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Wavelet Filtering in Shock Stochastic Systems with High Availability

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Abstract: For filtering problems in StSHA under nonGaussian ShD methodological and algorithmically WL support is developed. 3 types of filters are considered: KBF (WLKBF), LPF (WLLPF) and SOLF (WLSOLF). These filters have the following advantages: on-line regime, high accuracy and possibility of algorithmically description of complex ShD. Wavelet filter modifications are based on Galerkin method and Haar wavelet expansions. WLF unlike KBF, LPF and SOLF do not need to integrate system of ordinary differential Eqs. These filters must solve system of linear algebraic Eqs with constant coefficients. KBF (WLKBF) and SOLF (WLSOLF) are recommended for StSHA with additive ShD whereas LPF (WLLPF) are recommended for StSHA with parametric and additive ShD. Basic applications are: on-line identification and calibration of nonstationary processes in StSHA of ShD. Methods are illustrated by example of 3 dimensional differential linear information control system at complex ShD. Basic algorithms and error analysis for KBF (WLKBF) and LPF (WLLPF) are presented and 15 Figure; illustrate filters peculiarities for small and fin damping. These filters allow to estimate the accumulation effects for systematic and random errors. Results may be generalized for filtration, extrapolation with interpolation problems in StSHA and multiple ShD.

Keywords: Kalman-Bucy WLF, Linear Pugachev WLF, Shock Disturbances, Stochastic Systems with High Availability (StSHA), Suboptimal Linear WLF, Wavelet Filtering (WLF)

1. Introduction

In series [1–6] methodological support for on-line express analysis of stochastic systems with high availability (StSHA) functioning at shock disturbances (ShD) was presented. Special attention was paid to wavelet methods and software tools. Wavelet modifications of Kalan-Bucy filters (WLKBF) for nonstationary linear StSHA at complex ShD were given and illustrated. Wavelet modifications of linear mean square (m.s.) conditionally optimal (Pugachev) filter (WLLPF) for StSHA with parametric ShD are presented and illustrated. Comparative computer results were described. Instrumental accuracy of WLKBF and WLLPF was considered.

Let generalize [1] for KBF, LPF and suboptimal linearized filters (SOLF) in case of non Gaussian ShD. Section 2 is dedicated to KBF and WLKBF. LPF and WLLPF are described in Section 3. In Section 4 SOLF based on linearization by known exact shock distributions are considered. Basic Propositions 1-5 are illustrated by 3 dimensional information control system at deterministic and stochastic ShD.

2. Kalman-Bucy Filters at Shock Disturbances

Kalman-Bucy Filter (KBF) for linear nonstationary StSHA is widely used for on-line analysis and synthesis problems. KBF is based on the following proposition [7–11].

Proposition 1. Let nonstationary differential StSHA being described by the following Eqs:

$$\dot{X}_t = a_{0t}^{sh} + a_1^{sh} X_t + V_1^{sh}, \tag{1}$$

$$Z_t = \dot{Y}_t = b_1 X_t + V_2. \tag{2}$$

Here $\dot{X}_t, Y_t$ are states and observation vectors, $V_1^{sh} = V_1$ and V_2 are independent white noises (in strict sense) and nonGaussian in general case with intensity matrices $v_1^{sh} = v_1$ and v_2. Then at nonsingular observation noise ($|\det v_2| \neq 0$) KBF equations are as follows:

$$\dot{\hat{X}}_t = a_{0t}^{sh} + a_1^{sh} \hat{X}_t + \beta_t (Z_t - b_1 \hat{X}_t), \quad \hat{X}(t_0) = \hat{X}_0, \tag{3}$$

$$\beta_t = R_t b_1^T v_2^{-1}, \tag{4}$$

$$\dot{R}_t = a_1 R_t + R_t a_1^T + v_{1t}^{sh} - \beta_t v_2 \beta_t^T, \quad R(t_0) = R_0, \tag{5}$$

where $\hat{X}_t$ being mean square error estimate of X_t; R_t being error covariance matrix; β_t being matrix amplifier.

Remark. 1. Calculation of R_t and β_t does not need current observation and may be calculated a priori.

For getting Eqs for WLKBF let us exchange variable according to the following Eqs:

$$\bar{t} = \frac{t - t_0}{T - t_0}, \quad \bar{t} \in [0,1], \tag{6}$$

$$\hat{\bar{X}}(\bar{t}) = \hat{X}((T - t_0)\bar{t} + t_0), \quad \hat{\bar{X}}(0) = \hat{X}_0, \tag{7}$$

$$\begin{aligned} \bar{A}(\bar{t}) &= (T - t_0)[a_{0t}^{sh}((T - t_0)\bar{t} + t_0) + \\ &+ \beta_t((T - t_0)\bar{t} + t_0) Z_t((T - t_0)\bar{t} + t_0)], \quad \hat{\bar{X}}(0) = \hat{X}_0, \end{aligned} \tag{8}$$

$$\bar{A}(\bar{t}) = (T - t_0)[a_1 - \beta_t((T - t_0)\bar{t} + t_0) b_1]. \tag{9}$$

As a result, we have

$$\hat{\bar{X}}'(\bar{t}) = \bar{A}_{0\bar{t}} + \bar{A}_{\bar{t}} \hat{\bar{X}}(\bar{t}), \quad \hat{\bar{X}}(0) = \hat{X}_0. \tag{10}$$

Remark. 2. Further we put $t = \bar{t}$.

Following [10–14] we introduce Haar wavelets w_i and integral of w_i using formulae

$$w_1(t) = \varphi(t) = \varphi_{00}(t) = \begin{cases} 1 & at \quad t \in [0,1), \\ 0 & at \quad t \notin [0,1), \end{cases} \tag{11}$$

$$w_2(t) = \psi(t) = \psi_{00}(t) = \begin{cases} 1 & at \quad t \in [0,1/2), \\ -1 & at \quad t \in [1/2,1), \\ 0 & at \quad t \notin [0,1), \end{cases} \tag{12}$$

$$w_i(t) = \psi_{jk}(t) = \begin{cases} \sqrt{2^j} & at \quad t \in [k/l, (k+0.5)/l), \\ -\sqrt{2^j} & at \quad t \in [(k+0.5)/l, (k+1)/l), \\ 0 & at \quad t \notin [k/l, (k+1)/l). \end{cases} \tag{13}$$

Here $\varphi = \varphi(t)$ being scale function; $\psi = \psi(t)$ mother wavelet

$$\psi_{jk} = \psi_{jk}(t) = \sqrt{2^j}\psi(2^j t - k), \tag{14}$$

$k = 0,1,...,l-1$; $l = 2^j$; $j = 1,2,...,J$; $L = 2 \times 2^j$, $i = l + k + 1$; $i = 3,4,...,L$; J being maximal level of wavelet resolution.

