
Performance Analysis of Multi-code Keying Scheme for Spectral-Amplitude-Coding Optical CDMA Network

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Abstract: Multi-code keying scheme is proposed for spectral-amplitude coding (SAC) optical code-division multiple-access (OCDMA) network, and in this scheme users are assigned several signature sequences for spectral efficiency enhancement. The shifted prime codes can be used in this scheme since their code patterns are suitable for the implementation of compact encoding/decoding structures based on arrayed-waveguide gratings. The performance analysis shows that the proposed multi-code keying scheme obtains better spectral efficiency than previous two-code keying when number of codewords for each user is greater than two.

Keywords: Optical Code-Division Multiple-Access (OCDMA), Multi-code Keying, Arrayed Waveguide Grating (AWG), Prime Codes

1. Introduction

Optical code-division multiple-access (OCDMA) schemes were useful techniques for asynchronous, bursty transmission. Conventionally, each user was assigned one signature sequence for information encoding in OCDMA networks and this was the so-called on-off keying (OOK) scheme. Though the total number of users in this kind of OCDMA networks could be increased by using additional time slots or wavelengths for the encoding process, the network throughputs were still limited when the multiple access interference (MAI) between users' codewords and various noises were present [1, 2]. To improve the spectral efficiency [3], one way was to use multi-code keying scheme for information encoding [4, 5]. In this scheme, each user was assigned $M=2^m$ codewords that correspond to the m information bits of each symbol. For example, in the multi-code keying scheme proposed in [4], each user was assigned one codeword of one generalized multiwavelength prime code (GMWPC) family [2] and the time (or wavelength) shifted versions of this codeword were used by the same user for multi-code keying. Though in this multi-code keying scheme, no M -fold increase in code cardinality was needed to maintain the total number of network users the same as that for OOK ones, network synchronization was necessary to

distinguish the shifted versions from the same GMWPC codewords. In addition, the MAI induced by the non-zero cross-correlations of the wavelength/time codes and the beat noises affected the bit error rate (BER) performance seriously [2].

The spectral-amplitude-coding (SAC) scheme was a useful OCDMA technique with excellent ability for MAI elimination and recently it was used in several applications of communication networks such as radio-over-fiber systems and multi-protocol label switching (MPLS) networks [6-9]. Recently, shifted prime (SP) codes with low in-phase cross correlations for two-code keying was proposed [7] and it obtained better performance against beat noises than other SAC codes. Due to the special code patterns, these SP codes were also suitable for the application of multi-code keying schemes implemented by arrayed waveguide gratings (AWGs). In this letter, the cardinalities of original SP codes in [7] are enlarged, and the resulting codewords are applied to the SAC-based networks employing multi-code keying. The BER performance for multi-code keying scheme is also analyzed and compared to that of previous SAC schemes.

2. Multi-code Keying Scheme

Though the proposed scheme is also useful for other codes

such as quadratic congruence codes, only SP codes are discussed in the following for simplicity. The elements of prime sequences $C_{e,f}$ ($e, f=0, 1, \dots, p-1$) with parameter p (p is a prime) used to generate the SP codeword $X_{e,f}$ are obtained by using [7, Eq. (3)], and the i -element of $X_{e,f}$ is obtained by using [7, Eq. (1) and (2)]. The original cardinality of one SP code family can be enlarged by adding the following additional codewords $X_{p,f}$ to the original SP code family:

$$X_{p,f}(i) = \begin{cases} 1, & i = f * p, f * p + 1, \dots, f * p + p - 1, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

For the case of $M=p-1$, the f -th ($0 \leq f < p$) codeword of group $\#e$ $X_{e,f}$ is assigned as the m -th multi-code codewords of user $\#e$ for the transmission of symbol $d_e=m$ in the proposed multi-code keying scheme ($d_e \in \{1, 2, \dots, M\}$). Thus in one multi-code OCDMA network, the total number of supported users is $p+1$ and each user has p codewords for multi-code keying of the broadband light source (BLS). Table 1 shows the codewords in one SP code family for $p=5$. Note that the cross-correlation between $X_{e,f}$ and $X_{g,h}$ is still the same as that described in [7, Eq. (4)] after the codewords in Eq. (1) are added.

