International Journal of Astrophysics and Space Science
2018; 6(1): 18-27
http://www.sciencepublishinggroup.com/j/ijass
doi: 10.11648/j.ijass.20180601.12
ISSN: 2376-7014 (Print); ISSN: 2376-7022 (Online)

SciencePG
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MHD Fluid Flow of an Exponentially Varying Plasma Density in a Radiating and Slowly Rotating Hot Sphere

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To cite this article:

B. S. Tuduo, T. M. Abbey, K. D. Alagoa. MHD Fluid Flow of an Exponentially Varying Plasma Density in a Radiating and Slowly Rotating Hot Sphere. *International Journal of Astrophysics and Space Science*. Vol. 6, No. 1, 2018, pp. 18-27. doi: 10.11648/j.ijass.20180601.12

Received: June 15, 2017; **Accepted**: July 6, 2017; **Published**: February 11, 2018

Abstract: The study presents the effect of density variation on the flow structure of a plasma gas in a slowly rotating and radiating hot sphere. The problem which is solved by general perturbation method shows that the plasma temperature decreases to a minimum at a radial distance of 1.4 solar radii and then increased to a maximum value at a radial distance of 3.5 solar radii, for various radiation parameters, N^2. The sudden increase in temperature profile when the radial distance is 1.4 solar radii, indicates the heating up of the upper regions of the solar atmosphere.

Keywords: MHD Flow, Exponentially Varying Plasma Density, Radiating Hot Sphere

1. Introduction

MHD flows have found useful applications in several fields of study such as engineering (as in the case of MHD power generation, plasma confinement, liquid metal cooling of nuclear reactors, etc), medicine, geophysics and as well as astrophysics, where magnetic field is seen to play a dominant role in the activities of stars and their formation [1 – 6, 35]. Our Sun which is a star in the Milky-way galaxy with a surface temperature of $a \times 10^6$ degree Kelvin, has the ability to replenish the huge amount of energy lost, in the form of radiation, to its surrounding through nuclear fusion at the core. In the literature, the studies of MHD flows abound, but in most of them the plasma density is usually considered constant and the effect of compressibility assumed to be zero. According to [7], ignoring the effect of compressibility in such flow models is rather dangerous.

This is because experiment indicates that free convection motion in a flow regime is usually caused by changes in the local density due to variations in the hydrostatic pressure resulting in isotropic acceleration of the fluid.

In this regards, several reports in the literature have highlighted the import of density in the study of astrophysical plasma. For instance, [8] calculated the variation of the brightness and polarization of the k-corona under the assumption of isothermal corona with a hydrostatic description of density. Also, [9] emphasized the possible deduction of hydrostatic coronal temperature from the coronal electron density and the relation of the coronal white light, which is due to scattering, to free electron density. [10] and [11] respectively described the distribution of the inner coronal electron densities as an exponential function of radial distance from the solar core to obtain the coronal brightness.

Similarly, the astrophysical data presented by [12] show that the plasma density within the solar interior and its atmosphere changes from one layer to another as one moves away from the core. To this end [13] presented a model and show that the plasma density in the solar sphere has an exponential dependent function.

In all the studies above, none have included the effect of radiation and heat transfer on the flow model. This study therefore, is to address the effect of plasma density variation on the flow structure of an ionized gas in the present of radiation and convective heat transfer in a slowly rotating hot sphere.

The study is divided into four main sections. Section 2 presents the physics and the mathematical formulations of the problem, leading to the governing equations of the problem. On the other hand, sections 3 and 4, respectively handle the method of solutions and the analysis, and the discussion of the various results of the problem with respect to some realistic values of the flow parameters.

2. Problem Formulations

The problem considers the flow of a plasma gas whose density varies radially in a slowly rotating and radiating hot sphere. The surface temperature of the solar sphere according to observations is $T_s = a \times 10^6$ degree Kelvin, where a is a constant integer. Several findings in the literature and astrophysical observations show that unlike pulsars, the Sun rotates slowly about the azimuthal with an angular velocity, Ω_0 such that its magnetic Reynolds number R_m is far less than one $(i.e., R_m << 1)$. This implies that the induced magnetic field B' can be neglected and the flow is seen to be influenced by the applied magnetic field, B_0, which manifest as a result of the dynamo action within the interior of the astrophysical object. Similarly, the analysis of the astrophysical data presented by [12, 14 - 16] reveal that the plasma density distribution in the Sun is as follows: (the core (1.6×10^5 kg/m^3), the radiative zone (2×10^{-3} kg/m^3), the convective zone (5×10^{-4} kg/m^3), the photosphere (8×10^{-5} kg/m^3), the chromospheres (5×10^{-6} kg/m^3) and the corona (1×10^{-11} kg/m^3)).

Following these, [13] indicates that the plasma density distribution in the solar globe can be illustrated as;

$$\rho(r) = 98.8\, Av_o.(\rho_{mass})\left[10^{\exp[-11.1r+o.15/r]} + 10^{\exp[-11.1r/(1+0.02/r)]}\right]$$

where, Av_o and ρ_{mass} are the Avogadros number and proton mass, respectively.

Also, due to nuclear processes within the core, the Sun as a star has a core temperature in the order of 10^7 degree Kelvin in relation to its surface with temperature of just about 5,800 degree Kelvin. That is, the core temperature T_c is far greater than the surface temperature T_s ($T_c >> T_s$). This is responsible for radiative heat transfer.

