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Bayesian Multiple Linear Regression Model for GDP in Nepal

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Abstract: Gross Domestic Product (GDP) known as the pulse of economy for any country depends on multiple factors like export-import, inflation rate and unemployment rate etc. Statistical assessment of GDP demands fresh concepts to explain GDP through its covariates in order to improve and strengthen estimation process. Descriptive analysis for the considered data set from world bank for GDP and covariates is presented through Heatmaps. Identification and relevance of possible set of covariates is done by Ordinary Least Square (OLS) regression followed by Step-wise regression. We propose an alternative statistical algorithm implemented as Bayesian Inference through Integrated Nested Laplace Approximation (INLA) which bridges the gap of accuracy in estimates as opposed to frequentist OLS regression for explaining GDP of country Nepal. Effect of changing prior parameters is assessed through Deviance Information Criterion (DIC). Different scenarios for prior distribution for regression parameters were analyzed to identify most suitable choice of parameter for normal distribution. The comparison of Bayesian and frequentist modelling results is done using several criteria such as Mean Square Error (MSE), Mean Absolute Deviation (MAD) and Mean Absolute Percent Error (MAPE). Bayesian estimation approach is a more efficient method for parametric estimation as compared to OLS classical method. GDP of Nepal was found to have strongest relationship with unemployment rate of Nepal as evident from both classical and Bayesian model.

Keywords: Bayesian, GDP, Nepal, INLA, Regression, DIC

1. Introduction

One of the most used statistical techniques for modelling time-series and cross-section data is linear regression analysis. Simple linear regression analysis is used when just one independent variable is utilised, whereas multiple linear regression analysis is used when numerous independent variables are employed. Dummy regression analysis is utilised when the scale of the data used for the independent variables is the same as the scale of the categorical data. A statistical method known as multiple linear regression is used to characterise the concurrent relationships of many factors with a single continuous outcome. According to Eberly, L. E. important procedures in using this strategy include estimation and inference, variable selection during model construction, and model fit evaluation [4]. The parameters of a regression model can be estimated using a variety of techniques. The Frequentist or Classic method, which employs OLS (Ordinary Least Square) or MLE (Maximum Likelihood Estimation), is the approach that researchers utilise the most frequently (Maximum Likelihood Estimation). The least squares method, often known as the OLS method, minimises the amount of errors in the regression equation. As a result, the regression model's parameters are determined by minimising the equation's error function. When performing MLE, the probability density function (or PDF) of the data is maximised. There are therefore traditional presumptions from the findings of regression modelling that must be met when utilising both methodologies. Error independence, identity, and normal distribution are among requirements. There are other approaches, such as the Bayesian approach, that can be employed in addition to these two to estimate the regression model parameters. In terms of how parameters are viewed, frequentist and Bayesian techniques differ. Permai, S. D asserted that frequentist point of view views parameters as a single value, but the Bayesian approach views them as random variables [13].

By appointing a prior distribution, the Bayesian inferential mechanism creates posterior means rather than means that represent both the data and the likely value of the parameter of interest. By designating priors as normal distributions to the regression parameters, the Bayesian process enables us to more effectively capture the relationship between the outcome and its covariates in the setting of linear regression. Asserted by Robert, CP. early in the 1990s, simulation-based inference was introduced to the statistical world via the concept of Markov chain Monte Carlo (MCMC) [15], which marked a significant advance in Bayesian inference. By building a Markov chain with the target posterior as the stationary distribution, MCMC offered a general recipe for producing samples from posteriors. This made it theoretically possible to extract and compute any desired information. Popular user-friendly MCMC programmes like WinBUGS explained by Spiegelhalter, DJ. JAGS explained by Plummer, M**.** and the newest endeavour by Gelman, A. which use Hamiltonian Monte Carlo, have all been made possible by additional significant advancements [20, 14, 7]. With the aid of these and comparable tools, Bayesian statistics has quickly gained acceptance and is now widely used in all the top academic publications across all areas of statistics. Rue, H. introduced an alternative tool to approximate Bayesian estimates which is more fast and computationally effective called as Integrated Nested Laplace Approximation (INLA) [16]. Computing high-dimensional integrals is the main step in Bayesian inference. The Laplace method or approximation, explained in Laplace, P. dates back to Pierre-Simon Laplace, is a traditional approximate method [9]. This straightforward concept computes the integral analytically while approximating the integrand with a second-order Taylor expansion centred on the mode. We obtain the strategy of INLA to do approximate Bayesian inference for latent Gaussian models by creating a layered version of this traditional concept and combining it with contemporary numerical approaches for sparse matrices (LGMs).

It has been customary to model and estimate GDP using economic factors, but in this study, we go beyond that practise by incorporating the essence of a brand-new descriptive visualisation in the form of a heatmap and modelling using Bayesian masking of INLA in the context of Nepal. Some recent studies include the work of Stani, S. who found that GDP growth rate, exports, and imports have positive associations whereas unemployment and inflation rates and foreign direct investment (FDI) had negative ones [22]. In their 2017 study, Urrutia, J. D. found that while interest rates and the unemployment rate are adversely correlated with GDP, total trade, capital formation, and exchange rates are all positively correlated [25]. In her research, Akter, M. found that whereas imports have a negative impact on GDP, exports have a favourable impact [1]. According to research by Javed, H. Z. export is statistically significant and positively correlated with GDP [8]. Eric, O. used multiple regression modelling to examine the effect of export and trade liberalisation on GDP and found that these factors had a positive effect on GDP [5]. Siregar, D. I. used a multiple regression model to investigate how export and GDP affected currency rate and found that both factors had a favourable impact [19]. Padder, A. H. in their study on Indian economy from 1990-2020, reported a non-significant relation between GDP and unemployment growth along with absence of Granger causality between the variables [12]. Thaba, T. K. also reported similar results in their studies of South Africa's economy [23]. Sharma, S explained that unemployment, with its economic and social effects is one of the most irresistible constraints faced by Nepalese policy makers [17]. The study by Singh, D. A. revealed the patterns of trade between India and Nepal after 1990 [18]. They discovered that Nepal has become more dependent on its neighbour for trade as a result of the growing share of exports and imports from India in its overall commerce. On the basis of the empirical findings, that report makes no recommendations for ways to lessen trade dependence on India.

The present paper is organized in five sections. Section 2 describes the materials and methods utilized in the analysis. It explains the concept of Heatmap and Bayesian regression modelling through INLA. Section 3 provides the snapshot of complete steps involved in the analysis through flowchart and the results of the complete analysis. Section 4 and section 5 are devoted the discussion and conclusion of the study.

2. Materials and Methods

2.1. Data

The World Bank Data (2022) [26], the Central Bureau of Statistics (CBS) [3], and the Ministry of Culture, Tourism, and Civil Aviation of the Government of Nepal are the primary sources of secondary data for Nepal used in this study [10]. Data is available from 1990 through 2020. In this paper, we suggest examining the effects of covariates such as the labour force participation rate, the inflation rate, the unemployment rate, exports and imports, population growth rate, the number of tourists arriving (in thousands), gross capital formation (in billions of dollars), and cereal production (in millions of metric tonnes) on the gross domestic product (in billions of dollars). Data for individual indicators is presented in supplementary file table S1.

2.2. Notations

Independent Variables	
X_1	Labour force rate
X_2	Inflation rate
X_3	Unemployment rate
X_4	Export (in billion US$)
X_5	Import (in billion US$)
X_6	Population growth rate
X_7	Number of tourist arrival (in hundred thousand)
X_8	Gross capital formation (in billion US$)
X_9	Cereal production (in million metric tons)
Dependent Variable	
y	Gross domestic product (in billion US$)

2.3. Material and Methods

2.3.1. Heatmap

By summarising massive data sets, the data visualisation approach known as the heatmap enhances descriptive analysis. Specifically, when there is a wealth of data available for many time periods, locations, or qualities. Each data point received after standardisation is given a relative score, and each score is then further assigned to a colour based on our preference. Heatmap scales down each feature under study by standardisation over the span (span can be in years or set of places depending on the data). To show the relative intensity of percentages of all variables over the course of the 31 years from 1990 to 2020, we created a heatmap for the full set of nine covariates and GDP. We decided to display low scores or low percentages in blue, and comparatively high scores or percentages in orange/red.

2.3.2. Multiple Linear Regression

A multiple linear regression model is an extension of the simple linear regression model for data with multiple predictor variables and one outcome $(x_{i1}, x_{i2}, x_{i3} \ldots . x_{im-1}, y_i)$ for $i = 1,2,3 \ldots m$ units of observation each of which consists of a measurement of set of predictors $(x_1, x_2, x_3 \ldots . x_{m-1})$ and a measurement of response variable y.

