

To Measure the Risk of Projects Financed from Structural Funds by a Fuzzy Logic System

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Abstract: To measure the risk of projects financed from structural funds is very difficult because there involved a great number of risks during the whole project process. Accordingly, a fuzzy logic system was applied to measure the risk of projects financed from a structural fund. First, the systematic structure of risk is also investigated, and the risk activities are analyzed for reflecting the finance problems, where the financing risk consists of basic risk element, project risk, and financing agreement in the second level. Second, a fuzzy risk measurement method is illustrated for risk management of projects. For each systematic part, the fuzzy logic system can be used to analyze and quantify different risks. At last, an experimental analysis was presented to verify the proposed model and some practical instructions are also indicated, as well as some interesting conclusions and future research directions.

Keywords: Projects Financed, Structural Funds, Fuzzy Logic System, Minimax

1. Introduction

Risk measurement has gained more and more attention in project management to improve the investment efficiency. Wallenius (2008) built a multiple criteria decision making, multi-attribute utility theory from recent accomplishments and what lies ahead [1]. Luce (2014) considered an inductive flux usage and its optimization in tokamak operation [2]. Jongman (2014) concerned increasing flood exposure in the Netherlands with implications for risk financing [3]. Pratt (2016) described the statistical analyses of hydrophobic interactions and made a mini-review [4]. Fishburn (2010) discussed the system tradeoffs in gamma-ray detection utilizing SPAD arrays and scintillators [5]. Roncoroni (2016) considered an energy finance and economics from the viewpoints of analysis and valuation, risk management, and the future of energy [6]. Smith (2012) developed a prospective randomized comparison of regular and diet lemonade upon risk for urinary stone formation [7]. Linn (2007) gave a complexity and the character of stock returns

with empirical evidence and a model of asset prices based on complex investor learning [8].

Many researchers indicated that the risk measurement is very difficult to make in projects financed from structural funds. Bertsimas (2015) presented a forum-tenure analytics and a model for predicting research impact [9]. Gonsalvez (2016) extended the shared risks in supply chain self-financed for incremental sales [10]. Bolos (2016) evaluated a business intelligence instrument for detection and mitigation of risks related to projects financed from structural funds [11]. Card (2015) put forward an inference on causal effects in a generalized regression kink design [12]. Csoka (2015) implied a corporate financing under moral hazard and the default risk of buyers [13]. Criscuolo (2015) indicated the environmental policies and risk finance in the green sector with cross-country evidence [14].

To solve these kinds of uncertain problems and risk measurement, fuzzy mathematics was applied. Bolos (2015) introduced the development of fuzzy logic system to identify the risk of projects financed from structural funds [15]. Frisari (2015) illustrated a de-risking concentrated solar

power in emerging markets from the role of policies and international finance institutions [16]. Jensen (2015) made a research on personal finance and life insurance under separation of risk aversion and elasticity of substitution [17]. Abdulkadiroglu (2014) talked about the elite illusion based on the achievement effects at Boston and New York exam schools, [18]. Verguet (2015) proposed a health gain and financial risk protection afforded by public financing of selected interventions in Ethiopia with an extended cost-effectiveness analysis [19]. Di Iasio (2015) offered a special issue of Quantitative Finance on 'Interlinkages and Systemic Risk' Foreword [20]. Guerra (2014) concerned an interval and fuzzy average internal rate of return for investment appraisal [21]. Lee (2015) researched the financing and risk management of renewable energy projects with a hybrid bond [22]. Jongman (2014) studied the increasing stress on disaster-risk finance due to large floods [23]. Tsetlin (2015) built a generalized almost stochastic dominance [24].

But to measure the risk of projects financed from structural funds is different from traditional risk measurement [8, 12], and still can not be easily solved by traditional crisp value. Here a fuzzy logic system is applied to measure the risk of projects financed from a structural fund. First, the systematic structure of risk is also investigated, and the defect is analyzed for reflecting the finance problems where the financing risk consists of basic risk element, project risk, and financing agreement in the second level. Second, a fuzzy risk measurement method is illustrated for risk management of

projects. For each systematic part, the fuzzy logic system can be used to analyze and quantify different risks. At last, experimental analysis is presented to verify the proposed model where the analytical results and other common methods are also provided, and some interesting conclusions and future research work are indicated.

2. Risk of Projects Financed from Structural Funds

2.1. Project Model from Structural Funds

The classification of the structural fund should be concise and clear in the subject structure, such as the index of investment, to ensure liquidity and moderate leverage. From this point of view, the classification of funds is more suitable for passive investment where investors need to grasp market trends. From the perspective of the development history of project financing, financing options for the implementation of the project financing risk field has an important influence and implement the innovation of the way. Project model from structural funds is shown in figure 1. Considering a Shortest Path problem in a directed graph $G = (V, E)$ it can be found that a shortest path between any two nodes in G . There are two fuzzy cases in this problem and the costs of them are specified as triangular fuzzy intervals. For simplicity it can be considered a sample path.

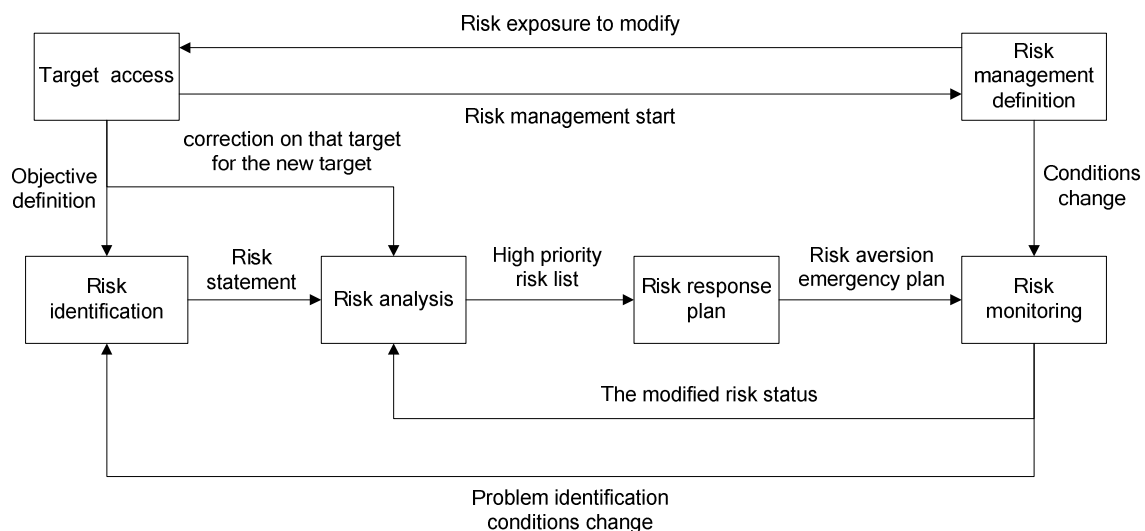


Figure 1. Project model from structural funds.

