

Investigation of the Effect of Interference of Photon and Z^0 Boson Exchanges on the Energy Dependence of Muon Pair Production in Electron Positron Annihilation

Atsue Tersoo^{1,2,*}, Oyewande Oluwale Emmanuel²

¹Department of Physics, Faculty of Science and Education, Federal University Dutsinma, Katsina State, Nigeria

²Department of Physics, Faculty of Science, University of Ibadan, Ibadan, Nigeria

Email address:

tatsue@fudutsinma.edu.ng (A. Tersoo)

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Abstract: Experimental data from literature has been used to evaluate accurately the cross – section from electron positron annihilation into muon pairs via photon exchange. Cross – section for the same process via Z^0 boson exchange was evaluated separately, after which the two amplitudes were summed together and their cross – section evaluated again. The center of mass energies ranging from 2GeV up to and above the Z^0 peak were used to study the behavior of the cross – sections at each stage. The three results were compared to observe the effect of the interference between photon/ Z^0 boson on the center of mass energy dependent of the process.

Keywords: Electron Positron Annihilation, Z^0 Boson, Photon Exchange, Center of Mass Energy, Cross – Section

1. Introduction

Electron positron annihilation ($e^+e^- \rightarrow \mu^+\mu^-$) is a very important process in high energy physics. As such, it is widely used for experimental purposes at an electron positron collider. This process occurs often in Quantum Electrodynamics (QED) as well as in the weak interaction. In QED, the first order diagram to this process is an s-channel photon exchange. This implies an annihilation of electron and positron in a virtual photon which then decays into charged fermion – antifermion pairs. These charged fermions can either be leptons (electrons, muons, tauons) or quark – antiquark pairs. The cross section for this process exhibits a forward backward symmetry which is the fact that, electromagnetic interaction is preserved under parity transformation^[1].

As we have already mentioned, annihilation is also possible in weak interactions. This is done through the exchange of a virtual Z^0 - boson. Z^0 – Boson is one of gauge bosons that mediate the weak interactions. It is electrically neutral and has a mass of $M_Z \approx 91.188 \pm 0.002 \text{ GeV}$. Because of its large mass; it is unstable and quickly decays into lighter fermion pairs when exchanged in annihilation process^[2].

Electromagnetic and weak interactions are combined into a single theory called the ‘electroweak theory’ by Glashow,

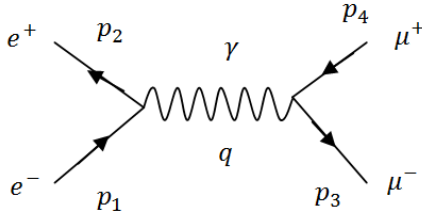
Weinberg and Salam between 1960 and 1967 which is a unification of the two forces^[3–6]. Therefore, we can describe the electron positron annihilation process as an electroweak process. Researchers used electroweak processes at large electron positron (LEP) collider to estimate the mass and width of Z^0 boson and its coupling to fermions^[7]. The precise measurement of the Z^0 boson mass improves the accuracy of the standard model predictions since the mass is a fundamental parameter of the standard model.

Several researchers use this process in search for physics beyond the standard model which they do by measuring the forward backward asymmetry. For instance, the most sensitivity case of forward – backward asymmetry for $e^+e^- \rightarrow f\bar{f}$ reaction^[8], the analysis of event topology for which angular distributions were used to measure forward – backward asymmetry at 7TeV center of mass energy in proton – proton collision for muon production^[9] and measurement of the production cross – section and asymmetries from the results of the four LEP experiments on the experiment of the fermion pairs at the highest energy (up to 209GeV)^[10].

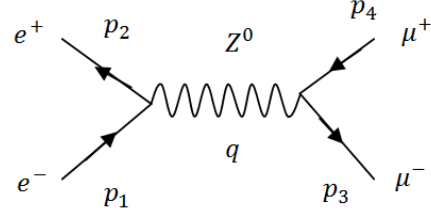
Generally, in fermion pair production from electron positron collision, there is an exchange of photon as well as Z^0 – boson. The photon and Z^0 – boson interfere in the course of measuring the differential cross section. And thus, there exist a

forward backward asymmetry in the differential cross section which is due to the interference of the photon and the Z^0 – boson exchanges. The asymmetry is said to violet parity around 90° ^[3, 11-12].