Table 1. SP codewords for $p=5$.

e	f	$C_{e,f}$	$X_{e,f}$
0	0	0 0 0 0 0	10000 10000 10000 10000 10000
0	1	1 1 1 1 1	01000 01000 01000 01000 01000
0	2	2 2 2 2 2	00100 00100 00100 00100 00100
0	3	3 3 3 3 3	00010 00010 00010 00010 00010
0	4	4 4 4 4 4	00001 00001 00001 00001 00001
1	0	0 1 2 3 4	10000 01000 00100 00010 00001
1	1	1 2 3 4 0	01000 00100 00010 00001 10000
1	2	2 3 4 0 1	00100 00010 00001 10000 01000
1	3	3 4 0 1 2	00010 00001 10000 01000 00100
1	4	4 0 1 2 3	00001 10000 01000 00100 00010
.	.	.	.
.	.	.	.
5	0	.	11111 00000 00000 00000 00000
5	1	.	00000 11111 00000 00000 00000
5	2	.	00000 00000 11111 00000 00000
5	3	.	00000 00000 00000 11111 00000
5	4	.	00000 00000 00000 00000 11111

By observing [7, Eq. (4)], the last codeword $X_{e,p}$ of user $\#e$ can be chosen as the common reference codeword for MAI elimination in the multicode keying scheme, and the following formula can be used to compute the decoding result for m -th codeword of user $\#e$ in the decoder ($0 \leq m \leq M-1, 0 \leq f \leq p-1$):

$$z_m = X_{e,f} \odot X_{g,h} - X_{e,p} \odot X_{g,h} = \begin{cases} p, & e=g, f=h, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

where \odot is the dot-product of two vectors [7] and $X_{g,h}$ is the interfering codeword sent by user $\#g$. In the decoder, the

largest value of z_m is used by the decision unit to decide which codeword is transmitted.

Since the codewords for SP codes assigned to the same user have cyclic relationship, they can be generated by the encoder branches at the AWG encoder in [7] and decoded by the decoder with similar structure. Figure 1 shows the multi-code encoders for three kinds of users, and the explanation of the first two encoders can be found in [7]. Since wavelength chips $\lambda_{0+bp}, \lambda_{1+bp}$ and λ_{2+bp} of the BLS representing $X_{p,b}$ appear at the output port $\#b$ of the coarse AWGs in Figure 1(c), the last encoder in Figure 1 generates all the codewords of user $\#p$ and one $M \times 1$ optical switch can be used to select these codewords according to the symbols of user $\#p$. The multi-code decoder for user $\#e$ ($0 < e < p$) is shown in Figure 2, which uses cascaded AWGs the same as that in the corresponding encoder to produce the value of $X_{e,f} \odot X_{g,h}$ at the input of photo-diode $\#f$. Since the output of the photo-diode $\#p$ is used for the computation of $Z_0 \sim Z_{M-1}$, it is amplified electrically with gain M before the distribution. The multi-code decoder for user $\#0$ and $\#p$ can be obtained from their corresponding encoder in Figure 1 in a similar manner.

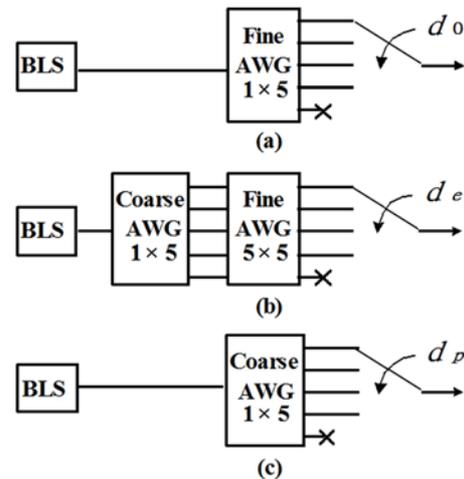


Figure 1. The multi-code encoder for (a) user #0 (b) user $\#e$ ($0 < e < p$) (c) user $\#p$.

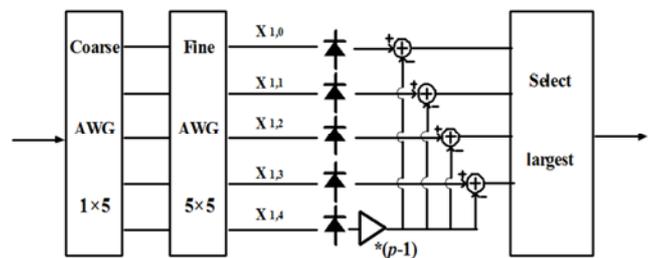


Figure 2. The multi-code decoder for user $\#e$ ($0 < e < p$).

3. System Performance

In the SAC OCDMA networks with flat power spectral density (PSD) of light sources in the coded bandwidth, the BER performances are mainly affected by the phase-induced intensity noise (PIIN) [6] and thermal noise. In the following analysis, the splitting loss during signal transmission is

neglected in order to avoid the influence of number of users. In addition, the light sources in the encoders are assumed to be unpolarized with equal magnitude $P_{sr}/\Delta\nu$ over bandwidth $\Delta\nu$ Hz. To clarify the performance evaluation process, the BER formulas for SAC multi-code keying scheme are deducted first, and then the noise variances for SP codes are computed and applied to these BER formulas.

