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Quantum Forces in a Relativistic Entangled "Space-Time" State in Complex Hamiltonian Dynamics

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Abstract: Based on complex quantum Hamilton-Jacobi theory and its natural complex spacetime configuration, we realized new states for both positive and negative values of energy and momentum, and we discuss this complex configuration in a relativistic entangled "space-time" state and the resultant 12 extra wave functions than the four solutions of Dirac equation for a free particle. Accordingly, we calculate the quantum forces between particles and antiparticles in a relativistic entangled "space-time" state.

Keywords: Complex Hamiltonian Dynamics, Entanglement, Quantum Force

1. Introduction

The origin of complex spacetime stems from complex time, as first proposed by El Naschie [1], according to a special case of E^{∞} theory [2-4] and then applied by C. D. Yang in a series of papers [5-14]. The complex spacetime proposed by Yang is:

$$x^{\mu} = x_R^{\mu} + ix_I^{\mu},\ \ x_R^{\mu}, x_I^{\mu} \in \mathbb{R};\ \ x^{\mu} = (ct, x, y, z) \qquad (1)$$

mentioning that the boundary between classical and quantum mechanics could be broken, since the quantum operators are derived from Hamilton equation of motion [5, 6]. Moreover, it has been shown that extending special relativity to the complex domain leads to relativistic quantum mechanics and considering both relativistic and quantum effects, the Klein-Gordon equation could be derived as a special form of the Hamilton-Jacobi (H-J) equation. It was also mentioned that the entangled state, causing the faster-than-light links, is a consequence of an entangled energy, plus a quantum potential, i.e. $E^2 + 2m_0c^2Q$, resulting in a constant quantity. This quantity if used in the relativistic H-J equations, may describe quantum multiple trajectories.

The basis of our work here is on a paper of Yang [13] in which after characterizing the complex time t involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies could be realized. However, in a previous work [15] we have realized new states for both positive and negative values of energy and momentum, and we have discussed the complex spacetime in a relativistic entangled "space-time" state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle, and then introducing a new physical interpretation, we have distinguished physical and unphysical particles and antiparticles, due to the concept of Krein space quantization.

In this paper, first we will have a review on quantum motion for relativistic free particles and consequences of complex spacetime in a relativistic entangled "space-time" state. Then, we will calculate the quantum forces between particles and antiparticles in a relativistic entangled "space-time" state.

2. Quantum Motion for Relativistic Free Particles

We begin with the relation (3.6) in Ref. [13] by C. D. Yang:

$$\psi(t,x,y,z) = \left(C_0^{+} e^{i(k_0/\hbar)t} + C_0^{-} e^{-i(k_0/\hbar)t}\right) \left(C_1^{+}e^{ik_1x} + C_1^{-}e^{-ik_1x}\right)\left(C_2^{+}e^{ik_2y} + C_2^{-}e^{-ik_2y}\right)\left(C_3^{+}e^{ik_3z} + C_3^{-}e^{-ik_3z}\right) \qquad (2)$$

the relation (4.1) of Ref. [13]:

$$\psi(t,x,y,z)=\left(C_0^+\,e^{i\frac{k_0}{\hbar}t}+C_0^-\,e^{-i\frac{k_0}{\hbar}t}\right)e^{-i(k_1x+k_2y+k_3z)} \tag{3}$$

which can be rewritten in terms of energy and momentum as:

$$\psi(t,\vec{r})=\left(C_0^+\,e^{i\frac{E}{\hbar}t}+C_0^-\,e^{-i\frac{E}{\hbar}t}\right)e^{-i\frac{\vec{p}.\vec{r}}{\hbar}} \tag{4}$$

the relation (4.13) of Ref. [13]:

$$E=cp_0=m_0c^2\frac{dt}{d\tau}=k_0\frac{C_0^+\,e^{i\frac{k_0}{\hbar}t}-C_0^-\,e^{-i\frac{k_0}{\hbar}t}}{C_0^+\,e^{i\frac{k_0}{\hbar}t}+C_0^-\,e^{-i\frac{k_0}{\hbar}t}} \tag{5}$$

indicating that the local time t implies a complex energy E:

$$E\frac{dE}{dt}=-\frac{\hbar}{2i}\frac{d^2E}{dt^2} \tag{6}$$

the relation (4.18) of Ref. [13]:

$$\frac{\hbar}{i}\frac{dE}{dt}+E^2=k_0^2 \tag{7}$$

showing that the energy E solved from Eq. (7) is of complex value and is given exactly by Eq. (5). And finally, the relations (4.19) and (4.20) of Ref. [13]:

$$Q(t)=\frac{\hbar}{2m_0ci}\frac{dp_0}{dt}=\frac{k_0^2}{2m_0c^2}\frac{4\alpha}{\left(\alpha e^{i\frac{k_0}{\hbar}t}+e^{-i\frac{k_0}{\hbar}t}\right)^2} \tag{8}$$

$$E^2(t)+2m_0c^2Q(t)=k_0^2=cte \tag{9}$$

In which, both E^2 and Q are time dependent, but their summation appears to be a constant. Now, we can write the Eq. (9) as:

$$\begin{aligned}E(t)&=\pm\sqrt{k_0^2-2m_0c^2Q(t)}\\&=\pm\sqrt{(m_0c^2)^2+c^2p^2-2m_0c^2Q(t)}\equiv\pm E_\pm\end{aligned} \tag{10}$$

It is clear that for any time t, there are two momenta $(p\rangle 0\,,\,p\langle 0)$ and two energies $(E_+\rangle 0\,,\,E_-\langle 0)$, and we can see that in this general form of energy, the quantum potential $Q(t)$ is nonzero. Also, we should notice that in Eqs. (3.13)-(3.16) of Ref. [13]:

$$\psi_1(t,x,y,z)=Ce^{i(Et-p.r)/\hbar},\ \psi_2(t,x,y,z)=C\,e^{i(-Et-p.r)/\hbar}$$

$$\psi_3(t,x,y,z)=Ce^{i(Et+p.r)/\hbar},\ \psi_4(t,x,y,z)=Ce^{i(-Et+p.r)/\hbar} \tag{11}$$

the derived four wave functions were the eigenfunctions solved from the Dirac equation for a free particle and there was considered no entanglement and $Q(t)=0$; we call it space-time state. In Eq. (3) or (4), the entanglement of time was considered and the wave function was written only for positive values of energy and momentum with $Q(t)\neq 0$; we call it space-entangled time state.

In the next section, we will realize another states for both values of positive and negative energy and momentum due to Eq. (10).

3. Complex Spacetime in a Relativistic Entangled "Space-Time" State

In this section, we will have a review on a previous work [15]. First we rewrite again the Eqs. (4) and (2) in the following form, respectively:

$$\psi(t,\vec{r})=\left(C_0^+\,e^{i\frac{Et}{\hbar}}+C_0^-e^{-i\frac{Et}{\hbar}}\right)e^{i\frac{\vec{p}.\vec{r}}{\hbar}} \tag{12}$$

$$\psi(t,\vec{r})=\left(C_0^+\,e^{i\frac{Et}{\hbar}}+C_0^-\,e^{-i\frac{Et}{\hbar}}\right)\left(C^+\,e^{i\frac{\vec{p}.\vec{r}}{\hbar}}+C^-\,e^{-i\frac{\vec{p}.\vec{r}}{\hbar}}\right) \tag{13}$$

Then, by using Eq. (9) we can write them down in terms of both positive and negative values of energy and momentum (i.e. E_+, E_-, $+p$, $-p$):

$$\psi(t,\vec{r})=\left(C_0^+\,e^{i\frac{E_\pm t}{\hbar}}+C_0^-e^{-i\frac{E_\pm t}{\hbar}}\right)e^{i\frac{(\pm\vec{p}).\vec{r}}{\hbar}} \tag{14}$$

$$\psi(t,\vec{r})=\left(C_0^+\,e^{i\frac{E_\pm t}{\hbar}}+C_0^-\,e^{-i\frac{E_\pm t}{\hbar}}\right)\left(C^+\,e^{i\frac{(\pm\vec{p}).\vec{r}}{\hbar}}+C^-\,e^{-i\frac{(\pm\vec{p}).\vec{r}}{\hbar}}\right) \tag{15}$$

Now, we can realize four states:

a) *Space-time* state $(\psi_1,\psi_2,\psi_3,\psi_4)$; $Q(t)=0$

b) *Space-entangled time* state $(\psi_5,\psi_6,\psi_7,\psi_8)$; $Q(t)\neq 0$

c) *Entangled space-time* state $(\psi_9,\psi_{10},\psi_{11},\psi_{12})$; $Q(t)\neq 0$

d) *Entangled space-entangled time* state $(\psi_{13},\psi_{14},\psi_{15},\psi_{16})$; $Q(t)\neq 0$

leading to the following 16 wave functions [15]:

$$\psi_1=Ce^{\frac{i}{\hbar}(Et-\vec{p}.\vec{r})};\ E>0,\ p>0$$

$$\psi_2=Ce^{\frac{i}{\hbar}(Et-(-\vec{p}).\vec{r})};\ E>0,\ p<0$$

$$\psi_3=Ce^{\frac{i}{\hbar}((-E)t-\vec{p}.\vec{r})};\ E<0,\ p>0$$

$$\psi_4=Ce^{\frac{i}{\hbar}((-E)t-(-\vec{p}).\vec{r})};\ E<0,\ p<0$$

$$\psi_5=\left(C_0^+\,e^{\frac{i}{\hbar}E_+t}+C_0^-\,e^{-\frac{i}{\hbar}E_+t}\right)C^-e^{-\frac{i}{\hbar}\vec{p}.\vec{r}};\ E_+>0,\ p>0$$

$$\psi_6 = (C_0^+ e^{\frac{i}{\hbar}E_+t} + C_0^- e^{-\frac{i}{\hbar}E_+t})\, C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \;;\; E_+ > 0,\ p < 0$$

$$\psi_7 = \left(C_0^+ e^{\frac{i}{\hbar}E_-t} + C_0^- e^{-\frac{i}{\hbar}E_-t} \right) C^- e^{-\frac{i}{\hbar}\vec{p}.\vec{r}} \;;\; E_- < 0,\ p > 0$$

$$\psi_8 = \left(C_0^+ e^{\frac{i}{\hbar}E_-t} + C_0^- e^{-\frac{i}{\hbar}E_-t} \right) C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \;;\; E_- < 0,\ p < 0$$

$$\psi_9 = C_0^+ e^{\frac{i}{\hbar}E_+t} \left(C^+ e^{\frac{i}{\hbar}\vec{p}.\vec{r}} + C^- e^{-\frac{i}{\hbar}\vec{p}.\vec{r}} \right);\ E_+ > 0,\ p > 0$$

$$\psi_{10} = C_0^+ e^{\frac{i}{\hbar}E_+t} \left(C^+ e^{\frac{i}{\hbar}(-\vec{p}).\vec{r}} + C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \right);\ E_+ > 0,\ p < 0$$

$$\psi_{11} = C_0^+ e^{\frac{i}{\hbar}E_-t} \left(C^+ e^{\frac{i}{\hbar}\vec{p}.\vec{r}} + C^- e^{-\frac{i}{\hbar}\vec{p}.\vec{r}} \right);\ E_- < 0,\ p > 0$$

$$\psi_{12} = C_0^+ e^{\frac{i}{\hbar}E_-t} \left(C^+ e^{\frac{i}{\hbar}(-\vec{p}).\vec{r}} + C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \right);\ E_- < 0,\ p < 0$$

$$\psi_{13} = \left(C_0^+ e^{\frac{i}{\hbar}E_+t} + C_0^- e^{-\frac{i}{\hbar}E_+t} \right)\left(C^+ e^{\frac{i}{\hbar}\vec{p}.\vec{r}} + C^- e^{-\frac{i}{\hbar}\vec{p}.\vec{r}} \right); E_+ > 0, p > 0$$

$$\psi_{14} = \left(C_0^+ e^{\frac{i}{\hbar}E_+t} + C_0^- e^{-\frac{i}{\hbar}E_+t} \right)\left(C^+ e^{\frac{i}{\hbar}(-\vec{p}).\vec{r}} + C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \right); E_+ > 0, p < 0$$

$$\psi_{15} = \left(C_0^+ e^{\frac{i}{\hbar}E_-t} + C_0^- e^{-\frac{i}{\hbar}E_-t} \right)\left(C^+ e^{\frac{i}{\hbar}\vec{p}.\vec{r}} + C^- e^{-\frac{i}{\hbar}\vec{p}.\vec{r}} \right);\ E_- < 0, p > 0$$

$$\psi_{16} = \left(C_0^+ e^{\frac{i}{\hbar}E_-t} + C_0^- e^{-\frac{i}{\hbar}E_-t} \right)\left(C^+ e^{\frac{i}{\hbar}(-\vec{p}).\vec{r}} + C^- e^{-\frac{i}{\hbar}(-\vec{p}).\vec{r}} \right);\ E_- < 0,\ p < 0$$

We can describe the wave functions ψ_1 , ψ_2 , ... , ψ_{16} after doing the multiplications as the following:

a) ψ_1 : *particle* with positive momentum,
   ψ_2 : *particle* with negative momentum,
   ψ_3 : *antiparticle* with positive momentum,
   ψ_4 : *antiparticle* with negative momentum.

b) ψ_5 , ψ_7 : *entangled particle-antiparticle* with positive momenta,
   ψ_6 , ψ_8 : *entangled particle-antiparticle* with negative momenta.

c) ψ_9 , ψ_{10} : *entangled particle-particle* with opposite momenta,

d) ψ_{11} , ψ_{12} : *entangled antiparticle-antiparticle* with opposite momenta.

e) ψ_{13} , ψ_{14} , ψ_{15} , ψ_{16} : entanglement of (*entangled particle-antiparticle* with positive momenta) to (*entangled particle-antiparticle* with negative momenta) or entanglement of (*entangled particle-particle* with opposite momenta) to (*entangled antiparticle-antiparticle* with opposite momenta).