If, therefore, $\rho'(r')$ is the density of the plasma as a function of r', and $V' = (u', v', w')$ are respectively the velocity components in the orthogonal (r', θ, φ) directions of the spherical coordinate system; p', the pressure; T', the temperature; q_r', the radiative heat transfer flux vector; μ, the dynamical viscosity; χ, the permeability; C_p, the specific heat capacity at constant pressure; κ, the thermal conductivity; σ, the Stefan-Boltzmann constant; γ, thermal expansivity coefficient; α_r, the absorption coefficient and T_∞, is the temperature of the medium at equilibrium with $T_w = T_s$, the wall or surface temperature which is kept constant, then the mathematical equations governing the flow of the plasma in the stratified density layers of the solar body, considering the Boussinesq approximation and following the method adopted by [1, 3, 17 - 18] can be presented as follows;

$$\nabla \bullet \left(\rho' V'\right) = 0 \tag{1}$$

$$\rho'\left[\left(V' \bullet \nabla\right) V'\right] = -\nabla \bullet P' + \mu \nabla^2 V' - \frac{\mu}{\chi} V' - \sigma B_0^2 V' + \rho' \beta g \left(T' - T_w\right) \tag{2}$$

$$\rho' C_p\left[\left(V \bullet \nabla\right) T'\right] = K \nabla^2 T' - \nabla q_r' \tag{3}$$

and

$$\nabla \bullet \nabla q_r' - 3\sigma\alpha^2 q_r' - 4\,\sigma\alpha T'^3 \nabla T' = 0 \tag{4}$$

where, the operator ∇ has its usual meaning and g is the acceleration due to gravity. Equations (1) to (4) are respectively the continuity, force, energy and the generalized Rosseline radiative heat transfer flux [19]. These equations, in other words, are the expressions for the conservation of mass, momentum and energy within the system.

As was observed in the case of [13] and [20], the plasma gases in the intergalactic and interplanetary layers are seen to be rarefied. Such that the optical property, α_r, of the plasma is far less than one (i.e., $\alpha_r \ll 1$). That is, the gas in this region is mostly regarded as optically thin. Hence, the generalized Rosseline radiative heat flux integro-differential equation for the optically thin limit can be expressed as;

$$\nabla q_r' = 16\alpha\sigma\left(T'^4 - T_\infty^4\right) \tag{5}$$

Furthermore, from the statistical data presented by [13], as well as the models of [21] and [22], the temperature difference between adjacent layers of the plasma is not much compared to each other, thus;

$$T' = T_\infty + \phi \tag{6}$$

where, ϕ is a small temperature correction factor, such that, $O(T') >> \phi >> O(T_\infty)$, then equation (5), the heat transfer flux vector equation reduced to;

$$\nabla q_r' = 16\sigma\alpha T_\infty^3 (T_\infty + \phi) \tag{7}$$

and the heat transfer equation becomes;

$$\rho' C_p\left[\left(V' \bullet \nabla\right)\phi\right] = K \nabla^2 \phi - 16\sigma\alpha T_\infty^3 \phi \tag{8}$$

Equations (1), (2) and (8) shall be solved subject to the following boundary conditions;

$T' = T_c$, and $u', v', w' = 0$ on r = 0.25 $R_\otimes$ (near the core)

and

$T' = T_w$, and $u', v', w' = 0$ on r = 1.0 $R_\otimes$ (the surface); where, $R_\otimes$ is the solar radius.

Now, introducing the following non-dimensional relations;

$$r = \frac{r'}{R_0}, (u, v, w) = \frac{(u', v', w')}{\Omega_0 R_0}, \rho(r) = \frac{\rho'(r)}{\rho_0}, \Theta = \frac{\phi - T_\infty}{T_w - T_\infty},$$

and eliminating the pressure gradients in the r and θ - directions, Equations (1), (2) and (8) in the spherical coordinate system (r, θ, φ) becomes;

$$\rho\frac{\partial u}{\partial r}+\frac{2\rho u}{r}+u\frac{\partial\rho}{\partial r}+\frac{\rho v}{r}\cot\theta+\frac{\rho}{r}\frac{\partial v}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\rho w)=0 \tag{9}$$

$$\operatorname{Re}\rho\left[u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial\theta}+\frac{w}{r\sin\theta}\frac{\partial u}{\partial\varphi}-\frac{v^2+w^2}{r}\right]=\left[\nabla^2-\chi^2-M^2-\frac{1}{r^2\sin^2\theta}\right]u-\frac{2}{r^2}\frac{\partial v}{\partial\theta}-\frac{2v}{r^2}\cot\theta-\frac{2}{r^2\sin\theta}\frac{\partial w}{\partial\varphi}+\rho G_r\Theta \tag{10a}$$

$$\operatorname{Re}\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial\theta}+\frac{w}{r\sin\theta}\frac{\partial v}{\partial\varphi}+\frac{uv}{r}-\frac{w^2}{r}\cot\theta\right]=\left[\nabla^2-\chi^2-M^2-\frac{1}{r^2\sin^2\theta}\right]v+\frac{2}{r^2}\frac{\partial u}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial w}{\partial\varphi}+\rho G_r\Theta \tag{10b}$$

$$\operatorname{Re}\rho\left[u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial\theta}+\frac{w}{r\sin\theta}\frac{\partial w}{\partial\varphi}+\frac{uw}{r}+\frac{vw}{r}\cot\theta\right]=\left[\nabla^2-\chi^2-M^2-\frac{1}{r^2\sin^2\theta}\right]w+\frac{2}{r^2\sin\theta}\frac{\partial u}{\partial\varphi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v}{\partial\varphi}+\rho G_r\Theta \tag{10c}$$

$$\operatorname{Re}\Pr\rho\left[u\frac{\partial\Theta}{\partial r}+\frac{v}{r}\frac{\partial\Theta}{\partial\theta}+\frac{w}{r}\frac{\partial\Theta}{\partial\varphi}\right]=\left(\nabla^2-N^2\right)\Theta \tag{11}$$

where, $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\varphi^2}$,

$\operatorname{Re}=\frac{\rho_0\,\Omega_0\,R_0^2}{\mu}$; the rotational Reynolds number,

$\chi^2=\frac{R_0^2}{k}$; the porosity parameter,

$M^2=\frac{\sigma B_0^2\,R_0^2}{\mu}$; the magnetic parameter (the Hartmann number),

$G_r=\frac{\rho_0\ R_0}{\Omega_0\,\mu}\gamma g\left(T'-T_\infty\right)$; the Grashof number,

$\Pr=\frac{\mu C_p}{\kappa}$; the Prandtl number, and

$N^2=\frac{16\alpha_r\sigma R_0^2\,T_\infty}{\kappa}$; the radiation or heat transfer parameter.

with,

$\Theta=\Theta_c$ and $u,\ v,\ w=0$, on $r=0.25\ R_\otimes$ (near the core)

and,

$\Theta=\Theta_w$ and $u,\ v,\ w=0$, on $r=1.0\ R_\otimes$ (the surface).

