



Methodology Article

A Capital Asset Pricing Model's (CAPM's) Beta Estimation in the Presence of Normality and Non-normality Assumptions

Ezekiel Oseni^{1,*}, Razak Olawale Olanrewaju²

¹Department of Banking and Finance, Faculty of Business Administration, University of Lagos, Lagos, Nigeria

²Department of Statistics, Faculty of Science, University of Ibadan, Ibadan, Nigeria

Email address:

zikoseni@yahoo.com (E. Oseni), rasakiolawale@gmail.com (R. O. Olanrewaju)

*Corresponding author

To cite this article:

Ezekiel Oseni, Razak Olawale Olanrewaju. A Capital Asset Pricing Model's (CAPM's) Beta Estimation in the Presence of Normality and Non-normality Assumptions. *International Journal of Finance and Banking Research*. Vol. 3, No. 3, 2017, pp. 44-52.

doi: 10.11648/j.ijfbr.20170303.12

Received: April 10, 2017; **Accepted:** April 14, 2017; **Published:** June 16, 2017

Abstract: This study describes the approach for estimating the beta-risk of the Capital Asset Price Model (CAPM) when the normality (Gaussian) assumption of both the error term and the excess return on an asset holds, and also when their normality assumption is violated or failed due to outliers or excessive skewness and excessive kurtosis. The student-t distribution was used as an alternative distribution to capture these anomalies. The monthly All-share Index (ASI) of 12 crucial Market Portfolios / Sectors derived from Nigeria Stock Exchange (NSE) were subjected to both the Gaussian error innovation and Student-t error innovation in this study. However, it was noted that estimates of portfolios' beta-risk and its standard error for Gaussian and student-t were approximately the same when the sector follows a normal distribution while the standard errors of portfolio beta-risk estimates will be smaller under student-t innovation than that of Gaussian innovation when the sector does not follow normal distribution due to these anomalies. Furthermore, it was discovered that building & construction, manufacturing, quarry & mining, communication, transportation, education and utilities sectors have been having lower volatility, that is, in boosting the economy over the last 15 years.

Keywords: Beta-Risk, Capm, Expected Returns, Gaussian Innovation, Student-T Innovation, Systematic Risk

1. Introduction

Risk identification and quantification are two of the major recipes to managing risk associated with business decisions. Scholars like (Sharpe, 1964; Markowitz, 1965 and Ross, 1976) came up with theories and techniques for managing risks and uncertainty. One of the early risk management models is the Capital Asset Price Model (CAPM) which has remained a principal ornament for modeling modern financial economics. The origin of the model was arguably introduced by Treynor (1965) while the likes of Sharpe (1964) and Lintner (1965) and Markowitz (1965) made some independent contributions to furthering the concepts of the model. A close variation to the CAPM is the arbitrage pricing theory propounded by Ross

(1976) which believes that the expected return of a financial instrument is a function of many macroeconomic variables with each of the variables having their own level of probability, (beta) coefficient (β).

The concept of CAPM which bothers more on systematic or market risk is based on some assumptions upon which the model can guarantee optimal risk measurement and risk-return rate on investment securities and portfolios. The systematic risk is a non-diversifiable for every investor regardless of the market or sector within an economy. The systematic risk is unavoidable, it is a risk that is common to all investors unlike unsystematic risk that is associated with a particular market or sector. The unsystematic risk is avoidable when you move to a different market, sector or economy. The

bedrock of the model is on certain economic and non-economic assumptions. The assumptions include existence of rational and risk-averse investors, availability of huge investment opportunities, cost-less information, tax-less economy, free market entry and exit, homogenous expectations by all market players, absence of information asymmetry and price sacrosanct to the extent that no one can influence asset prices (Cheema, 2010; Bouchaddekh, Bouri and Kefi, 2014; Myers and Turnbull, 1977). There is no doubt that none of the assumptions is in consonance with reality and this has accounted for the major pitfall of the model as it has not been tested with live situations.

However, in spite of the supposedly impracticability of the model in live situations where information is free, taxes are paid, heterogeneous expectations and high presence of information asymmetry in the market, the contributions of CAPM Model to developing other risk measurement models that are more sensitive to economic realities cannot be overemphasized. This study has therefore decided to estimate the risk-free rate (R_f) in a risk-less market where the CAPM underlying assumptions subsist as well as under the reality of economic situations where the CAPM assumptions become a non-normality.

This study would shift the frontier of academic knowledge as it concerns the theories of asset pricing under the normal and non-normal (reality economics) assumptions while it would also add to theoretical references for academic researches. For investment analysts, the study would assist in broadening their knowledge in asset pricing under different assumptions and economic scenarios.

2. Literature Review

The alternative description of CAPM as defined by Markowitz (1965) is the Portfolio Model. The essence of the model according to Francioni and Schwartz, 2017; Gencay, Selcuk and Whitcher, 2005; Santis, 2010) is that it provides a platform for risk measurement while also establishing the correlation between future market risk and return. The essence of the model is the relationship between expected risk and return. In fact, the model according to Demircioglu (2015), and Pamane and Vikpossi (2014) describes a direct relationship between market expected return and risks. The practicability of the model to support reality has not been supported with many empirical evidences and the model is best described as a theoretical exercise because of its many unrealistic assumptions (Nyangara, Nyangara, et al., 2016; Ward and Muller, 2013; Fama and French, 2004). According to Leonard et al. (2012), he defined the model as one that gives a precise prediction of the relationship between the risk of an asset or stock and its expected return. He stressed further that the two vital functions of the model are to provide a benchmark rate of return for evaluating possible investments and one that helps in educational guess as to the expected return on assets that have not yet been traded in the marketplace.