In structural funds, risk evaluation is different from un-structural funds where there are several possible outcomes and there is relevant past experience to enable statistical evidence to be protected for predicting the possible outcome. The attitude is often vague to deal with risk for three kinds of decision makers, there are risk averse, risk seeker and risk neutral, respectively. Although they have some different risk activities to solve those problems in risk classification, there are great difference and uncertainties in research techniques (such as desk research, field work and focus group) or

modelling techniques (such as expected value, sensitivity analysis, Monte Carlo simulation and decision tree), which make the risk analysis and aversion behavior inaccurate. What's more, from the point of view, the shortage of traditional crisp risk control for nonlinear, time-varying, and uncertain risk projects, are often difficult to be established as accurate mathematical models.

The possibility distributions between $\tilde{A}(x)$ and $\tilde{Z}(x)$ where observe fuzzy intervals are not triangular, provide us with some information about the shortest path x . For instance, the

maximum cost of the shortest path x can be gotten under the case which is in an interval $[4, 12]$. Especially, it is known to all that the maximum over all cases will not surpass 12, which can be expressed as $P(A(x) \leq 12) = 1$. In the same way, it is sure that the maximum deviation of x will not exceed 10. Similarly, $\rho_{\tilde{Z}(x)}(0) = 9/11$ means that the possibility that x will be a shortest path (its deviation will be equal to zero) regardless of all cases that will occur at 9/11. According to Mammalian algorithm, taking a ζ -cut of $\tilde{A}(x)$ (or $\tilde{Z}(x)$), a range of values of the maximum cost can be gotten whose possibility of occurrence exceeds ζ . So, the right bounds of $\tilde{Z}^\zeta(x)$ and $\tilde{\Delta}^\zeta(x)$ for $\zeta \in [0, 1]$ can be taken into account.

In order to choose a solution, an additional criterion must be provided. Let X and Y be the universes of input variables x and y , respectively, and Z be the universe of output variable

z. Let $\tilde{G} = (g, \beta_g)$ be a fuzzy goal given by decision makers. G (the fuzzy goal) is a fuzzy set in R that is described as a preference of the decision makers for the values of the maximum cost (deviation) over all cases. So, the values in $[0, g]$ are all accepted while the values in $[g + \beta_g, \infty)$ are not accepted. The acceptance degree decreases linearly from 1 to 0 in the interval, and the following two kinds of optimization problems can be considered:

$$\begin{aligned} \text{Fuzzy Minimax } P: \max N(\tilde{A}(x) \in \tilde{G}) \\ x \in X \\ \text{Fuzzy Minimax } P: \max N(\tilde{Z}(x) \in \tilde{G}) \\ x \in X \\ \tilde{G}: (a) N(\tilde{A}(x) \in \tilde{G}) = 0; \\ (b) N(\tilde{A}(x) \in \tilde{G}) = 1 - \zeta^*; \\ (c) N(\tilde{A}(x) \in \tilde{G}) = 1. \end{aligned} \quad (1)$$

There is a similar concept that applied to fuzzy linear programming in this. If all costs under all cases are all precisely known and $\tilde{G} = (0, M)$ for a sufficiently is larger than M , then Fuzzy Minimax P is equal to Minimax P and Fuzzy Minimax P is equal to Minimax P . So, both fuzzy problems are generalization of the classical robust ones. For a fuzzy logic system, because its inputs are crisp quantity, they must be changed into fuzzy sets. This also means that fuzzy problems are not easier than their classical robust counter.

2.2. Risks in Projects

Project risk is in the stage of construction and operation, which is determined by the external and internal conditions of technical and economic risk. Let \tilde{G}^d be a complement of the fuzzy goal \tilde{G} , that is to say it has a membership function

of the form 1 to $\rho_{\tilde{G}}(y)$. So, \tilde{G}^d represents the degree of dissatisfaction of the values of the maximal cost (deviation) over all cases. Now the Fuzzy Minimax P is equal to minimizing the value of $\Pi(\tilde{A}(x) \in \tilde{G}^d)$ and Fuzzy Minimax P is equal to minimizing the value of $\Pi(\tilde{Z}(x) \in \tilde{G}^d)$ over all $x \in X$.

A wide class of discrete optimization problems can be put forward as the following 0–1 programming problem:

$$\begin{aligned} \text{Min } cx^T \\ P: x \in X \subseteq \{0, 1\}^n \end{aligned} \quad (2)$$

Where $x = [x_1, \dots, x_n]$ is a binary vector and $c = [c_1, \dots, c_n]$ is a real vector of costs. It is assumed that the set of X that feasible solution can be described by a linear system, namely:

$$X = \{x \in \{0, 1\}^n : Ax = b\} \quad (3)$$

Where A is a matrix and b is a vector that was fixed with coefficients. According to slack variables, $F(x, c)$ can be used to search the total cost that $F(x, c) = cx^T$ is as the solution x .

Besides, $F^*(c)$ will be denoted as the total cost of an optimal solution $\Delta(x, c) = F(x, c) - F^*(c)$ and x is under cost over all cases and it is optimal under c only if $\Delta(x, c) = 0$.

Formulation (1) is polynomial solvable for instance Minimum Cut, Minimum Spanning Tree, Shortest Path and Minimum Assignment while other ones are NP-problems, for instance, Traveling Salesperson or 0–1 Knapsack.