The multiple linear regression model is expressed as,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_m x_{im} + \varepsilon_i \qquad (1)$$

$$\mathrm{y} = \beta \mathrm{X} + \varepsilon \qquad (2)$$

where, y = vector of dependent variable

X = matrix of independent variables

β = vector of regression model parameters

ε = vector of errors

In this linear model (1), the relationship between y and $(x_1, x_2, x_3 \ldots . x_m)$ is modeled using the linear predictor function $\mu = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m$ and a disturbance term or error variable ϵ. The unknown model parameters $(\beta_0, \beta_1, \beta_2, \ldots \ldots, \beta_m x_m)$ are estimated from the data. The model becomes a simple linear regression when m=1. Linearity here is with respected to the unknown parameters. Sometimes the predictor function contains a nonlinear function of a predictor. For example, a polynomial regression of degree 3 is expressed as, $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \varepsilon_i$. This model remains linear since it is linear in the parameter vector. y is the n- dimensional column vector, x is a n(m+1) matrix, β is a (m+1)-dimensional column vector of parameters, and ϵ is a n- dimensional column vector of error term.

$$\begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{pmatrix} = \begin{bmatrix} 1 & x_{11} & \cdot & \cdot & \cdot & x_{1m} \\ \cdot & & & & & \\ \cdot & x_{21} & \cdot & \cdot & \cdot & x_{2m} \\ \cdot & \cdot & & & & \cdot \\ 1 & x_{n1} & \cdot & \cdot & \cdot & x_{nm} \end{bmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_m \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{pmatrix}$$

Its matrix nature is written in the form of

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{1i} & \ldots & \sum_{i=1}^{n} x_{mi} \\ \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{1i}^2 & \ldots & \sum_{i=1}^{n} x_{1i} x_{mi} \\ \vdots & \vdots & & \ldots \vdots \\ \sum_{i=1}^{n} x_{mi} & \sum_{i=1}^{n} x_{1i} x_{mi} & \ldots & \sum_{i=1}^{n} x_{mi}^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{1i} y_i \\ \vdots \\ \sum_{i=1}^{n} x_{mi} y_i \end{bmatrix}$$

We begin our distribution by assuming that the errors are independent and normally distributed with mean zero and constant variance, i.e., the error term ε in (2) is assumed to be distributed as $\mathrm{N}(0, \sigma^2 I)$ with an unknown variance parameter σ^2. In frequentist statistics, the parameters can be estimated using the maximum likelihood estimation (MLE) or the least squares method. The likelihood function for the model (2) is.

$$\mathrm{L}(\beta, \sigma^2 / \mathrm{X}, \mathrm{y}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-1}{2\sigma^2} \{y_i - (\beta_0 + \beta_1 x_{i1} + +\beta_{m-1} x_{i,m-1})\}^2\right] \qquad (3)$$

In matrix form, this is

$$\mathrm{L}(\beta, \sigma^2 / \mathrm{X}, \mathrm{y}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-1}{2\sigma^2} (\mathrm{y} - \mathrm{X}\beta)^{\mathrm{T}} (\mathrm{y} - \mathrm{X}\beta)\right] \qquad (4)$$

The log-likelihood function is,

$$\mathrm{L}(\beta, \sigma^2 / \mathrm{X}, \mathrm{y}) = -\frac{n}{2} \log(2\pi) - \frac{n}{2} \log(\sigma^2) - \frac{1}{2\sigma^2} (\mathrm{y} - \mathrm{X}\beta)^{\mathrm{T}} (\mathrm{y} - \mathrm{X}\beta) \qquad (5)$$

Simplifying the partial derivative of the log likelihood function with respect to β and σ^2 are respectively gives,

$$\frac{\partial \log L}{\partial \beta} = -\frac{1}{2\sigma^2} \mathrm{X}^{\mathrm{T}} (\mathrm{y} - \mathrm{X}\beta) \qquad (6)$$

$$\frac{\partial logL}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{2\sigma^3} (\mathrm{y} - \mathrm{X}\beta)^{\mathrm{T}} (\mathrm{y} - \mathrm{X}\beta) \qquad (7)$$

Assuming $X^T X$ is of full rank and setting (6) and (7) to zero, we obtain the maximum likelihood estimators

$$\hat{\beta} = (\mathrm{X}^{\mathrm{T}} \mathrm{X})^{-1} \mathrm{X}^{\mathrm{T}} \mathrm{y} \qquad (8)$$

and

$$\hat{\sigma}^2 = \frac{1}{n} (\mathrm{y} - \mathrm{X}\hat{\beta})^{\mathrm{T}} (\mathrm{y} - \mathrm{X}\hat{\beta}) \qquad (9)$$

Where, $\hat{\beta}$ is an unbiased estimator of β. However, $\hat{\sigma}^2$ is not unbiased estimator of σ^2.

2.3.3. Stepwise Regression

A technique for fitting regression models that involves adding or eliminating predictor (explanatory) variables is known as stepwise regression. Based on some predetermined criteria, a variable is taken into consideration for addition to or deletion from the set of explanatory variables in each stage. In general, stepwise regression is a method for choosing independent variables, and it entails a sequence of stages intended to identify the independent variables that will be most helpful to include in a regression model. Nine explanatory factors that could have an impact on Nepal's GDP are used in this study. The next step, after fitting the multiple linear regression model, is to choose the best set of covariates by removing those that have the least impact on GDP. One of the common methods for choosing variables is step-wise regression, which offers the best set of variables with significance levels of 10%, 5%, 1%, and 0.1%. We used R 4.1.3's step-wise regression function to find the most likely set of covariates.

2.3.4. Bayesian Multiple Linear Using INLA

The next step was to fit the multiple linear regression model first for all the considered set of nine covariates and then with those covariates which came significant in stepwise regression by Bayesian approach. In linear regression model using ordinary least squares estimation method, error assumption is normally distributed that is $\varepsilon \sim \mathrm{N}(0, \sigma^2)$ as well as the variables $(y/X, \beta, \sigma^2)$ are also normally distributed. In Bayesian analysis, the inverse of the variance parameter plays an important role and is called precision, $\tau = 1/\sigma^2$. We shall use the precision τ in manipulating the distributions. Based on the assumption of the model, we have

$$y/\beta, \tau \sim \mathrm{N}(X\beta, \tau^{-1}I).$$

We further assume β and τ are independent. Therefore, the joint posterior distribution of the unknown parameters, thus is

$$\pi(\beta, \tau/X, y) \propto \mathrm{L}(\beta, \tau | X, y)\ p(\beta)\ p(\tau).$$

where p (β) and p (τ) are the priors for the parameters β and τ. Close form of the posterior distributions of β and τ are only available under certain restricted prior distributions. How to define the prior distribution is a significant issue in Bayesian analysis. If prior knowledge of the parameters was known, it should have been included in the prior distribution. If we don't know anything beforehand, we want to be able to guarantee that a prior distribution will only have a little impact on the inference. Since many real applications don't have parameter information, noninformative prior distributions, such p (β) $\propto$ 1, have long been interesting. The main disadvantage of a noninformative prior is that it is not invariant to parameter modification. The model is assumed to be a latent Gaussian model in INLA, hence we must give it β a Gaussian prior. For the hyperparameter τ, we often assume a diffuse prior, a probability distribution with an extremely large variance. A typical prior choice for β and τ is

$$\beta \sim \mathrm{N}_{m+1}(C_0, V_0),\ \tau \sim \mathrm{Gamma}(a_0, b_0).$$

Here the prior of β is m+1 dimensional multivariate normal with known C_0 and V_0. We often assume that V_0 is diagonal, which is equivalent to specifying separate univariate normal priors on the regression coefficients. The precision τ follows a dispersed gamma distribution with a known shape parameter a_0 and a known rate parameter b_0 (that is, we have mean a_0/b_0 and variance a_0/b_0^2). In linear regression, the gamma prior is conditionally conjugate for τ since the conditional posterior distribution, $p(\tau/X, y)$, is also in that class.