It is rational for every investor to desire a commensurate

rate of return for every element of risk taken (Dawson, 2014; Aduda and Muimi, 2011) but determining the return under realistic situations (risk presence) as opposed to the assumptions of the model (risk absence) remains a mirage (Mullins, 1982). The model according to Mullins (1982) is of good use for estimating costs of capital (equity) especially when it is used in conjunction with other risk-return estimation techniques. In fact, it is a preferred method in estimating the cost of equity than the usual dividend growth model because of its ability to emphasize on the company's systematic risks (Perold, 2004). In the stock markets, Lee, Cheng and Chong (2016) argued that investment managers engages CAPM to determine stock returns while at the same time diversifying their investment portfolios to eliminate or reduce the unsystematic risks for the purpose of maximizing profitability and shareholders' wealth. In a similar vein, Nwani (2015) affirmed what Miller (1999) stated that CAPM through its empirical evidence has been contributing immensely to the development of finance by providing unflinching insights into the form of risk involved in an asset or stock pricing. He elaborated that CAPM assumes that the expected return on any asset/stock is positively and linearly related to its market beta-risk (systematic risk), which according to the CAPM is the only relevant measure of undiversifiable risk of the asset, so the elucidating power of the CAPM's systematic risk (also known as beta risk) and abnormal return cannot be jettisoned in offering significant explanation to variations in assets/stocks in various equity market. The CAPM provides the required return based on the perceived level of systematic risk of an investment.

Furthermore, it is clear that all potential return arises with risk and that low-risk (low levels of uncertainty) associated with low potential returns while high risk goes along with high potential risk. The two risks associated with the CAPM are; the unsystematic risk and the systematic risk otherwise known as the Market beta-risk or the non-diversifiable risk. The former can be eliminated through diversification while the latter is the market risk and cannot be eliminated via diversification due to factors like Interest rates, recession, inflation, unstable energy prices, changes of policies, political unstable, business cycle etc.

The Beta-risk (market risk) of the CAPM has been the index used by researchers in classifying the risk associated with an asset/stock to know whether the beta risk is aggressive, tracks the market, conservative, independent or perfectly hedged. This risk has always been estimated using cross-sectional regression analysis via Ordinary Least Square (OLS) where both the dependent variable (the excess return on an asset) and the error term are normally distributed. Jensen et al. (1972) in their study indicated that the expected excess return on an asset might not be strictly proportional to its systematic risk, they are of sufficient belief that cross-sectional estimates and analysis of the beta factor might be subjected to measurement bias error due to the traditional form of the model (that is, the joint probability distribution of all security returns).

Several scholars (Bajpai and Sharma, 2015; Novak, 2015; Muthama, Munene and Tirimba, 2014; Oke, 2013; Brown and

Walter, 2013; Masood, et al. 2012; Hassan, et.al. 2012; Nel, 2011; Fama and French, 2004) made attempts over the years to empirically test the validity of CAPM in estimating rate of return in a systematic (non-diversifiable) market. For instance, Hassan et al. (2012) examined 80 quoted non-financial institutions' monthly stock returns covering the period of 2005 to 2009 on the Dhaka Stock Exchange to estimate risk-return rate using the CAPM model. The findings which were contrary to CAPM hypothesis showed a positive relationship between Beta (β) and return. The interactions between the variables were found to be insignificant. The findings of the likes of Hassan et al. (2012) possibly influenced Novak (2015) to modify the CAPM which in turn affected the presumptions about the correlation between the Beta (β) and return. The elimination of unrealistic assumptions of CAPM makes the interaction between the variables significant contrary to what obtained when the CAPM model was strictly adopted by Hassan et al. (2012).

In a similar study, Oke (2013) applied the CAPM to the Nigerian stock market using weekly stock returns of 110 companies from the Nigerian Stock Exchange (NSE). To enhance the precision of the beta risk estimates and statistical problems that might arise from measurement errors in individual beta estimates, combined the securities with portfolios such that the results will invalidate the CAPM's prediction that higher beta risk is associated with a higher level of return and the abnormal return should equal zero when estimating Security Market Line (SML). Also, Osamwonyi and Asein (2012) examined the market risk defined by CAPM as an explanatory variable for security returns in most capitalized firms in the Nigeria Capital Market. His findings confirmed a positive linear relationship between market betas and security returns for the selected Nigerian firms after subjecting the CAPM to Independent and identically normal distribution. In a similar vein, Nwani (2015) subjected the UK Stock Returns to CAPM beta risk via the OLS cross-sectional regression analysis and deduced that the stock returns in the UK equity market are not significantly sensitive to the systematic (market) risk. Like Nwani (2015), Tumala and Yaya (2015) came up with an alternative method of estimating beta risk using Logistic Smooth Threshold Model (LSTM). They examined the Nigeria market Sector returns using the LSTM in order to conquer the problem of high positive skewness coefficients and high negative skewness coefficients characterized by some sector equity returns. They discovered that Petroleum, Finance, and Food and Beverages sector equities yielded higher investment risk within their study period.

The aforesaid bias measurement in the systematic (market risk) is always related to the violation of normality assumption(s) in the error term and excess return on an asset of the CAPM. These assumptions violation might be due to excessive kurtosis or excessive skewness (that is, non bell-shaped distributions), outliers (due to recession, hike in exchanges, policy changes etc.). Subjecting the CAPM to Gaussian distribution, the assumption violations will surely distort the estimates of the beta risk (market risk), so, a more

robust distribution is needed in capturing these violations. The Student-t distribution theoretical framework will be used in this research work as an alternative distribution in the presence of assumption(s) violation.

3. Theoretical Analysis

Hurn et al. (2015) and Oke (2013) provided a coherent and updated framework by defining beta- risk (systematic risk) that can be derived from CAPM defined by ((Sharpe, 1964), (Treynor, 1965) and (Mossin, 1966) as

$$\beta = \frac{\text{cov}(r_{it} - r_{ft}, r_{mt} - r_{ft})}{\text{var}(r_{mt} - r_{ft})}$$

Where;

β = The beta risk or systematic risk

r_{it} = Return on security i

r_{ft} = Rate of return on risk-free security

r_{mt} = The rate of return on market index

$\text{var}(r_{mt} - r_{ft})$ = The variance of the market returns

$\text{cov}(r_{it} - r_{ft}, r_{mt} - r_{ft})$ = The covariance between asset i and the market portfolio.