Define integrals p_i by formulae

$$p_i(t) = \int_0^t w_i(t)dt \quad (i = 1,2,...,L), \tag{15}$$

where

$$p_1(t) = \begin{cases} t & \text{at} \quad t \in [0,1), \\ 0 & \text{at} \quad t \notin [0,1), \end{cases} \quad p_i(t) = \begin{cases} \sqrt{2^j} t - (k/l) & \text{at} \quad t \in [k/l, (k+0.5)/l), \\ -\sqrt{2^j} t - ((k+1)/l) - t & \text{at} \quad t \in [(k+0.5)/l, (k+1)/l), \\ 0 & \text{at} \quad t \notin [k/l, (k+1)/l) \end{cases} \quad (i = 2,3,...,L).$$

For every component $\hat{\bar{X}}_h$ $(h = 1,2,...,p)$ for $\hat{\bar{X}}(\bar{t})$ Eq (10) gives the following expression:

$$\hat{\bar{X}}_h'(\bar{t}) = \bar{A}_{h0} + \sum_{k=1}^{p} \bar{A}_{hk} \hat{\bar{X}}_k. \tag{16}$$

Let us expand the drerivative $\hat{\bar{X}}_h'$ into Haar series:

$$\hat{\bar{X}}_h'(\bar{t}) = \sum_{i=1}^{L} c_{hi} w_i, \tag{17}$$

$$c_{hi}(\bar{t}) = \int_0^1 \hat{\bar{X}}_h' w_i d\tau. \tag{18}$$

Then we get for Eq (16) solution

$$\hat{\bar{X}}_h(\bar{t}) = \sum_{i=1}^{L} c_{hi} p_i + \hat{X}_{0h}. \tag{19}$$

After substitution Eq (17) and Eq (19) into Eq (16) we have the following expression for c_{hi}:

$$\sum_{i=1}^{L} c_{hi} w_i = \bar{A}_{h0} + \sum_{k=1}^{p} \bar{A}_{hk} \left(\sum_{i=1}^{L} c_{hi} p_i + \hat{X}_{0h} \right) \quad (20)$$

After projecting Eq (20) on basis w_i, taking into consideration w_i orthonormality we come to $(L \times p)$ dimensional system of linear algebraic equations (SLAEq):

$$c_{hs} = \sum_{k=1}^{p} \sum_{i=1}^{L} c_{hi} (\bar{A}_{hk} p_i, w_s) + \hat{X}_{0h} \sum_{i=1}^{L} (\bar{A}_{hk}, w_s) + (\bar{A}_{h0}, w_s) \quad (s = 1, 2, ..., L). \quad (21)$$

Putting

$$\bar{A}_{hk} p_i = \sum_{j=1}^{L} g_j^{hki} w_j, \quad \bar{A}_{hk} = \sum_{j=1}^{L} q_j^{hk} w_j, \quad \bar{A}_{h0} = \sum_{j=1}^{L} \rho_j^h w_j, \quad (22)$$

$$g_j^{hki} = (\bar{A}_{hk} p_i, w_j) = \int_0^1 \bar{A}_{hk} p_i w_j d\tau;$$

$$q_j^{hk} = (\bar{A}_{hk}, w_j) = \int_0^1 \bar{A}_{hk} w_j d\tau; \quad (23)$$

$$\rho_j^h = (\bar{A}_{h0}, w_j) = \int_0^1 \bar{A}_h w_j d\tau,$$

we rewrite Eqs (22) in the final form:

$$c_{hs} = \sum_{k=1}^{p} \sum_{i=1}^{L} c_{hi} g_s^{hki} + \hat{X}_{0h} \sum_{i=1}^{L} q_s^{hk} + \rho_s^h \quad (s = 1, 2, ..., L). \quad (24)$$

Thus we have Proposition 2 [6].

Proposition 2. At conditions

(i) scalar functions $\hat{\bar{X}}_h, \hat{\bar{X}}_h, \bar{A}_{hk}, \bar{A}_{h0} \quad (h, k = 1, 2, ..., p)$ belong to space $L^2[0,1]$,

(ii) Haar functions w_i, p_i are defined in space $L^2[0,1]$.

Then Eqs for WLKBF being (19) at conditions (24).

Note that Proposition 2 is the basis of corresponding algorithm for calculating m.s. estimation of state StSHA vector described by Eqs (1), (2):

1) R_t and β_t off-line calculation according to Eqs (4), (5).

2) Off-line definition of Haar w_i, p_i in space $L^2[0,1]$ according to Eqs (11)–(14) with maximal level of wavelet resolution J.

3) Off-line calculation of p_i according to Eq (15).

4) Off-line reduction of Eq (3) to Eq (10).

5) Assign values of observation $Z(t) \quad (t \in [t_0, T])$ at points $t_j = t_0 (j-1)\Delta t$ for $j = 1, 2, ..., L$; $L = 2 \cdot 2^j$; $\Delta t = (T - t_0)/(L-1)$.

6) On-line composition and solution SLAEq (24) for coefficients c_{hs}.

7) On-line calculation of m.s. estimate $\hat{\bar{X}}_h$ for every component $\bar{X}_t$.

8) On-line transition from $\bar{t} \in [1,0]$, $t \in [1,0]$ and calculation of m.s. estimate $\hat{X}(t)$ according to formula $\hat{\bar{X}}(\bar{t}) = \hat{X}((T - t_0)\bar{t} + t_0)$.

3. Linear Pugachev Filter at Shock Disturbances

Let us consider the following StSHA described by Eqs with shock parametric noises:

$$\dot{X}_t = a_{0t} + a_{1t} X_t + a_t Y_t + \left(c_{10t} + \sum_{r=1}^{n_Y} c_{1rt} Y_r + \sum_{r=1}^{n_X} c_{1,n_y+r,t} X_r \right) V, \quad X(t_0) = X_0, \quad (25)$$

$$\dot{Y}_t = b_{0t} + b_t Y_t + b_{1t} X_t + \left(c_{20t} + \sum_{r=1}^{n_Y} c_{2rt} Y_r + \sum_{r=1}^{n_X} c_{2,n_y+r,t} X_r \right) V, \quad Y(t_0) = Y_0, \quad (26)$$

where V being vector white noise. In this case LPF is defined by the following proposition [6].