Even though the posterior under these priors is intractable, it is simple to create a blocked Gibbs sampling technique explained by Gelman, A. that is appropriate for MCMC implementation [6]. The procedure specifically alternates between the following two conditional distributions:

$$\pi(\beta/X, y, \tau) \propto \mathrm{L}(\beta, \tau | X, y)\ p(\beta),$$

$$\pi(\tau/X, y, \beta) \propto \mathrm{L}(\beta, \tau | X, y)\ p(\tau).$$

Instead of MCMC simulations, the INLA approach provides approximations to the posterior marginals of the parameters which are both very accurate and extremely fast to compute. The marginal posterior π (τ| X, y) is approximated using

$$\pi(\tau/X, y) \propto \frac{\pi(\beta, \tau, X, y)}{\tilde{\pi}(\beta | \tau, X, y)}\bigg| \beta = \beta * (\tau) \tag{10}$$

which is the Gaussian approximation to the full conditional distribution of β evaluated in the mode $\beta^*(\tau)$ for a given τ. Expression (10) is equivalent to the Laplace approximation (Tierney, L.) of marginal posterior distribution and it is exact when $\pi(\beta/\tau, X, y)$ is Gaussian [24]. Posterior marginals for the model parameters, $\tilde{\pi}(\beta_j/\tau, X, y)$, j = 0, 1, ….,m, are then approximated via numerical integration as:

$$\tilde{\pi}(\beta_j/X, y) = \tilde{\pi}(\beta_j/\tau, X, y)\tilde{\pi}(\tau | X, y)\, d\tau$$

$$\approx \textstyle\sum_k \tilde{\pi}(\beta j | \tau_k, X, y)\ \tilde{\pi}(\tau_k | X, y)\Delta_{k,}$$

where the sum is over values of made up of τ with area weights Δ_k. The INLA procedure's approximate posterior marginals can then be utilised to calculate summary statistics that are of importance, such as posterior means, variances, and quantiles. By-products of the primary computations, such as the deviance information criteria explained by Spiegelhalter, D. J. and marginal likelihoods, etc., are computed by INLA and are valuable for comparing and validating models [21].

2.3.5. Comparison Criteria for OLS and Bayes Estimates

(i). Mean Squared Error (MSE)

Mean Squared Error (MSE), a measurement of statistical model error. Between the observed and predicted values, it evaluates the average squared difference. The MSE is equal to 0 when a model is error-free. Its value increases when model error does as well. The mean squared error (sometimes referred to as the mean squared deviation, or MAD) is represented as follows:

$$MSE = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n} \tag{11}$$

(ii). Mean Absolute Deviation (MAD)

The sum of simple errors is used as a metric for evaluating performance technique for estimators. Mean Absolute Deviation (MAD) averages the purported inaccuracy to determine the prediction's accuracy (the absolute value of each error). When assessing prediction errors using the same unit as the original series, MAD is helpful. It is expressed as the mean absolute difference between the values that were observed and those that were fitted.

$$\mathrm{MAD} = \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{n} \quad (12)$$

(iii). Mean Absolute Percentage Error (MAPE)

The absolute error for each period is divided by the values that are clearly visible for the period to produce the mean absolute percentage error (MAPE). Averaging those predetermined percentages is the next step. This method is helpful when determining whether a prediction is accurate depends on the size or size of a prediction variable. MAPE is a measure of the amount of prediction error in relation to the actual value and is expressed as:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{|y_i - \hat{y}_i|}{|y_i|}\right] \times 100 \quad (13)$$

2.3.6. Paired T-test

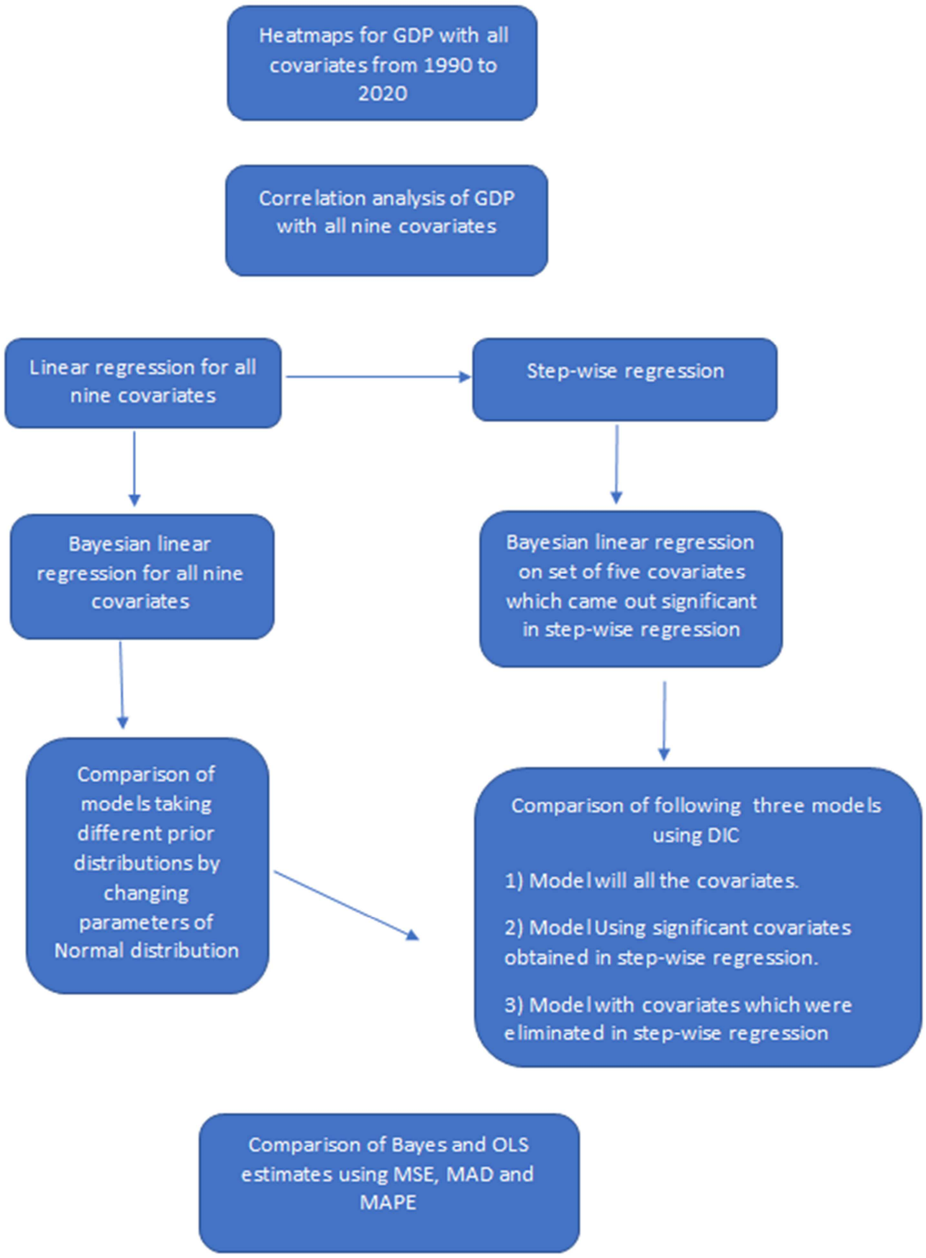
To assess and determine whether there was a significant difference between the observed and fitted values, a paired t-test was utilised. When two samples are correlated, the paired sample t-test is a statistical method used to compare the two population means. The before and after investigations employ the paired sample t-test. The formula is written as follows to calculate the parameter:

$$t = \frac{\bar{y}}{\sqrt{\frac{s^2}{n}}} \quad (14)$$

Where $\bar{y}$ is the mean difference between two samples (observed values and fitted values obtained by, s^2 is the sample variance, n is the sample size and t is a paired sample t test with n-1 degree of freedom.

3. Results

Roadmap of our analysis presented as flowchart.