The beta-risk (β) expresses concisely and succinctly the risk attributes of an asset in terms of its CAPM, that is, the beta-risk indicates the linear relationship that exist between a securities' required rate of return and it's CAPM. Stan et al. (2015) classified the degree of this risk as follows:

Aggressive if $\beta > 1$, Tracks the Market if $\beta = 1$, Conservative if $0 < \beta < 1$

Independence if $\beta = 0$, Imperfect Hedge if $-1 < \beta < 0$, and Perfect Hedge if $\beta = -1$

Sharpe (1994) defined a CAPM model to be, $r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + \mu_t$

Where μ_t is a white noise (disturbance term) i.e. $\mu_t \sim N(0, \sigma^2)$ (Normality assumption holds).

The slope parameter β represents the asset's beta risk. The intercept parameter α represents the abnormal return to the asset over and above the asset's exposure to the excess return on the market.

This can be re-written as

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2) \quad (1)$$

Where;

$y_t = (r_{it} - r_{ft})$ Is the excess return on an Asset, $x_t = (r_{mt} - r_{ft})$ is the excess on the market Portfolio

β_0 Measures the abnormal returns, β_1 Represents the beta risk of the asset, σ^2 is the disturbance variance that measures the idiosyncratic risk of the asset.

It follows that;

$\varepsilon_t \sim N(0, \sigma^2)$ (Normality), from (1) equation above,

$$y_t - \beta_0 - \beta_1 x_t \sim N(0, \sigma^2)$$

$$\Rightarrow y_t \sim N(\beta_0 + \beta_1 x_t, \sigma^2)$$

Estimating of the Beta-Risk under the Normality Assumption (Gaussian innovation)

Given the normal distribution as;

$$f(y / \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{1}{2\sigma^2} [(y - \mu)^2] \quad \text{for } -\infty < y < \infty \quad (2)$$

The distribution of r_t with parameter vector is $\eta = \{\beta_0, \beta_1, \sigma^2\}$ for

$y = r_t - r_{ft}$ and $\mu = \beta_0 - \beta_1 x_t$, $x_t = r_{mt} - r_{ft}$ is given as

$$f(r_t / r_{mt}, r_{ft}, \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{1}{2\sigma^2} \left[- \left((r_t - r_{ft}) - \beta_0 - \beta_1 (r_{mt} - r_{ft}) \right)^2 \right], -\infty < r_t < \infty$$

But $y = y_t = r_t - r_{ft}$ and $\mu = \beta_0 - \beta_1 x_t$

$$f(y_t / x_t; \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{1}{2\sigma^2} \left[- (y_t - \beta_0 - \beta_1 x_t)^2 \right], -\infty < y_t < \infty$$

The maximum likelihood estimator of $\eta = \{\beta_0, \beta_1, \sigma^2\}$

Hurn et al.(2015) proposed a working document for parameter estimation for the beta risk when the normality assumption holds.

$$L(\eta) = f(y_1 / x_1; \eta) \times f(y_2 / x_2; \eta) \times f(y_3 / x_3; \eta) \times \dots \times f(y_N / x_N; \eta)$$

$$T = \ln L(\eta) = \frac{1}{N} \sum_{t=1}^N \log f(y_t / x_t; \eta) \quad (3)$$

$$T = -\frac{1}{2} \log 2\pi - \frac{1}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \frac{1}{N} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t)^2$$

So,

$$\begin{aligned} \frac{\partial T}{\partial \beta_0} &= \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) \\ \frac{\partial T}{\partial \beta_1} &= \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) x_t \\ \frac{\partial T}{\partial \sigma^2} &= -\frac{1}{2\sigma^2} + \frac{1}{2N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t)^2 \end{aligned}$$

$$H(\hat{\eta}) = \begin{pmatrix} -\frac{1}{\hat{\sigma}^2} & -\frac{1}{\hat{\sigma}^2} \frac{1}{N} \sum_{t=1}^N x_t & -\frac{1}{\hat{\sigma}^4} \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t) \\ -\frac{1}{\hat{\sigma}^2} \frac{1}{N} \sum_{t=1}^N x_t & -\frac{1}{\hat{\sigma}^2} \frac{1}{N} \sum_{t=1}^N x_t^2 & -\frac{1}{\hat{\sigma}^4} \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t) x_t \\ -\frac{1}{\hat{\sigma}^4} \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t) & -\frac{1}{\hat{\sigma}^4} \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t) x_t & \frac{1}{2\hat{\sigma}^2} - \frac{1}{\hat{\sigma}^6} \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2 \end{pmatrix}$$

$H(\hat{\eta})$ conforms with the negativity of the second

In a matrix form, we have

$$K(\eta) = \begin{pmatrix} \frac{\partial T}{\partial \beta_0} \\ \frac{\partial T}{\partial \beta_1} \\ \frac{\partial T}{\partial \sigma^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) \\ \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) x_t \\ -\frac{1}{2\sigma^2} + \frac{1}{2N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t)^2 \end{pmatrix}$$

Maximizing $\beta_0, \beta_1, \& \sigma^2$ by equating $K(\hat{\eta}) = 0$ gives

$$K(\hat{\eta}) = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\sigma}^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) \\ \frac{1}{N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t) x_t \\ -\frac{1}{2\sigma^2} + \frac{1}{2N\sigma^2} \sum_{t=1}^N (y_t - \beta_0 - \beta_1 x_t)^2 \end{pmatrix} = 0,$$

$$\text{where } 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$K(\hat{\eta}) = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\sigma}^2 \end{pmatrix} = \begin{pmatrix} \bar{y} - \hat{\beta}_1 \bar{x} \\ \frac{\sum_{t=1}^N (y_t - \bar{y})(x_t - \bar{x})}{\sum_{t=1}^N (x_t - \bar{x})^2} \\ \frac{1}{N} \sum_{t=1}^N (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2 \end{pmatrix}$$