Traditionally, the values of all objective function are precisely known in deterministic case, there is only one exactly cost realization c which wish to be worked out. But it can be supposed that a known cost realization exactly from a finite set $\Gamma = \{c^1, \dots, c^k\}$ that will occur but it is impossible to predict which one it is. The vector $c^j = [c_1^j, \dots, c_n^j]$ is a cost realization under j -th case and contains k cases. It can be defined as

$$A(x) = \max_{j=1, \dots, k} F(x, c^j) \quad (4)$$

and

$$Z(x) = \max_{j=1, \dots, k} \Delta(x, c^j) \quad (5)$$

So, $A(x)$ donates the maximal cost and $Z(x)$ expresses the maximal deviation over all cases. There is

$$\text{Minimax } P: \min A(x) \quad (6)$$

$$x \in X$$

$$\text{Minimax } P: \min Z(x) \quad (7)$$

$$x \in X$$

Observing in both cases, the worst performance can be minimized in all cases. Unfortunately, minimax problem of P

can be difficult to calculate even though it can be solved by using the polynomial. However, if the k is a small number as in mixed integer linear programming, there is

$$\begin{aligned} \min t \\ \text{Min max P: } c^j x^T \leq t, j=1, \dots, k \\ x \in X \\ t \geq 0 \end{aligned} \quad (8)$$

$$\begin{aligned} \min t \\ \text{Minmax Regret P: } c^j x^T - F^*(c^j) \leq t, j=1, \dots, k \\ x \in X \\ t \geq 0 \end{aligned} \quad (9)$$

Observing the model above, minimax deviation P can be effectively structure hypothesis that the polynomial can be used to solve it. Otherwise, the amount of $F^*(c^j)$ can be difficult to be calculated.

3. Fuzzy Risk Measuring Method

3.1. Assumptions for Fuzzy Risk Measuring

For the sake of simplicity, it is assumed that all \tilde{c}_i^j are trapezoid fuzzy intervals, namely $\tilde{c}_i^j = (\underline{c}_i^j, \bar{c}_i^j, \alpha_i^j, \beta_i^j)$. This assumption is not a serious limitation, because in most application, it is sufficient application of trapezoidal fuzzy interval. Notice that if $\alpha_i^j = \beta_i^j = 0$ \tilde{c}_i^j is a closed interval and \tilde{c}_i^j is a closed interval, $k=1$ can be gotten. It is assumed that all cost is specified for the traditional closed interval.

Every vague scene may contain infinite possibility of implementation costs. Through the j -th fuzzy case the following joint probability distribution costs realizations $c = [c_1, \dots, c_n]$ can be described under j :

$$\pi^j(c) = \prod_{i=1}^n \tilde{c}_i^j = c_1 \Lambda \dots \Lambda c_n = c_n = \min_{i=1, \dots, n} \rho_{\tilde{c}_i^j}(c_i) \quad (10)$$

The value of $\pi^j(c)$ is the cost possibility with case j , and $\tilde{F}(x, c^j)$ is a fuzzy possibility distribution cost of solution x under cost (deviation) over all cases. According to the possibility theory, there is

$$\rho_{\tilde{F}(x, c^j)}(y) = \sup_{\{c: \tilde{F}(x, c) = y\}} \pi^j(c) \quad (11)$$

It is very easy to calculate $\tilde{F}(x, \tilde{c}^j)$ when all costs are precisely fuzzy intervals. Formula (3) can be applied, and there is

$$\sum_{i=1}^n (\bar{c}_i^j + \beta_i^j) x_i - (\sum_{i=1}^n \beta_i^j \zeta x_i) \leq g + \zeta \beta_g, j=1, \dots, k \quad (12)$$

And a fuzzy deviation $\tilde{\Delta}(x, c^j)$ can also be taken as a solution x under case j with possibility distribution function.

$$\mu_{\tilde{\Delta}(x, c^j)}(y) = \sup_{\{c: \tilde{\Delta}(x, c) = y\}} \pi^j(c) \quad (13)$$

On the contrary, the fuzzy interval $\tilde{\Delta}(x, \tilde{c}^j)$ is not trapezoidal, and can be defined as the fuzzy counterparts of $A(x)$ and $Z(x)$:

$$\tilde{A}(x) = \max_{j=1, \dots, k} \tilde{F}(x, c^j) \quad (14)$$

$$\tilde{Z}(x) = \max_{j=1, \dots, k} \tilde{\Delta}(x, c^j) \quad (15)$$

The $\rho_{\tilde{A}(x)}$ and $\rho_{\tilde{Z}(x)}$ are the biggest cost membership function and maximum deviation probability distribution function of x divided by all the solution case, respectively,

and $\prod_{Z(x)}(\tilde{Z}(x) = y) = \rho_{\tilde{Z}(x)}(y)$ and $\prod_{A(x)}(\tilde{A}(x) = y) = \rho_{\tilde{A}(x)}(y)$.

Generally, if the cost of the fuzzy scene is known, the deterministic real numbers $\tilde{A}(x)$ and $\tilde{Z}(x)$ equal to $A(x)$ and $Z(x)$, respectively.

3.2. Fuzzy Risk Management Architecture

In this paper, a binary variable $x_i \in \{0, 1\}$ with every edge $e_i \in E$ can be associated. Due to the shortest path is a special case of a network flow problem, setting X is very easy to be described by a linear system. There are two cases of cost problem. Under case one path: $s \rightarrow 1 \rightarrow 2 \rightarrow t$ is an optimal way and its cost is 5, and under case two path: $s \rightarrow 2 \rightarrow t$ is an optimal way and its cost is 6. Path $s \rightarrow 1 \rightarrow t$ is equal to the solution x that is the optimal minimal path and $A(x) = 8$. This path is also the optimal minimax deviation path and $Z(x) = 2$. Observing that path x is optimal either under case one or two. It is a compromise solution that has the best performance in the worst case. The minimax deviation approach to risk evaluation was discussed in [6, 9, 17]. Supposing that one needs to choose p among n available. So, variable $x_i \in \{0, 1\}$ indicates whether item i is chosen or not. The set X in this problem can be described as $x_1 + x_2 + \dots + x_n = p$. Obviously, matrix $A = [1, 1, \dots, 1]$ in this problem is totally unimodular. The sub problem (24) takes the following form:

$$\begin{aligned} \min \sum_{i=1}^n [\underline{c}_i^j(\zeta)(1-x_i) + \bar{c}_i^j(\zeta)x_i] y_i \\ u_0^j - u_i^j \leq (\bar{c}_i^j - \underline{c}_i^j + \alpha_i^j + \beta_i^j)x_i + \underline{c}_i^j - \alpha_i^j(1-\zeta) - (\alpha_i^j + \beta_i^j)t_i \\ y_1 + y_2 + \dots + y_n = p \\ 0 \leq y_i \leq 1, i=1, \dots, n \end{aligned} \quad (16)$$