Proposition 3. Let StSHA at ShD is describe by Eqs:

$$\dot{\hat{X}}_t = a_{0t} + a_{1t} \hat{X}_t + a_t Y_t + \beta_t [Z_t - (b_{0t} + b_t Y_t + b_{1t} \hat{X}_t)], \quad \hat{X}(t_0) = \hat{X}_0. \quad (27)$$

Probabilistic moments of first and second order of $[Y_1 ... Y_{n_Y} \; X_1 ... X_{n_X}]^T$ satisfy the following Eqs:

$$\dot{m}_t = a_{0t} + a_{1t} m_t, \quad m_{t_0} = m_0, \quad (28)$$

$$\dot{K}_t = \bar{a}_t K_t + K_t \bar{a}_t^T + \bar{c}_{0t} v_t \bar{c}_{0t}^T + \sum_{r=1}^{n_Y+n_X} (\bar{c}_{0t} v_t \bar{c}_{rt}^T + \bar{c}_{rt} v_t \bar{c}_{0t}^T) m_{rt} + \sum_{r,s=1}^{n_Y+n_X} \bar{c}_{0t} v_t \bar{c}_{st}^T (m_{rt} m_{st} + K_{rs}), \quad K_{t_0} = K_0, \quad (29)$$

where

$$\overline{a}_t = \begin{bmatrix} b_t & b_{1t} \\ a_t & a_{1t} \end{bmatrix}, \quad \overline{a}_{0t} = \begin{bmatrix} b_{0t} \\ a_{0t} \end{bmatrix}, \quad \overline{c}_{rt} = \begin{bmatrix} c_{2tt} \\ c_{1rt} \end{bmatrix}, \tag{30}$$

Error covariance matrix R_t satisfy Eq

$$\begin{aligned} \dot{R}_t &= a_t R_t + R_t a_{1t}^T - \left[R_t b_{1t}^T + \left(c_{10t} + \sum_{r=1}^{n_Y+n_X} c_{1rt} m_{rt} \right) \times v_t \left(c_{20t}^T + \sum_{r=1}^{n_Y+n_X} c_{2rt}^T m_{rt} \right) + \sum_{r,s=1}^{n_Y+n_X} c_{1rt} v_t c_{2st}^T K_{rs} \right] \times \\ &\times æ_{11t}^{-1} \left[R_t b_{1t} + \left(c_{20t} + \sum_{r=1}^{n_Y+n_X} c_{2rt} m_{rt} \right) v_t \left(c_{10t}^T + \sum_{r=1}^{n_Y+n_X} c_{1rt}^T m_{rt} \right) + \sum_{r,s=1}^{n_Y+n_X} c_{2rt} v_t K_{rs} \right] + \left(c_{10t} + \sum_{r=1}^{n_Y+n_X} c_{1rt} m_{rt} \right) \\ &\times v_t \left(c_{20t}^T + \sum_{r=1}^{n_Y+n_X} c_{1rt}^T m_{rt} \right) + \sum_{r,s=1}^{n_Y+n_X} c_{1rt} v_t c_{1st}^T K_{rs}, \end{aligned} \tag{31}$$

where $æ_{11t}$ and β_t as follows

$$æ_{11t} = \left(c_{20t} + \sum_{r=1}^{n_Y+n_X} c_{2rt} m_{rt} \right) \times v_t \left(c_{20t}^T + \sum_{r=1}^{n_Y+n_X} c_{2rt}^T m_{rt} \right) + \sum_{r,s=1}^{n_Y+n_X} c_{2rt} v_t c_{2st}^T K_{rs}, \tag{32}$$

$$\beta_t = \left[R_t b_{1t}^T + \left(c_{10t} + \sum_{r=1}^{n_Y+n_X} c_{1rt} m_{rt} \right) \times v_t \left(c_{20t}^T + \sum_{r=1}^{n_Y+n_X} c_{2rt}^T m_{rt} \right) + \sum_{r,s=1}^{n_Y+n_X} c_{1rt} v_t c_{2st}^T K_{rs} \right] æ_{11t}^{-1}. \tag{33}$$

Remark. 3. LPF as KBF does not depend on current observations and the basic calculations may be performed a priori. For WLLPF we have the following Eqs [6]:

$$\hat{\overline{X}}'(\overline{t}) = \tilde{A}_{0\overline{t}} + \tilde{A}_{\overline{t}} \hat{\overline{X}}(\overline{t}), \quad \hat{\overline{X}}(\overline{t}) = \hat{X}_0(\overline{t}), \tag{34}$$

$$\hat{\overline{X}}_h' = \tilde{A}_{h0} + \sum_{k=1}^{n_X} \tilde{A}_{hk} \hat{\overline{X}}_k, \tag{35}$$

$$\hat{\overline{X}}_h = \sum_{i=1}^{L} \tilde{c}_{hi} p_i + \hat{X}_{oh}, \tag{36}$$

$$\tilde{c}_{hs} = \sum_{k=1}^{p} \sum_{i=1}^{L} \tilde{c}_{hi} \tilde{g}_s^{hki} + \hat{X}_{oh} \sum_{i=1}^{L} \tilde{q}_s^{hk} + \tilde{\rho}_s^h \ (h = 1,2,...,n_X, s = 1,2,...,L), \tag{37}$$

$$\tilde{A}_{hk} p_i = \sum_{j=1}^{L} \tilde{g}_j^{hki} w_j, \quad \tilde{A}_{hk} = \sum_{j=1}^{L} \tilde{q}_j^{hk} w_j, \quad \tilde{A}_{0h} = \sum_{j=1}^{L} \tilde{\rho}_j^h w_j, \tag{38}$$

$$\tilde{g}_j^{hki} = \int_0^1 \tilde{A}_{hk} p_i w_j d\tau, \quad \tilde{q}_j^{hk} = \int_0^1 \tilde{A}_{hk} w_j d\tau, \quad \tilde{\rho}_j^h = \int_0^1 \tilde{A}_{h0} w_j d\tau, \tag{39}$$

$$\tilde{A}_0(\overline{t}) = (T - t_0)[a_{0t}((T - t_0)\overline{t} + t_0) + \tilde{\beta}_t((T - t_0)\overline{t} + t_0) Z_t((T - t_0)\overline{t} + t_0)], \tilde{A}(\overline{t}) = (T - t_0)[a_1 - \tilde{\beta}_t((T - t_0)\overline{t} + t_0) b_1] \tag{40}$$

$$\dot{R}_{\bar{t}} = a_1 R_{\bar{t}} + R_{\bar{t}} a_1^T - \left[R_{\bar{t}} b_1^T + \left(\tilde{c}_{10} + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{m}_r \right) \times \tilde{v}_{\bar{t}} \left(\tilde{c}_{20}^T + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{2r}^T \tilde{m}_r \right) + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{v}_{\bar{t}} \tilde{c}_{2s}^T \tilde{K}_{rs} \right] \tilde{\text{æ}}_{11}^{-1} \times$$

$$\times \left[R_{\bar{t}} b_1 + \left(\tilde{c}_{20} + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{2r} \tilde{m}_r \right) \tilde{v}_{\bar{t}} \left(\tilde{c}_{10}^T + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{1r}^T \tilde{m}_r \right) + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_{2r} \tilde{v}_{\bar{t}} \tilde{K}_{rs} \right] + \left(\tilde{c}_{10} + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{m}_r \right) \quad (41)$$

$$\times \tilde{v}_{\bar{t}} \left(\tilde{c}_{20}^T + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{1r}^T \tilde{m}_r \right) + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{v}_{\bar{t}} \tilde{c}_{1s}^T \tilde{K}_{rs},$$

$$\tilde{\beta}_{\bar{t}} = \left[R_{\bar{t}} b_1^T + \left(\tilde{c}_{10} + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{m}_r \right) \tilde{v}_{\bar{t}} \left(\tilde{c}_{20}^T + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{2r}^T \tilde{m}_r \right) + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_{1r} \tilde{v}_{\bar{t}} \tilde{c}_{2s}^T \tilde{K}_{rs} \right] \tilde{\text{æ}}_{11}^{-1}, \quad (42)$$