To get the variance-covariance for $\hat{\eta}$, otherwise known as Hessian matrix.

$$H(\hat{\eta}) = \begin{pmatrix} \frac{\partial^2 T}{\partial^2 \beta_0} & \frac{\partial^2 T}{\partial \beta_0 \partial \beta_1} & \frac{\partial^2 T}{\partial \beta_0 \partial \sigma^2} \\ \frac{\partial^2 T}{\partial \beta_1 \partial \beta_0} & \frac{\partial^2 T}{\partial^2 \beta_1} & \frac{\partial^2 T}{\partial \beta_1 \partial \sigma^2} \\ \frac{\partial^2 T}{\partial \sigma^2 \partial \beta_0} & \frac{\partial^2 T}{\partial \sigma^2 \partial \beta_1} & \frac{\partial^2 T}{(\partial \sigma^4)} \end{pmatrix}$$

derivative of an estimable parameter via maximum likelihood. In practice, the return distributions for stocks and assets are

not symmetric instead assets do exhibit fat tails (higher kurtosis or higher skewness and outliers i.e. extreme positive values or extreme negative values), but assets do possess extreme positive values. These extreme values can come to play due to change in policies and implementation, recession, hike in exchange rates, unforeseen circumstances etc. in a country. These exhibition of fat tails and outliers by stocks and assets do affect the assumptions of the Independently and Identically Normal Distribution of the error term of the CAPM model which definitely result in distorting the estimates of beta-risk (non-diversifiable risk) ($\hat{\beta}_1$) and the abnormal return ($\hat{\beta}_0$). The ideal solution is to replace the normality assumption with a fat-tailed distribution (student-t-distribution) that will capture and accommodate outliers and at the same time reduces the distortion of the estimates of beta-risk in the presence of outlier(s). So, a robust version of the CAPM that will replace the normal distribution with standardized mean zero and variance is needed.

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad \varepsilon_t \sim iid st(0, \sigma^2, \nu)$$

The notation $st(0, \sigma^2, \nu)$ represents the standardized student-t-distribution given by

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu\sigma^2}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\varepsilon_t^2}{\nu\sigma^2}\right)^{-\left(\frac{\nu+1}{2}\right)}, \quad \varepsilon_t \in \mathbb{R} \quad (4)$$

The parameter " ν " is the degree of freedom parameter which captures the effects outliers in the tails of the distribution.

4. Estimating of the Beta-Risk Under the Assumption Violation (Student-t Innovation)

Felipe et al (2004) used an EM algorithm in estimating the variance in student-t-distribution via E-step and M-step. The same Iterative procedure will be employed but for estimating β_0, β_1 & σ^2 . The responses $Y_t, t=1, \dots, N$, are independent random variables distributed as $N(x_t^T \beta, \sigma^2, \nu)$. For any noted change-point detection.

We considered $k = p, \dots, N - p$,

$$Y_1 = (Y_1, Y_2, \dots, Y_k)^T, \quad Y_2 = (Y_{k+1}, \dots, Y_N)^T,$$

$$X_1 = (X_1^T, X_2^T, \dots, X_k^T)^T, \quad X_2 = (X_{k+1}^T, \dots, X_N^T)^T$$

The distribution in terms of y_t gives

$$f(y_t / x_t; \eta) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu\sigma^2}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{(y_t - \beta_0 - \beta_1 x_t)^2}{\nu\sigma^2}\right)^{-\left(\frac{\nu+1}{2}\right)}, \quad y_t \in \mathbb{R} \quad (5)$$

To derive the maximum likelihood estimator of

$\eta = \{\beta_0, \beta_1, \sigma^2, \nu\}$, $\log L(\eta)$ is based on

$$\begin{aligned} \log L(\eta) &= \frac{1}{N} \sum_{i=1}^N \log f(y_i / x_i; \eta) \\ &= \log \Gamma\left(\frac{\nu+1}{2}\right) - \frac{1}{2} \log \sigma^2 - \frac{1}{2} \log(\pi\nu) \\ &\quad - \log \Gamma\left(\frac{\nu}{2}\right) - \left(\frac{\nu+1}{2}\right) \frac{1}{N} \sum_{i=1}^N \log \left(1 + \frac{(y_i - \beta_0 - \beta_1 x_i)^2}{\nu\sigma^2}\right) \end{aligned} \quad (6)$$

Since equation (6) for log-likelihood is a non-linear in the parameters that has no close-form an iterative solution would be adopted to compute $\hat{\eta}$.

Equation (6) can be re-written as

$$\log L(\eta) = \log h(\nu) - \frac{1}{2} \log \sigma^2 - \frac{\nu+1}{2} \sum_{i=1}^N \log \left(1 + \frac{(y_i - \beta_0 - \beta_1 x_i)^2}{\nu\sigma^2}\right) \quad (7)$$

Where,

$$h(\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}, \quad \eta = (\beta_0^T, \beta_1^T, \sigma^2)^T$$

$$\text{Letting } d_i^2 = d_i^2(\beta_0, \sigma^2) = \frac{(y_i - X_i^T \beta_0)^2}{\sigma^2}, \quad i = 1, \dots, k$$

$$d_i^2 = d_i^2(\beta_1, \sigma^2) = \frac{(y_i - X_i^T \beta_1)^2}{\sigma^2}, \quad i = k+1, \dots, N$$