A. The deduction of BER for multi-code keying scheme:

Without lost of generality, suppose the desired user is user #1 and the codeword $X_{1,0}$ is transmitted. Since the photocurrent value of the 0-th input at the decision device z_0 is contributed by both the signal and the noises, it is a nonzero-mean Gaussian random variable with mean $I_0=RP_{sr}/p$ and variance σ^2 , where R is the responsivity of the photo-diodes. However, the photocurrent value of the m -th input at the decision device z_m ($0 < m < M$) is a zero-mean Gaussian random variable with variance σ^2 . Assuming that z_0 has taken a value γ , the probability that z_m has value larger than z_0 is

$$P(z_m > \gamma | z_0 = \gamma) = Q(\gamma/\sigma'), \quad (3)$$

where $Q(\cdot)$ is the well-known Q-function. Thus the probability of correct decision for the decision device is

$$P(C | I_0, z_0 = \gamma) = P(z_1 < \gamma, z_2 < \gamma, \dots, z_{M-1} < \gamma | z_0 = \gamma) = [1 - Q(\gamma/\sigma')]^{M-1}. \quad (4)$$

By averaging over z_0 , the probability of correct decision for the decision device is

$$\begin{aligned} P(C | I_0) &= \int_{-\infty}^{\infty} P(C | I_0, z_0 = \gamma) f(\gamma; I_0, \sigma) d\gamma = E[(1 - Q(\gamma/\sigma'))^{M-1}] \\ &= \int_{-\infty}^{\infty} [1 - Q(\gamma/\sigma')]^{M-1} f(\gamma; I_0, \sigma) d\gamma, \end{aligned} \quad (5)$$

where $f(\gamma; I_0, \sigma)$ is the probability density function of the Gaussian distribution with mean I_0 and variance σ^2 :

$$f(\gamma; I_0, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{\gamma - I_0}{\sigma})^2}, \quad (6)$$

Thus the symbol error probability is [10]

$$P_s(e) = 1 - P(C | I_0) = 1 - E[(1 - Q(\gamma/\sigma'))^{M-1}] = \int_{-\infty}^{\infty} \{1 - [1 - Q(\gamma/\sigma')]^{M-1}\} f(\gamma; I_0, \sigma) d\gamma \quad (7)$$

By using the method in [8], the Johnson bound can be used to estimate the upper bound of $P_s(e)$:

$$P_s(e) = 1 - E[(1 - Q(\gamma/\sigma'))^{M-1}] \leq 1 - \{1 - E[Q(\gamma/\sigma')]\}^{M-1} \quad (8)$$

From Craig's formula [11], Q-function can be upper bounded by $Q(\gamma/\sigma') \leq \frac{1}{2} e^{-\frac{1}{2}(\frac{\gamma}{\sigma'})^2}$. Therefore, it is found that

$$P_s(e) \leq 1 - \{1 - E[Q(\gamma/\sigma')]\}^{M-1} \leq 1 - \left\{1 - \frac{1}{2} E[e^{-\frac{\gamma^2}{2\sigma'^2}}]\right\}^{M-1} \leq 1 - \{1 -$$

$$\frac{e^{A+B}}{2\sqrt{1+\sigma^2/\sigma'^2}}\}^{M-1}, \quad (9)$$

where

$$E[e^{-\frac{\gamma^2}{2\sigma'^2}}] = \frac{e^{A+B}}{\sqrt{1+\sigma^2/\sigma'^2}}, \quad A = \frac{I_0^2}{2\sigma^2(1+\sigma^2/\sigma'^2)}, \quad B = -\frac{I_0^2}{2\sigma'^2}. \quad (10)$$

Finally the BER can be obtained from $P_s(e)$:

$$P_b(e) = \frac{M/2}{M-1} P_s(e) \quad (11)$$

B. The deduction of noise variances for SP codes:

Assumed that bit synchronization is achieved and the desired user is active. Therefore, the noises in the photo-diodes of the decoder are contributed by the desired users and K' interfering users. Since one codeword in each group of the SP code family should be used as the reference codeword for MAI elimination, the number of codewords that can be adopted as one user's signature sequences is $p-1$ for each code group. For the case of $M=p-1$, the variance of PIIN at the PD #0 of the decoder is [6]

$${}^{(0)}\sigma_p^2 = BR^2 \int_0^\infty G_0^2(\nu) d\nu = BR^2 \frac{P_{sr}^2}{p^2 \Delta\nu} [p + K'(3 + \frac{K'-1}{p})], \quad (12)$$

where $G_m(\nu)$ is the single sideband PSD of the source received by photodiodes # m , and B is the noise-equivalent bandwidth of the photo-detectors. Due to the code characteristics, the wavelengths of interfering signal are uniformly distributed at these PDs, and thus the variances of PIIN at the PD #1~ M of the decoder is ($1 \leq m \leq M$)

$${}^{(m)}\sigma_p^2 = BR^2 \int_0^\infty G_m^2(\nu) d\nu = BR^2 \frac{P_{sr}^2}{p^2 \Delta\nu} [K'(1 + \frac{K'-1}{p})] \quad (13)$$

