thermal conductivity ratioβ_1. While it decrease due to the increase of the Prandtl number, viscosity ratio α_1 thermal conductivity ratioβ_2 Grashof number Gr. The coefficient of the skin-friction at the lower plate decreases with increase of viscosity ratio magnetic parameter and modified Grashof number Gc. While it increases with the increase of Schmidt number Sc, Prandtl number Pr, thermal conductivity ratioβ_1 and chemical parameter.

Table 1. *Show the values of coefficient of skin-friction at the upper plate and lower plate for various values of physical parameters where K=F=A=* $\gamma_1=\gamma_2=1$, *ω=10 and ωt=30.*

E	Pr	Gr	Gc	Kc	M	Sc	β_1	β_2	α_1	α_2	Cf (u)	Cf (l)
0	1	5	5	1	1	0.78	1	1	1	1	-24.96	54.23
0.025	1	5	5	1	1	0.78	1	1	1	1	-1.81	1.27
0.025	3	5	5	1	1	0.78	1	1	1	1	-3.06	3.42
0.025	1	10	5	1	1	0.78	1	1	1	1	-3.59	1.99
0.025	1	5	10	1	1	0.78	1	1	1	1	-1.83	2.17
0.025	1	5	5	3	1	0.78	1	1	1	1	-1.49	2.62
0.025	1	5	5	1	3	0.78	1	1	1	1	1.91	1.12
0.025	1	5	5	1	1	0.87	1	1	1	1	-1.83	2.69
0.025	1	5	5	1	1	0.78	3	1	1	1	8.10	53.18
0.025	1	5	5	1	1	0.78	1	3	1	1	-3.60	1.97
0.025	1	5	5	1	1	0.78	1	1	3	1	-8.10	-1.14
0.025	1	5	5	1	1	0.78	1	1	1	3	-1.81	1.32

Table 2: depict that the Nusselt number at the upper plate and the lower plate in case of unsteady flow is greater than that of the mean flow. Nusselt number at the upper plate increases due to increase in Schimdt number Sc, chemical parameter Kc, and Prandtl number Pr. While the Nusselt number decrease with an increase in thermal conductivity ratio β_2. Whereas at the lower plate, the Nusselt number decreases with increase in Schmidt number Sc and thermal conductivity ratio β_1. While it increases with increase in prandtl number and thermal conductivity ratio β_2.

Table 2. *Show the values of Nusselt number at the upper plate and lower plate for various values of physical parameters where K=F=A=*$\gamma_1=\gamma_2=1$*, ω=10 and ωt=30.*

E	Pr	Gr	Gc	Kc	M	Sc	β_1	β_2	α_1	α_2	Nu (u)	Nu (l)
0	1	5	5	1	1	0.78	1	1	1	1	0.07	-1.30
0.025	1	5	5	1	1	0.78	1	1	1	1	-0.08	-2.19
0.025	3	5	5	1	1	0.78	1	1	1	1	-0.03	-1.19
0.025	1	10	5	1	1	0.78	1	1	1	1	-0.08	-2.19
0.025	1	5	10	1	1	0.78	1	1	1	1	-0.08	-2.19

E	Pr	Gr	Gc	Kc	M	Sc	β_1	β_2	α_1	α_2	Nu (u)	Nu (l)
0.025	1	5	5	3	1	0.78	1	1	1	1	-0.06	-2.19
0.025	1	5	5	1	3	0.78	1	1	1	1	-0.08	-2.19
0.025	1	5	5	1	1	0.87	1	1	1	1	-0.02	-4.63
0.025	1	5	5	1	1	0.78	3	1	1	1	-0.09	-2.22
0.025	1	5	5	1	1	0.78	1	3	1	1	-0.15	-1.15
0.025	1	5	5	1	1	0.78	1	1	3	1	-0.08	-2.19
0.025	1	5	5	1	1	0.78	1	1	1	3	-0.08	-2.19

Table 3: Shows that the Sherwood number at the upper and lower plate in case of unsteady flow is higher than that of the mean flow. Furthermore in case of unsteady flow, the Sherwood number at the upper plate decreases with an increase in chemical parameter and Schmidt number. While it increases with increase in Prandtl Pr number and Grashof number Gr.

Table 3. *Show the values of Sherwood number at the upper plate and lower plate for various values of physical parameters where K=F=A=γ_1=γ_2=1, ω=10 and ωt=30.*

E	Pr	Gr	Gc	Kc	M	Sc	β_1	β_2	α_1	α_2	Sh (u)	Sh (l)
0	1	5	5	1	1	0.78	1	1	1	1	-3.82	3.05
0.025	1	5	5	1	1	0.78	1	1	1	1	-3.75	3.01
0.025	3	5	5	1	1	0.78	1	1	1	1	-3.75	4.02
0.025	1	10	5	1	1	0.78	1	1	1	1	-3.75	3.75
0.025	1	5	10	1	1	0.78	1	1	1	1	-3.75	3.02
0.025	1	5	5	3	1	0.78	1	1	1	1	-32.9	3.11
0.025	1	5	5	1	3	0.78	1	1	1	1	-3.75	3.34
0.025	1	5	5	1	1	0.87	1	1	1	1	-4.61	3.02
0.025	1	5	5	1	1	0.78	3	1	1	1	-3.75	3.02
0.025	1	5	5	1	1	0.78	1	3	1	1	-3.75	3.02
0.025	1	5	5	1	1	0.78	1	1	3	1	-3.75	3.02
0.025	1	5	5	1	1	0.78	1	1	1	3	-3.75	3.02

6. Conclusion

The unsteady MHD free convective three phase flow through porous medium sandwiched between viscous fluids with heat and mass transfer has been studied. After separating the harmonic and non -harmonic terms, the governing equations of flows in non-dimensional form are converted into ordinary linear differential equations with constant coefficients, which are solved under perturbation method with boundary and interface conditions.

It can be concluded that;

- The fluid velocity decreases with an increase in Prandtl number, Radiation parameter, Hartmann number and Schmidt number.
- Some of these governing parameters had little effect on the velocity profile while others had significant effect on this velocity profile.
- The same was seen on the temperature profile and concentration profile.
- Finally the governing parameters had effects on the flow and this study does aid in the practical usage of such flow or when confronted with such a flow.

List of Symbols

U, V Velocity components
T Time
P Pressure
B_0 Coefficient of electromagnetic field
F Thermal Radiation parameter
θ Dimensionless temperature
K Permeability of porous medium
ϑ Kinematic viscosity
μ Fluid viscosity
A Real positive constant
g Acceleration due to gravity
Cp Specific heat at constant pressure
Tw1 Fluid temperature at upper wall
Tw2 Fluid temperature at lower wall
Cw1 Fluid concentration at upper wall
Cw2 Fluid concentration at lower wall
Gr Grashoff number
Re Reynolds number
M^2 Hartmann number
Pr Prandtl number
Sc Schmidt number
α Ratio of viscosity
β Ratio of thermal conductivity
γ Ratio of thermal diffusivity
ω Frequency parameter
ε Coefficient of periodic parameter
ωt Periodic frequency parameter
Subscripts 1, 2, 3: Region I, II and III

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