This brings to conclusion the physics and the mathematical formulations of the problem. Next to be considered is the analysis and method of solution.

3. Method of Solution

The formulations show that the density of the plasma can be expressed as;

$$\rho'(r)=\rho_0\,Exp(-\varepsilon_0'\,r) \tag{12}$$

such that in a non-dimensional form, we have;

$$\rho(r)=Exp(-\varepsilon_0\,r) \tag{13}$$

where, $\varepsilon_0=\frac{\varepsilon_0'}{R_0}=-\frac{In[\rho(r)]}{r}$ is the density parameter and is a measure of the ratio of logarithmic decrement of the density with radial distance, r.

Generally, the flow equations expressed in (9) to (11) are non-linear and highly coupled so that their analytical solutions are intractable. Therefore, as suggested by [5, 23 – 24], problem of this nature can be tackled analytically by adopting general perturbation method of solution about a small parameter whose value is far less than one. In this analysis, since the Sun is a slowly rotating star, its Reynolds number R_e is far less than one (i.e., $O(\operatorname{Re})<<1$). Therefore, the flow variables can be expressed as;

$$f(r,\theta)=f_0(r,\theta)+\operatorname{Re}f_1(r,\theta)+... \tag{14}$$

where, $O(f_0)$ and $O(f_1)$ are the zero and first order variables. Considering that the flow is symmetrical about the φ-axis so that, $\frac{\partial f}{\partial\varphi}=0$; give the following orders of approximations in R_e;

$$\rho\frac{\partial u_0}{\partial r}+\frac{2\rho}{r}u_0+u_0\frac{\partial\rho}{\partial r}+\frac{\rho}{r}\frac{\partial v_0}{\partial\theta}+\frac{\rho}{r}v_0\cot\theta=0 \tag{15}$$

$$\left(\nabla^2-\chi^2-M^2-\frac{1}{r^2\sin^2\theta}\right)\left[\frac{1}{r}\frac{\partial}{\partial r}(r\,v_0)-\frac{1}{r}\frac{\partial u_0}{\partial\theta}\right]+\rho G_r\left(\frac{\partial\Theta_0}{\partial r}-\frac{\partial\Theta_0}{\partial\theta}\right)+G_r\Theta_0\left(\frac{\partial\rho}{\partial r}+\frac{\rho}{r}\right)=0 \tag{16}$$

$$\left(\nabla^2-\chi^2-M^2-\frac{1}{r^2\sin^2\theta}\right)w_0+\rho G_r\,\Theta_0=0 \tag{17}$$

$$\left(\nabla^2 - N^2\right)\Theta_0 = 0 \tag{18}$$

for the order-$O(f_0)$ equations and

$$\rho\frac{\partial u_1}{\partial r} + \frac{2\rho}{r}u_1 + u_1\frac{\partial \rho}{\partial r} + \frac{\rho}{r}\frac{\partial v_1}{\partial \theta} + \frac{\rho}{r}v_1\cot\theta = 0 \tag{19}$$

$$\rho\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[u_0\frac{\partial v_0}{\partial r} + \frac{v_0}{r}\frac{\partial v_0}{\partial \theta} + \frac{u_0 v_0}{r} - \frac{w_0^2}{r}\cot\theta\right]\right) - \frac{1}{r}\frac{\partial}{\partial \theta}\left[u_0\frac{\partial u_0}{\partial r} + \frac{v_0}{r}\frac{\partial u_0}{\partial \theta} - \frac{v_0^2 + w_0^2}{r}\right]\right\} == \left(\nabla^2 - \chi^2 - M^2 - \frac{1}{r^2\sin^2\theta}\right) \tag{20}$$
$$\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_1) - \frac{1}{r}\frac{\partial u_1}{\partial \theta}\right] + \rho G_r\left(\frac{\partial \Theta_1}{\partial r} - \frac{\partial \Theta_1}{\partial \theta}\right) + G_r\Theta_1\left(\frac{\partial \rho}{\partial r} + \frac{\rho}{r}\right)$$

$$\rho\left[u_0\frac{\partial w_0}{\partial r} + \frac{v_0}{r}\frac{\partial w_0}{\partial \theta} + \frac{u_0 w_0}{r} + \frac{v_0 w_0}{r}\cot\theta\right] = \left(\nabla^2 - \chi^2 - M^2 - \frac{1}{r^2\sin^2\theta}\right)w_1 + \rho G_r\Theta_1 \tag{21}$$

$$\rho\Pr\left[u_0\frac{\partial \Theta_0}{\partial r} + \frac{v_0}{r}\frac{\partial \Theta_0}{\partial \theta}\right] = \left(\nabla^2 - N^2\right)\Theta_1 \tag{22}$$

for the order-$O(f_1)$ equations.