$$\tilde{\text{æ}}_{11}^{-1} = \left(\tilde{c}_{20} + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{2r} \tilde{m}_r \right) \tilde{v}_{\bar{t}} \left(\tilde{c}_{20}^T + \sum_{r=1}^{n_Y+n_X} \tilde{c}_{2r}^T \tilde{m}_r \right) + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_{2r} \tilde{v}_{\bar{t}} \tilde{c}_{2s}^T \tilde{K}_{rs}, \quad (43)$$

$$\tilde{m}_{\bar{t}}' = a_{0\bar{t}} + \tilde{a}_1 \tilde{m}_{\bar{t}}, \quad \tilde{m}(0) = m_0, \quad (44)$$

$$\tilde{K}_{\bar{t}}' = \tilde{a} \tilde{K}_{\bar{t}} + \tilde{K}_{\bar{t}} a^T + \tilde{c}_0 \tilde{v}_{\bar{t}} \tilde{c}_0^T + \sum_{r=1}^{n_Y+n_X} (\tilde{c}_0 \tilde{v}_{\bar{t}} \tilde{c}_r^T + \tilde{c}_r \tilde{v}_{\bar{t}} \tilde{c}_0^T) \tilde{m}_r + \sum_{r,s=1}^{n_Y+n_X} \tilde{c}_r \tilde{v}_{\bar{t}} \tilde{c}_s^T (\tilde{m}_r \tilde{m}_s + \tilde{K}_{rs}), \quad K(0) = K_0, \quad (45)$$

$$\tilde{a} = \overline{a}_{\bar{t}}, \quad \tilde{a}_0 = \overline{a}_{0\bar{t}}, \quad \tilde{c}_r = \overline{c}_{r\bar{t}}. \quad (46)$$

Here we use wave for functions depending on dimensionless time $\bar{t}$.

Using Proposition 3 in case $a_t = 0$, $b_t = 0$, $b_{0t} = 0$ for Eqs (25)-(27) we get the following proposal [6].

Proposition 4. At conditions of Proposition 3 and conditions:

(iii) scalar functions $\hat{\tilde{X}}_h', \hat{\tilde{X}}_h, \tilde{A}_{hk} p_i, \tilde{A}_{h0}$ $(h,k = 1,2,...,n_X)$ belong to space $L^2[0,1]$;

(iv) Haar functions w_i, p_i belling to space $L^2[0,1]$ WLLPF is defined by Eqs (36), (37).

From Proposition 4 the corresponding algorithm follow.

1) Off-line reduction of Eqs (25), (2) to dimensionless form.
2) Off-line calculation of probability moments $\tilde{m}_{\bar{t}}$ and $\tilde{K}_{\bar{t}}$ of random vector $[\overline{Y}_1 ... \overline{Y}_{n_Y} \ \overline{X}_1 ... \overline{X}_{nX}]^T$ by integrating Eqs (44), (45).
3) Off-line calculation of error covariance matrix $R_{\bar{t}}$ by Eq (41) integrating.
4) Off-line calculation of parameter $\tilde{\text{æ}}_{11}$ and optimal amplifier $\tilde{\beta}_t$ according to formulae (42) and (43).
5) Off-line introduction in space $L^2[0,1]$ Haar wavelet defined by Eqs (11)-(14) with maximal level of wavelet resolution J.
6) Off-line calculation p_i for Haar wavelets according to formulae (15).
7) On-line assign values Z_t $(t \in [t_0, T])$ in points $t_j = t_0 + (j-1)\Delta t$ for $j = 1,2,...,L$; $L = 2 \bullet 2^j$; $t \in [t_0, T]$.
8) On-line composition and solution SLAEq (37) for determination of coefficients ξ_{sh}.
9) On-line computation of error m.s. estimations $\hat{\tilde{X}}_h$ for every component $\overline{X}_t$ according to formula (36).
10) On-line transition from $\bar{t} \in [1,0]$ to $t \in [t_0, T]$ and calculation $\hat{X}(t)$ to: $\hat{\tilde{X}}(\bar{t}) = \hat{X}((T - t_0)\bar{t} + t_0)$.

4. Suboptimal Linearized Filters at Shock Disturbances

If the noise V in Eqs (25)-(26) is autocorrelated and connected with white noise V^{sh} by linear Eqs of shape filter such StSHA with parametric ShD during small sock time comparable with time of StSHA inertia are reduced to bilinear StSHA of the form [7–10]:

$$\dot{X}_t = a_{0t} + a_{1t}X_t + a_t Y_t + \left(c_{10t} + \sum_{r=1}^{n_Y} c_{1rt}Y_r + \sum_{r=1}^{n_X} c_{1,n_y+r,t}X_r \right) N_t^{sh}, \quad X(t_0) = X_0,$$

$$\dot{Y}_t = b_{0t} + b_t Y_t + b_{1t}X_t + \left(c_{20t} + \sum_{r=1}^{n_Y} c_{2rt}Y_r + \sum_{r=1}^{n_X} c_{2,n_y+r,t}X_r \right) N_t^{sh}, \quad Y(t_0) = Y_0,$$

$$\dot{N}_t^{sh} = c_{0t} + c_{1t}N_t^{sh} + c_t V_t^{sh}, \quad N_t^{sh}(t_0) = N_{0t}^{sh}. \tag{47}$$

For given exact distribution (ED) and using equivalent linearization of bilinear functions $E_{ED}X_{ht}N_{0t}^{sh}$ we reduce Eqs (47) to Eqs for KBF (Section 2) and for WLKBF for the composed vector $U_t = [X_t^T Y_t^T N_t^{shT}]^T$ (Section 3). So we have the following Proposition.

Proposition 5. Let bilinear StSHA (47) at known exact distribution of ShD may be reduced to linear autocorrelated stochastic system with additive noises. Then KBF and WLKBF are defined by Eqs of Proposition 1 and 2 for the corresponding state vector U_t.

5. Example

At first let us consider KBF for the following system:

$$\dot{X}_1 = X_2, \dot{X}_2 = -\omega_c^2 X_1 - 2\varepsilon\omega_c X_2 + S + n_t^{sh} + V_1^{sh}, \dot{X}_3 = X_1,$$

$$Z_1 = \dot{Y}_1 \equiv 0, \quad Z_2 = \dot{Y}_2 = X_2 + V_2, \quad Z_3 = \dot{Y} \equiv 0, \tag{48}$$

where $X = [X_1 X_2 X_3]^T$ and $Y = [0\ Y_2\ 0]^T$ are state and observation vectors, V_1^{sh} and V_2 are scalar independent Gaussian noises with intensities v_1^{sh} and v_2 $(v_2 \neq 0)$. Note that in case of Eq (48) KBF and LPF coincide. So using Poposition 1 we have the following vector Eqs for KBF:

$$\dot{\hat{X}}_t = a_{0t}^{sh} + \beta_t Z_t + (a_1^{sh} - \beta_t b_1)\hat{X}_t, \quad \hat{X}_t(t_0) = \hat{X}_0. \tag{49}$$

$$a_{0t}^{sh} = \begin{bmatrix} 0 \\ S + n_t^{sh} \\ 0 \end{bmatrix}, \quad a_1^{sh} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_c^2 & -2\varepsilon\omega_c & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{50}$$

$$\beta_t = R_t b_1^T v_2^{-1} = \begin{bmatrix} 0 & -(1/v_2)R_{12} & 0 \\ & -(1/v_2)R_{22} & 0 \\ 0 & -(1/v_2)R_{23} & 0 \end{bmatrix}, \tag{51}$$

$$\dot{R}_t = a_1 R_t + a_1^T R_t + v_1^{sh} - \beta_t v_2 \beta_t^T, \quad R(t_0) = R_0. \tag{52}$$