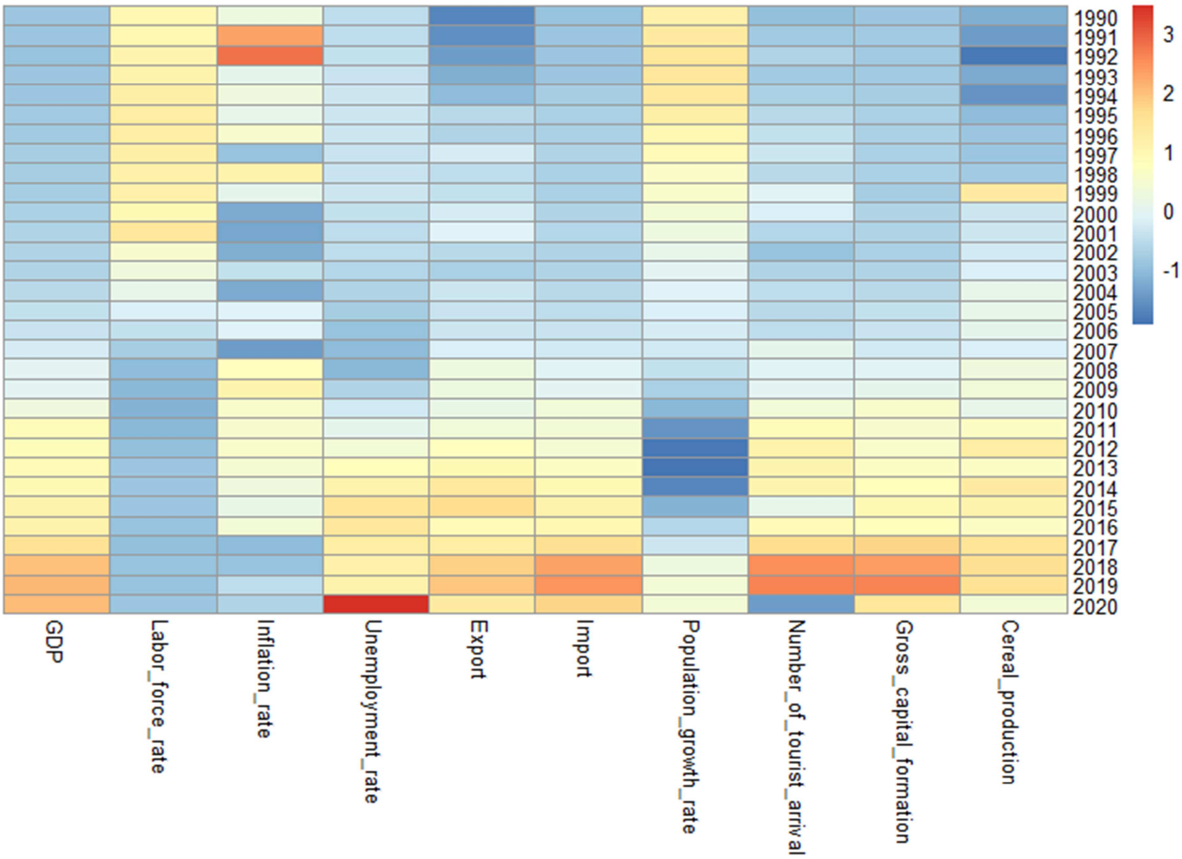


Figure 1. *Heatmap for complete data set.*

Figure 1 represents a heatmap to visualise complete data set representing comparative view of percentages of all nine covariates and GDP from 1990 to 2020. We can observe from figure 1 that except labour force rate which shows a decreasing trend, all the other covariates and GDP shows increasing trend since 1990 to 2020 in Nepal. Inflation rate and population growth rate also show a fluctuating trend as we have some red blocks initially and then blue ones for both of them. Number of tourist arrival was increasing since 1990 at a constant rate but we can see that a sudden blue block appears for the year 2020 which can be attributed to COVID-19.

Table 1 displays the estimated correlation coefficient and the corresponding p-value for each explanatory variable. It can be seen that unemployment rate, export, import, number of tourist arrival, gross capital formation, and cereal production, have a strong positive linear relationship with gross domestic production with their coefficients lying between the range of 0.70 to 0.99. On the other hand, labour force rate has a strong negative, inflation rate has a weak negative and population growth rate has a moderate negative linear relationship with gross domestic product. It can be seen that except the unemployment rate all other variables have a p- value less than α, where $\alpha = 0.05$. Therefore, the covariates labour force rate, population growth rate, export, import, number of tourist arrival, gross capital formation, and cereal production have a significant association with GDP.

Table 1. *Correlation test between GDP rate and covariates.*

Covariates	**Correlation coefficient**	**p-value**
labor force rate (X_1)	-0.8058299	0.00000
inflation rate (X_2)	-0.1687651	0.36410
unemployment rate (X_3)	0.8368669	0.00000
export (X_4)	0.9315724	0.00000
Import (X_5)	0.9874333	0.00000
population growth rate (X_6)	-0.5507095	0.00000
number of tourist arrival (X_7)	0.7473330	0.00000
gross capital formation (X_8)	0.9768696	0.00000
cereal production (X_9)	0.8009717	0.00000

Figure 2 to Figure 10 provide correlation plot of GDP with each of nine covariates visualised in R.