Assigning dual variable u^j to the equality constraint and dual variables u_1^j, \dots, u_n^j $y_i \leq 1, i=1, \dots, n$, to constraints, the following dual model can be gotten:

$$\max pu_0^j - u_1^j - \dots - u_n^j$$

$$u_0^j - u_i^j \leq (\bar{c}_i^j(\zeta)x_i + \underline{c}_i^j(\zeta)(1-x_i))$$

$$u_i^j \geq 0, i = 1, \dots, n \quad (17)$$

Consequently, $\Phi(u^j) = pu_0^j - u_1^j - \dots - u_n^j$ and $D(c_x^j(\zeta))$ can be set and described by the constraints of the dual model, and can now be designed using formulation (14). There is

Min ζ

$$\sum_{i=1}^n (\bar{c}_i^j + \beta_i^j)x_i - \sum_{i=1}^n \beta_i^j t_i + \sum_{i=1}^n u_i^j \leq g + \beta_g \zeta - pu_0^j, j = 1, \dots, k$$

$$\sum_{i=1}^n x_i = p$$

$$u_i^j \geq 0, i = 1, \dots, n, j = 1, \dots, k$$

$$u_0^j - u_i^j \leq \bar{c}_i^j(\zeta)(1-x_i), i = 1, \dots, n, j = 1, \dots, k$$

$$\zeta \in [0, 1]$$

$$x_i \in \{0, 1\}, i = 1, \dots, n \quad (18)$$

It holds

$$\begin{aligned} \bar{c}_i^j(\zeta)x_i + \underline{c}_i^j(\zeta)(1-x_i) &= [\bar{c}_i^j + (1-\zeta)\beta_i^j]x_i + [\underline{c}_i^j - (1-\zeta)\alpha_i^j](1-x_i) \\ &= (\bar{c}_i^j - \underline{c}_i^j + \alpha_i^j + \beta_i^j)x_i - (\alpha_i^j + \beta_i^j)\zeta x_i - \alpha_i^j(1-\zeta) + \underline{c}_i^j \end{aligned} \quad (19)$$

Substituting t_i by A and x_i , and adding some additional constraints as in (17, 18), the final model can be gotten:

Min ζ

$$\sum_{i=1}^n (\bar{c}_i^j + \beta_i^j)x_i - \sum_{i=1}^n \beta_i^j \zeta x_i + \sum_{i=1}^n u_i^j \leq g + \beta_g \zeta - pu_0^j, j = 1, \dots, k$$

$$\sum_{i=1}^n x_i = p$$

$$t_i - x_i \leq 0, i = 1, \dots, n$$

$$\zeta - t_i + x_i \leq 0, i = 1, \dots, n$$

$$\zeta - t_i \geq 0, i = 1, \dots, n$$

$$u_i^j \geq 0, i = 1, \dots, n, j = 1, \dots, k$$

$$x_i \in \{0, 1\}, i = 1, \dots, n$$

$$t_i \geq 0, i = 1, \dots, n$$

$$\zeta \in [0, 1] \quad (20)$$

Let us illustrate the criterion $N(\tilde{A}(x) \in \tilde{G})$ as follows. And $N(\tilde{Z}(x) \in \tilde{G})$ will be similar, considering the Shortest Path problems again.

With possibility equal to 1, a case with completely

unacceptable cost of x may occur, so $N(\tilde{A}(x) \in \tilde{G}) = 0$. The fuzzy goal $\tilde{G} = (13, 3)$ can be set and the maximum cost is completely not in \tilde{G}^d , namely $\Pi(\tilde{A}(x) \in \tilde{G}^d) = 0$. In consequence, regardless of one case that will occur and regardless of the cost under this case, the cost will be acceptable. So, $N(\tilde{A}(x) \in \tilde{G}) = 1$. And $\Pi(\tilde{A}(x) \in \tilde{G}^d) = \zeta^*$, with possibility not less than ζ^* , the largest cost of x over all cases will not be greater than ζ^* . In consequence $N(\tilde{A}(x) \in \tilde{G}) = 1 - \zeta^*$.

4. Fuzzy Operational Objective Function

4.1. Fuzzy Minimax Problem

Let us build $\tilde{A}^\zeta(x) = [\underline{a}(x, \zeta), \bar{a}(x, \zeta)]$ with a ζ -cut of $\tilde{A}(x)$. $\tilde{F}^\zeta(x, \tilde{c}^j) = [\underline{f}^j(x, \zeta), \bar{f}^j(x, \zeta)]$ denotes a ζ -cut of $\tilde{F}(x, \tilde{c}^j)$. Using (12), it can be regarded a Fuzzy Minimax P problem.

$$\min \zeta$$

$$\bar{a}(x, \zeta) \leq \bar{g}(1 - \zeta)$$

$$x \in X$$

$$\zeta \in [0, 1] \quad (21)$$

If x^*, ζ^* is an optimal solution to (21), $N(\tilde{A}(x^*) \in \tilde{G}) = 1 - \zeta^*$. If (21) is incorrect, then $N(\tilde{A}(x) \in \tilde{G}) = 0$ will be accepted for all solutions $x \in X$. Now using the definition of $\tilde{A}(x)$ and formula (9), it can be gotten

$$\bar{a}(x, \zeta) = \max_{j=1, \dots, k} \bar{f}^j(x, \zeta).$$

So, it can be rewritten as follows:

$$\min \zeta$$

$$\bar{f}^j(x, \zeta) \leq \bar{g}(1 - \zeta), j = 1, \dots, k$$

$$x \in X$$

$$\zeta \in [0, 1] \quad (22)$$

Referring formulas (22) and (1), it can be obtained

$$\bar{f}^j(x, \zeta) = \sum_{i=1}^n \bar{c}_i^j x_i + (1 - \zeta) \sum_{i=1}^n \beta_i^j x_i = \sum_{i=1}^n (\bar{c}_i^j + \beta_i^j) x_i - \sum_{i=1}^n \beta_i^j \zeta x_i$$

and $\bar{g}(1 - \zeta) = g + \zeta \beta_g$. So, (16) is equivalent to

$$\min \zeta$$

$$\tilde{F}(x, \tilde{c}^j) = (\sum_{i=1}^n \bar{c}_i^j x_i, \sum_{i=1}^n \bar{c}_i^j x_i, \sum_{i=1}^n \alpha_i^j x_i, \sum_{i=1}^n \beta_i^j x_i)$$

$$\begin{aligned} x &\in X \\ \zeta &\in [0,1] \end{aligned} \quad (23)$$

It is easy to find that if all costs are specific values as $\alpha_i^j = \beta_i^j = 0$ and $\underline{c}_i = \bar{c}_i$ for all $i = 1, \dots, n$ and $j = 1, \dots, k$, and $\tilde{G} = (0, M)$ for an adequately large constant M , then model (23) will be equivalent to (21). So, Fuzzy Minimax P generalizes the Minimax P problem. However Model (23) is still not linear. It can be transformed into a linear model by importing $t_i = Ax_i$.