Note that though there are M codewords used by each user, one extra codeword is needed for MAI elimination and then there are $M+1$ photo-diodes in one user's multi-code decoder. The computation of variances mentioned above is based on the fact that each codeword of one interfering user contribute just one hit with each codeword of the desired user and, for one interfering codeword, the hits corresponding to different decoding vectors occur in different wavelengths. Thus the variances of noises at the input port #0 and # m ($m \neq 0$) of the "select largest" device are

$$\sigma^2 = {}^{(0)}\sigma_p^2 + {}^{(M)}\sigma_p^2 + \sigma_t^2 + \sigma_i^2, \quad (14)$$

and

$$\sigma'^2 = {}^{(m)}\sigma_p^2 + {}^{(p)}\sigma_p^2 + \sigma_t^2 + \sigma_i^2 = 2 {}^{(p)}\sigma_p^2 + \sigma_t^2 + \sigma_i^2, \quad (15)$$

respectively. The variance of thermal noise σ_t^2 is given in [6], and the variance of the thermal noise induced by electrical amplifier is given by

$$\sigma_i^2 = 4KT_0(F-1)B/R_L \quad (16)$$

Finally, the computing results in Eq. (14)-(16) can be substituted into Eq. (9) and (11) to obtain the symbol error probability and BER. For the case of $M < p-1$, the codewords belonging to one code group can be assigned to $\left\lfloor \frac{p-1}{M} \right\rfloor$ users since these users can use the same reference codeword for MAI elimination. Therefore, only $K' - \left\lfloor \frac{K'}{p} \right\rfloor$ interfering users have contributions to the variance of PIIN and the parameter K' in Eq. (12) and (13) should be replaced by $K' - \left\lfloor \frac{K'}{p} \right\rfloor$.

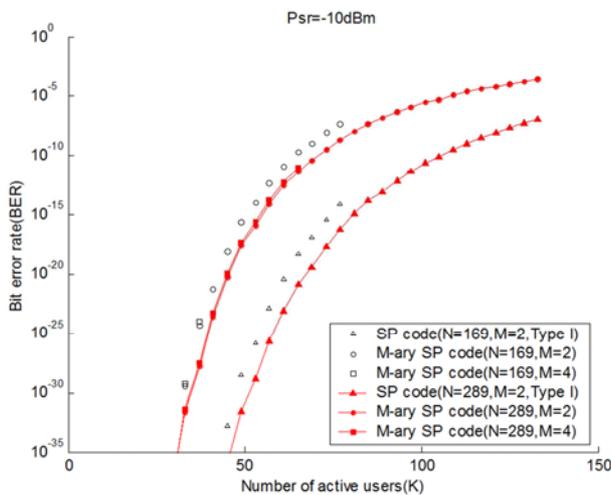


Figure 3. BER vs. number of active users.

The relationship between the number of active users K ($=K'-1$) and BER is shown in Figure 3, where the effective source powers P_{sr} at the decoder are set to -10dBm. The SP codes denoted as type I refer to the use of the decoding method in [4], and it was found that the BER performances of the two-code keying scheme in [4] were better than that of conventional SAC schemes. From Figure 3, it can be observed that the BERs of M -ary SP codes nearly change when M increases for the same code lengths. Therefore, when M is larger than 2, M -ary SP codes can be used for spectral efficiency improvement since the spectral efficiencies of M -ary SP codes increase when M increases. However, when two-code keying is adopted ($M=2$), it is better to use the scheme in [4] because of both better BER performance and simpler hardware requirement.

4. Conclusions

Multi-code keying scheme based on SAC OCDMA is proposed for spectral efficiency enhancement. Due to the code characteristics, compact multi-code encoder/decoder based on AWG can be used and this also alleviates the problem of splitting loss in the multi-code decoder. The performance

evaluation shows that the BER performances of the two-code keying scheme were better than that of conventional SAC schemes when SP codes denoted as type I are used. It can be observed that the BERs of M -ary SP codes nearly change when M increases for the same code lengths. Therefore, when M is larger than 2, M -ary SP codes can be used for spectral efficiency improvement since the spectral efficiencies of M -ary SP codes increase when M increases.

On the other side, when two-code keying is adopted ($M=2$), it is better to use the SP codes denoted as type I because better BER performance can be obtained and the decoding hardware requirement is alleviated.

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