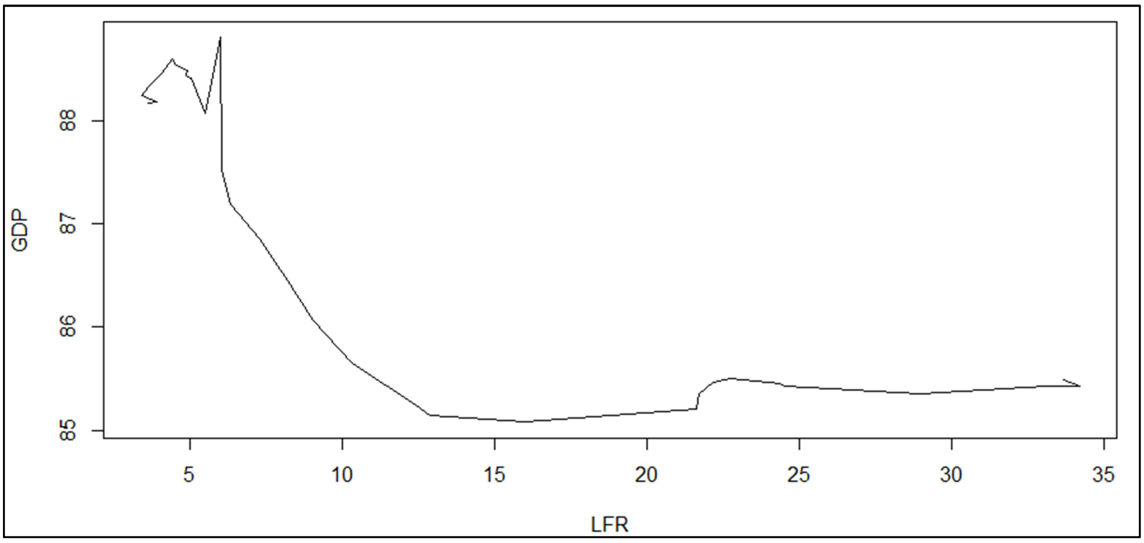


Figure 2. *Plot between GDP and Labuor Force Rate (LFR).*

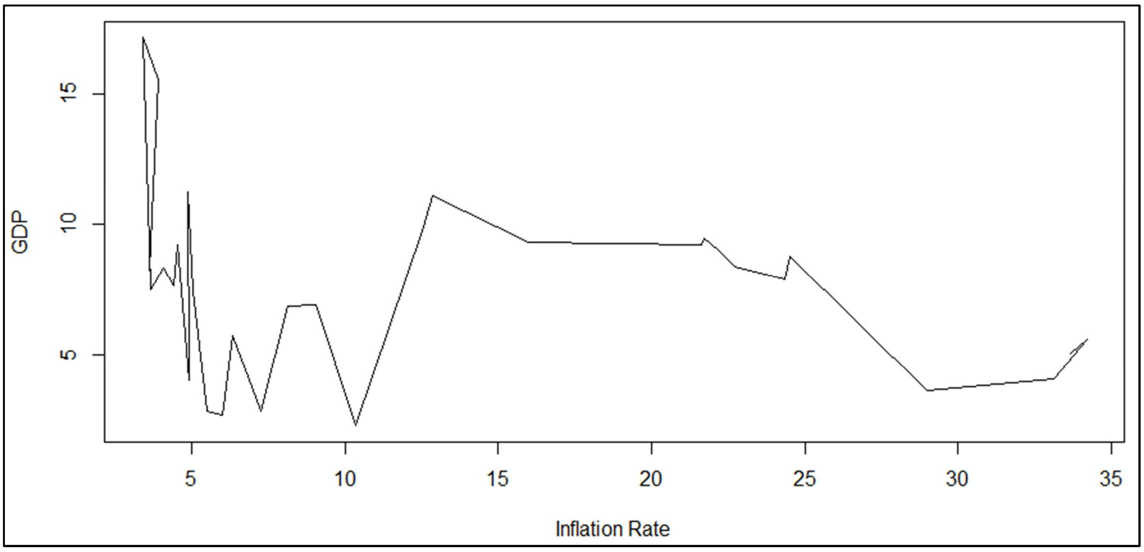


Figure 3. *Plot between GDP and Inflation Rate.*

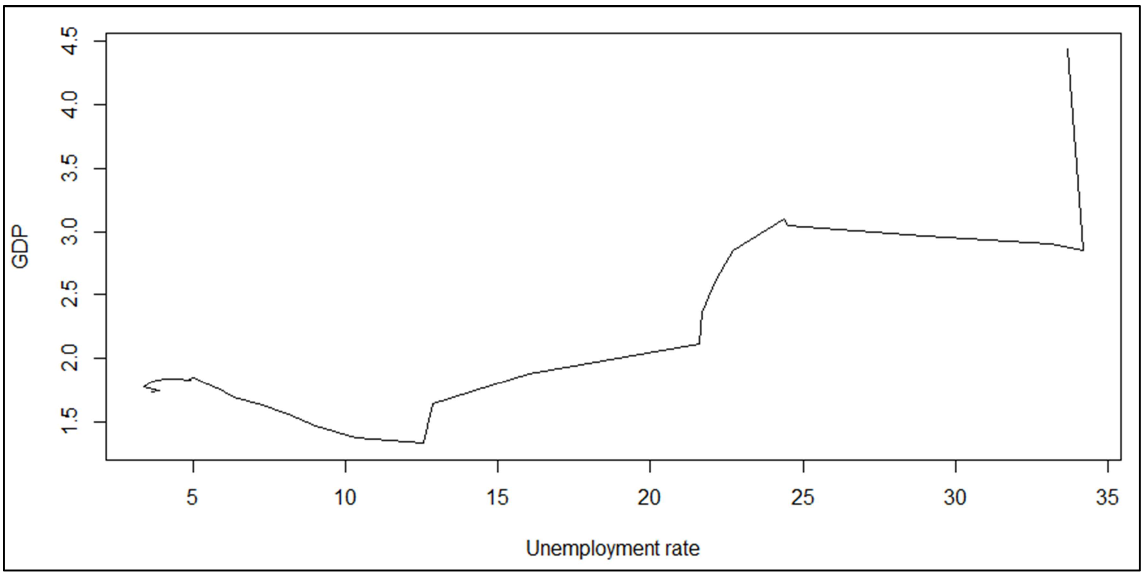


Figure 4. *Plot between GDP and Unemployment Rate.*

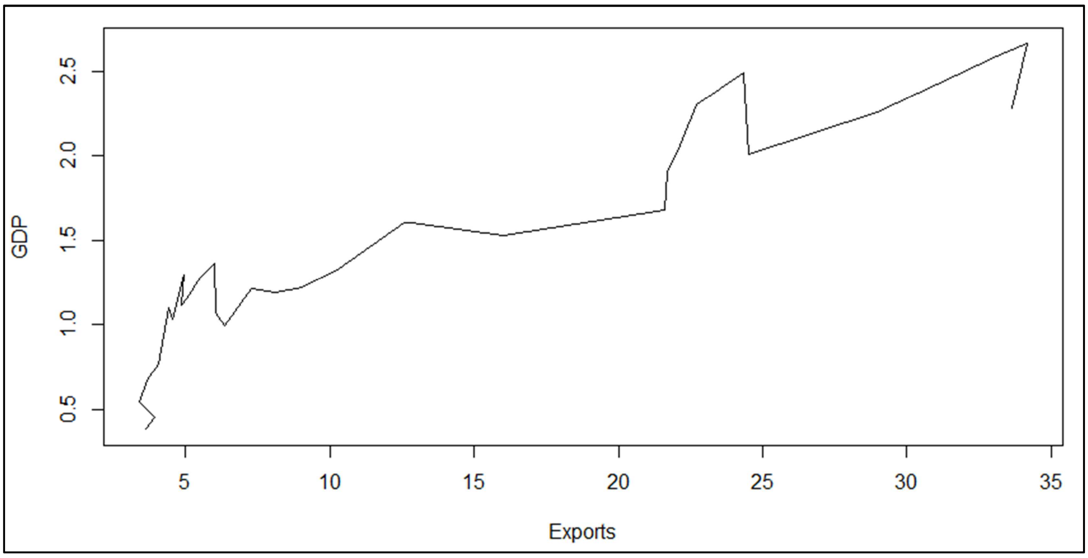


Figure 5. *Plot between GDP and Exports.*

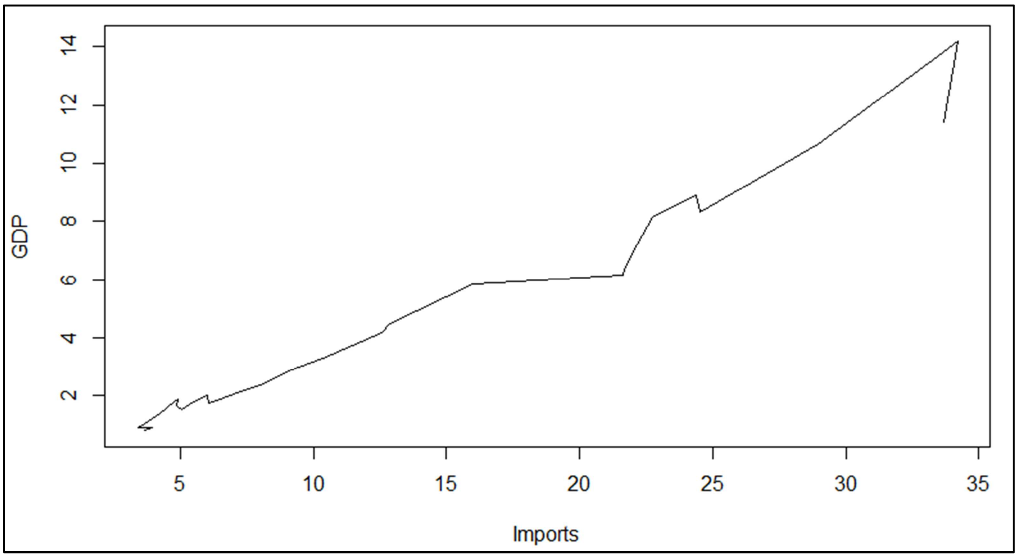


Figure 6. *Plot between GDP and Imports.*

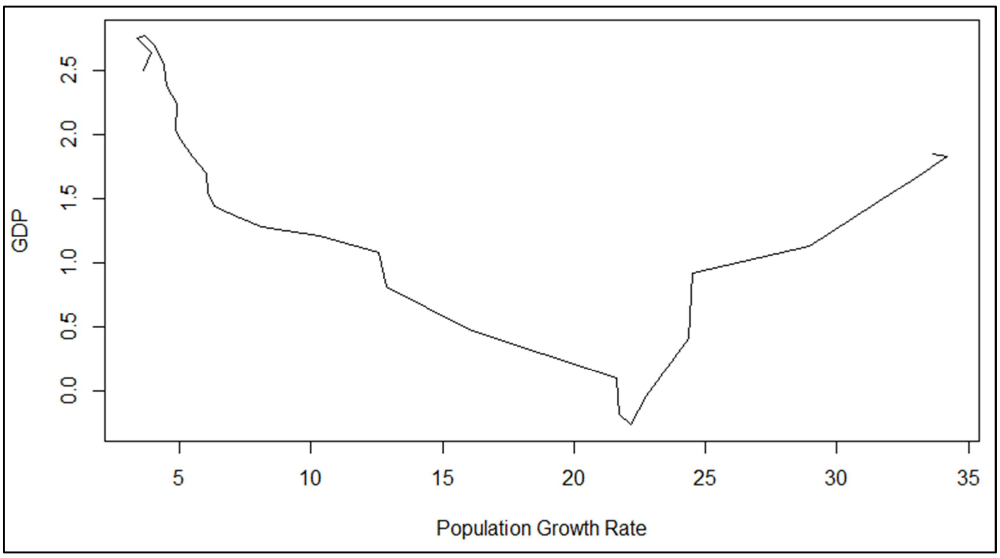


Figure 7. *Plot between GDP and Population Growth Rate.*

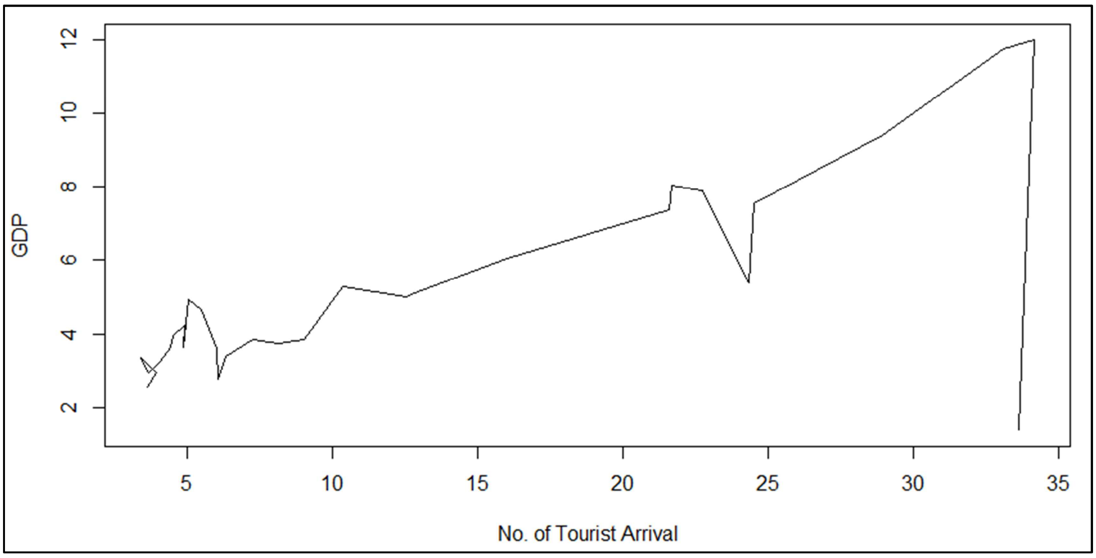


Figure 8. *Plot between GDP and NO. of Tourist Arrival.*

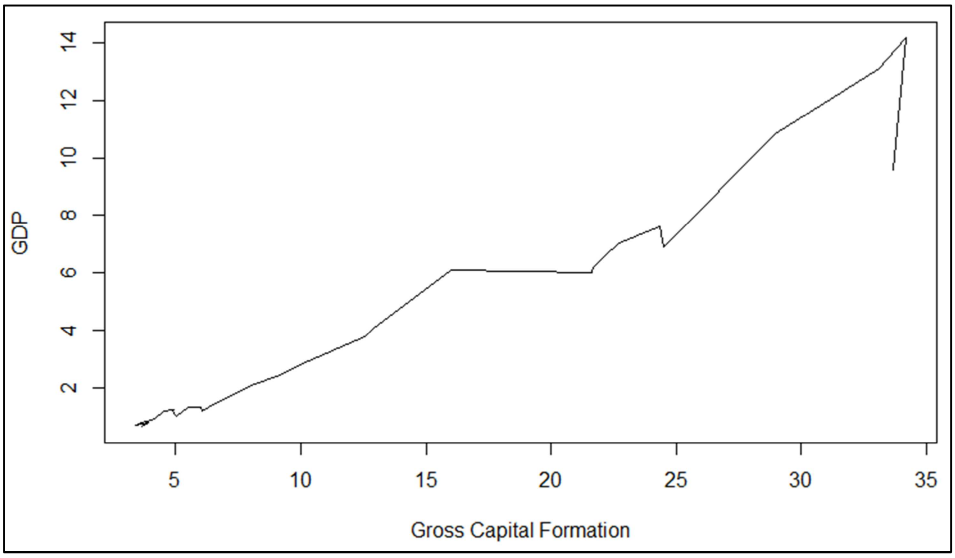


Figure 9. *Plot between GDP and Gross Capital Formation.*

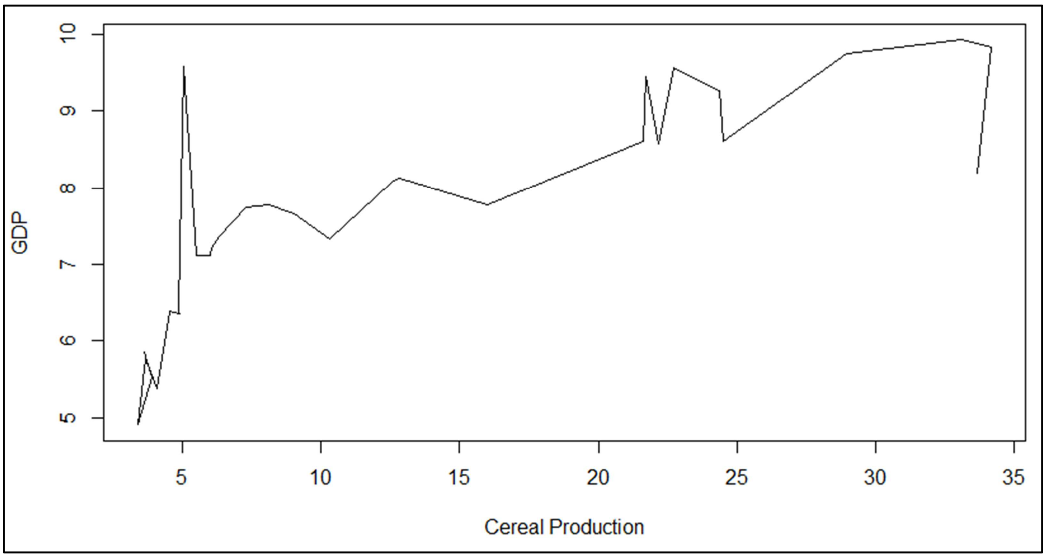


Figure 10. *Plot between GDP and Cereal Production.*

Table 2. *OLS estimates for multiple linear regression model.*

Covariates	Coefficient	Standard Error	t - value	P- value	95% C.I.
Intercept	82.08053	41.64808	1.970812	0.062069	(-4.53139, 168.6924)
labor force rate (X_1)	–0.93725	0.505251	–1.85502	0.077688	(-1.98797,0.113477)
inflation rate (X_2)	–0.03046	0.058279	–0.52272	0.606643	(-0.15166, 0.090734)
unemployment rate (X_3)	3.358927	1.068221	3.144413	0.004895 **	(1.13744, 5.580413)
export (X_4)	–1.8212	1.564206	–1.16429	0.257362	(-5.07414, 1.431748)
Import (X_5)	1.56400	1.077385	1.451663	0.161372	(-0.67654, 3.804544)
population growth rate (X_6)	–1.03954	0.633021	–1.6422	0.115441	(-2.35598, 0.276895)
number of tourist arrival (X_7)	0.109257	0.206477	0.529146	0.602254	(-0.32014, 0.53865)
gross capital formation (X_8)	0.365457	0.813353	0.449321	0.657803	(-1.326, 2.056918)
cereal production (X_9)	0.088043	0.283775	0.310256	0.759425	(-0.5021, 0.678184)
R-square: 0.994482, Adjusted R-square: 0.992117					
F-statistic: 420.5255 on 5 and 30 DF, p-value: < 1.2e-21					

Significance level: '***' 0.001 (0.1%) '**' 0.01 (1%) '*' 0.05 (5%) '.' 0.1 (10%)

Table 3. *Bayes estimates for multiple linear regression.*

Covariates	Posterior Mean	Posterior Std.	95% Credible Interval
Intercept	82.064	41.391	(0.088, 163.987)
Labor force rate (X_1)	–0.937	0.502	(-1.931, 0.057)
Inflation rate (X_2)	–0.030	0.058	(-0.145, 0.084)
Unemployment rate (X_3)	3.358	1.061	(1.256, 5.458)
Export (X_4)	–1.818	1.553	(-4.893, 1.257)
Import (X_5)	1.563	1.070	(-0.555, 3.680)
Population growth rate (X_6)	–1.039	0.629	(-2.285, 0.206)
Number of tourist arrival (X_7)	0.109	0.205	(-0.298, 0.515)
Gross capital formation (X_8)	0.366	0.808	(-1.234, 1.966)
Cereal production (X_9)	0.088	0.282	(-0.471, 0.647)