That is $t_i = \zeta$ if $x_i = 1$ and $t_i = 0$

If $x_i = 0$. The final model takes the following form:

$$\begin{aligned} \text{Min } &\zeta \\ \sum_{i=1}^n (\bar{c}_i^j + \beta_i^j) x_i - \sum_{i=1}^n \beta_i^j t_i &\leq g + \zeta \beta_g, j = 1, \dots, k \\ t_i - x_i &\leq 0, i = 1, \dots, n \\ \zeta - t_i + x_i &\leq 1, i = 1, \dots, n \\ \zeta - t_i &\geq 0, i = 1, \dots, n \\ x &\in X \\ \zeta &\in [0, 1] \\ t_i &\geq 0, i = 1, \dots, n \end{aligned} \quad (24)$$

The (24) is a mixed integer linear model and can be easily solved by software.

4.2. Fuzzy Minimax Problem

However, if P is polynomial solvable, then it can be formulated as a 0–1 linear constraint programming problem with a totally unimodular constraints matrix. Namely, the case for a wide class of network flow problems. As in the previous section, to solve this problem a decomposition of $\tilde{Z}(x)$ in ζ -cuts can be used.

Considering the formula $\tilde{Z}^\zeta(x) = [\underline{z}(x, \zeta), \bar{z}(x, \zeta)]$ in a ζ -cut of $\tilde{Z}(x)$, $\tilde{\Delta}^\zeta(x, \tilde{c}^j) = [\underline{\delta}^j(x, \zeta), \bar{\delta}^j(x, \zeta)]$ can be expressed as a ζ -cut of $\Delta(x, \tilde{c}^j)$. Using Fuzzy Minimax P in (12), the following mathematical problem can be formulated:

$$\begin{aligned} \text{Min } &\zeta \\ \bar{z}(x, \gamma) &\leq \bar{g}(1 - \gamma) \\ x &\in X \\ \zeta &\in [0, 1] \end{aligned} \quad (25)$$

If x, ζ is a group of optimal solutions to (38), then $N(\tilde{Z}$

$(x) \in \tilde{G} = 1 - \zeta$ can be gotten. If (24) is infeasible, then $N(\tilde{Z}(x) \in \tilde{G}) = 0$ will be accepted under all solutions $x \in X$. From (24, 25) and (9), it can be gotten $\bar{z}(x, \zeta) = \max_{j=1, \dots, k} \bar{\delta}^j(x, \zeta)$. So (25) is equivalent to the following formulation

$$\begin{aligned} \text{min } &\lambda \\ \bar{\delta}^j(x, \zeta) &\leq \bar{g}(1 - \zeta), j = 1, \dots, k \\ x &\in X \\ \zeta &\in [0, 1] \end{aligned} \quad (26)$$

Now, let us focus on the quantitative relation. Since it is the upper bound on of the ζ -cut of x under \tilde{c}^j , it can be calculated as follows:

$$\bar{\delta}^j(x, \lambda) = \sup_{\{c: \pi^j(c) \geq \zeta\}} \Delta(x, c) \quad (27)$$

However $\{c: \pi^j(c) \geq \zeta\} = (\tilde{c}_1^j)^\zeta \times \dots \times (\tilde{c}_n^j)^\zeta$, where $(\tilde{c}_i^j)^\zeta$ is a ζ -cut of \tilde{c}_i^j . Consequently, using the definition of $\Delta(x, c)$, it can be gotten

$$\bar{\delta}^j(x, \lambda) = \sup_{\{c \in (\tilde{c}_1^j)^\zeta \times \dots \times (\tilde{c}_n^j)^\zeta\}} \{F(x, c) - F^*(c)\} \quad (28)$$

It is clear that $(\tilde{c}_1^j)^\zeta \times \dots \times (\tilde{c}_n^j)^\zeta$ is a Cartesian vector in closed intervals $[\underline{c}_i^j(\zeta), \bar{c}_i^j(\zeta)]$ with $i = 1, \dots, n$. Let $c_x^j(\zeta) = [c_1, \dots, c_n]$ be a cost vector with $c_i = \underline{c}_i^j(\zeta)(1 - x_i) + \bar{c}_i^j(\zeta)x_i$. So, c_i is set to the upper bound $\bar{c}_i^j(\zeta)$ if $x_i = 1$, and to the lower bound $\underline{c}_i^j(\zeta)$ if $x_i = 0$. And $c_x^j(\zeta)$, $\bar{\delta}^j(x, \zeta) = F(x, c_x^j(\zeta)) - F^*(c_x^j(\zeta))$, there is:

$$\bar{\delta}^j(x, \zeta) = F(x, c_x^j(\zeta)) - \min_{y \in X} F(y, c_x^j(\zeta)) \quad (29)$$

Then x and A can be fixed in (29). Considering sub problem $\min \sum_{i=1}^n [\underline{c}_i^j(\zeta)(1 - x_i) + \bar{c}_i^j(\zeta)x_i] y_i$ which makes use of (14) and it can be rewritten by the definition of $c_x^j(\zeta)$ as follows:

$$\begin{aligned} \min \sum_{i=1}^n [\underline{c}_i^j(\zeta)(1 - x_i) + \bar{c}_i^j(\zeta)x_i] y_i \\ Ay^T = b \\ y_i \in \{0, 1\}, i = 1, \dots, n \end{aligned} \quad (30)$$