This paper is aimed at studying the dependent of the total cross – section on the center of mass energy as photon and Z^0 boson interfere in the muon pair production in electron positron annihilation process. We thus consider the process first as a QED process, a weak process secondary and we finally look at its combination as electroweak process.



(a) Photon exchange diagram (QED)



(b) Z^0 boson exchange diagram (Weak Interaction)

Figure 1. Feynman Diagrams for Photon and Z^0 Boson Exchanges in Muon Pair Production.

2.1. QED Process ($e^+e^- \rightarrow \mu^+\mu^-$ Through Photon Exchange)

The amplitude for this process is given as

$$\mathcal{M}_\gamma = -\frac{e^2}{q^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}(p_3) \gamma_\mu v(p_4) \quad (1)$$

Where e , is the electromagnetic coupling constant, p_1 and p_2 are the four momenta of the incoming electron and positron while p_3 and p_4 are the four momenta of the outgoing muon and anti – muon respectively, and q is the four momentum of the virtual photon. u and v are Spinors while \bar{u} and \bar{v} are their corresponding adjoint Spinors.

We square the amplitude, sum and average over all spins, taking into consideration that electron and positron beam are usually unpolarize and muon detectors are always blind to muon polarization^[14]. Thus, it is easy to arrive at

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_\gamma|^2 = \frac{e^4}{4q^4} \text{Tr} [\gamma^\mu p_1 \gamma^\nu p_2] \text{Tr} [\gamma_\mu p_3 \gamma_\nu p_4] \quad (2)$$

Where $p = \gamma^\mu p_\mu$

We apply trace theorems to (2) as can be found in [2, 14 - 15] and simplify to arrive at

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_\gamma|^2 = \frac{8e^4}{q^4} \Sigma_1(p) \quad (3)$$

$$\Sigma_1(p) = (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \quad (4)$$

The dot products in (4) are evaluated using kinematic variables for the process in the center of mass frame which thus simplifies (3) to.

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_\gamma|^2 = e^4 \chi(\theta) \quad (5)$$

Where,

Throughout in this work, we will use the natural units system in which the fundamental physical quantities are mass, velocity and angular momentum where we choose $\hbar = c = 1$.

2. Materials and Methods

We use Feynman rules as presented in [2, 13] in order to derive the amplitudes from the Feynman diagrams given in figure 1 below.

$$\chi(\theta) = 1 + \cos^2 \theta \quad (6)$$

The electromagnetic coupling e , constant is related to the fine structure constant α , as $e^2 = 4\pi\alpha$

Thus, we obtain the differential cross – section for QED process as

$$\frac{d\sigma_\gamma}{d\Omega} = \frac{\alpha^2}{4E_{cm}^2} \chi(\theta) \quad (7)$$

Where E_{cm} , is the center of mass energy and θ , is the angle between the incoming positron and the outgoing muon or the incoming electron and the outgoing anti muon.

The total cross – section is obtained by integrating (7) over all angles with $d\Omega = 2\pi d \cos \theta$

Thus, we have

$$\sigma_\gamma = \frac{4\pi\alpha^2}{3E_{cm}^2} \quad (8)$$

2.2. Weak Interaction Process ($e^+e^- \rightarrow \mu^+\mu^-$ Through Z^0 – Boson Exchange)

The amplitude for this is given by

$$\mathcal{M}_Z = -\frac{g_z^2}{4(q^2 - M_Z^2)} \bar{v}(p_2) \gamma^\mu (C_V^e - C_A^e \gamma^5) u(p_1) \bar{u}(p_3) \gamma_\mu (C_V^\mu - C_A^\mu \gamma^5) v(p_4) \quad (9)$$

Where M_Z , is the mass of Z^0 – boson. The factor $(1 - \gamma^5)$ is the one that ensures the vertex contains both vector and axial vector coupling with γ^μ representing the vector coupling whereas $\gamma^\mu \gamma^5$ representing the axial vector coupling [16]. C_V and C_A are the correction to vector weak charge and axial vector weak charge respectively.

