

Masses of Hadrons by Higgs like Mechanism from Harmonic Oscillator Model in Weak Interactions Mediated by W^\pm Bosons

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Abstract: When a W^\pm boson is emitted in the weak interaction, the coupling of the particle with the boson puts the particle into simple harmonic oscillation by the transient Coulomb force between the particle and the boson. If the W^\pm boson is displaced by some selected distance, hadrons appear with inertial masses by a Higgs like mechanism due to coupling of the quark field of a particle with the Higgs field via W^\pm boson. Masses of meson nonets, baryon octet and decuplet are constructed by using weak coupling constant corrected for screening effect. The mass differences between pairs of particles arising from the breaking of the isospin (I_z) symmetry in the Standard Model (SM) is explained considering a Higgs like mechanism in the harmonic oscillator (HO) model for hadrons. The hypercharge (Y) of the standard model is found to be related to the distance quantum number (N) at which a hadron appears. Zero point energies of hadrons predicted from this model are verifiable from Casimir effect.

Keywords: Masses of Hadrons, Higgs Like Mechanism, Harmonic Oscillator Model, Electroweak Radiative Corrections, Mass of the Proton

1. Introduction

In the Standard Model (SM) masses of fermions, W^\pm and Z bosons are generated by spontaneous symmetry breaking (SSB) in the Higgs field [1]. Currently Higgs mechanism is the accepted procedure in generating masses of fermions and intermediate vector bosons [2]. Coupling of the Higgs field to the electroweak field gives masses to the intermediate vector bosons (W^\pm , and Z) by Higgs mechanism. The SSB of the scalar Higgs field coupled to the states of fermions of opposite helicity gives masses to fermions [3, 4]. However, so far it has not been possible to apply Higgs mechanism to the determination of masses of all other particles.

In Quantum Chromodynamics hadron spectrum is obtained with limited success considering confined harmonic potential in the harmonic oscillator (HO) model [5, 6, 7]. In the weak interaction mediated by the W^\pm boson, coupling of the boson with the particle puts the particle into simple harmonic oscillation by the transient Coulomb force between the particle and the boson. Recently, masses of a few hadrons were obtained without vacuum polarization correction by the

Higgs like mechanism using a phenomenologically proposed mass-energy formula in the platform of the HO model [8]. In this work the mass-energy formula is derived and masses of neutrons, protons and all other hadrons are obtained applying vacuum polarization correction to the weak coupling constant for W^\pm boson mediated weak interactions. Zero point energies of hadrons are predicted from this model and it is suggested to be verified by Casimir effect.

2. Harmonic Oscillator (HO) Model for Hadrons

In the W^\pm mediated weak interaction, the emitted W^\pm boson pushes the charged particle at a distance $X_{op} = N \cdot \lambda_{op}$ by the Coulomb force F_c given by

$$F_c = (q^2/4\pi\epsilon_0)/(X_{op}^2) \quad (1)$$

When $N=1, 2, \dots$ and λ_{op} is the reduced Compton wavelength of the particle.

If the W^\pm boson is coupled to the particle and displaced by a

distance $X_w = D_{em} \lambda_w$, the restoring force on the particle is given by

$$F_{res} = -k \cdot X_w \quad (2)$$

Where, k is the restoring force constant, $D_{em} = 1, 2, \dots$, and λ_w is the reduced Compton wavelength of the W^\pm boson. When all the forces act in the same line, assuming the heavy W^\pm boson fixed, the general equation of simple harmonic motion for the particle is given by

$$m_{op} (d^2 X / dt^2) = F_{res} - F_c = -k \cdot [(X_w (1 - F_c / F_{res}))] = -k \cdot X_w' \quad (3)$$

If F_c is switched off after a short time $T < (m_{op} / k)^{1/2}$, then the particle just continues to oscillate at the system's natural frequency. When $F_c = F_{res}$ we get

$$k = (\alpha(0) \cdot M_w) / (D_{em} \cdot X_{op}^2) \quad (4)$$

The restoring force constant (k) arises due to electromagnetic coupling of the particle via $\alpha(0)$ with the boson and it is different for different particle. As $k = m_{op} \cdot \omega_{op}^2$ when $F_{res} = F_c$, we get,

$$[m_{op} \cdot (\omega_{op} \lambda_{op})^2] = (m_{op} \cdot c^2) = [(\alpha(0)_{em} \cdot M_w) / (N^2 \cdot D_{em})] \quad (5)$$

When, m_{op} is the rest mass of a "hadron seed"; $\alpha(0)_{em}^{-1} = 137.035999$ = fine structure constant; $M_w \approx 80.4$ MeV [9], c velocity of light and $N = 1, 2, \dots$ gives distance at which a particle appears. Equation (5) tells that when the restoring force of the particle for displacement X_w of the W^\pm boson equals the transient Coulomb force between the particle and the boson, the particle appears at a distance X_{op} .