Eq (52) may be written in scalar form

$$\begin{cases} \dot{R}_{11} = 2R_{12} - (1/v_2)R_{12}^2, \quad R_{11}(t_0) = R_{110}; \\ \dot{R}_{12} = R_{22} - \omega_c^2 R_{11} - 2\varepsilon\omega_c R_{12} - (1/v_2)R_{12}R_{22}, \\ R_{12}(t_0) = R_{120}; \\ \dot{R}_{13} = R_{11} + R_{23} - (1/v_2)R_{12}R_{23}, \quad R_{13}(t_0) = R_{130}; \\ \dot{R}_{22} = -2(\omega_c^2 R_{12} + 2\varepsilon\omega_c R_{22}) + v_1^{УД} - (1/v_2)R_{22}^2, \\ R_{22}(t_0) = R_{220}; \\ \dot{R}_{23} = R_{12} - \omega_c^2 R_{13} - 2\varepsilon\omega_c R_{23} - (1/v_2)R_{22}R_{23}, \\ R_{23}(t_0) = R_{230}; \\ \dot{R}_{33} = 2R_{13} - (1/v_2)R_{23}^2, \quad R_{33}(t_0) = R_{330}. \end{cases} \tag{53}$$

So we get the final Eqs for KBF:

$$\begin{aligned} \dot{\hat{X}}_1 &= [1 + (1/v_2)R_{12}]\hat{X}_2 - (1/v_2)R_{12}Z_2, \\ \dot{\hat{X}}_3 &= \hat{X}_1 + (1/v_2)R_{23}\hat{X}_2 - (1/v_2)R_{23}Z_2, \\ \dot{\hat{X}}_2 &= S + n_t^{sh} - \omega_c^2 \hat{X}_1 - [2\varepsilon\omega_c - (1/v_2)]\times \\ &\times R_{22}\hat{X}_2 - (1/v_2)R_{22}Z_2. \end{aligned} \tag{54}$$

Secondly we use Proposition 2 and notations

$$\dot{\hat{X}}_t = A_0 + AX_t, \quad \hat{X}(t_0) = \hat{X}_0, \tag{55}$$

$$A_0 = [A_{h0}]_h = \begin{bmatrix} -(1/v_2)R_{12}Z_2 \\ -(1/v_2)R_{22}Z_2 + S + n_t^{sh} \\ -(1/v_2)R_{23}Z_2 \end{bmatrix}, \tag{56}$$

$$A = [A_{hk}]_{h,k} = \begin{bmatrix} 0 & 1 + (1/v_2)R_{12} & 0 \\ -\omega_c^2 & -[2\varepsilon\omega_c - (1/v_2)]R_{22} & 0 \\ 1 & 0 & (1/v_2)R_{23} \end{bmatrix}, \tag{57}$$

$$\dot{\hat{X}}_t' = \bar{A}_0 + \bar{A}\hat{\bar{X}}, \quad \hat{\bar{X}}(0) = \hat{X}_0, \tag{58}$$

$$\bar{A}_0 = [\bar{A}_{h0}]_h = (T - t_0) \begin{bmatrix} -(1/v_2)R_{12}Z_2 \\ -(1/v_2)R_{22}Z_2 + S + n_t^{sh} \\ -(1/v_2)R_{23}Z_2 \end{bmatrix}, \tag{59}$$

$$\overline{A} = [\overline{A}_{hk}]_{h,k} = (T - t_0)\begin{bmatrix} 0 & 1+(1/\nu_2)R_{12} & 0 \\ -\omega_c^2 & -[2\varepsilon\omega_c - (1/\nu_2)]R_{22} & 0 \\ 1 & 0 & (1/\nu_2)R_{23} \end{bmatrix}. \quad (60)$$

So we get the final WLKBF Eqs:

$$\begin{cases} \overline{R}'_{11} = (T-t_0)[2R_{12} - (1/\nu_2)R_{12}^2], \quad \overline{R}_{11}(t_0) = R_{110}; \\ \overline{R}'_{12} = (T-t_0)[R_{22} - \omega_c^2 R_{11} - 2\varepsilon\omega_c R_{12} - \\ -(1/\nu_2)R_{12}R_{22}], \quad \overline{R}_{12}(t_0) = R_{120}; \\ \overline{R}'_{13} = (T-t_0)[R_{11} + R_{23} - (1/\nu_2)R_{12}R_{23}], \\ \overline{R}_{13}(t_0) = R_{130}; \\ \overline{R}'_{22} = (T-t_0)[v_t^{sh} - 2(\omega_c^2 R_{12} + 2\varepsilon\omega_c R_{22}) - \\ -(1/\nu_2)R_{22}^2], \quad \overline{R}_{22}(t_0) = R_{220}; \\ \overline{R}'_{23} = (T-t_0)[R_{12} - \omega_c^2 R_{13} - 2\varepsilon\omega_c R_{23} - \\ -(1/\nu_2)R_{22}R_{23}], \quad \overline{R}_{23}(t_0) = R_{230}; \\ \overline{R}'_{33} = (T-t_0)[2R_{13} - (1/\nu_2)R_{23}^2], \\ \overline{R}_{33}(t_0) = R_{330}. \end{cases} \quad (61)$$

$$\hat{\overline{X}}_h = \sum_{s=1}^{L} c_{hs} p_s \overline{X}_{0h}, \quad (62)$$

$$c_{hs} = \sum_{k=1}^{3}\sum_{i=1}^{L} c_{hi} g_s^{hki} + \hat{X}_{oh} \sum_{i=1}^{L} q_s^{hk} + \rho_s^h \quad (63)$$
$$(k = 1,2,...,p, \quad s = 1,2,...,L),$$

$$g_j^{hki} = \int_0^1 \overline{A}_{hk}(\tau) p_i(\tau) w_s(\tau) d\tau,$$
$$q_j^{hk} = \int_0^1 \overline{A}_{hk}(\tau) w_s(\tau) d\tau, \quad \rho_j^h = \int_0^1 \overline{A}_{h0}(\tau) w_s(\tau) d\tau. \quad (64)$$

Computer experiments "Figures 1-15" were realized for the following values of parameters: $\omega_c = 1$; $S = 1$; $\nu_2 = 1$, $j = 5$ and $\varepsilon = 0,7$; $\varepsilon = 0,1$. The following variants of ShD were considered.