The solutions to the flow variables in the orders $O(f_0)$ and $O(f_1)$ equations can be effected by starting with the temperature equations and using the following transformations;

$$\Theta_0(r,\theta) = \Theta_{00}(r)\sin\theta$$

$$w_0(r,\theta) = w_{00}(r)\sin\theta$$

$$u_0(r,\theta) = u_{00}(r)\cos\theta$$

$$v_0(r,\theta) = v_{00}(r)\sin\theta$$

$$\Theta_1(r,\theta) = \Theta_{10}(r)\sin 2\theta$$

$$w_1(r,\theta) = w_{10}(r)\sin 2\theta$$

$$u_1(r,\theta) = u_{10}(r)\ (2\cos^2\theta - \sin^2\theta)$$

and

$$v_1(r,\theta) = v_{10}(r)\sin 2\theta\ .$$

The solutions to the flow variables, obtained by the analysis of the resulting differential equations based on the principles stated in [25] and [26] for modified spherical Bessel and Bessel differential equations, becomes;

$$\Theta_0(r) = a_1\, i_1(Nr) + a_2\, k_1(Nr) \tag{23}$$

$$w_0(r) = b_1\, i_1(\beta r) + b_2\, k_1(\beta r) - \frac{G_r}{\beta}\int \rho(r)\ \Theta_0(r)\,dr \tag{24}$$

$$u_0(r) = d_1 + d_2\, r^{-3} + \frac{2}{3}\int \psi_0(r)\,dr + \frac{2}{3}r^{-3}\int r^3\,\psi_0(r)\,dr \tag{25}$$

$$v_0(r) = -d_{11} + \frac{d_{12}}{2}r^{-3} - \frac{2}{3}\int \psi_0(r)\,dr - \frac{r^{-3}}{3}\int r^3\,\psi_0(r)\,dr \tag{26}$$

$$\Theta_1(r) = a_{11}\, i_2(N r) + a_{22}\, k_2(N r) + N\int T_1(r)\,dr \tag{27}$$

$$w_1(r) = b_{11}\, i_2(\beta r) + b_{22}\, k_2(\beta r) + \beta\int f_1(r)\,dr \tag{28}$$

$$u_1(r) = d_{10}\, r + d_{20}\, r^{-4} + \frac{r}{5}\int r^{-1}\, h_1(r)\,dr - \frac{r^{-4}}{5}\int r^4\, h_1(r)\,dr \tag{29}$$

and

$$v_1(r) = -d_{21}\, r + \frac{2}{3}d_{22}\, r^{-4} - \frac{r}{5}\int r^{-1}\, h_1(r)\,dr - \frac{2r^{-4}}{15}\int r^4\, h_1(r)\,dr \tag{30}$$

where, $z = N r$ and $\eta = \beta r$ and $i_n(z)$, $k_n(z)$, $i_n(\eta)$ and $k_n(\eta)$ are respectively the modified spherical Bessel functions of the first and second kind of order n, with z and η as the arguments. Whereas,

$$\psi_0(r) = c_1\, i_1(\beta r) + c_2\, k_1(\beta r) + \beta\int Y_0(r)\,dr\ ,$$

$$Y_0(r) = -\frac{1}{\beta^2}\left\{\rho(r)\, G_r\frac{d\,\Theta_0(r)}{d\,r} + G_r\,\Theta_0(r)\left(\frac{d\rho}{d\,r} + \frac{\rho}{r}\right)\right\},$$

$$T_1(r) = \frac{\rho(r)\Pr}{2N^2}\left[u_0(r)\frac{d\,\Theta_0(r)}{d\,r} + \frac{v_0(r)\Theta_0(r)}{r}\right].$$

$$f_1(r) = \frac{\rho(r)}{2\beta^2}\left(\left[u_0(r)\frac{d\,w_0(r)}{d\,r} + \frac{u_0(r)\,w_0(r)}{r} + \frac{2\,v_0(r)\,w_0(r)}{r}\right] - 2\,G_r\,\Theta_1(r)\right)\quad h_1(r) = -3\psi_1(r)\ ,$$

$$\psi_1(r) = c_{11}\, i_2(\beta r) + c_{22}\, k_2(\beta r) + \beta\int Y_1(r)\,dr\ ,$$

and

$$Y_1(r)=\frac{\rho(r)}{2\beta^2}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r u_0(r)\frac{d v_0(r)}{d r}+v^2{}_0(r)+u_0(r)v_0(r)-w^2{}_0(r)\right)+\left(\frac{2u_0(r)}{r}\frac{d u_0(r)}{d r}+\frac{2v_0(r)u_0(r)}{r^2}+\frac{2v_0{}^2(r)}{r^2}+\frac{2w_0{}^2(r)}{r^2}\right)\right\}$$

$$-\frac{G_r}{\beta^2}\left[\rho(r)\frac{d\Theta_1(r)}{d r}+\Theta_1(r)\left(\frac{d\rho(r)}{d r}+\frac{\rho(r)}{r}\right)\right]$$

with

$$\beta^2=\left(\chi^2+M^2\right),$$

$$a_1=\frac{\Theta_0(r_1)k_1(Nr_2)-\Theta_0(r_2)k_1(Nr_1)}{i_1(Nr_1)k_1(Nr_2)-i_1(Nr_2)k_1(Nr_1)},a_2=\frac{\Theta_0(r_2)i_1(Nr_1)-\Theta_0(r_1)i_1(Nr_2)}{i_1(Nr_1)k_1(Nr_2)-i_1(Nr_2)k_1(Nr_1)},b_1=\frac{f_0(r_1)k_1(\beta r_2)-f_0(r_2)k_1(\beta r_1)}{i_1(\beta r_1)k_1(\beta r_2)-i_1(\beta r_2)k_1(\beta r_1)},$$

$$b_2=\frac{f_0(r_1)k_1(\beta r_2)-f_0(r_2)k_1(\beta r_1)}{i_1(\beta r_1)k_1(\beta r_2)-i_1(\beta r_2)k_1(\beta r_1)},$$

$$d_1=\frac{u_0(r_2)r_1^{-3}-u_0(r_1)r_2^{-3}}{r_2^{-3}-r_1^{-3}},\ d_2=\frac{u_0(r_1)-u_0(r_2)}{r_2^{-3}-r_1^{-3}},\ d_{11}=\frac{v_0(r_1)r_1^3-v_0(r_2)r_2^3}{r_2^3-r_1^3},$$