3. Harmonic Oscillator (HO) Model for Weak Interaction and Higgs like Mechanism

In beta decay a down quark (d) becomes an up quark (u) by flavor change in which charge changes by one unit and weak interaction takes place by the exchange of a W^{*-} boson [10]. This is shown as:

$$d \rightarrow u + W^{*-} \rightarrow u + e + \bar{\nu}_e \quad (6)$$

In this example, the particle containing the d -quark is displaced by the virtual W^{*-} boson. The Coulomb force between the charge of the d -quark constituting a particle and charge of the W^{*-} boson switched on for a very short time ($T < ((m_{op} / k)^{1/2})$), puts the composite particle into simple harmonic oscillation and this is designated as the Harmonic Oscillation (HO) model.

At low energy when neutral current (NC) is zero and there is no radiative decay, a gauge coupling constant α_w is suggested by Kushtan [11] as

$$(\alpha_w) = [(G_F \cdot M_w^2) / (4\pi \cdot (\sqrt{2}))] \approx 4.244 \times 10^{-3} \quad (7)$$

Where, $G_F / (\hbar)^3 = 1.166 / \text{GeV}^2$ [9].

This weak coupling constant (α_w) after correction for the screening effect (described later) has been applied

successfully in the weak interactions mediated by W^\pm boson as $\alpha(0)_{em}$ is related to the weak coupling constant α_w via the Fermi constant G_F .

W^\pm bosons acquire masses by SSB of the Higgs field so that

$$M_w = 1/2 \cdot g \cdot \langle v \rangle \quad (8)$$

Where, g is the coupling constant with the Higgs field; and $\langle v \rangle = 246 \text{ MeV}$ vacuum expectation value of the Higgs field [12]. For the weak mixing angle (θ_w), the Fermi constant is given as

$$G_F = (\sqrt{2} \cdot e^2) / (8 M_w^2 \cdot \sin^2 \theta_w) \quad (9)$$

From equations (7) and (9) the weak coupling constant α_w is related to the electromagnetic coupling constant $\alpha(0)$ as

$$\alpha_w = [\alpha(0) / (8 \sin^2 \theta_w)] \quad (10)$$

From equations (5), (8) and (10) we get,

$$(m_{op} \cdot c^2) = [(g_{op} \cdot \langle v \rangle) / (2 N^2 \cdot D_w)] \quad (11)$$

When, $D_w = [D_{em} / (8 \cdot \sin^2 \theta_w)]$, $D_w = 1, 2, 3, \dots$, and $g_{op} = (\alpha_w \cdot g)$ is the coupling constant of the particle.

Equations (5) and (11) show that the rest mass of any hadron is generated from a "seed boson" M_w (seed) $= (\alpha_w M_w) / (N^2 \cdot D_w)$ by a Higgs like mechanism when the quark in a particle is coupled to the Higgs field via W^\pm boson. The chiral symmetry due to the coupling of the quark of a particle to the W^\pm boson is spontaneously broken by the Higgs field, and thus the entire mass of a particle is generated. similar to the generation of a fermion mass ($m_f = [(g_f \cdot v) / \sqrt{2}]$) where, the chiral symmetry is spontaneously broken by the Yukawa coupling of the Higgs field to the fermion states of opposite helicity [3, 4] for the coupling constant g_f . From equations (10) and (11) we also get,

$$(D_{em} / D_w) = (\alpha_{em} / \alpha_w) = 8 \cdot \sin^2 \theta_w \quad (12)$$

Equation (12) presents a running weak mixing angle θ_w due to running values of α_{em} which runs from $(1/128.9)$ to $(1/137)$ and α_w varies due to radiation correction.