1) Deterministic ShD with n_t^{sh} and stochastic ShD with v_t^{sh} at $t_{sh} = 2,1875$, $\Delta n = 3$, $\Delta t = 3$.

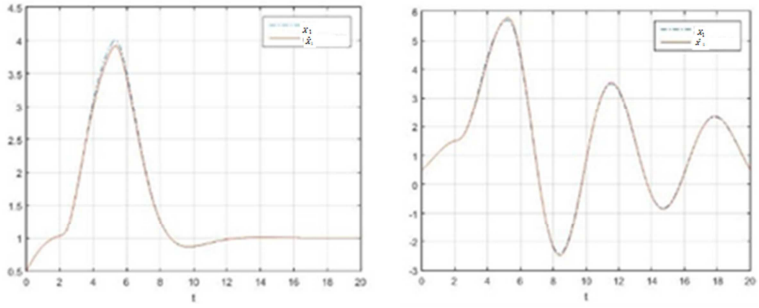
$$n_t^{sh} = \begin{cases} \Delta n, & \text{at} \quad t \in [t_{sh}; t_{sh} + \Delta t], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + \Delta t], \end{cases}$$
$$v_t^{sh} = \begin{cases} \Delta n, & \text{at} \quad t \in [t_{sh}; t_{sh} + \Delta t], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + \Delta t]. \end{cases}$$

2) Deterministic ShD n_t^{sh} and stochastic ShD v_t^{sh} at $t_{sh} = 2,1875$

$$n_t^{sh} = \begin{cases} 10, & \text{at} \quad t \in [t_{sh}; t_{sh} + 5], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + 5], \end{cases}$$
$$v_t^{sh} = \begin{cases} 6, & \text{at} \quad t \in [t_{sh}; t_{sh} + 7], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + 7]. \end{cases}$$

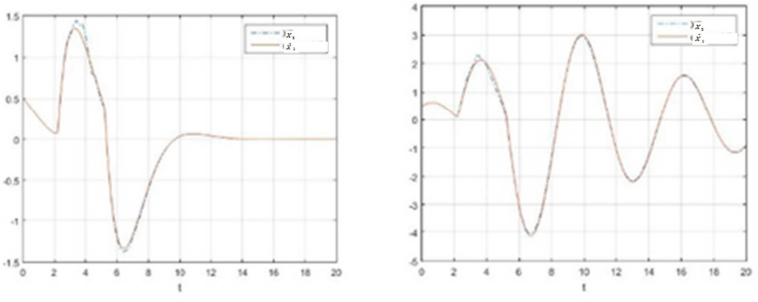
3) Deterministic ShD $n_t^{УД}$ and stochastic ShD $v_t^{УД}$ at $t_{УД} = 0,135$,

$$n_t^{sh} = \begin{cases} 10, & \text{at} \quad t \in [t_{sh}; t_{sh} + 1], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + 1], \end{cases}$$
$$v_t^{sh} = \begin{cases} 6, & \text{at} \quad t \in [t_{sh}; t_{sh} + 1], \\ 0, & \text{at} \quad t \notin [t_{sh}; t_{sh} + 1]. \end{cases}$$



(a) (b)

Figure 1. *Plot realization of* X_1 *and its estimation (variant 1) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*



(a) (b)

Figure 2. *Plot realization of* X_2 *and its estimation (variant 1) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

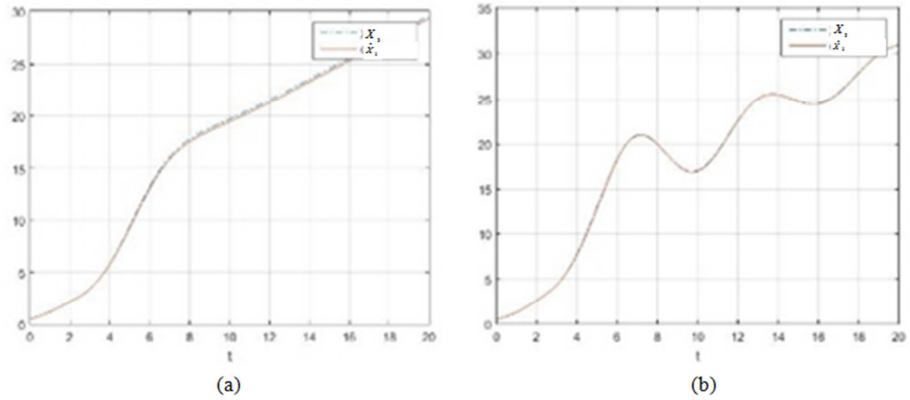


Figure 3. *Plot realization of* X_3 *and its estimation (variant 1) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

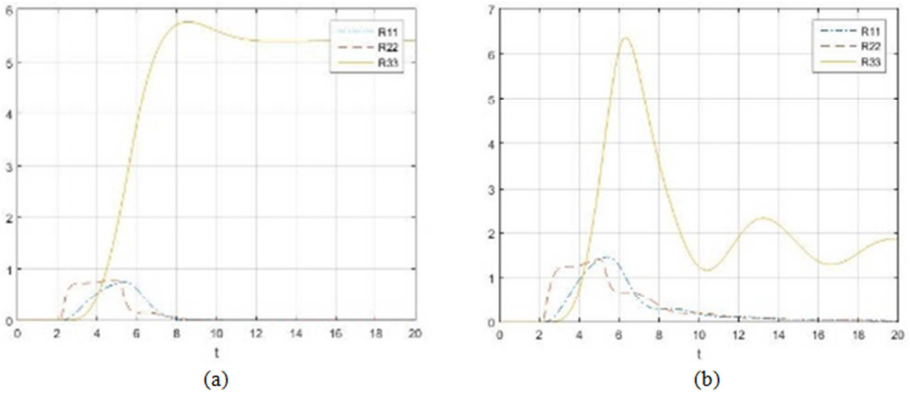


Figure 4. *Filter error variance plots* R_{11}, R_{22}, R_{33} *(variant 1) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

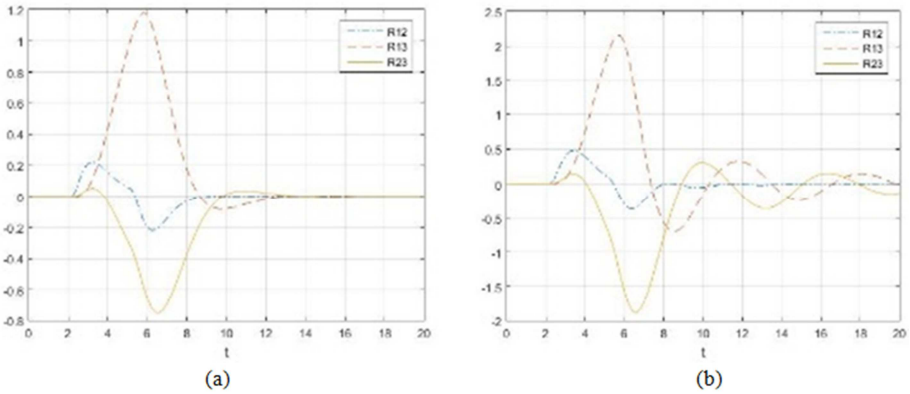


Figure 5. *Filter error variance plots* R_{12}, R_{13}, R_{23} *(variant 1) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