$$d_{12}=\frac{2r_1^3r_2^3\{v_0(r_1)-v_0(r_2)\}}{r_2^3-r_1^3},\ a_{11}=\frac{(\Theta_1(r_1)-T_1(r_1))k_2(Nr_2)-(\Theta_1(r_2)-T_1(r_2))k_2(Nr_1)}{i_2(Nr_1)k_2(Nr_2)-i_2(Nr_2)k_2(Nr_1)},$$

$$a_{22}=\frac{(\Theta_1(r_2)-T_1(r_2))i_2(Nr_1)-(\Theta_1(r_1)-T_1(r_1))i_2(Nr_2)}{i_2(Nr_1)k_2(Nr_2)-i_2(Nr_2)k_2(Nr_1)},$$

$$b_{11}=\frac{f_1s(r_2)k_2(\beta r_1)-f_1s(r_1)k_2(\beta r_2)}{i_2(\beta r_1)k_2(\beta r_2)-i_2(\beta r_2)k_2(\beta r_1)},b_{22}=\frac{f_1s(r_1)i_2(\beta r_2)-f_1s(r_2)i_2(\beta r_1)}{i_2(\beta r_1)k_2(\beta r_2)-i_2(\beta r_2)k_2(\beta r_1)},$$

$$d_{10}=\frac{u_1s(r_2)r_1^{-4}-u_1s(r_1)r_2^{-4}}{r_1r_2^{-4}-r_2r_1^{-4}},d_{20}=\frac{u_1s(r_1)r_2-u_1s(r_2)r_1}{r_1r_2^{-4}-r_2r_1^{-4}},$$

$$d_{21}=\frac{v_1s(r_1)r_1^4-v_1s(r_2)r_2^4}{r_2^5-r_1^5}$$

and

$$d_{22}=\frac{3r_1^4r_2^4\{v_1s(r_1)r_2-v_1s(r_2)r_1\}}{2\left(r_2^5-r_1^5\right)}.$$

While, r_1 and r_2 are respectively the non-dimensional radii at the boundaries and the other constants that emanated from the solutions of the equations are given as follows;

$$c_1=\frac{Y_0(r_1)k_1(\beta r_2)-Y_0(r_2)k_1(\beta r_1)}{i_1(\beta r_1)k_1(\beta r_2)-i_1(\beta r_2)k_1(\beta r_1)}$$

$$c_2=\frac{Y_0(r_1)k_1(\beta r_2)-Y_0(r_2)k_1(\beta r_1)}{i_1(\beta r_1)k_1(\beta r_2)-i_1(\beta r_2)k_1(\beta r_1)}$$

$$c_{11}=\frac{Y_1(r_2)k_2(\beta r_1)-Y_1(r_1)k_2(\beta r_2)}{i_2(\beta r_1)k_2(\beta r_2)-i_2(\beta r_2)k_2(\beta r_1)}$$

$$c_{22} = \frac{Y_1(r_1)\, i_2(\beta r_2) - Y_1(r_2)\, i_2(\beta r_1)}{i_2(\beta r_1)\, k_2(\beta r_2) - i_2(\beta r_2)\, k_2(\beta r_1)}$$

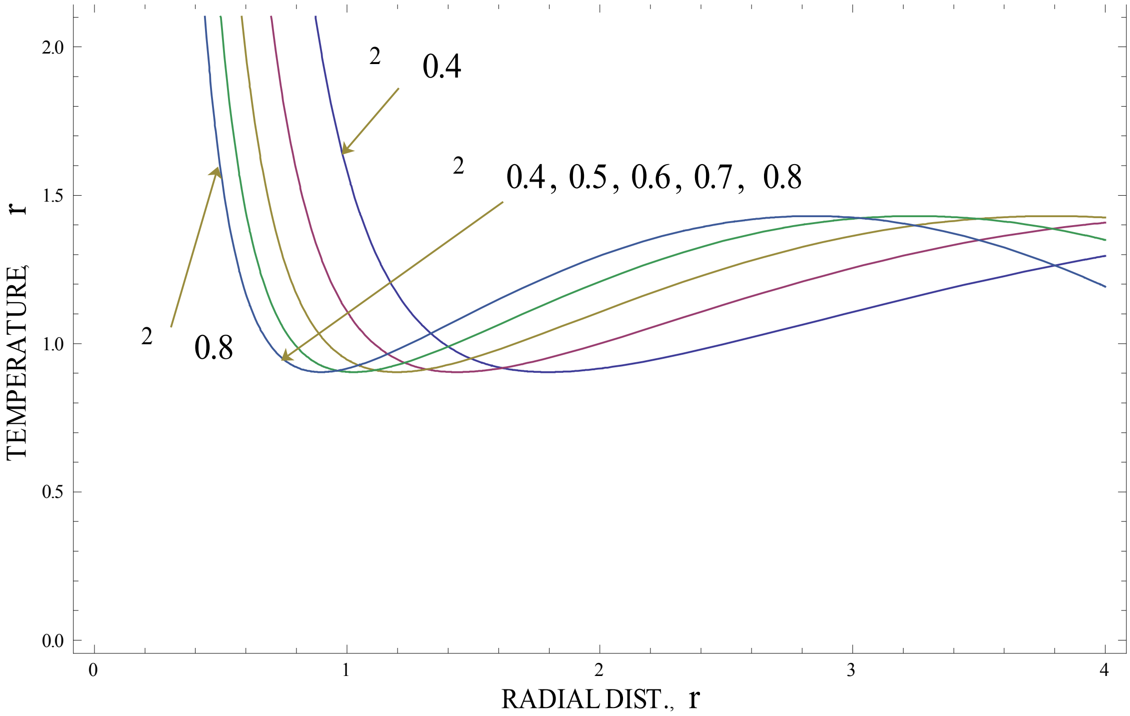

Figure 1. *Temperature profiles for various values of Radiation Parameter (N^2).*