For a 3-d simple harmonic oscillator, energy eigenvalues E_n at the n th level is given by [10]:

$$E_n = (m_{op})_n \cdot c^2 = (m_{op} \cdot c^2) (n + d/2) \quad (13)$$

When $n = 0, 1, 2, \dots$, and dimension $d = 1, 2$ and 3 corresponding to the dimensions of a 3-D simple harmonic oscillator. The dimension (d) in the three-dimensional harmonic oscillator is linked with the charge Q of a particle as

$$Q = \pm (N_q - d) \quad (14)$$

In case of mesons, for $d = 1$, the value of $Q = \pm 1$; for $d = 2$, the value of $Q = 0$. For baryons, when $d = 1$, the value of $Q = \pm 2$; when $d = 2$, the value of $Q = \pm 1$, and when $d = 3$, the value of $Q = 0$.

Isospin (I_z) of the empirical Gell-Mann Nishijima (GMN) formula is redefined for HO model as.

$$I_z = (N_q - d) - Y/2 \quad (15)$$

From equation (15), for $\Delta N_q=0$ we get,

$$\Delta I_z = -\Delta d - \Delta Y/2 \quad (16)$$

For $\Delta(N_q)=0$ and $\Delta Y=0$, when $\Delta I_z=\pm 1$, there is break in the local gauge symmetry causing change in the dimension (d) and charge (Q). When there is a change in $\Delta Q=(-/+1)$ implying no physical change, change in dimension $\Delta d=\pm 1$ causes change in the alignment of isospins.

4. Vacuum Polarization and Screening Effect Correction to Weak Coupling Constant

Vacuum polarization [13] diminishes the coupling constant due to screening of the charges.

Screening effect corrected weak coupling constant $(\alpha)_{sc}$ is given as

$$(\alpha_{sc}) = \alpha_w \cdot (1 - \Delta\alpha_{lep} - \Delta\alpha_{had} - \Delta\alpha_{tau}) = \alpha_w (1 - \Delta\alpha) \quad (17)$$

Where, $\Delta\alpha = \sum (f_r \cdot \Delta\alpha_r)$ for $r=e, \mu, \tau, had$; f_r 's are decay fractions, and, $\Delta\alpha_e = 174.7 \times 10^{-4}$ is the contribution from the electron, $\Delta\alpha_\mu = 92.0 \times 10^{-4}$ contribution from the muon, and $\Delta\alpha_{had} = 277.7 \times 10^{-4}$ contribution from the hadron [14, 15, 16]. Weighted mean values of $\Delta\alpha_r$'s are used with decay fractions (f_r) as the weights [17]. For screening effect corrections, $\alpha(e)_{sc} \cdot M_w = 335.28 \text{ MeV}$, $\alpha(\mu)_{sc} \cdot M_w \approx 338.30 \text{ MeV}$, $\alpha(H)_{sc} \cdot M_w = 331.74 \text{ MeV}$; and for equal contributions from e and H we get $\alpha(e+H)_{sc} \cdot M_w \approx 325.715 \text{ MeV}$. For K^\pm the weighted correction gives $\alpha(e+\mu+H)_{sc} \cdot M_w = 335.62 \text{ MeV}$. For the decay process of the neutron as $n \rightarrow p + e + \bar{\nu}$ (100%) the weighted mean coupling constant correction $\Delta\alpha(e+H)_{sc}$ takes care of correction due to the screening effect from the electron and the proton, and for this we get $\alpha(e+H)_{sc} \cdot M_w \approx 325.715 \text{ MeV}$. As the proton does not decay, screening effect correction is not needed in calculating mass of the proton and for this $\alpha_w \cdot M_w$

$\approx 341.19 \text{ MeV}$. In the decay process $\pi^0 \rightarrow 2\gamma$ (98.8%) + $\nu e^+ e^-$ (1.17%), the $(e^+ e^-)$ pair is considered to contribute to the screening effect, and $\Delta\alpha$ (e+photon) corrects screening effect, while for photon as decay product, no correction for screening effect is made as $\Delta\alpha$ (photon)=0. Masses of Δ 's are calculated assuming weak interaction.