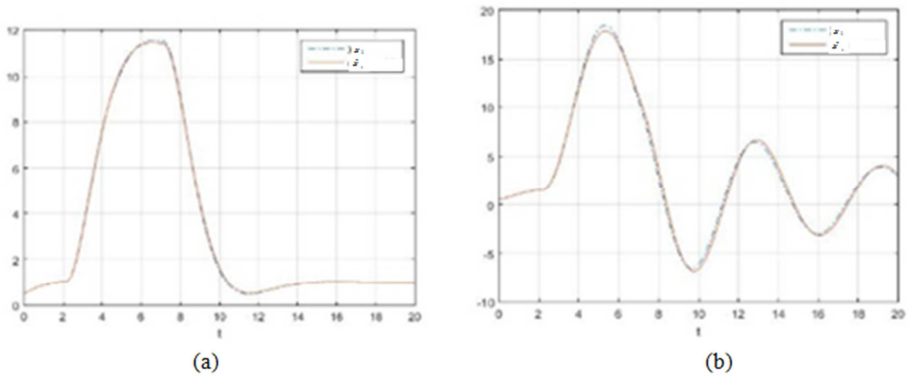


Figure 6. *Plot realization of* X_1 *and its estimation (variant 2) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

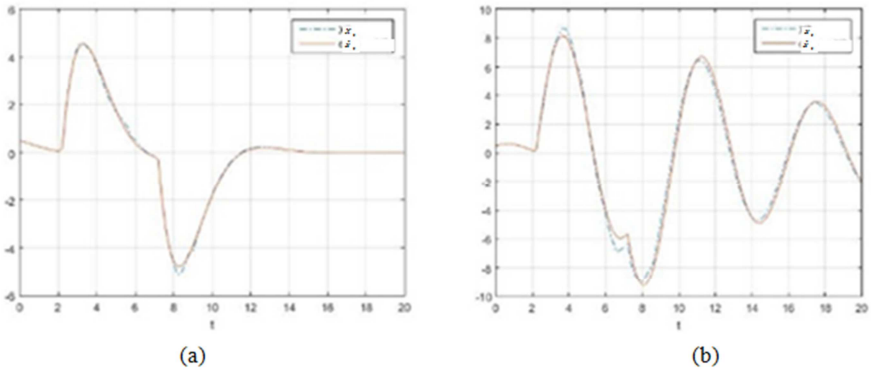


Figure 7. *Plot realization of* X_2 *and its estimation (variant 2) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

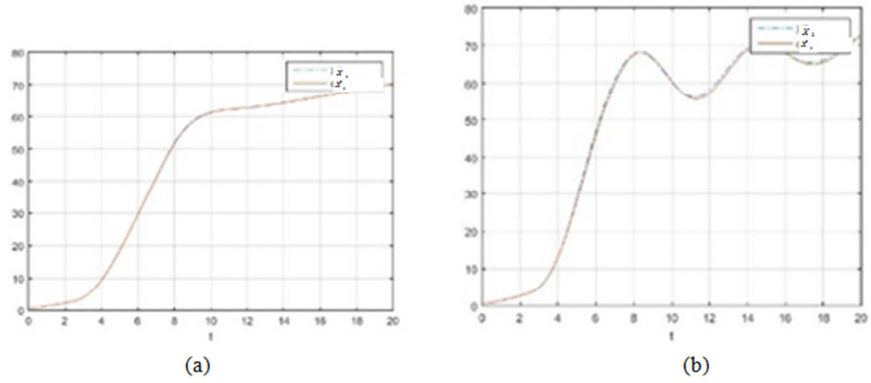


Figure 8. *Plot realization of* X_3 *and its estimation (variant 2) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

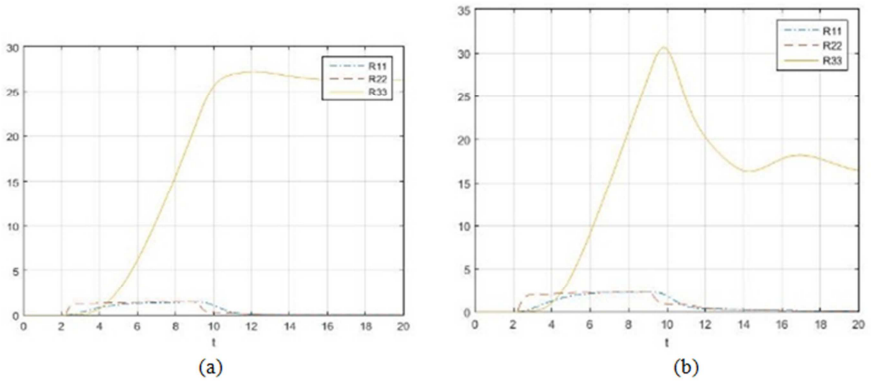


Figure 9. *Filter error variance plots* R_{11}, R_{22}, R_{33} *(variant 2) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

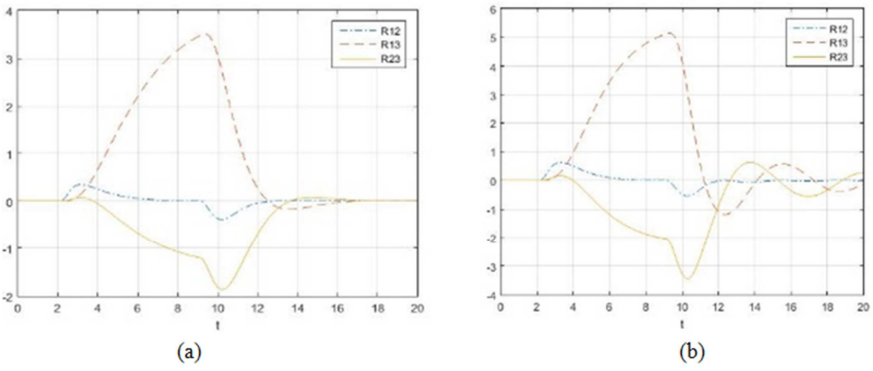


Figure 10. *Filter error variance plots* R_{12}, R_{13}, R_{23} *(variant 2) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

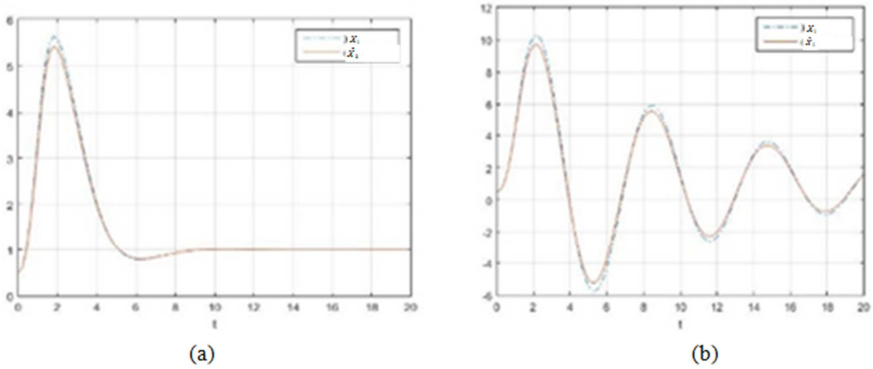


Figure 11. *Plot realization of* X_1 *and its estimation (variant 3) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