Where the constraints $y_i \in \{0, 1\}$ can be replaced in (30) with $0 \leq y_i \leq 1$ without the cost changing in (29). Then there is:

$$\min_{y \in X} F(y, c_x^j(\lambda))$$

$$A y^T = b$$

$$0 \leq y_i \leq 1, i = 1, \dots, n \quad (31)$$

Then a dual model can be constructed to (31) which has a dual variable vector u^j with the constraints of (30). Inspired by the target of the dual vectors $\varphi(u^j)$, the dual model can be expressed by the following formula

$$\max_{u^j \in D(c_x^j(\zeta))} \varphi(u^j)$$

And a more completed duality theorem follows

$$\min_{y \in X} F(y, c_x^j(\zeta)) = \max_{u^j \in D(c_x^j(\zeta))} \Phi(u^j) \quad (32)$$

Using (32) and (31) there is

$$\bar{\delta}^j(x, \zeta) = F(x, c_x^j(\zeta)) - \max_{u^j \in D(c_x^j(\zeta))} \varphi(u^j)$$

and model (31) changes into the following form:

$$\min_{\zeta} F(x, c_x^j(\zeta)) - \max_{u^j \in D(c_x^j(\zeta))} \varphi(u^j) \leq \bar{g}(1 - \zeta), j = 1, \dots, k$$

$$x \in X$$

$$\zeta \in [0, 1] \quad (33)$$

Taking no account of the maximum operator in (33), the following equivalent model temporary is:

$$\min_{\zeta}$$

$$F(x, c_x^j(\zeta)) - \varphi(u^j) \leq \bar{g}(1 - \zeta), j = 1, \dots, k$$

$$x \in X$$

$$u^j \in D(c_x^j(\zeta)), j = 1, \dots, k$$

$$\zeta \in [0, 1] \quad (34)$$

Since $F(x, c_x^j(\zeta)) = \sum_{i=1}^n \bar{c}_i^j(\zeta) x_i = \sum_{i=1}^n [\bar{c}_i^j + (1 - \zeta) \beta_i^j] x_i$
 $= \sum_{i=1}^n (\bar{c}_i^j + \beta_i^j) x_i - \sum_{i=1}^n \beta_i^j \zeta x_i$ and $\bar{g}(1 - \zeta) = g + \zeta \beta_g$, it can be gotten

$$\min_{\zeta} \sum_{i=1}^n (\bar{c}_i^j + \beta_i^j) x_i - \sum_{i=1}^n \beta_i^j \zeta x_i - \varphi(u^j)$$

$$\leq g + \zeta \beta_g, j = 1, \dots, k \quad (35)$$

$$x \in X$$

$$u^j \in D(c_x^j(\zeta)), j = 1, \dots, k$$

$$\zeta \in [0, 1]$$

In model (35) some nonlinear formulas form the ζx_i appeared. Making (35) into linear by replacing all the expressions that has additional variables t_j . After such modification, formula (35) will be a complex problem mixed integer linear one. Then let us do some observation on the obtained model.

For instance, if all precise costs can be taken under different uncertain cases, which is $\alpha_i^j = \beta_i^j = 0$ and $\bar{c}_i = \underline{c}_i$

for all $i = 1, \dots, n$ and $j = 1, \dots, k$, and $\tilde{G} = (0, M)$ is of a sufficiently large constant M , then the model (35) will be equivalent to model (21, 22). So, Fuzzy Minimax P will be same as the Minimax P problem. That is $\alpha_i^1 = \beta_i^1 = 0$ for all $i = 1, \dots, n$. Not only that, $\tilde{G} = (0, M)$ for a large constant M . The model (35) turns into the following form:

$$\min \zeta$$

$$\sum_{i=1}^n c_i x_i - \varphi(u^1) \leq \zeta M$$

$$x \in X$$

$$u^1 \in D(c_x^1(\zeta))$$

$$\zeta \in [0, 1] \quad (36)$$

which is equivalent to

$$\min \sum_{i=1}^n c_i x_i - \varphi(u^1)$$

$$x \in X$$

$$u^1 \in D(c_x^1(\zeta)) \quad (37)$$

Model (37) is a complex problem which is a mixed integer linear formulation for the Fuzzy Minimax P with interval costs. Model (37) was supported by [24] for the Shortest Path model and in [21] for Credit Risk with machine learning methods.

5. Experimental Analysis

5.1. Problem Description

In the Chinese High-speed rail project, a large amount of money is badly needed where the structural fund is used. Because the structural fund uses the structured classification technology, more short-term funds can be fed into investment with lower cost to produce higher yields in longer duration of the capital. In this way, more and more investors can be easily involved in to obtain higher investment returns. Here the risk project is simplified to demonstrate the uncertainty of structural funds. Supposing that the sample models, which is

a part of a risk communication network and the weights are project cost between the points of this network. There are two time cases, which may lead to two risk events (for instance two possible traffic risks) that have a global influence on the network. It is not sure which risk will happen and both of them must be taken into account while choosing a solution. The fuzzy case based on project finance allows us to model a fuzzy statistical logic system that corresponds to some unpredictable risks having a global influence on the costs, while a second fuzzy natural type of uncertainty may also appear. Although in every fuzzy case risk costs are calculable, some costs as traveling times are rarely precisely known in the fuzzy model. Assuming that a traveling time will fall within an interval of statistical possible values. Thus it is reasonable to have a second type of uncertainty connected with an uncertain nature of the costs in this fuzzy statistical logic system. The risk measurement parameters in project example are shown in Table 1.

Table 1. The risk measurement parameters in project example.

Monitor	Evaluation	Emergency	Evaluation	Danger	Evaluation
(1, 3, 10)	1	(1, 3, 10)	1	(1, 3, 10)	1
(1, 3, 20)	3	(1, 3, 20)	4	(1, 3, 20)	6
(1, 3, 30)	9	(1, 3, 30)	10	(1, 3, 30)	21
(1, 6, 10)	2	(1, 6, 10)	4	(1, 6, 10)	8
(1, 6, 20)	7	(1, 6, 20)	20	(1, 6, 20)	15
(1, 6, 30)	12	(1, 6, 30)	31	(1, 6, 30)	47
(1, 9, 10)	25	(1, 9, 10)	39	(1, 9, 10)	85
(1, 9, 20)	46	(1, 9, 20)	132	(1, 9, 20)	153
(1, 9, 30)	101	(1, 9, 30)	178	(1, 9, 30)	247

While fuzzy cases could model the statistical uncertainty and the correlations between the costs, the risk intervals model a local uncertainty connected with the fuzzy nature of a single risk cost. If $k = 1$ and the risk costs are specified as closed intervals, the fuzzy interval uncertainty representation and this type of problems is widely discussed in literature. However, if $k = 1$, the structural statistical uncertainty would be omitted. So, it is reasonable to take both types of uncertainty risks analysis into account. Namely, there are several fuzzy cases and under every case the costs may be uncertain. So, in this approach a fuzzy case is a vector of uncertain costs.

