
Quadrupole Moments for Oxygen Isotopes in *p*-Shell and *psd* Shell with Using Different Effective Charges

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Abstract: Quadrupole Q moments and effective charges are calculated for ^{13}O , ^{15}O , ^{17}O , ^{18}O and ^{19}O neutron rich nuclei using shell model calculations. Excitations out of major shell space are taken into account through a microscopic theory which is called core-polarization effects. The simple harmonic oscillator potential is used to generate the single particle matrix elements of $^{13,15,17,18,19}\text{O}$. The results showed a rapprochement between theoretical calculations with different effective charges and experimental data for most of the proposed isotopes in this work.

Keywords: Effective Charges and Quadrupole Moments of *p* and *psd*-shell Isotopes ($Z=8$): *p* and *psd*- Shell Model

1. Introduction

Theoretically, studying unstable nuclei is one of the powerful ways to clarify the interaction between hadrons in the nucleus. From the data for the electric and magnetic moments we can study the nuclear wave functions. Quadrupole-moments (Q -moments) give a measure of the deviation of the nuclear charge distribution from spherical symmetry [1].

Electromagnetic observables will provide useful information to study the structure of nuclei, not only ground states but also excited states. Namely, these observables are expected to pin down precise information of deformations and unknown spin parities of both stable and unstable nuclei since the deformation is intimately related to observables such as Q moments and $E2$ transitions [2].

Shell model calculations are carried out within a model space in which the nucleons are restricted to occupy a few orbits. If appropriate effective operators are used taking into account the effect of the larger model space, the shell model provides a reasonable description of these observables [3]. A well-known example is the effective charge for the electric quadrupole ($E2$) observables [4].

The effective charges have been used commonly in the shell model calculations to study Q moments. The conventional approach to supply this added ingredient to shell model wave functions is to redefine the properties of

valence nucleons from those exhibited by actual nucleons in free space to model-effective values [5]. Effective charges are introduced for evaluating $E2$ transitions in shell-model studies to take into account effects of model-space truncation. A systematic analysis has been made for observed $B(E2)$ values with shell-model wave functions using a least-squares fit with two free parameters gave standard proton and neutron effective charges, $=1.3, 0.5(e)$ [6], in *sd*-shell nuclei.

An interpretation of effective charge in valence nuclear models was proposed in which they are seen as proportional to derivatives of the "collectivity" with charges in proton or neutron number [7].

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which allows one particle-one hole ($1p-1h$) excitations of the core and also of the model space to describe these Q properties. These effects provide a more practical alternative for calculating nuclear collectivity. These effects are essential in describing transitions involving collective modes such as $E2$ transition between states in the ground-state rotational band, such as in ^{18}O [8].

The effective charges have been used commonly in the shell model calculations to study Q moments. In this paper, shell model calculations are performed with two effective

interactions, Cohen-Kurath (CK) interactions in region p model space (1965) [9, 10] and Millener-Kurath PSDMK interaction(1975) in region psd model space (1975) [11].

2. Theory

The one-body electric multipole transition operator with multipolarity J for a nucleon is given by [12]

$$\hat{O}_{JM}(\vec{r})_k = r_k^J Y_{JM}(\Omega_K) \quad (1)$$

The reduced single-particle matrix element of the transition operator given in eq. (1) in spin space is given by

$$\langle j' || \hat{O}_{Jt_z} || j \rangle = e(t_z) \langle j' || Y_J || j \rangle \langle n'l' | r^J | n l \rangle \quad (2)$$

where;

$e(t_z) = \frac{1+\tau_z(k)}{2}$ is the electric charge of the k -th nucleon, where $t_z = 1/2$ for a proton and $t_z = -1/2$ for a neutron with

$$\tau_z(k) = 2t_z(k)$$

$$\tau_z |p\rangle = |p\rangle \text{ and } \tau_z |n\rangle = -|n\rangle$$

The reduce matrix element of the spherical harmonics part Y_J is given by [12]

$$\begin{aligned} \langle j' || Y_J || j \rangle &= (-1)^{j+1/2} \sqrt{\frac{(2j+1)(2J+1)(2j'+1)}{4\pi}} \\ &\times \begin{pmatrix} j' & J & j \\ 1/2 & 0 & -1/2 \end{pmatrix} \times \frac{1}{2} [1 + (-1)^{l+J+l'}] \end{aligned} \quad (3)$$

and the radial part of the matrix element of a HO potential is:

$$\langle n'l' | r^J | n l \rangle = \int_0^\infty dr r^J r^2 R_{n'l'}(r) R_{n l}(r) \quad (4)$$

Eq. (4) can be written as:

$$\langle n'l' | r^J | n l \rangle = \int_0^\infty dr r^\mu R_{n'l'}(r) R_{n l}(r) \quad (5)$$

where $\mu = J + 2$.