5. Results and Discussions

5.1. Construction of Hadron Masses from Weak Decay Processes

Masses of meson nonets, baryon octet and decuplet are constructed by using weighted mean coupling constants of the decay products. From equations (13), (14) and (17), the masses of some hadrons, their zero point energies (ZPE) and PDG values of masses [17] are given in Table 1. The ZPE is explained as contribution to the total mass from the zero point energy and the HO model predicts 9.09% contribution to the total mass from ZPE for the proton. Zero point energies predicted from this model could be confirmed by studying Casimir effect [18]. In the calculation of masses of hadrons, it is found that particles with the same values of hypercharge (Y) and isospin (I) appear at the same distance $X_{op}=N$. λ_{op} , and then hypercharge (Y) of the SM is given as:

$$Y = (2-N) \quad (18)$$

From equation (18) for pions when $Y=0$ and $I=0, \pm 1$, we get $N=2$ and all pions appear at the same distance $X_{op}=2$. λ_{op} in the HOM; for kaons when $Y=1$ and $I=\pm 1/2$, we get $N=1$ and all kaons appear at the same distance $X_{op}=1$. λ_{op} . This redefinition of Y is found to be obeyed in the analysis of mass spectrum of hadrons. In the HO model the change in hypercharge ($\Delta Y=\pm 1$) for $\Delta N_q=0$ and $\Delta d=0$ is explained as change in the position at which the particle appears as $\Delta N=(-/+1)$.

Table 1. Masses of meson nonet, and baryon octet and decuplet and their zero point energies. ZPE = $(d/2) \cdot \hbar \omega$. PDG values of masses are from ref. [17].

Name (quark)	Screening Correction	$(\alpha_{sc}) \times M_w \text{ MeV}$	$N^2 \times D_w$	$(n+d/2)$	Mass MeV	Observed Mass MeV [17]	ZPE MeV
π^\pm (ud)	$\Delta\alpha(\mu)$	338.35	$2^2 \times 10$	(16+1/2)	139.57	139.57 ± 0.00035	4.23
π^0 (uu-dd)/ $2^{1/2}$	$\Delta\alpha$ (1.17% $\nu e e$ +photon)	341.00	$2^2 \times 12$	(18+2/2)	134.979	134.977 ± 0.0005	7.10
K^\pm (us)	$\Delta\alpha$ (e+ μ +H)	335.571	$1^2 \times 52$	(76+1/2)	493.677	493.677 ± 0.016	3.23
K_s^0 (ds+ds)/ $2^{1/2}$	$\Delta\alpha$ (H)	331.74	$1^2 \times 2$	(2+2/2)	497.610	497.611 ± 0.013	165.9
K_L^0 (ds-ds)/ $2^{1/2}$	$\Delta\alpha$ (e+ μ +H)	328.52	$1^2 \times 68$	(102+2/2)	497.611	497.611 ± 0.013	4.83
(p^\pm) (ud)	$\Delta\alpha$ (H)	331.74	$2^2 \times 10$	(93+1/2)	775.44	775.40 ± 0.40	4.15
(p^0) (uu, dd)	$\Delta\alpha$ (H)	331.74	$2^2 \times 20$	(186+2/2)	775.44	775.49 ± 0.34	4.15
(ω^0) (self)	$\Delta\alpha$ (H) $\times 0.99$	331.76	$2^2 \times 16$	(150+2/2)	782.75	782.65 ± 0.12	5.18
(η^0) (self)	$\Delta\alpha$ (H) $\times 0.9923$	331.76	$2^2 \times 10$	(65+2/2)	547.40	547.86 ± 0.017	8.29
p (uud)	No Decay	341.19	$1^2 \times 4$	(10+2/2)	938.2725	938.2720 ± 0.00002	85.30
n (udd)	$\Delta\alpha$ (e+H)	325.715	$1^2 \times 13$	(36+3/2)	939.5625	939.5654 ± 0.00002	12.53
Λ^0 (uds)	$\Delta\alpha$ (H) $\times 0.997$	331.76	$2^2 \times 10$	(133+3/2)	1115.54	1115.68 ± 0.006	4.15
Σ^+ (uus)	$\Delta\alpha$ (H) $\times 0.9988$	331.76	$2^2 \times 3$	(42+2/2)	1188.81	1189.37 ± 0.07	27.65
Σ^0 (uds)	$\Delta\alpha$ (H)	331.74	$2^2 \times 4$	(56+3/2)	1192.19	1192.64 ± 0.02	10.37
Σ^- (dds)	$\Delta\alpha$ (H) $\times 0.9985$	331.76	$2^2 \times 7$	(100+2/2)	1196.70	1197.45 ± 0.08	11.85
Ξ^0 (uss)	$\Delta\alpha$ (H) $\times 0.996$	331.76	$3^2 \times 4$	(141+3/2)	1313.22	1314.86 ± 0.20	4.60
Ξ^- (dss)	$\Delta\alpha$ (H) $\times 0.999$	331.76	$3^2 \times 6$	(214+2/2)	1320.90	1321.71 ± 0.07	6.14
Δ^0 (udd)	$\Delta\alpha$ (H) $\times 0.994$	331.76	$1^2 \times 19$	(69+3/2)	1231.00	1232.00 ± 2.00	4.15
Δ^{++} (uuu)	$\Delta\alpha$ (H) $\times 0.994$	331.76	$1^2 \times 19$	(70+1/2)	1231.00	1232.00 ± 2.00	4.15
Δ^+ (uud)	$\Delta\alpha$ (H) $\times 0.994$	331.76	$1^2 \times 7$	(25+2/2)	1232.25	1232.00 ± 2.00	47.39
Ω^- (sss)	$\Delta\alpha$ (H)	331.74	$4^2 \times 3$	(241+2/2)	1672.52	1672.45 ± 0.29	6.91