So, equation (2) can be re-written as

$$\begin{aligned} \log L(\eta) &= \log h(\nu) - \frac{1}{2} \log \sigma^2 \\ &\quad - \frac{\nu+1}{2} \frac{1}{N} \left[\sum_{i=1}^k \log \left\{1 + \frac{d_i^2(\beta_0, \sigma^2)}{\nu}\right\} + \sum_{i=k+1}^N \log \left\{1 + \frac{d_i^2(\beta_1, \sigma^2)}{\nu}\right\} \right] \end{aligned} \quad (8)$$

While the associated score functions are given by,

$$U(\beta_0) = -\frac{1}{2\sigma^2} - \frac{\nu+1}{2} \frac{1}{N} \left[\sum_{i=1}^k \log \left(-\frac{\frac{\partial}{\partial \beta_0} \left(\frac{d_i^2(\beta_0, \sigma^2)}{\nu} \right)}{1 + \frac{d_i^2(\beta_0, \sigma^2)}{\nu}} \right) \right],$$

$$U(\beta_1) = -\frac{1}{2\sigma^2} - \frac{\nu+1}{2} \frac{1}{N} \left[\sum_{i=1}^k \log \left(-\frac{\frac{\partial}{\partial \beta_1} \left(\frac{d_i^2(\beta_1, \sigma^2)}{\nu} \right)}{1 + \frac{d_i^2(\beta_1, \sigma^2)}{\nu}} \right) \right],$$

$$U(\sigma^2) = -\frac{1}{2\sigma^2} - \frac{\nu+1}{2} \frac{1}{N} \left[\sum_{i=1}^k \log \left(-\frac{\frac{\partial}{\partial \sigma^2} \left(\frac{d_i^2(\beta_0, \sigma^2)}{\nu} \right)}{1 + \frac{d_i^2(\beta_0, \sigma^2)}{\nu}} \right) + \sum_{i=1}^k \log \left(-\frac{\frac{\partial}{\partial \sigma^2} \left(\frac{d_i^2(\beta_1, \sigma^2)}{\nu} \right)}{1 + \frac{d_i^2(\beta_1, \sigma^2)}{\nu}} \right) \right]$$

With $V_1 = \text{diag}(v_1, v_2, \dots, v_k)$, $V_2 = \text{diag}(v_{k+1}, \dots, v_N)$.

So, developing estimates procedure using the EM algorithm, the observed log-likelihood in function equation (3) becomes

$$\log L(\eta) = L_k(Y/v; \eta) \\ = \log h(v) - \frac{1}{2} \log \sigma^2 - \frac{v+1}{2} \frac{1}{N} \left[\sum_{i=1}^k \log(1 + U(\beta_0)) + \sum_{i=k+1}^N \log(1 + U(\beta_i)) \right] \quad (9)$$

The $(r+1)^{\text{th}}$ iteration of the algorithm of the expectation maximize consists of two steps as follows,

E-Step: The conditional expectations $v_i^{(r)}$ should be obtained via

$$E(U_i / Y_i; \eta^{(r)}) = v_i^{(r)} = \frac{v+1}{v + d_i^2(\eta^{(r)})}$$

This based on the following weights function,

$$d_i^2(\eta^{(r)}) = \begin{cases} \frac{(Y_i - X_i^T \beta_0^{(r)})^2}{\sigma^{2(r)}}, & i = 1, 2, \dots, k \\ \frac{(Y_i - X_i^T \beta_i^{(r)})^2}{\sigma^{2(r)}}, & i = k+1, 2, \dots, N \end{cases}$$

Where the independent random variables $U_i \sim \text{Gamma}\left(\frac{v+1}{2}, \frac{v+d_i^2}{2}\right)$

M-Step: Using the estimates to be obtained at E-step likelihood estimates to obtain

$$\beta_0^{(r+1)} = (X_1^T V_1^{(r)} X_1)^{-1} X_1^T V_1^{(r)} Y_1$$

$$\beta_1^{(r+1)} = (X_2^T V_2^{(r)} X_2)^{-1} X_2^T V_2^{(r)} Y_2$$

$$\sigma^{2(r+1)} = \frac{1}{N} \{U_{V_1^{(r)}}(\beta_1^{(r)}) + U_{V_2^{(r)}}(\beta_2^{(r)})\}$$

With

$$V_1 = \text{diag}(v_1, v_2, \dots, v_k), \quad V_2 = \text{diag}(v_{k+1}, \dots, v_N)$$

The algorithm proceeds between E and M step until the sequence $\eta^{(r)}$ converges. It is to be noted that the degree of freedom will be $v = N - 2$.

5. Experimental Work

The data used in this research work was the monthly All-share Index (ASI) of Market Portfolios / Sectors indexes derived from the Nigeria Stock Exchange (NSE) from 1:2000 to 12:2015. Twelve crucial Sectors were considered based on the constant supply of their indexes by NSE, impact on the Nigerian economy and how crucial to the Nigeria market. These Sectors are agriculture, building and construction, wholesale and retail, manufacturing, quarry & mining, petrol & gas, transportation, communication, education, health, utilities, finance and insurance.