The radial integral eq. (5) can solved analytically for a HO radial wave functions as [13].

$$\begin{aligned} &\int_0^\infty dr r^\mu R_{n'l'}(r) R_{n l}(r) \\ &= \frac{2^3}{2^2 2} \sqrt{(n'-1)!(n-1)!} \sqrt{\Gamma(n'+l'+\frac{1}{2}) \Gamma(n+l+\frac{1}{2})} \\ &\times \sum_{k'=0}^{n'-1} \sum_{k=0}^{n-1} \frac{(-1)^{k'+k}}{(n'-k'-1)!(n-k-1)!k'!k! \Gamma(k'+l'+\frac{3}{2}) \Gamma(k+l+\frac{3}{2})} \\ &\times \frac{b^{2m+1}}{b^{3+l'+l+2k'+2k}} \Gamma(m+\frac{1}{2}) \end{aligned} \quad (6)$$

where;

$$m = \frac{1}{2}(\mu + l' + l + 2k' + 2k)$$

$$\frac{b^{2m+1}}{b^{3+l'+l+2k'+2k}} = b^{\mu-2}$$

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n-1)!!}{2^n} \sqrt{\pi}$$

The reduced electric matrix element between the initial and final nuclear states is [12]

$$M(EJ) = \langle J_f || \sum_k e(k) \hat{O}_J(\vec{r})_k || J_i \rangle \quad (7)$$

where $e(k)$ is the electric charge for the k -th nucleon. Since $e(k) = 0$ for neutron, there should appear no direct contribution from neutrons; however, this point requires further attention: The addition of a valence neutron will induce polarization of the core into configurations outside the adopted model space. Such core polarization effect is included through perturbation theory which gives effective charges for the proton and neutron. The reduced electric matrix element can be written in terms of the proton and neutron contributions

$$M(EJ) = \sum_{t_z} e(t_z) \langle J_f || \hat{O}_J(\vec{r}, t_z) || J_i \rangle \quad (8)$$

where $\langle J_f || \hat{O}_J(\vec{r}, t_z) || J_i \rangle$ is the electric matrix element which is expressed as the sum of the products of the one-body density matrix (*OBDM*) times the single-particle matrix elements,

$$\langle J_f || \hat{O}_J(\vec{r}, t_z) || J_i \rangle = \sum_{jj'} \text{OBDM}(J_i, J_f, J, t_z, j, j') \langle j' || \hat{O}_J(\vec{r}, t_z) || j \rangle \quad (9)$$

with j and j' label single-particle states for the shell model space.

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation to describe EJ excitation: these are called *CP* effects. The reduced matrix elements of the transition operator is expressed as a sum of the model space (*MS*) contribution and the *CP* contribution, as follows:

$$M(EJ) = \sum_{t_z} [e(t_z) \langle J_f || \hat{O}_J(\vec{r}, t_z) || J_i \rangle_{MS} + e \langle J_f || \Delta \hat{O}_J(\vec{r}, t_z) || J_i \rangle_{CP}] \quad (10)$$

Similarly, the *CP* electric matrix element is expressed as the sum of the products of the *OBDM* times the single-particle matrix elements [14],

$$\langle J_f || \Delta \hat{O}_J(\vec{r}, t_z) || J_i \rangle = \sum_{jj'} \text{OBDM}(J_i, J_f, J, t_z, j, j') \langle j' || \Delta \hat{O}_J(\vec{r}, t_z) || j \rangle \quad (11)$$