The next section would show how to extend the classical fuzzy discrete case based approach to take a local uncertainty risks loss into account, and a fuzzy intervals to model the imprecise costs under all cases. The possibility theory could be applied to extend more robust criteria.

5.2. Results and Analysis

In this section, the results of some computational tests would be presented. Assuming that P is the Risk evaluation problem. Distinctly, the computation times may depend on a particular risk problem P , which solves the fuzzy problems by a particular solver applied. The aim, however, was to identify the factors that make the most effect on the computation times and risks loss. A family of Fuzzy Minimax or Risk evaluation could be denoted as (n, k, d) , and the fuzzy language membership functions are shown in figure 2 and 3.

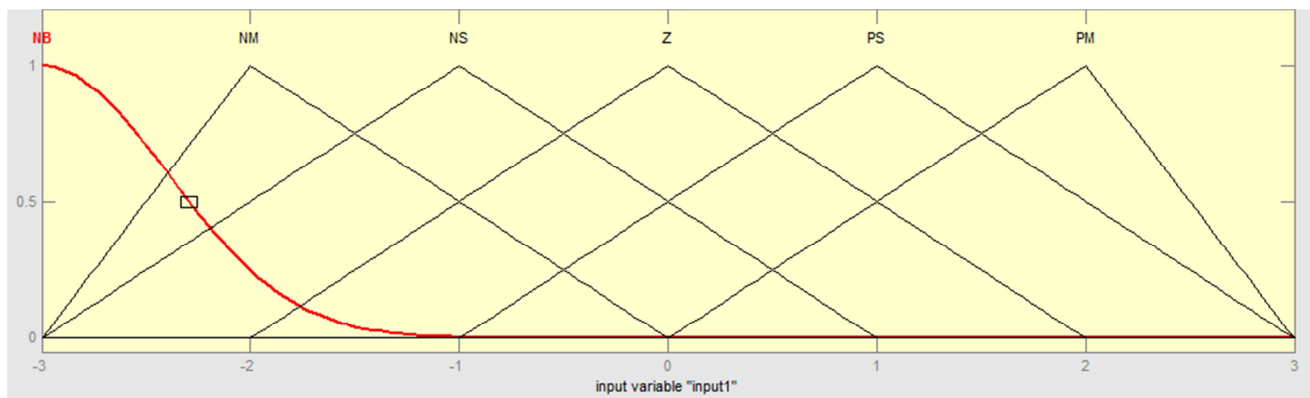


Figure 2. Fuzzy languages of input.

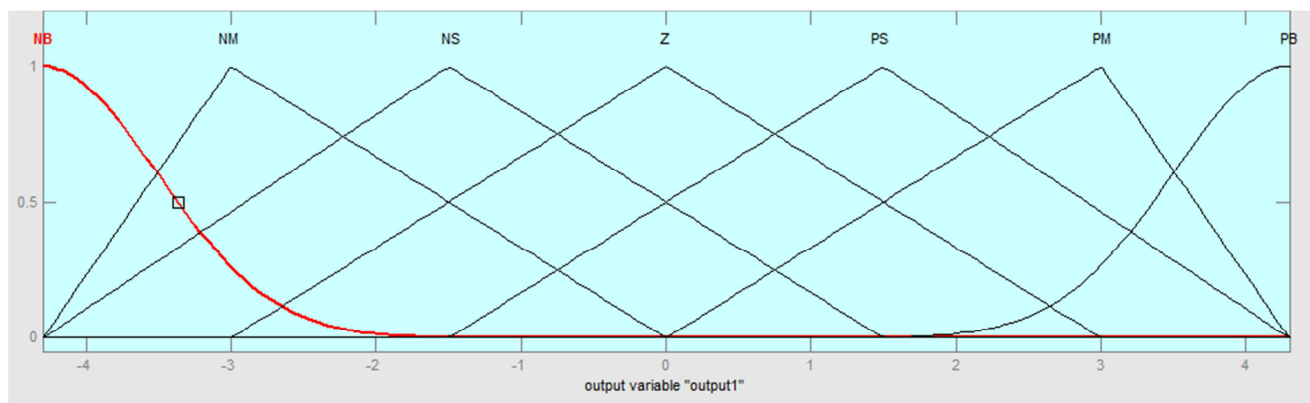


Figure 3. Fuzzy languages of output.

So, it is assumed that these impact factors will be independent for all particular problems P .

- n is the binary number of risk and can be calculated by $p = \lceil n/2 \rceil$. Thus, a selection of $\lceil n/2 \rceil$ is sought among n ;
- k is the number of fuzzy cases;
- d is an uncertain parameter of the costs under uncertain environment. Namely, random fuzzy cost can be

obtained through $\tilde{c}_i^j = (\underline{c}_i^j, \bar{c}_i^j, \alpha_i^j, \beta_i^j)$ so that $(\tilde{c}_i^j)^0 = [\underline{c}_i^j - \alpha_i^j, \bar{c}_i^j + \beta_i^j]$ is entirely contained in the interval $[0, 100]$ and the length of (\tilde{c}_i^j) is d . If $d = 0$, all risks costs are expressed as crisp numbers and the greater d result in the more imprecise costs under the fuzzy cases.

The fuzzy rules are shown in figure 4.