We square the amplitude, sum and average over all spins to get

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_z|^2 = \frac{g_z^4}{64(q^2 - M_Z^2)^2} \text{Tr} \Xi \times \text{Tr} \Theta \quad (10)$$

$$\begin{aligned} \Xi = & (C_V^e)^2 \gamma^\mu \gamma^\nu p_1 p_2 - 2C_V^e C_A^e \gamma^\mu \gamma^\nu \gamma^5 p_1 p_2 \\ & + (C_A^e)^2 \gamma^\mu \gamma^5 \gamma^\nu \gamma^5 p_1 p_2 \end{aligned} \quad (11)$$

$$\begin{aligned} \Theta = & (C_V^\mu)^2 \gamma_\mu \gamma_\nu p_3 p_4 - 2C_V^\mu C_A^\mu \gamma_\mu \gamma_\nu \gamma^5 p_3 p_4 \\ & + (C_A^\mu)^2 \gamma_\mu \gamma^5 \gamma_\nu \gamma^5 p_3 p_4 \end{aligned} \quad (12)$$

where g_z , is the neutral weak coupling constant and is related to the electromagnetic coupling constant by

$$g_z = \frac{e}{\sin \theta_w \cos \theta_w} \quad (13)$$

where θ_w , is the weak mixing angle with experimental value of 28.75° . We apply trace theorems again to evaluate (11) and (12) which then simplifies (10) to

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_z|^2 = \frac{g_z^4}{2(q^2 - M_Z^2)^2} [\zeta \Sigma_1(p) - 4\varphi \Sigma_2(p)] \quad (14)$$

$$\zeta = [(C_V^e)^2 + (C_A^e)^2] [(C_V^\mu)^2 + (C_A^\mu)^2] \quad (15)$$

$$\varphi = C_V^e C_A^e C_V^\mu C_A^\mu \quad (16)$$

$$\Sigma_2(p) = (p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3) \quad (17)$$

Applying the kinematic variables to evaluate the dot products in (17) and (4) simplifies (14) to

$$\frac{1}{4} \sum_{spins} |\mathcal{M}_z|^2 = \frac{g_z^4 E^4}{2(q^2 - M_Z^2)^2} [\zeta \chi(\theta) + 8\varphi \cos \theta] \quad (18)$$

The differential cross – section is therefore given as

$$\frac{d\sigma_z}{d\Omega} = \frac{g_z^4 E^4}{64\pi^2 E_{cm}^2 [(2E)^2 - M_Z^2]^2} \{\zeta \chi(\theta) + 8\varphi \cos \theta\} \quad (19)$$

The total cross – section is obtained from (19) taking into account (13) and the relationship between the electromagnetic coupling constant and the fine structure constant as

$$\sigma_z = \frac{\pi \alpha^2}{12(\sin \theta_w \cos \theta_w)^4} \frac{E_{cm}^2}{[E_{cm}^2 - M_Z^2]^2} \zeta \quad (20)$$

2.3. The Electroweak Process

For this process, we sum the amplitude in (1) for QED process and the amplitude in (9) for the weak interaction process to obtain the electroweak amplitude given below

$$\mathcal{M}_\gamma + \mathcal{M}_z \quad (21)$$

i.e.

$$\mathcal{M}_\gamma = -\frac{e^2}{q^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}(p_3) \gamma_\mu v(p_4)$$

And

$$\begin{aligned} \mathcal{M}_z = & -\frac{g_z^2}{4(q^2 - M_Z^2)} \bar{v}(p_2) \gamma^\mu (C_V^e \\ & - C_A^e \gamma^5) u(p_1) \bar{u}(p_3) \gamma_\mu (C_V^\mu - C_A^\mu \gamma^5) v(p_4) \end{aligned}$$

From (1) and (9) respectively

Next we square, sum and average over all spins, we have

$$\begin{aligned} \frac{1}{4} \sum_{spins} |\mathcal{M}_\gamma + \mathcal{M}_z|^2 = \\ \frac{1}{4} \sum_{spins} [|\mathcal{M}_\gamma|^2 + 2|\mathcal{M}_\gamma \mathcal{M}_z| + |\mathcal{M}_z|^2] \end{aligned} \quad (22)$$