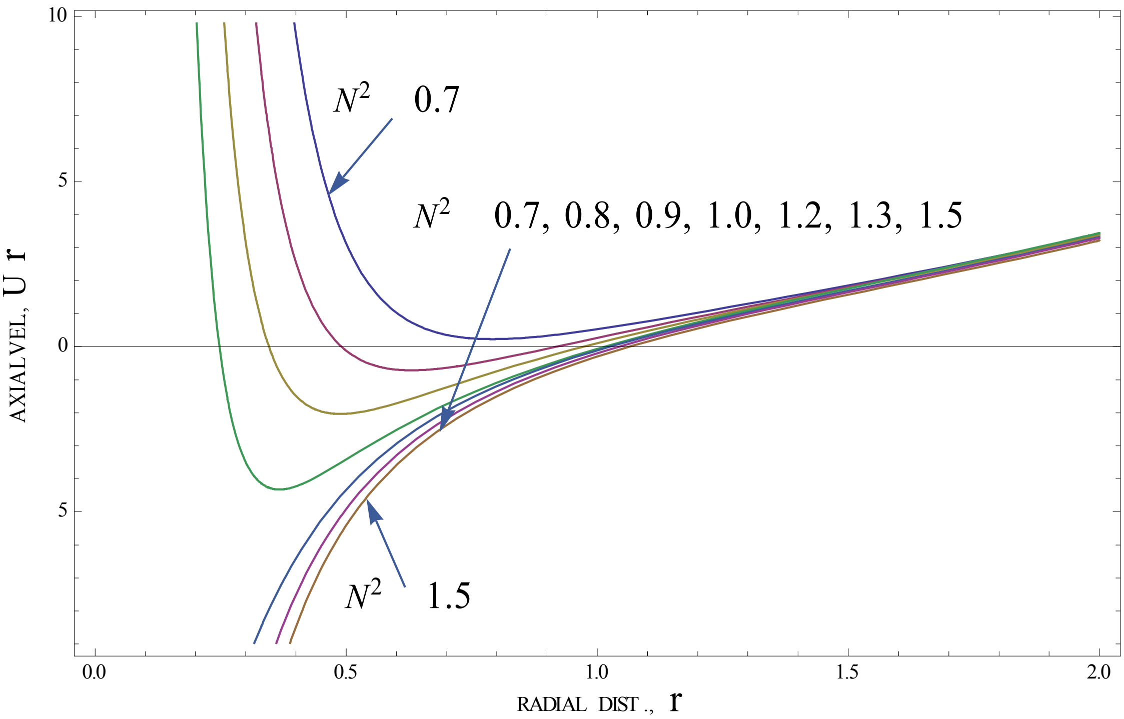

Figure 2. *Axial Velocity profiles for various Radiation Parameter (N^2).*

4. Discussion

The primary aim of this study is to investigate the effect of density variation on the dynamical variables, such as temperature and velocity; of a plasma gas in a radiating and slowly rotating hot sphere. To this end, Section 2 presents the mathematical model of the problem incorporating the density as an exponential function of the radial distance, *r*. The results of the analysis using the Wolframs Mathematica software (version 9) are presented in Figures 1 to 7 for the following realistic computational parameters; ($\boldsymbol{\varepsilon_0}$ = 0.0, 0.1, 0.3, 0.5, 0.7, 1.0, 2.0, 5.0 and 10.0); (N^2 = 0.4, 0.5, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 and 1.5); (M^2 = 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0); (Re = 0.001, 0.01, 0.1) and (Pr = 0.71).

The analysis shows that variation of the plasma density has tremendous effects on the profiles of the plasma flow variables. For example, the temperature of the plasma is seen to decease to a minimum value at the point where $r = 1.4\ R_{\otimes}$ and then suddenly increase to a maximum value when $r = 3.5\ R_{\otimes}$ from where it decreases as r tends to infinity ($r \to \infty$). Similar observation is noticed in the case of the velocity profiles (Figure 2). On the other hand, the model shows that the flow experience a velocity reversal when the magnetic field parameter is greater than 0.8 (i.e., $M^2 > 0.8$). That is, the magnitude of the flow velocities according to Figures 1, 5 and 6 increase with magnetic field strength and thermal radiation parameter but decreases with increase in the density parameter. The general picture of the temperature distribution of the flow model is presented in 3D format in Figure 7.

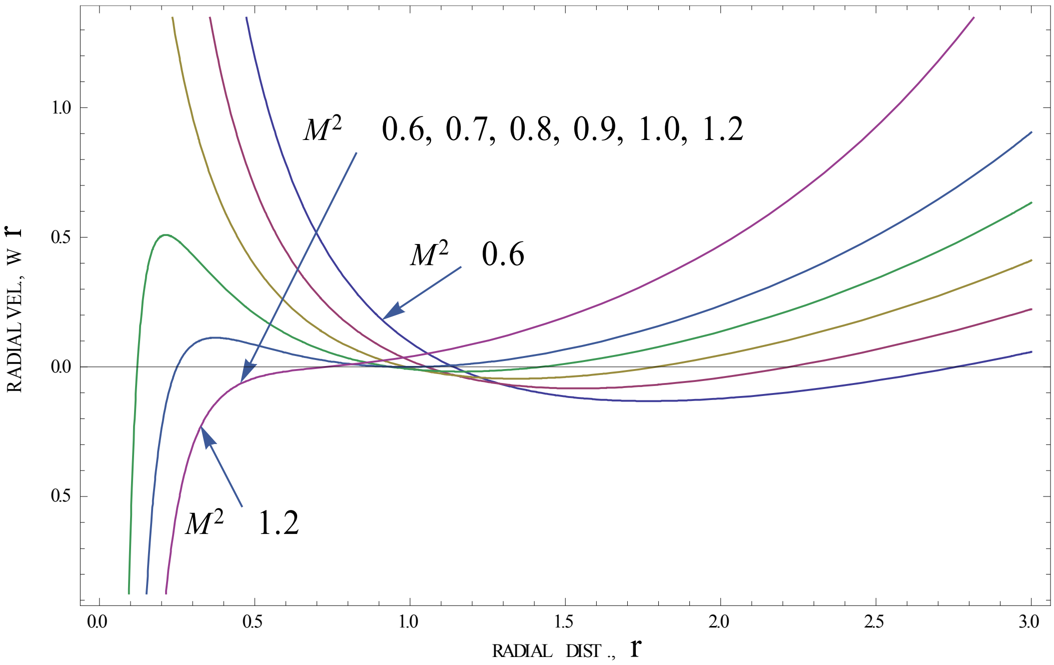

Figure 3. *Radial Velocity profiles for various values of Magnetic Parameter (M^2).*

The region with low ions density indicates that it is dominated by the neutral atoms. In this case, the flow experience ambipolar effects. This is seen to account for the reduction in the velocity of the plasma. [27 – 30] suggest that this is true in the case of the *D* and *E* layers of the ionosphere.