The single- particle matrix element of the *CP* term is

$$\langle j' || \Delta \hat{O}_J || j \rangle = \langle j' || \hat{O}_J \frac{\hat{Q}}{E_i - H_0} V_{res} || j \rangle + \langle j' || V_{res} \frac{\hat{Q}}{E_i - H_0} \hat{O}_J || j \rangle \quad (12)$$

where the operator \hat{Q} is the projection operator onto the space outside the model space. The single particle *CP* terms given in eq. (10) are written as [12]

$$\langle j' || \Delta \hat{O}_J || j \rangle = \sum_{j_1 j_2 \lambda} \frac{(-1)^{j+j_2+\lambda}}{\varepsilon_j - \varepsilon_{j'} - \varepsilon_{j_1} + \varepsilon_{j_2}} (2\lambda + 1) \begin{Bmatrix} j' & j & J \\ j_2 & j_1 & \lambda \end{Bmatrix} \\ \times \sqrt{(1 + \delta_{j_1 j'}) (1 + \delta_{j_2 j})} \langle j' j_1 | V_{res} | j j_2 \rangle_\lambda \langle j_2 | \hat{O}_J | j_1 \rangle \quad (13)$$

+terms with j_1 and j_2 exchanged with an overall minus sign, where the index j_1 runs over particle states and j_2 over hole states and ε is the single-particle energy. For the residual two-body interaction V_{res} , the two-body M3Y interaction of Bertsch *et al.* [15] is adopted.

The electric matrix element can be represented in terms of only the model space matrix elements by assigning effective charges ($e^{eff}(t_z)$) to the protons and neutrons which are active in the model space,

$$M(EJ) = \sum_{i_z} e^{eff}(t_z) \langle J_f || \hat{O}_2(\vec{r}, t_z) || J_i \rangle_{MS} \quad (14)$$

They formulated an expression for the effective charges to explicitly include neutron excess via [16]

$$e^{eff}(t_z) = e(t_z) + e\delta e(t_z), \delta e(t_z) = Z/A - 0.32(N - Z)/A - 2t_z[0.32 - 0.3(N - Z)/A]. \quad (15)$$

The electric quadrupole moment in a state $|J = 2 M = 0\rangle$ for $J_i = J_f$ is [12]:

$$Q(J = 2) = \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{16\pi}{5}} M(EJ) \quad (16)$$

3. Results and Discussion

The quadrupole moment gives a useful measure of how the core is polarized especially if the valence nucleons are neutrons which do not directly participate to the electric quadrupole moment. Shell model calculations are performed with NuShellX [17] with the p model space for neutron number $N \leq 8$, which covered the orbits $1s_{1/2}, 1p_{1/2}$ and $1p_{3/2}$ and psd model space for $N > 8$. Results based on the p -shell interactions CK (Cohen-Kurath) for P -shell model space [9, 10] and the interaction $PSDMK$ [11] for psd model space with valence (frozen) protons are restricted to the p shell and $N-8$ neutrons to the psd shell. The 8 neutrons are frozen in s and p shells (full p valence space for $Z-8$ protons and full psd valence space for $N-20$ neutrons).

The Q -moments for the p -shell and sd -shell nuclei using the matrix elements calculated with the shell model and harmonic oscillator wave functions.

The one body matrix elements (OBME) are calculated with harmonic oscillator parameter and the radial wave functions for the single-particle matrix elements are calculated with the HO potential. The size parameters b is calculated for each nucleus with mass number A as:

$$b = \sqrt{\frac{\hbar}{M_P \omega}}, \hbar\omega = 45A^{-1/3} - 25A^{-2/3} \text{ Mp} = \text{mass of proton} \quad [18].$$

The quadrupole moments are calculated for N isotopes with mass number $A = 13, 15, 17, 18, 19$ and with neutron number $N = 5, 7, 9, 10, 11$, respectively. The proton and neutron effective charges Bohr- Mottelson are calculated according to equation (15) and tabulated in Table 1.

Figure 1 and Table 1 shows the quadrupole moments in O isotopes with three sets of effective charges, one Bohr-Mottelson effective charges (B-M) [16], standard effective charges (ST) for proton and neutron $ep = 1.36$ and $en = 0.45$ [19], respectively and also try the conventional effective charges for p shell and psd shell, $ep = 1.3$ and $en = 0.5$ [6], to calculate Q -moments and comparison with experimental values are taken from Refs. [20, 21].

The results of the Q -moments are displayed in Table 1 in comparison with the experimental values. The Q moment for ^{13}O with set effective charges are the calculated values overestimate the measured values; this value shows a small prolate deformation.