Table 2 and Table 3 present the results from the OLS and Bayesian approach respectively and are found to be close. In table 2 the estimated coefficients of covariates unemployment rate, import, number of tourist arrival, gross capital formation and cereal production display a positive influence on GDP and the estimated coefficients of covariates labor force rate, inflation rate, export and population growth rate display a negative influence on GDP. The estimated coefficient of labor force rate is -0.93, which indicates that the per unit change in labour force rate changes GDP growth rate by -0.93, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of inflation rate is -0.03, which indicates that the per unit change in inflation rate changes GDP growth rate by -0.03, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of unemployment rate is 3.358, which indicates that the per unit change in unemployment rate changes GDP growth rate by 3.358, keeping the all the other explanatory variables unchanged in the model. The estimated coefficient of export is -1.818, which indicates that the per unit change by export changes GDP growth rate by -1.818, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of import is 1.563, which indicates that the per unit change in import changes GDP growth rate by 1.563, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of population growth rate is -1.039, which indicates that the per unit change in population growth rate changes GDP growth rate by -1.039, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of number of tourist arrival is 0.109, which indicates that the per unit change in number of tourist arrival changes GDP growth rate by 0.109, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of gross capital formation is 0.366, which indicates that the per unit change in gross capital formation changes GDP growth rate by 0.366, keeping all the other explanatory variables unchanged in the model. The estimated coefficient of cereal production is 0.088, which indicates that the per unit change in cereal production changes GDP growth rate by 0.088, keeping all the other explanatory variables unchanged in the model. The statistical tests that check the quality of the economic model considered; it can be seen that the values of the tests R^2, respectively adjusted R^2 are high, which lead us to claim that the modal used has a degree of minimal risk analysis of economic forecasting. The report of determination shows that all of the explanatory variables simultaneously influence the total variation of the variable result is 99.74%. We can say that the modal is statistically significant after applying the test F-statistic, whose value is greater than the value of the table.