The first and the third terms of (22) have been evaluated in (5) and (18) respectively. Now evaluating the second term leads to

$$\begin{aligned} \frac{1}{4} \sum_{spins} 2|\mathcal{M}_\gamma \mathcal{M}_z| = \frac{e^2 g_z^2}{8q^2(q^2 - M_Z^2)} \text{Tr} [\gamma^\mu p_1 \gamma^\nu (C_V^e - \\ C_A^e \gamma^5) p_2 \text{Tr} \gamma_\mu p_4 \gamma_\nu C_V^\mu - C_A^\mu \gamma^5 p_3] \end{aligned} \quad (23)$$

Then, we apply trace theorems and simplify after which we substitute the kinematic variables to arrive at

$$\frac{1}{4} \sum_{spins} 2|\mathcal{M}_\gamma \mathcal{M}_z| = \frac{2E^2 e^2 g_z^2}{[(2E)^2 - M_Z^2]} [\xi_1 \chi(\theta) + 2\xi_2 \cos \theta] \quad (24)$$

Where,

$$\xi_1 = C_V^e C_V^\mu \text{ and } \xi_2 = C_A^e C_A^\mu$$

The beam energy E is related to the center of mass energy as $E_{cm} = 2E$.

Making use of (13) and the relation between the beam energy and the center of mass energy, we let

$$F(s) = \frac{E_{cm}^2}{4(\sin \theta_w \cos \theta_w)^2 (E_{cm}^2 - M_Z^2)} \quad (25)$$

Thus, putting together (5), (18) and (24), we obtain the total squared amplitude for the electroweak process considering (25) as

$$\begin{aligned} \frac{1}{4} \sum_{spins} |\mathcal{M}_\gamma + \mathcal{M}_z|^2 = e^4 [\chi(\theta) + F(s) [\xi_1 \chi(\theta) + \\ 2\xi_2 \cos \theta + F^2(s) \zeta \chi(\theta) + 8\varphi \cos \theta] \end{aligned} \quad (26)$$

We obtain the differential cross – section for the electroweak process as

$$\begin{aligned} \frac{d\sigma_{\gamma/z}}{d\Omega} = \frac{\alpha^2}{4E_{cm}^2} \{ \chi(\theta) + F(s) [\xi_1 \chi(\theta) + 2\xi_2 \cos \theta] + \\ F^2(s) \zeta \chi(\theta) + 8\varphi \cos \theta \} \end{aligned} \quad (27)$$

The total cross – section for the process is therefore obtain from (27) as

$$\sigma_{\gamma/z} = \frac{4\pi \alpha^2}{3E_{cm}^2} \{1 + F(s) \xi_1 + F^2(s) \zeta\} \quad (28)$$

3. Results and Discussion

The differential cross – section in (7) was used to obtain the total cross – section in (8) for the quantum electrodynamics (QED) process of muon pair production in electron positron annihilation. A plot of the total cross – section against the center of mass energies presented in figure 2, shows an inverse relation to the square of the center of mass energy. This is an illustration of a typical behavior of QED.

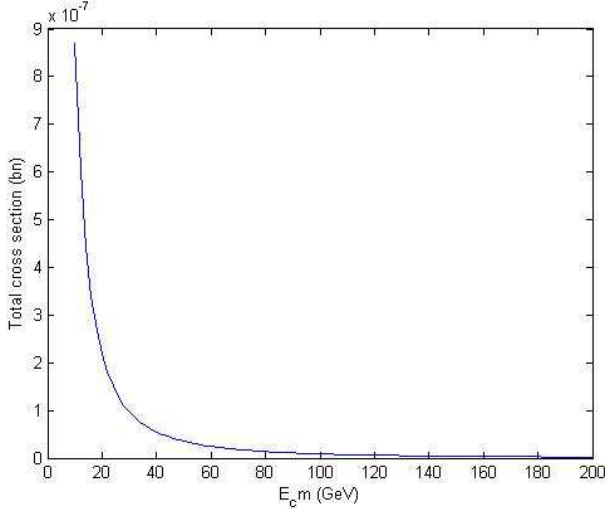


Figure 2. A plot for QED Process (photon exchange).