It can be seen that the entanglement of two particles or two antiparticles could be done only with opposite momenta, but the entanglement of particle and antiparticle could be done only with the same momenta. Moreover, we can deduce from (b) and (c) that the entanglement of time causes the entanglement of particle with antiparticle, and the entanglement of space causes the particle-particle or antiparticle-antiparticle entanglement. However, we encounter to a confliction, here. Empirical experiments demonstrate the quantum correlation at a distance of a particle-antiparticle system like kaon and antikaon system which are entwined. Therefore, at this stage, we want to introduce a parallel approach to correct all the being results. It seems as if something is missed here.

Due to the theory of Dirac, antiparticles are believed to be particles of negative energy. But, due to the fact that antiparticles are detectable, so the physical antiparticles must be of positive energies. Moreover, according to above results, there are both positive and negative energy states. However, it seems that taking the negative energies as antiparticles, is not covering all the underlying physics [11]. So, we propose that it is rational to accept that positive energy belongs to physical particles and negative energy belongs to unphysical particles. Then, we can say that the solutions of Dirac equation describe both physical particles and antiparticles with positive energy and both unphysical particles and antiparticles with negative energy. Consequently, we should modify the descriptions of wave functions ψ_1 , ψ_2 , ... , ψ_{16} according to the following:

a) ψ_1 : *physical particle* or *antiparticle* with $E_+ > 0,\ P > 0$.
   ψ_2 : *physical particle* or *antiparticle* with $E_+ > 0, P < 0$.
   ψ_3 : *unphysical particle* or *antiparticle* with $E_- < 0, P > 0$.
   ψ_4 : *unphysical particle* or *antiparticle* with $E_- < 0, P < 0$.

b) ψ_5 , ψ_6 : *entangled physical (particle-particle* or *antiparticle-antiparticle* or *particle-antiparticle* or *antiparticle-particle* with $E_+ > 0$ and $P > 0$, or $P < 0$

ψ_7, ψ_8 : *entangled unphysical (particle-particle or antiparticle-antiparticle* or *particle-antiparticle* or *antiparticle-particle*) with $E_- < 0$ and $P > 0$, or $P < 0$

c) ψ_9, ψ_{10} : entangled physical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with $E_+ > 0$ and opposite momenta.

ψ_{11}, ψ_{12} : entangled unphysical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle) with $E_- < 0$ and opposite momenta.

ψ_{13}, ψ_{14} : entanglement of two entangled physical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle) with $E_+ > 0$ and the same or opposite momenta.

ψ_{15}, ψ_{16} : entanglement of two entangled unphysical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle) with $E_- < 0$ and the same or opposite momenta.

As we can see, the wave functions ψ_9, ψ_{10} in part (cc) can describe an entangled particle-antiparticle with $E_+ > 0$ and opposite momenta which are in accordance with experimental results i.e. an EPR system like entwined kaon and antikaon system [31].

As a result, we see that negative energies could be as important as positive ones.

4. Quantum Forces Between Particles and Antiparticles in a Relativistic Entangled "Space-Time" State

In this section, we want to study the quantum forces between different states of particle and antiparticle. Before doing that, we first mention that in relation (4.19) of Ref. [13], i.e. the relation (8) of the present paper, there has been only considered the time contribution, since there has been used only k_0 and t. Therefore, α should be α_0, i.e. the time contribution:

$$Q(t) = \frac{\hbar}{2m_0 ci}\frac{dp_0}{dt} = \frac{k_0^2}{2m_0 c^2}\frac{4\alpha_0}{\left(\alpha e^{i\frac{k_0}{\hbar}t} + e^{-i\frac{k_0}{\hbar}t}\right)^2}$$