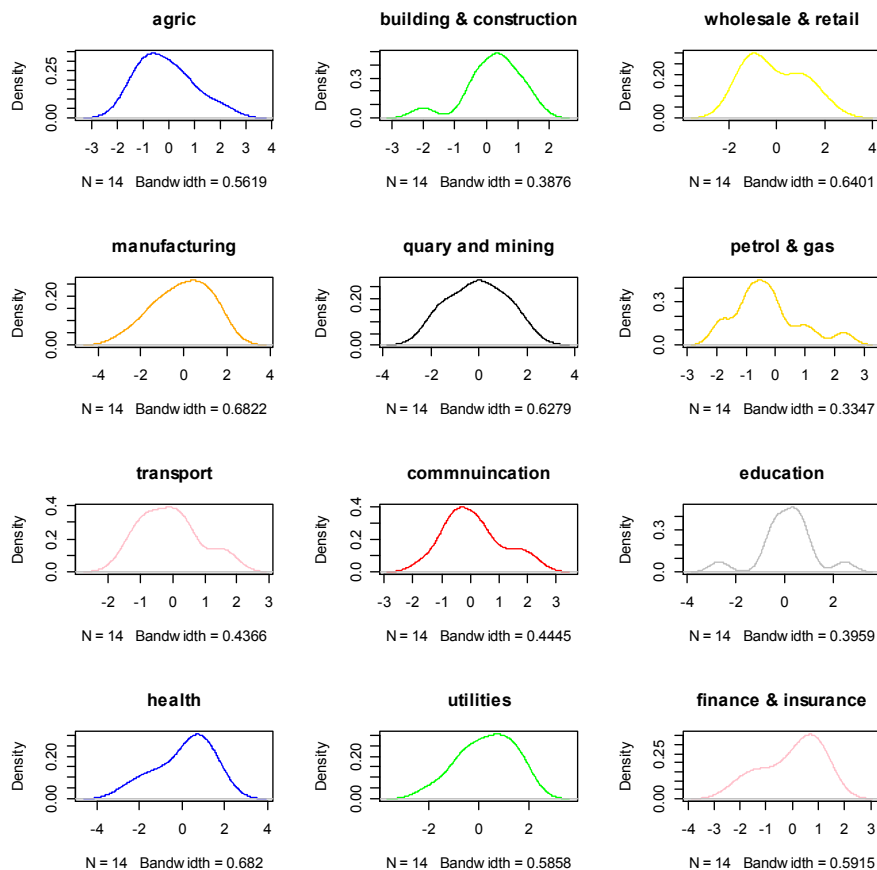


Figure 1. The Kernel Density Curves of each of the Sectors

It was noted that from Figure 1 above that among the sectors considered only Agriculture sector, Building & Construction sector, Manufacturing sector, Quarry & Mining sector, Communication Sector, Education sector, Transportation sector, and Utilities sector equity returns had bell-shapes, that is, they are approximately normally distributed (Mesokurtic in nature) while other sectors like Wholesale & Retail, Petrol & Gas, Health, and Finance & Insurance are not normally distributed.

It can be deduced from table 1 above that the sectors return series of Health Sector, Utilities Sector, and Finance & Insurance Sector are negatively skewed that is, skewed to the

left with thin-tailed (leptokurtic) while other sectors that seemed not normal are positively skewed. The P-value of the Jarque-Bera tests pointed-out that the return series of Agriculture sector, Building & Construction sector, Manufacturing sector, Quarry & Mining sector, Transportation sector, Education sector, Communication sector and Utilities Sector are normally distributed while the return series of Wholesale & Retail sector, Petrol & Gas sector, Health sector and Finance & Insurance sector are non-normally distributed. This conclusion conformed to the afore-noted normally distributed sectors and non-normally distributed sectors from the kernel density curves in Fig. 1.

Table 1. Descriptive Statistics and Jarque-Bera Test for the Market Portfolios.

Sectors	Mean	S.D	Skewness	Kurtosis	Jarque-Bera	P-Value
Agriculture	261.7	64.4428	0.1738	1.6765	1.0923	0.5792**
Building & Construction	11.600	5.3918	0.6389	2.2099	1.3167	0.5177**
Wholesale & Retail	106.50	48.0506	0.4855	1.8604	1.9975	0.3201
Manufacturing	25.40	8.5143	0.2956	1.8231	1.0118	0.603**
Quarry & Mining	2.013	0.8899	0.6703	2.2100	1.4125	0.8935**
Petrol & Gas	122.0	9.5561	0.1611	2.1619	2.4703	0.2905
Transport	17.09	4.6554	0.2955	1.9303	0.8712	0.6469**
Communication	22.990	2.2454	1.1284	3.0269	2.9712	0.7264**
Education	1.216	0.47336	0.4809	1.9547	1.1771	0.5551**
Health	0.2829	0.1024	-0.5505	1.9422	2.487	0.3066
Utilities	20.70	4.8598	-0.5582	2.2623	0.289	0.7132**
Finance & Insurance	24.31	4.2578	-0.0569	2.1011	2.4788	0.2871

** Significant at 5% Level

From the estimated coefficients of beta-risk (systematic risk) in Table 2 below, it was noted that the beta-risk estimates for both the Gaussian error and student-t error innovations for Agricultural sector and Financial & Insurance sector were valued as (1.0868 and 1.4390) and (1.0868 and 1.2220) respectively ($\hat{\beta}_1$'s > 1), this implies that the two sectors were nimble and operative, that is, the two sectors have been in a state of working order. In other words, the security's price in the two sectors seemed theoretically more volatile than the market (economy) around 8% and 50% respectively with the possibility of a higher rate of return over 15 years. While Building & Construction sector, Manufacturing sector, Quarry & Mining sector, Communication sector, Transportation sector, Education sector, Health sector, and Utilities sector with their beta-risk estimates under Gaussian error and student-t innovations were also valued to be (0.5901, 0.7930, 0.6288, 0.2243, 0.9839, 0.6981, 0.7586, 0.9687) and (0.5901, 0.7930, 0.6289, 0.2243, 0.9837, 0.6981, 0.6919, 0.9687) respectively ($\hat{\beta}_1$'s < 1). It seemed that the security's price in the seven sectors theoretically are less volatile than the market

(economy) around 41%, 31%, 38%, 78%, 2%, 41%, 70% and 4% respectively, that is, the fund's excess return is anticipated to underachieve the benchmark by these percentages aforestated in boasting the economy (markets) and perform better by these percentages during receding economy (markets). In others, their systematic risks are fairly efficient in explaining the relationship between the economy risk and return.