5.2. The Mass Difference

From equation (13) the mass difference (MD) between two particles for $\Delta Y=0$ and $\Delta I_z=\pm 1$ is given as

$$MD=[(\alpha_{sc}.M_w). [(n+d/2)/(N^2D_w)]]_H-[(\alpha_{sc}.M_w)[(n+d/2)/(N^2D_w)]]_L \quad (19)$$

Where, H and L stand for the heavier and the lighter particle respectively.

In the HO model masses of particles arise due to coupling of the quark field of a particle with Higgs field via W^\pm boson. In the SM mass difference (MD) between pairs of particles arising from change in $\Delta I_z=\pm 1$ for $\Delta Y=0$ is attributed to the electromagnetic interaction i.e. for $\Delta Q=\pm 1$, and in the HO model breaking of gauge symmetry changes dimension $\Delta d \leftrightarrow \Delta Q$ of the 3-dimensional HO.

Table 2. Mass Differences between pairs of particles arising due to isospin symmetry breaking for $\Delta Q=(-/+)$ 1 in the SM and $\Delta d = \pm 1$ in HO Model.

Names	Mass Difference from HO Model MeV	Observed Mass Difference MeV [17]
$(n^\pm - p^\pm)$	4.59	4.59
$(k^0 - k^\pm)$	3.93	3.93
$(\rho^0 - \rho^\pm)$	0.00	0.09
$(m_n - m_p)$	1.29	1.29
$(\Xi^- - \Xi^0)$	7.68	6.84
$(\Sigma^0 - \Sigma^\pm)$	3.38	3.27
$\Sigma^- - \Sigma^0$	4.51	4.81
$(\Delta^+ - \Delta^0)$	1.25	X
$(\Delta^+ - \Delta^{++})$	1.25	X

The MD's between pairs of particles for the breaking of the I_z symmetry for $\Delta(N_q)=0$ and $\Delta Y=0$ in the SM and $\Delta d=\pm 1$ in the HO model are given in Table 2. In the HO model the MD is attributed to the change in the alignment of the isospins between the two particles.

6. Conclusions

The mass spectrum of hadrons from the Harmonic Oscillator (HO) model completely agrees with the experimental values at low energy level and shows for the first time how neutrons, protons and all other hadrons get their masses by a Higgs like mechanism in the weak interaction mediated by W^\pm boson.

The HO model revalidates the PDG data for the decay processes of particles and radiation corrections of the weak coupling constant.

The model gives hypercharge (Y) of the Standard Model (SM) with a physical interpretation. Particles with the same value of Y in the SM appear at the distance $X_{op}=N.\lambda_{op}$ with the same value of N when $Y=(2-N)$. according to the HO model.

While the stride of Lattice Quantum Chromodynamical (LQCD) models throws more light in the progress of hadron physics [19, 20, 21], the Harmonic Oscillator (HO) model confirms that masses of neutrons, protons and all other hadrons are acquired by the Higgs like mechanism.

The vacuum quark condensates corresponding to the low energy QCD vacuum is interpreted as the “smeared collective effects” containing both perturbative and non-perturbative parts. Perturbative part gives the quark scalar condensate [22]. With 1-loop perturbative calculation a recent study reports that u, d, and s quark masses contribute 9% as quark scalar condensate to the proton mass [23, 24], and this is in agreement with 9.09% contribution from the zero point energy to the proton mass in the HO model. Rest of the mass of the

proton comes from the dynamics of quarks and gluons in the LQCD model and from the excitation of the “hadron seed” in the HO model.

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