In other words, due to the relation (3.18) of Ref. [13], the general relation for quantum force is:

$$Q\,(t,x,y,z) = -\frac{\hbar^2}{2m_0}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\ln\,\psi(t,x,y,z)$$

or

$$Q\,(t,\vec{r}) = -\frac{\hbar^2}{2m_0}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \vec{r}^2}\right)\ln\,\psi(t,\vec{r}) \tag{16}$$

which incorporates both time & space, and only the time part of the relation (16) was affected on the relation (4.1) of Ref [13]:

$$\psi(t,\vec{r}) = \left(C_0^+\,e^{i\frac{E_+ t}{\hbar}} + C_0^- e^{-i\frac{E_+ t}{\hbar}}\right)e^{-i\frac{\vec{p}.\vec{r}}{\hbar}} \quad ; \quad \left(E_+ > 0\,,\ p > 0\right) \quad ; \quad \alpha_0 = \frac{C_0^+}{C_0^-} \tag{17}$$

leading to the relation (8). Now, if we consider the effect of the space part of the relation (16) on the relation (13), the most general form of the quantum potential and the associated quantum force, considering $C^+ \neq 0$ and $C^- \neq 0$, is obtained:

$$Q(t,\vec{r}) = \frac{k_0^2}{2m_0c^2}\frac{4\alpha_0}{\left(\alpha_0\,e^{i\frac{k_0}{\hbar}t} + e^{-i\frac{k_0}{\hbar}t}\right)^2} - \frac{p^2}{2m_0}\frac{4\alpha_i}{\left(\alpha_i\,e^{i\frac{p.r}{\hbar}} + e^{-i\frac{p.r}{\hbar}}\right)^2};\ (i = 1,2,3) \tag{18}$$

or with $E \equiv k_0$:

$$Q(t,\vec{r}) = \frac{2}{m_0}\left[\frac{\left(\frac{E^2}{c^2}\right)\alpha_0}{\left(\alpha_0\,e^{i\frac{E}{\hbar}t} + e^{-i\frac{E}{\hbar}t}\right)^2} - \frac{p^2\alpha}{\left(\alpha\,e^{i\frac{p.r}{\hbar}} + e^{-i\frac{p.r}{\hbar}}\right)^2}\right];\ (\alpha_i \equiv \alpha) \tag{19}$$

$$F=-\frac{dQ}{dr}=\frac{4p^3\alpha}{im_0\hbar}\left[\frac{\alpha e^{i\frac{p.r}{\hbar}}-e^{-i\frac{p.r}{\hbar}}}{\left(\alpha e^{i\frac{p.r}{\hbar}}+e^{-i\frac{p.r}{\hbar}}\right)^3}\right] \tag{20}$$

For which considering $C^-=0$, we get:

$$Q(t,\vec{r})=\frac{2}{m_0}\left[\frac{\left(\frac{E^2}{c^2}\right)\alpha_0}{\left(\alpha_0 e^{i\frac{E}{\hbar}t}+e^{-i\frac{E}{\hbar}t}\right)^2}-\frac{p^2}{\alpha e^{2i\frac{p.r}{\hbar}}}\right] \tag{21}$$

$$F=-\frac{dQ}{dr}=\frac{4p^3}{im_0\hbar\alpha}e^{-2i\frac{p.r}{\hbar}} \tag{22}$$

and considering $C^+=0$, we get:

$$Q(t,\vec{r})=\frac{2}{m_0}\left[\frac{\left(\frac{E^2}{c^2}\right)\alpha_0}{\left(\alpha_0 e^{i\frac{E}{\hbar}t}+e^{-i\frac{E}{\hbar}t}\right)^2}-\frac{p^2\alpha}{e^{-2i\frac{p.r}{\hbar}}}\right] \tag{23}$$

$$F=-\frac{dQ}{dr}=-\frac{4p^3\alpha}{im_0\hbar}e^{2i\frac{p.r}{\hbar}} \tag{24}$$

It should be noted that in obtaining the quantum force, $F=-\frac{dQ}{dr}$, only the space part of quantum potential, Q, plays the role and since the positive or negative energies originate from the time part of quantum potential, so there is no need to talk about the kind of energy in discussing quantum forces.

5. Conclusion

Based on complex quantum Hamilton-Jacobi theory, complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics, so that relativistic quantum mechanics could be obtained extending special relativity to the complex domain.

In this paper, along with a previous work [15], distinguishing physical and unphysical particles and antiparticles and introducing unphysical particle and antiparticle with negative energy, as the complement of the sets of solutions for Dirac equation, in accordance to the concept of Krein quantization in which negative energy states are applied to achieve a naturally renormalized theory [17-30], we calculated the quantum forces between particles and antiparticles in a relativistic entangled "space-time" state.

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