For the weak interaction process, we obtained the differential cross – section given in (19) from which we obtained the total cross – section given in (20). From (20), there is a factor of $\frac{1}{E_{cm}^2 - M_Z^2}$ which appears as a result of the weak interaction propagator. There appears a Z^0 pole in this factor which implies that, the total cross – section blows up when the center of mass energy hit the mass of the Z^0 boson. According to [2], massive spin 1 particles are unstable. Therefore, the finite lifetime of such particles has effect on its mass. Thus, we have to modify the propagator to account for this. This is given below

$$\frac{1}{E_{cm}^2 - M_Z^2} \rightarrow \frac{1}{E_{cm}^2 - M_Z^2 + (M_Z \Gamma_Z)^2}$$

The parameter Γ_Z , is the decay width of Z^0 boson with an experimental value of 3.719 GeV . Therefore, total cross – section in (20) is modify to

$$\sigma_Z = \frac{\pi \alpha^2}{12(\sin \theta_w \cos \theta_w)^4} \frac{E_{cm}^2}{[E_{cm}^2 - M_Z^2]^2 + (M_Z \Gamma_Z)^2} \zeta \quad (29)$$

Figure 3 shows a plot of this total cross – section as a function of the center of mass energy. The behavior of this curve indicates a peak of the total cross – section around the Z^0 pole. This peak is called the Z^0 peak.

Electroweak process has been considered by combining the QED and the weak interaction processes together. Thus, we added the QED (photon exchange) amplitude and weak interaction (Z^0 boson exchange) amplitude together and used it to obtain the differential cross – section given in (27). We

then obtained the total cross – section from (27) which is the result presented in (28). In the process of obtaining the total cross – section from the differential cross – section, the term containing $\cos \theta$ in (27) integrates to zero, but it rather induces the forward backward asymmetry. This effect is as a result of the interference between photon exchange and Z^0 boson exchange.

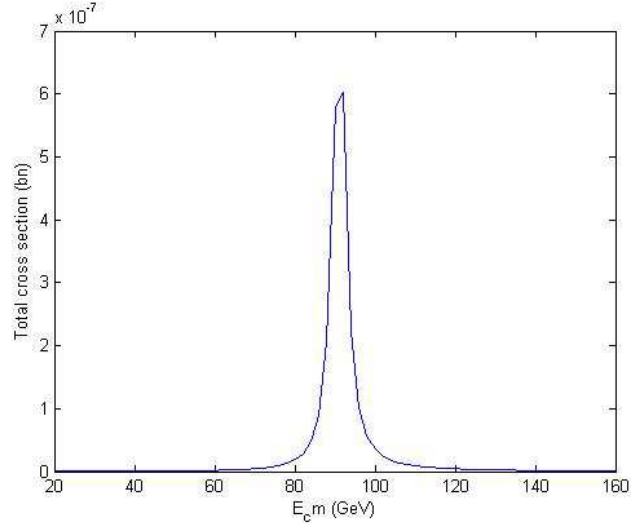


Figure 3. A plot for weak interaction process (Z^0 boson exchange).

A parity violating amplitude arising from the Z^0 boson exchange diagram exist in the addition of the photon exchange diagram according to [1]. We can see that the Z^0 boson exchange diagram contains both vector and axial vector couplings and [2] tells us that, any theory that adds a vector to an axial vector is bound to violate parity conservation.

A modification in the propagator presented above also has effect on (25) which is an important factor in (27) and (28). Thus, we can put (25) in this form

$$F(s) = \frac{E_{cm}^2}{4(\sin \theta_w \cos \theta_w)^2 ([E_{cm}^2 - M_Z^2] + (M_Z \Gamma_Z)^2)} \quad (30)$$

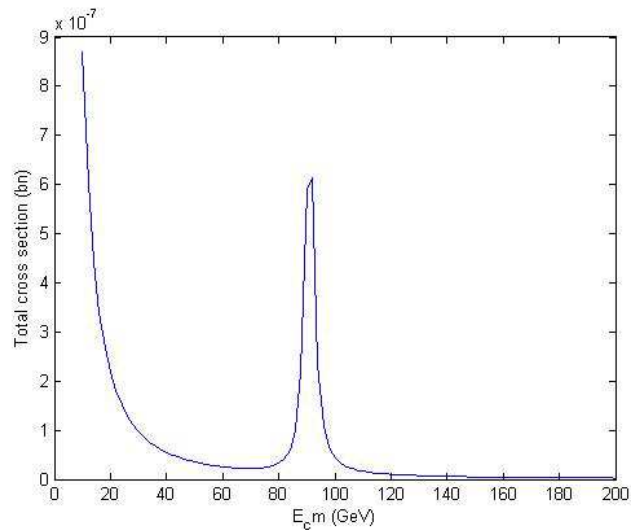


Figure 4. The curve representing the electroweak process (photon and Z^0 boson exchanges).