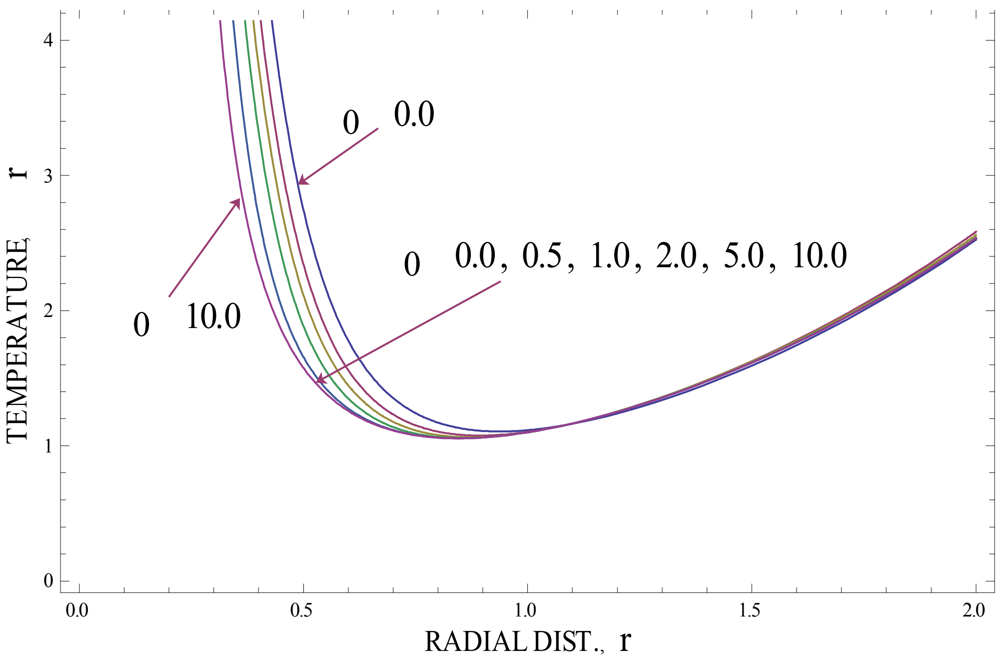

Figure 4. *Temperature profiles for various values of Density Parameter (ε_0).*

It is observed that when the density parameter, ε_0 is low the fluid tends to be populated by ions and electrons and their interactions lead to increase in the kinetic energy of the plasma and a subsequent increase in the temperature as can be seen from Figure 7. [20, 31 – 33] indicate that such high temperature and low pressure within the gas lead to gas rarefication, resulting in the production of rarefaction waves which flow in the reverse direction. The reversal carries along with it energetic particles at high temperature resulting in the heating up of the upper solar atmosphere [12, 34]. Generally, the analysis aids our understanding of astrophysical interiors and its atmospheres as well as the interactions in the *D* and *E* layers of the Earth's ionosphere as noted by [35].