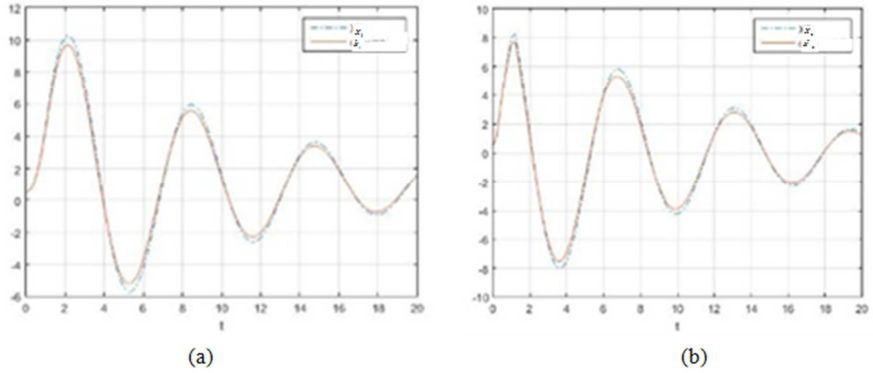


Figure 12. *Plot realization of* X_2 *and its estimation (variant 3) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

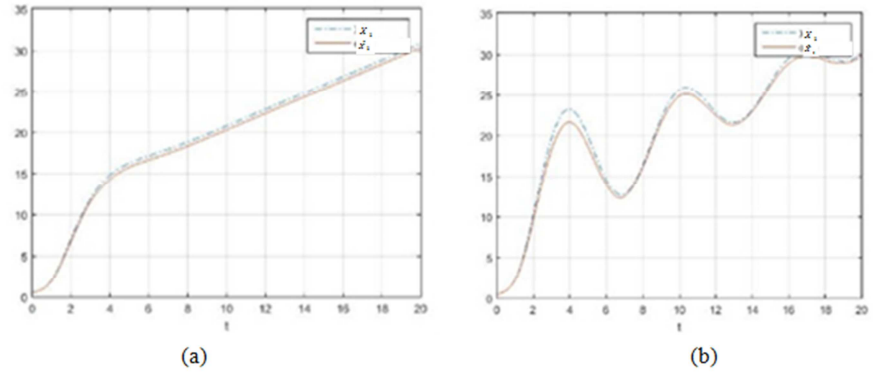


Figure 13. *Plot realization of* X_3 *and its estimation (variant 3) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

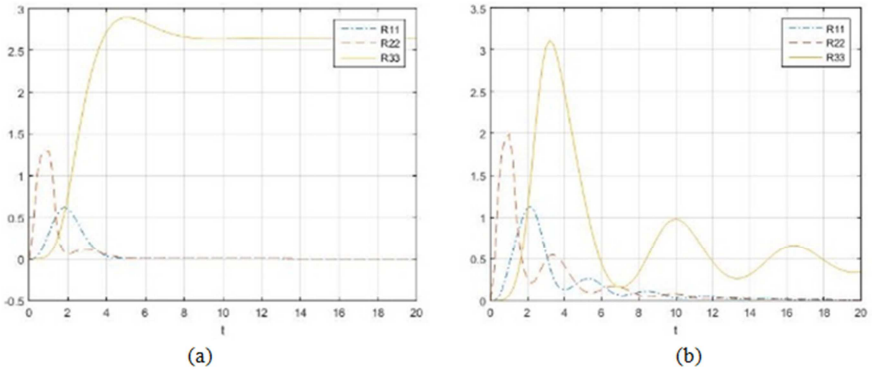


Figure 14. *Filter error variance plots* R_{11}, R_{22}, R_{33} *(variant 3) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

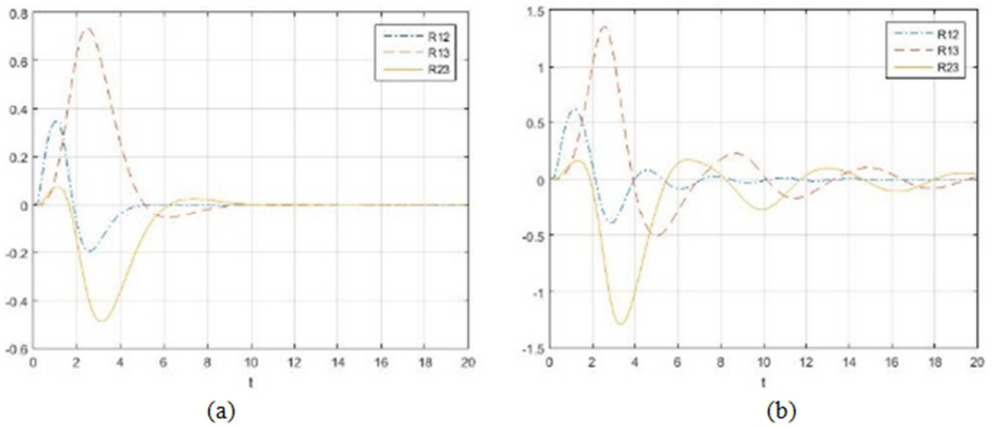


Figure 15. *Filter error variance plots* R_{12}, R_{13}, R_{23} *(variant 3) for* $\varepsilon = 0,7$ *(a) and* $\varepsilon = 0,1$ *(b).*

6. Conclusion

For filtering problems in StSHA under nonGaussian ShD methodological and algorithmically WL support is developed. 3 types of filters are considered: KBF (WLKBF), LPF (WLLPF) and SOLF (WLSOLF). These filters have the following advantages: on-line regime, high accuracy and possibility of algorithmically description of complex ShD. Wavelet filter modifications are based Galerkin method and Haar wavelet expansions. WLF unlike KBF, LPF and SOLF do not need to integrate system of ordinary differential Eqs. These filters must solve system of linear algebraic Eqs with constant coefficients.

KBF (WLKBF) and SOLF (WLSOLF) are recommended for StSHA with additive ShD whereas LPF (WLLPF) are recommended for StSHA with parametric and additive ShD.

Basic applications are on-line identification and calibration of nonstationary processes in StSHA at ShD.

Methods are illustrated by example of 3 dimensional differential linear information control system at complex ShD. Basic algorithms and error analysis for KBF (WLKBF) and LPF (WLLPF) are presented on 15 figures illustrate filters popularities for small and big damping. These filters allow to estimate the accumulation effects for systematic and random errors.

Results may be generalized for filtration, extrapolation and interpolation problems in StSHA with multiple ShD.

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