Therefore, (30) takes care of (28) which is the total cross – section for the electroweak process. A plot of (28) as a function of center of mass energy is shown in figure 4. We observed this figure in comparison with figures 2 and 3. The curve in figure 3 shows an almost zero total cross – sections between center of mass energy range of 60GeV and below. In figure 4 which is a combination of the two processes, the total cross – section for photon exchange (QED) dominates at center of mass energies between 15GeV and 60GeV . The effect of interference appears shortly and vanishes at center of mass energies close to the mass of Z^0 boson and the effect of QED is almost negligible compare to the Z^0 boson contribution. The Z^0 boson dominates at center of mass energy equal to the mass of Z^0 boson and beyond this region, the behavior of the two is almost alike.

4. Conclusion

We have considered the effect of the interference of photon and Z^0 boson exchanges on the center of mass energy dependent of muon pair production in electron positron annihilation. This was done by computing the total cross – section of the process which is center of mass energy dependent using experimental data from literatures. We consider the fact that, at each final state of the muon – antimuon, three different contributions were involved. The electromagnetic interaction contribution was dominant for center of mass energies below the mass of Z^0 boson, the interference of the electromagnetic and weak interactions appear shortly and vanished as the center of mass energy approached the mass of Z^0 boson. At the Z^0 peak, the weak interaction contribution dominates.

We can then conclude that, there is a clear effect of the interference of photon and Z^0 – boson exchanges on the center of mass energy dependence of the process. This interference effect may be seen as a contributing factor for the unification of the electromagnetic and weak interactions at sufficiently high energies.

References

- [1] D. H. Perkins (2000). Introduction to high energy physics (4th Ed). The pitt building, UK: Cambridge University press.
- [2] D. Griffiths (2008). Introduction to elementary particles (2nd Rev. Ed). Weinheim: Wiley–VCH Verlag GmbH & Co.
- [3] A. Bettini (2014). Introduction to elementary particles physics (2nd ed.). New York: Cambridge University press.
- [4] S. L. Glashow (1961). Partial symmetries of weak interactions. Nuc. Phys. 22, 579 – 588.
- [5] S. Weinberg (1967). A model of leptons. Phys. Rev. Lett. 19.
- [6] A. Salam (1968). Weak and electromagnetic interactions, in 8th Nobel Symposium. Ed. N. Svartholm, Stockholm: Almqvist & Wiksell.
- [7] M. Davier (1994). Testing the electroweak interaction: from PETRA to LEP. Phys. B1.50 Nr. 7/8687 – 693.
- [8] B. S. Berin (2015). Search Forward-Backward Asymmetry Due to the New Z' Gauge Boson in e^+e^- Collision. *Gazi University Journal of Science: GU J Sci Part:A* 3(2):27-29 (2015).
- [9] C. G. Giulio (2012). Measurement of the Z/γ^* forward backward asymmetry in muon pairs with the ATLAS experiment at the LHC. Journal of physics: conference series 383(2012)012005.
- [10] I. Boyko (2005). Fermion pair production at LEP2. Proceedings of science. PoS(HEP2005)303.
- [11] G. Abbiendi, C. Ainsley and others (2001). Angular analysis of the muon pair asymmetry at LEP1; OPAL collaboration. Physics letters B 516 (2001) 1 – 20.
- [12] C. G. Giulio (2012). Measurement of the Z/γ^* forward backward asymmetry in muon pairs with the ATLAS experiment at the LHC. Journal of physics: conference series 383(2012)012005.
- [13] D. McMahon (2008). Quantum field theory demystified; a self teaching guide. New York: nMcGraw Hill.
- [14] M. E. Peskin and D. V. Schroeder (1995). An introduction to quantum field theory. Reading, Massachusetts: Perseus Books publishing L.L.C.
- [15] W. Greiner and J. Reinhardt (2009). Quantum electrodynamics (4th ed.) Berlin Heidelberg: Springer – Verlag.
- [16] E. Leader (2001). Cambridge monographs on particle physics, Nuclear physics and cosmology. Edited by Ericson T. and Landshoff P.V. The pitt building, UK: Cambridge University press.