Table 4. *Comparing models with different choice of priors for regression parameters.*

Estimated Coefficients	Different choices of prior distributions with varying parameters							
	N (0,0.01)	N (0,0.1)	N (0,1)	N (0,2)	N (0,3)	N (1, 0)	N (1,0.1)	N (2,0.1)
$\widehat{\beta_0}$	70.308	69.919	56.250	48.534	48.121	70.228	72.958	75.993
$\widehat{\beta_1}$	-0.795	-0.792	-0.633	-0.543	-0.539	-0.794	-0.832	-0.872
$\widehat{\beta_2}$	-0.026	-0.019	0.017	0.037	0.045	-0.027	-0.018	-0.016
$\widehat{\beta_3}$	3.135	3.072	2.278	1.752	1.453	3.138	3.177	3.282
$\widehat{\beta_4}$	-1.986	-1.674	-0.592	-0.202	-0.025	-2.032	-1.480	-1.285
$\widehat{\beta_5}$	1.703	1.606	1.503	1.500	1.482	1.720	1.488	1.369
$\widehat{\beta_6}$	-1.182	-1.137	-1.039	-0.962	-0.859	-1.190	-1.070	-1.003
$\widehat{\beta_7}$	0.070	0.048	-0.109	-0.197	-0.236	0.073	0.056	0.064
$\widehat{\beta_8}$	0.324	0.405	0.602	0.690	0.744	0.311	0.474	0.542
$\widehat{\beta_9}$	0.096	0.088	0.089	0.131	0.168	0.098	0.090	0.092
DIC	94.08	93.29	94.32	97.19	99.58	94.22	93.30	93.42

In table 4 we present the comparative analysis of model by changing parameters of prior distributions. We ran the same model eight times by changing mean and variance of Normal distribution for each of the regression coefficient β_1, β_2,…, β_9 and captured its effect through DIC. The DIC came out best or lowest when prior for each regression coefficient was chosen as N (0,0.1).

Table 5. *Estimates of regression coefficients using stepwise regression method.*

Covariates	Estimate	Standard Error	t - value	Pr (>| t |)
Intercept	58.0541	27.1315	2.140	0.04232 *
Labor force rate (X_1)	-0.6345	0.3229	-1.965	0.06059
Unemployment rate (X_3)	2.5394	0.4529	5.607	7.82e-06 ***
Export (X_4)	-2.1311	0.9961	-2.139	0.04236 *
Import (X_5)	2.2144	0.1996	11.094	3.79e-11 ***
Population growth rate (X_6)	-1.3825	0.4336	-3.188	0.00383 **

Significance level: '***' 0.001 (0.1%) '**' 0.01 (1%) '*' 0.05 (5%) '.' 0.1 (10%) ' ' 1 (100%)

Table 5 provides the analysis of step-wise regression in which we are able to find the intensity with which considered covariates influence GDP of Nepal. Step-wise regression eliminated the non-significant covariates like inflation rate, number of tourist arrival, gross capital formation and cereal production. The covariates which have significant influence on GDP of Nepal are displayed in table 5 with p-values with varying level of significance respectively. Unemployment rate, import and population growth rate significantly influenced GDP of Nepal.

Table 6. *Posterior estimates assuming N (0,125) as a prior for each regression coefficient.*

Covariates	Posterior Mean	Posterior Std.	95% Credible Interval
Intercept	56.504	25.417	(6.214, 106.503)
Labor force rate (X_1)	-0.618	0.302	(-1.212, -0.021)
Unemployment rate (X_3)	2.486	0.441	(1.609, 3.353)
Export (X_4)	-1.953	0.930	(-3.774, -0.103)
Import (X_5)	2.202	0.186	(1.835, 2.568)
Population growth rate (X_6)	-1.355	0.406	(-2.154, -0.552)
DIC	86.20		

Table 6 present the Bayes estimates for coefficients of covariates which came out significant in stepwise regression taking N (0,0.1) as prior for each regression coefficient. We considered only N (0,0.1) for estimation because DIC came lowest for N (0,0.1) as discussed in analysis of table 4. We can observe a sharp fall in DIC from 93.29 in table 4 to 86.2 in table 6 indicating relevance of considering significant covariates as obtained by step-wise regression.

Table 7. *Model Comparison for significant, non-significant and all covariates.*

Model	Fitted Models	DIC
(i)	$\hat{y} = 69.919 - 0.792X_1 - 0.019X_2 + 3.072X_3 - 1.674X_4 + 1.606X_5 - 1.137X_6 + 0.048X_7 + 0.405X_8 + 0.088X_9$	93.29
(ii)	$\hat{y} = -0.094 + 0.061X_2 - 0.705X_7 + 2.767X_8 + 0.692X_9$	133.07
(iii)	$\hat{y} = 56.504 - 0.618X_1 + 2.486X_3 - 1.953X_4 + 2.202X_5 - 1.355X_6$	86.20

Table 7 presents the model comparison estimated by INLA for three models described as follows. The full model considering all possible nine covariates. The model containing only those covariates which got eliminated as a result of step-wise regression. The model containing only those covariates which were significant and selected as covariates as a result of step-wise regression. The use of step-wise regression reflects its credibility by fall in DIC from 133.07 to 86.20.

Table 8. *Estimates of MSE, MAD and MAPE.*

Estimation Method	MSE	MAD	MAPE
OLS	0.5662605	0.4733400	4.128021%
Bayesian (INLA)	0.5662592	0.4732924	4.125480%

Table 8 displays the estimates of performance metrices used for comparing accuracy of estimation process through Bayes and classical ideologies. It can be seen that all three metrices MSE, MAD and MAPE are very close to each other but are slightly less for Bayesian estimation of regression parameters through INLA.

Table 9. *T-test between observed and fitted values.*

	OLS Estimates		Bayes Estimates	
	Actual value	**Fitted value**	**Actual value**	**Fitted value**
Mean	13.20903226	13.20903226	13.2090323	13.20901029
Variance	106.041369	105.4562331	106.041369	105.4545354
Observations	31	31	31	31
df	30		30	
t Stat	3.11247E-13		0.0001599	
P(T<=t) one-tail	0.5		0.49993674	
t-value one-tail	1.697260887		1.69726089	
P(T<=t) two-tail	1		0.99987347	
t-value two-tail	2.042272456		2.04227246	

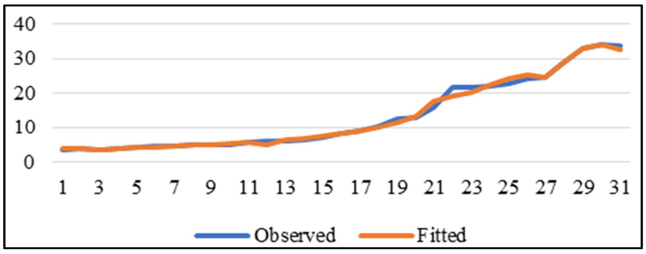


Figure 11. *Observed vs Fitted Values.*

In table 9 we computed the differences in fitted and observed values under both circumstances i.e., under classical and Bayesian framework and then carried out t-test for detecting that whether there exists a significant difference between fitted and observed values. From table 9 we can see that our null hypothesis that there is no significant difference is not rejected. Observed and predicted values are presented in supplementary file as table S2. Graph of observed and fitted values is displayed in Figure 11 and it can be inferred that adopted model fits well to the data.

Furthermore, the graph of the two values reveals that the values are closely related to each other indicating that they are closely identical to each other.