It was noted that the estimated standard errors for the non-diversifiable risks in Table 2 for student-t error innovation for the seemed non-normally distributed sectors of Wholesale & Retail, Petrol & Gas, Health, and Finance (0.0124, 0.9539, 0.0579, 0.1311) respectively are smaller than that of the Gaussian error innovation (0.0114, 0.8832, 0.0379, 0.0100) respectively. This was due to the fact that normal error type model was fitted to non-normality error innovation type model. Also, the Akaike Information Criteria (AIC) values were also smaller in student-t innovation compared to the Gaussian innovation in these seven sectors, meaning these four sectors had been fluctuating over the past 15 years.

Table 2. Estimates of the Beta-Risk for each Sector with Gaussian and Student-t Innovations

Sectors	Distribution	Abnormal return ($\hat{\beta}_0$)	Beta-Risk ($\hat{\beta}_1$)	$\hat{\sigma}$	AIC	Log-Lik	R.S.E
Agriculture	Gaussian	0.4071 0.1311**	1.0868 0.0237**	0.0022	-62.1783	35.0892	0.0213
	Student-t	0.40712 0.1311**	1.0868 0.0237**	0.0022	-62.1785	35.3459	0.0213
Building & Construction	Gaussian	5.0381 0.0297**	0.5901 0.0124**	0.0250	-63.14305	35.57	0.0206
	Student-t	5.0381 0.0295**	0.5901 0.0114**	0.0230	-63.0031	35.5715	0.0203

Sectors	Distribution	Abnormal return ($\hat{\beta}_0$)	Beta-Risk ($\hat{\beta}_1$)	$\hat{\sigma}$	AIC	Log-Lik	R.S.E
Wholesale & Retail	Gaussian	0.7541 0.0888**	0.5953 0.0193**	0.0056	-51.07046	29.54	0.0317
	Student-t	3.5100 0.0883**	0.4310 0.0069**	0.0031	-53.07046	30.6781	0.0300
Manufacturing	Gaussian	3.9035 0.0497**	0.7930 0.0155**	0.0200	-65.0925	36.55	0.0192
	Student-t	3.9035** 0.0410**	0.7930 0.0143**	0.0153	-66.0926	36.9463	0.0185
Quarry & Mining	Gaussian	6.0412 0.0126**	0.6288 0.0170**	0.0265	-56.16228	32.08114	0.0264
	Student-t	6.0412 0.0116**	0.6289 0.0175**	0.0230	-58.16228	33.1201	0.0260
Petrol & Gas	Gaussian	1.8568 4.5803**	0.9518 0.9539**	0.2726	-0.2825	7.1650	0.2725
	Student-t	1.7840 4.5803**	0.7930 0.8832**	0.1223	-0.5825	9.1651	0.2271
Transport	Gaussian	3.6676 0.0799**	0.9839 0.0283**	0.0283	-56.3580	31.17901	0.0282
	Student-t	3.6674 0.0798**	0.9837 0.0237**	0.0245	-56.6917	36.34587	0.0253
Communication	Gaussian	5.8570 0.0095**	0.2243 0.0054**	0.0141	-72.2228	40.1111	0.0120
	Student-t	5.8570 0.0014**	0.2243 0.0051**	0.0100	-72.2212	40.1110	0.0118
Education	Gaussian	6.3383 0.0052**	0.6981 0.0134**	0.002	-65.57863	36.78932	0.0189
	Student-t	6.3383 0.0049**	0.6981 0.0124**	0.0173	-67.57864	36.9972	0.0182
Health	Gaussian	7.2889 0.0518**	0.7586 0.0579**	0.0480	-41.2175	23.6088	0.0484
	Student-t	6.7860 0.0302**	0.6919 0.0379**	0.0316	-43.6001	25.8001	0.0301
Utilities	Gaussian	3.5195 0.3118**	0.9687 0.1036**	0.0995	-19.33355	13.66678	0.09846
	Student-t	3.5195 0.3001**	0.9687 0.0959**	0.0964	-19.33355	13.66678	0.004914
Finance & Insurance	Gaussian	1.8559 0.4170**	1.4390 0.1311**	0.0854	-25.33826	15.66913	0.08534
	Student-t	1.6780 0.3412**	1.2220 0.0100**	0.0354	-28.33826	17.66913	0.0085

NOTE: Standard Errors estimates are asterisked.6. Conclusion

It was found that not in all cases that the error term in the CAPM model for estimating the beta-risk and the abnormal return is independently, identically and normally distributed. Subjecting the error term to student-t distribution encompasses and accommodates both fat-tailed and bell-shaped distributed variables.

The student-t error innovation approximately gives the same estimate and standard error of such estimate if the sectors/assets are independently, identically and normally distributed while subjecting the variable under consideration to Gaussian whereas it follows independent and identical student-t distribution, the estimates under student-t will be more stable. Student-t approaches the true parameter with a smaller standard error compared to indexes of the estimates from normally distributed error innovation.

References

- [1] Aduda, J. and Muimi P. (2011). Test for investor rationality for companies listed at the Nairobi stock exchange. *Journal of Modern Accounting and Auditing*, Vol. 7 (8): 827-840.
- [2] Bajpai, S. and Sharma, A. K. (2015). An empirical testing of capital asset pricing model in India. *Procedia – Social and Behavioral Sciences* 189:259-265.
- [3] Bouchaddekh, T, Bouri, A. and Kefi, M. K. (2014), Capital asset pricing model with frictions: Application to the Tunisian stock market. *International Journal of Economics and Financial Issues (IJEFI)*. Vol. 4 (3): 2146-4138.
- [4] Brown, P. and Walter, T. (2013). The CAPM: Theoretical validity, empirical intractability and practical applications. *ABACUS, A Journal of Accounting, Finance and Business Studies*, Vol. 49 (S1): 44-50.
- [5] Cheema, A. K. (2010). Test of capital asset pricing model on stocks at Karachi stock exchange. *Paradigms: A Research Journal of Commerce, Economics and Social Sciences*, Vol. 4 (1): 80 -97.
- [6] Dawson, P. C. (2015). The capital asset pricing model in economic perspective. *Journal of Applied Economics*, Vol. 47 (6): 569 – 598.
- [7] Demircioglu, E. (2015). Testing of capital assets pricing model (CAPM) in cement sector & power generation and distribution sector in Turkey. *International Journal of Advanced Multidisciplinary Research and Review* Vol. 3(4): 1 – 25.