For $A = 15$ ($N = 7$), no experimental value is available, and the calculated Q -moment is $-5.89, -2.96$ and $-3.92 e fm^2$ with effective charges B-M, standard effective charges ST and conventional effective charges, respectively. The calculated for ^{17}O were underestimate the measured values, this value shows a small oblate deformation, while calculated Q -moment for ^{18}O were overestimate the measured values and shown a small oblate deformation.

The calculated values for ^{19}O agree very well with the measured values and within the experimental error for ^{19}O . The calculated and measured Q moments are shown in Figure 1 as a function of neutron number N .

4. Conclusions

The results that obtained from this work have been deduced by using the shell model calculations. The main conclusions that may be drawn from the calculations are briefly summarized as:

(a)- The p model space for neutron number $N \leq 8$, which covered the orbits $1s_{1/2}, 1p_{1/2}$ and $1p_{3/2}$ and psd model space for $N > 8$.

(b)- Two effective interactions, Cohen-Kurath (CK) interactions in region p model space and Millener-Kurath $PSDMK$ interaction in region psd model space.

(c)- In our calculations employed different effective charges.

(d)- Calculations of Q moments with standard effective charges and conventional effective charges are better than of B-M effective charge.

Table 1. Quadrupole moments in units efm^2 of calculated with HO potential for Oxygen (O) isotopes ($Z=8$). Experimental Quadrupole moments are taken from Ref. [20, 21]. Quadrupole moments calculated with effective charges of B-M model [16], standard effective charges for proton and neutron 1.36, 0.45 [19], respectively and convention effective charges proton and neutron 1.3, 0.5 [6], respectively are presented.

A, N ₈ O	J_i^π	$b(fm)$	e_p	e_n	$Q_{theo. B-M, ST}$	$Q_{theo. ep=1.3, en=0.5}$	$Q_{exp.}$
13, 5	$3/2^-$	1.68	1.3	1.08	3.49, 1.45	1.62	+1.1±0.08
15, 7	$5/2^+$	1.713	1.21	0.89	-5.89, -2.96	-3.92	
17, 9	$5/2^+$	1.739	1.15	0.75	-4.53, -2.7	-3.0	-2.5±0.02
18, 10	2^+	1.751	1.12	0.7	-3.21, -2.07	-2.3	-3.6±0.09
19, 11	$5/2^+$	1.763	1.1	0.64	-0.41, -0.28	-0.32	-0.37±0.04 ^a

a [21] Prog. Theo. Phys. 102, 5 (1999).

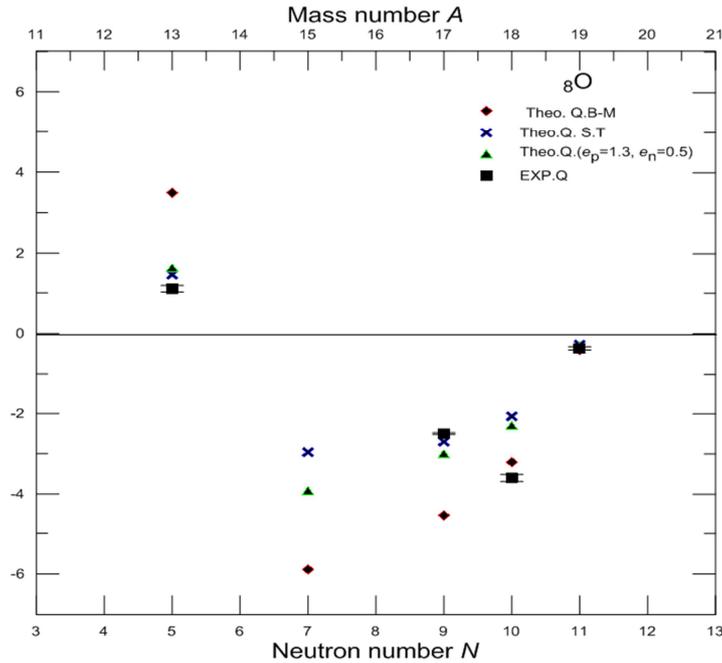


Figure 1. Experimental [20, 21] and theoretical quadrupole moments with versus Neutron and mass number for O isotopes.

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