4. Discussion

Statistical modelling solves two purposes. First, identifies the relevant variables or covariates which have significant impact on outcome variable. Second, explains the variation in outcome variable with the help of selected covariates. OLS estimates are always a tradition to be performed for obtaining prima facie evidence for the relationship between outcome and exposures but with increase in computational proficiency and considering modern day modelling requirements we need to capture relationships as accurately as possible which is well achieved by Bayesian approximations. Rue, H. have developed a novel computational method for Bayesian inference called as INLA which has been winning the race against MCMC with respect to computational ease [16]. In the present research, we have explained the GDP of Nepal though Labor force rate, Inflation rate, Unemployment rate, Export (in billion US$), Import (in billion US$), Population growth rate, number of tourist arrival (in hundred thousand), Gross capital formation (in billion US$), and Cereal production (in million metric tons) from world Bank data of a span of 31 years (1990-2020) and validated the relationship though classical multiple linear regression and Bayesian linear regression using INLA.

Countable studies and research work has been carried out for GDP modelling and processes of determining GDP influencers with both classical and Bayesian ideologies. Alshuwaiee, W. M. in a time series study of oil-based economy for Kuwait had discussed about various macroeconomic factors such as GDP, inflation rate, oil prices and their impact on unemployment rate of Kuwait [2]. Also, Nurmakhanova, M. in his time series paper gave emphasis on the economy of Kazakhstan and discuss the relationship between real GDP, oil prices, exchange rates by employing Bayesian model [11]. Present research elaborates the process of finding most appropriate covariates for GDP in Nepal through essence of regression modeling under Bayesian Paradigm.

It was observed that unemployment rate was found to be positively associated with GDP of Nepal (Table 2 and Table 3) and was significant in both classical and Bayesian sense. There might be various reasons for this but some of them include unequal distribution of income and resources, dependency on scientific equipment in agricultural activities and inclusion of foreign labors from China, India, South Korea and Japan in cement factories, hydropower plants and other developmental projects. This also reflects lack of trained and skill manpower in Nepal for which government policies need to be introspected.

The model analysis using Bayesian approximation was fitted again eight times using eight priors as normal distribution with altered mean and precision for each of the regression coefficients. All eight models were compared on the basis of DIC (Table 4). It was observed that DIC (=93.29) was lowest when we chose N (0,0.1) as a prior distribution for each of the nine regression coefficients. We also identified the most relevant variables influencing GDP of Nepal through step-wise regression approach (Table 5) and then again fitted the model based on selected covariates through INLA (Table 6) and compared the full model, model with selected covariates and the model with eliminated covariates through DIC (Table 7). There was a sharp fall in DIC (=86.20) for the model with selected covariates through step-wise regression. Bayesian approximations are based on both prior and data and therefore provide better estimates as compared to classical ones, this was also justified by comparison metrics like MSE, MAD and MAPE (Table 8).

5. Conclusion

The present research demonstrated the dynamics of GDP variation in Nepal from 1990-2020 through heatmaps as a descriptive measure and through regression modeling for adjusting effect of determinants. Bayesian regression implemented through INLA exposed the relationship of GDP with possible covariates more evidently than classical regression by exhibiting shorter credible intervals and lower MSE, MAD and MAPE. Step-wise regression further solved the purpose of identifying most relevant economic indicators for GDP in Nepal which were fitted again to reflect their relevance by Bayesian measure of model complexity and adequacy, popularly known as DIC. Different scenarios for prior distribution for regression parameters were analyzed to identify most suitable choice of parameter for normal distribution. Fitted and observed realizations were validated to have in-significant difference by t-test.

Data Availability

Links for various data sources have been listed below.

http://data.worldbank.org/country/nepal.

https://www.tourism.gov.np/files/publication_files/299.pdf

https://data.worldbank.org/indicator/AG.PRD.CREL.MT?locations=NP

https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?locations=NP

https://www.tourism.gov.np/files/NOTICE%20MANAGER_FILES/Nepal_%20tourism_statics_2019.pd

https://www.macrotrends.net/countries/NPL/nepal/tourism-statistics

https://data.worldbank.org/indicator/NE.GDI.TOTL.CN?locations=NP

https://www.worldometers.info/gdp/nepal-gdp/

https://www.tourism.gov.np/files/publication_files/299.pdf

Declarations

Ethical Approval

Not Applicable.

Conflict of Interest

The authors declare no competing interests.

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Appendix

Independent variables	
X_1	Labour force rate
X_2	Inflation rate
X_3	Unemployment rate
X_4	Export (in billion US$)
X_5	Import (in billion US$)
X_6	Population growth rate
X_7	Number of tourist arrival (in hundred thousand)
X_8	Gross capital formation (in billion US$)
X_9	Cereal production (in million metric tons)
Dependent variable	
y	Gross domestic product (in billion US$)

Table S1. *Data for GDP and independent variables.*

S. No.	y	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	3.63	88.16	8.24	1.73	0.38	0.79	2.5	2.549	0.657	5.847
2	3.92	88.18	15.56	1.75	0.45	0.91	2.64	2.929	0.794	5.519
3	3.4	88.24	17.15	1.78	0.54	0.88	2.75	3.343	0.704	4.902
4	3.66	88.34	7.51	1.82	0.67	1.05	2.77	2.935	0.826	5.773
5	4.07	88.46	8.35	1.84	0.77	1.28	2.69	3.265	0.911	5.375
6	4.4	88.59	7.62	1.84	1.1	1.52	2.54	3.634	1.109	6.078
7	4.52	88.53	9.22	1.84	1.03	1.61	2.38	3.936	1.23	6.378
8	4.92	88.48	4.01	1.83	1.29	1.85	2.24	4.218	1.246	6.35
9	4.86	88.44	11.24	1.83	1.11	1.65	2.03	3.637	1.206	6.389
10	5.03	88.41	7.45	1.85	1.15	1.5	1.96	4.915	1.031	9.59
11	5.49	88.07	2.84	1.8	1.28	1.78	1.83	4.636	1.336	7.116
12	6.01	88.8	2.69	1.75	1.36	2	1.7	3.612	1.342	7.12
13	6.05	87.51	3.03	1.73	1.07	1.72	1.55	2.755	1.225	7.221
14	6.33	87.19	5.71	1.69	0.99	1.81	1.44	3.381	1.355	7.367
15	7.27	86.85	2.82	1.63	1.21	2.14	1.35	3.853	1.784	7.754
16	8.13	86.48	6.84	1.56	1.19	2.4	1.28	3.754	2.151	7.774
17	9.04	86.07	6.92	1.47	1.22	2.83	1.25	3.859	2.428	7.664
18	10.33	85.65	2.27	1.38	1.33	3.28	1.21	5.267	2.962	7.337
19	12.55	85.22	9.91	1.33	1.6	4.17	1.08	5.003	3.803	8.077
20	12.85	85.15	11.09	1.64	1.6	4.46	0.81	5.099	4.072	8.122
21	16	85.08	9.33	1.88	1.53	5.83	0.48	6.028	6.124	7.771
22	21.62	85.21	9.23	2.11	1.68	6.15	0.1	7.362	5.999	8.615
23	21.7	85.35	9.46	2.36	1.9	6.33	-0.19	8.031	6.208	9.457
24	22.16	85.46	9.04	2.63	2.06	7.22	-0.27	7.976	6.577	8.58
25	22.73	85.5	8.36	2.85	2.3	8.15	-0.04	7.901	7.043	9.563
26	24.36	85.45	7.87	3.1	2.49	8.88	0.41	5.389	7.619	9.266
27	24.52	85.43	8.79	3.05	2.01	8.32	0.92	7.53	6.926	8.614
28	28.97	85.36	3.63	2.97	2.26	10.67	1.13	9.408	10.814	9.759
29	33.11	85.43	4.06	2.9	2.59	13.45	1.68	11.731	13.095	9.93
30	34.19	85.43	5.57	2.85	2.66	14.18	1.83	11.972	14.146	9.844
31	33.66	85.49	5.05	4.44	2.28	11.41	1.85	1.382	9.558	8.193

Table S2. *Observed and Fitted Values.*

S. No.	Observed value	Fitted value
1	3.63	3.98948
2	3.92	3.793094
3	3.4	3.42237
4	3.66	3.841331
5	4.07	4.063493
6	4.4	4.069598
7	4.52	4.615004
8	4.92	4.86871
9	4.86	4.844829
10	5.03	5.177402
11	5.49	5.668396
12	6.01	5.045611
13	6.05	6.294882
14	6.33	6.908056
15	7.27	7.565035
16	8.13	8.195479
17	9.04	9.027029
18	10.33	10.12491
19	12.55	11.50771
20	12.85	13.42521
21	16	17.7838
22	21.62	19.2335
23	21.7	20.3411
24	22.16	22.39291
25	22.73	24.1422
26	24.36	25.28189
27	24.52	24.49477
28	28.97	29.17867
29	33.11	33.14265
30	34.19	34.19053
31	33.66	32.84967

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