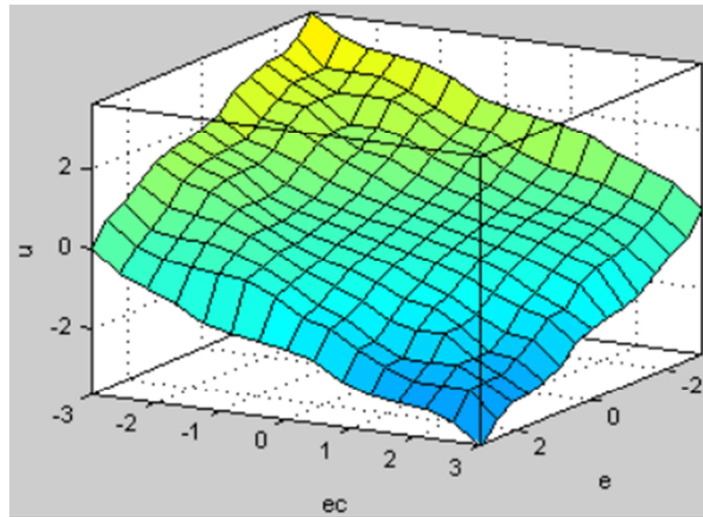


Figure 4. Fuzzy rules.

The risk measurement of $\tilde{G} = (g, \beta_g)$ depends on different environments. In Fuzzy Minimax Minimum Selecting Risk is fixed $g = 0.3 * p$ and $\beta_g = 0.7 * p$ assuming p is an upper limit of the max cost. That means the maximum cost of any solution under any cases cannot exceed p , and the

evaluation of Fuzzy Risk depends on the degree of uncertainty d .

Fund in project process were generated for every family of problems (n, k, d) . The results of the tests are shown in figure 5, taking 20 weeks for example.

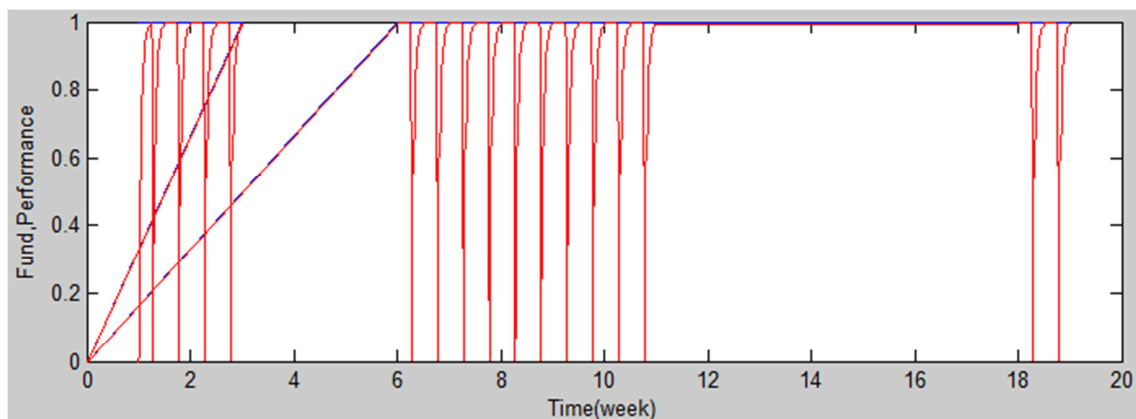


Figure 5. Fund in project process.

The computational results indicated that the risks measurement problem could be solved with fund and performance. The computation times increased with the number of fuzzy case increasing. However, the most crucial factor affecting computation times seems to be the degree of uncertainty according to the results. Observing that the computation times grow grossly fast with d if n and k are

constant. Especially, a Fuzzy Risk problem could be solved from family (1, 2, 10) within about 78s while solving a problem from family (1, 9, 30) may require numerous hours. So those problems even larger than shown in Table 1 would be solved under the fuzzy assumption that the degree of uncertainty of risk costs is small. The risks in project process are measured in Figure 6.

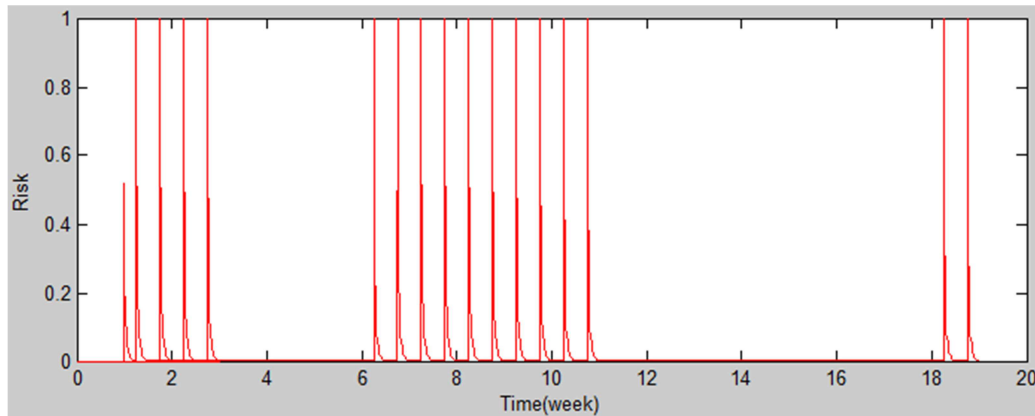


Figure 6. Risks in project process.

Hence, the tests for a risk evaluation are visualized, and the fuzzy computation method may be different for traditional crisp one. However, the general measurements obtained from the proposed method remain still effective for most cases. Particularly, the uncertainty of fuzzy risk costs plays a crucial factor in application.

For application instruction, some things should be noticed before its application.

First, this article gives a comprehensive risk management system, which employs the risk administration as the core of a project.

Second, the various evaluation methods should be used in the risk management neatly to analyze and evaluate the different perspectives of a project, then to summarize and calculate the results to obtain the possibility of a risk.

Third, the whole process of risk management focuses on the strategic target and the final results of a project, and pays close attention to its overall effect of migration, in addition, the process can improve the traditional risk management whose focus is on the project implementation of the risk of various specific phases and activities.

6. Conclusions

In this paper, the measurement of the risk of projects financed from structural funds is studied from the view point of a fuzzy logic system which is different from traditional crisp method. As there are a lot of uncertainties in the projects, the cost can also not be easily evaluated. Assuming the uncertain costs under uncertain cases may be measured by fuzzy logic system, a possibility interpretation of the practical project problems can be described. The example can provide an effective solution for uncertain risk project models. For future research, as the shortcomings of classical crisp approaches, more computational method should be researched, and their fuzzy counterparts should be further investigated. However, the fuzzy numbers play an important role in risk evaluation, and different fuzzy number models should be further studied by some available software. For practical problems some application system and management information system should be designed as a next research

direction.

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