- [8] Fama, E. F. and French, K. R. (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives*, Vol. 18 (3): 25–46.
- [9] Francioni, R. and Schwartz, R. A. (2017). Equity markets in transition: The value chain, price discovery, regulation, and beyond. 1st ed; *Springer*.
- [10] Gençay, R; Selçuk, F. and Whitcher, B. (2005). Multiscale systematic risk. *Journal of International Money and Finance*, Vol. 24 (1): 55 – 70.
- [11] Hassan, M. Z. et al. (2012). Relationship between risk and expected returns: Evidence from the Dhaka stock exchange. *Proc. Econ. Finance*, Vol.2: 1-8.
- [12] Hurn, S., Martin, V., Phillips, P. and Jun Yu (2015). Financial econometric modeling. *Current Working Document*. http://mysmu.edu/faculty/yujun/MSFE_FEC/PartI.PDF (Downloaded 21/12/2016).
- [13] Jensen, M. C and Scholes, M. (1972). The capital asset pricing model: Some empirical tests. *Studies in the Theory of Capital Markets*: 5-54.
- [14] Lee, H, Cheng, F and Chong, S. (2016). Markowitz portfolio theory and capital asset pricing model for Kuala Lumpur stock exchange: A case revisited. *International Journal of Economics and Financial Issues*, Vol. 6(S3): 59-65.
- [15] Leonard, F; et al (2012). The Capital Asset Pricing Model (CAPM). Investment and Valuation of Firms. http://www.uhu.es/45151/temas/Unit%205_1_paper_CAPM.pdf (Downloaded 15/12/2016).
- [16] Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*. 47 (1): 13–37.
- [17] Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, Vol 7(1):77–99.
- [18] Masood, Y. et al. (2012). Capital asset pricing model: Empirical evidence from Pakistan. *MPRA. MPRA Paper No. 41961*. https://mpra.ub.uni-muenchen.de/41961/1/MPRA_paper-41961.pdf (Downloaded 14/01/2017).
- [19] Miller, M. H. (1999). The history of finance. *The Journal of Portfolio Management* Vol. 25: 95-101.
- [20] Mossin, Jan. (1966). Equilibrium in a capital asset market. *Econometrica*, Vol. 34(2): 768-83.
- [21] Mullins, D. W. (1982). Does the capital asset pricing model work? *Havard Business Review*, January Issue. <https://hbr.org/1982/01/does-the-capital-asset-pricing-model-work>. (Downloaded 27/01/2017).
- [22] Muthama, A. K; Munene, M. G. and Tirimba. O. I. (2014). Empirical tests of capital asset pricing model and its testability for validity verses invalidity. *International Journal of Innovation and Applied Studies*, Vol. 9 (2): 600-614.
- [23] Myers S C, Turbull S M (1977). Capital budgeting and the capital asset pricing model: Good news and bad news. *Journal of Finance*, 32(2): 321–333.
- [24] Nel, W. S. (2011). The application of the Capital Asset Pricing Model (CAPM): A South African Perspective. *African Journal of Business Management*. Vol. 5(13):5336-5347.
- [25] Novak, J. (2015), Systematic risk changes, negative realized excess returns and time- varying CAPM beta. *Finance a Uver - Czech Journal of Economics and Finance*, 65(2): 167-190.
- [26] Nwani, C. (2013). CAPM beta and the UK stock returns. *International Journal of Science and Research (IJSR)*. Vol. 4 (2): 1117-1123.
- [27] Nyangara, M. et al. (2016). An empirical test of the validity of the capital asset pricing model on the Zimbabwe Stock Exchange. *International Journal of Economics and Financial Issues*, 2016, Vol. 6(2): 365-379.
- [28] Oke, B. O (2013). Capital asset pricing model (CAPM): Evidence from Nigeria. *Research Journal of Finance and Accounting*, Vol.4 (9): 17-27.
- [29] Osamwonyi, I. O and Asein, E. I (2012). Market risk and returns: Evidence from the Nigerian capital market. *Asian Journal of Business Management*, Vol 4(4):367-372.
- [30] Pamane, K. and Vikpossi, A. E. (2014). An analysis of the relationship between risk and expected return in the BRVM stock exchange: Test of the CAPM. *Research in World Economy*, Vol. 5 (1): 3-28.
- [31] Perold, A. F. (2004). The capital asset pricing model. *Journal of Economic Perspectives*, Vol. 18 (3): 3–24.
- [32] Ross, S. A. (1976), The arbitrage theory of capital asset pricing, *Journal of Economic Theory*. Vol.13: 341-360.
- [33] Santis, R. A. (2010). The geography of international portfolio flows, international CAPM, and the role of monetary policy frameworks. *International Journal of Central Banking*, Vol. 6 (2):147 – 197.
- [34] Sharpe, William F. (1964). Capital Asset Prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*. Vol.19 (3):425–442.
- [35] Treynor, J. L. (1965). How to rate the performance of mutual funds. *Harvard Business Review*. Vol.43:63–75.
- [36] Tumala, M. M. and Yaya, O. S. (2015). Estimating bull and bear betas for the Nigerian stock market using logistic smooth threshold model. *CBN Journal of Applied Statistics*. Vol. 61 (b): 263-283. <http://www.investopedia.com/terms/b/beta.asp>.
- [37] Ward, M. and Muller, C. (2013). Empirical testing of the CAPM on the JSE. *Investment Analysts Journal*, No. 76: 1-12.