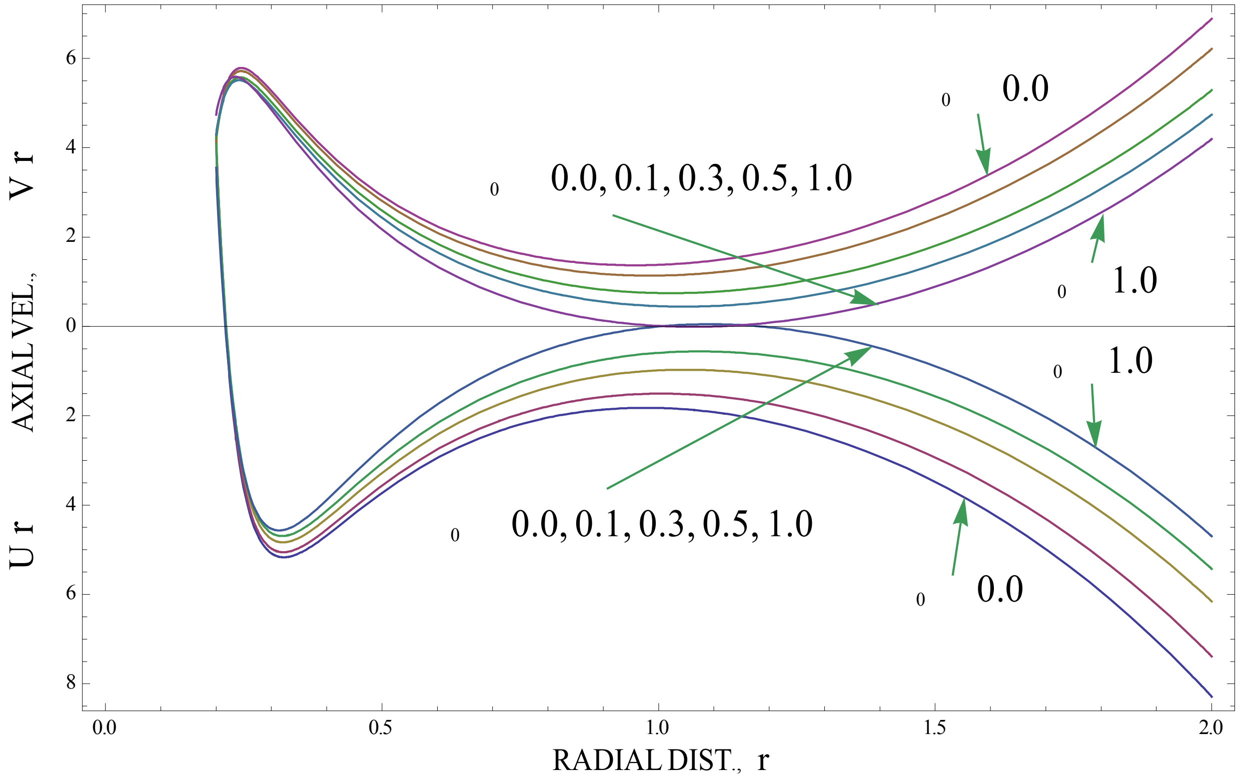

Figure 5. *Axial Velocity profiles for various values of Density Parameter (ε_0).*

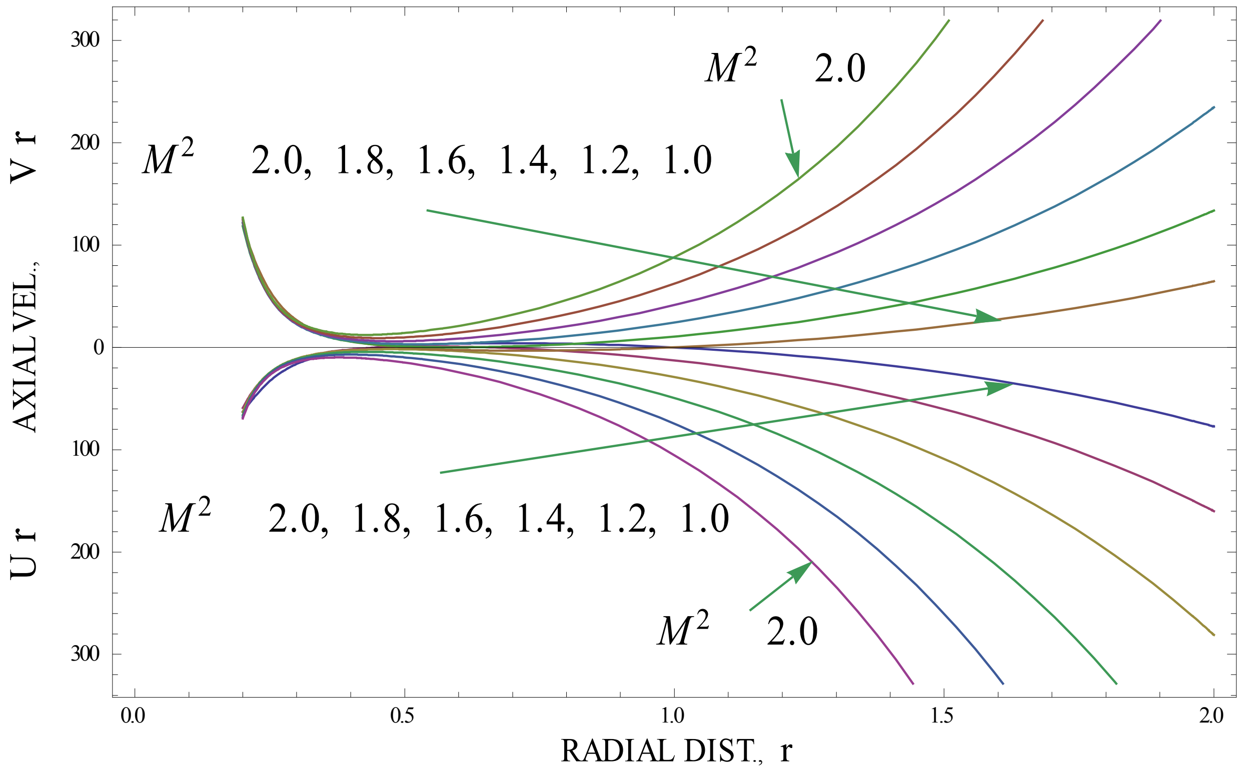

Figure 6. *Axial Velocity profiles for various Magnetic Parameter (M^2).*

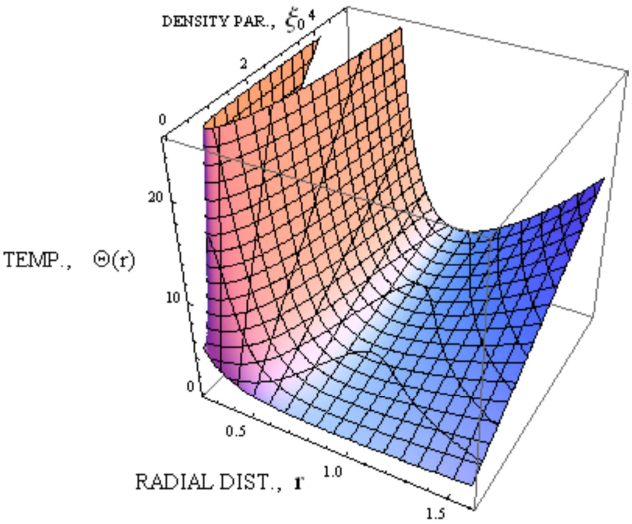

Figure 7. *3D Temperature profile with Density Parameter (ε_0).*

5. Conclusion

The rarefaction waves produced by the interactions of ionized species of the plasma at low density parameters lead to flow reversal. Energetic fluid elements carried along in this reversed flow heat up the plasma which could account for the high temperatures observed in upper solar atmosphere. The reduction in temperature above the solar sphere leads to increased presence of neutral atoms in the lower chromosphere. In this case, the flow experience ambipolar effects resulting from the interactions of the ionized species with the neutrals [35]. Generally, variation in the plasma density is seen to have an important effect on these interactions and the flow profiles of the plasma.

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