

Effect of Ambipolar Diffusion on the Flow of a Two-Component Plasma Gas Model in the Earth's Planetary Ionosphere

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Abstract: The paper presents analytical study on the effect of ambipolar diffusion on the flow of a two-component plasma gas in the Earth's Planetary Ionosphere as a model to examine the ions-neutral and electrons-neutral atom interactions. The problem which consists of a set of partial non-linear differential equations was addressed using a plane wave and perturbation method of solutions. The result indicates that plasma frequency and electron-density in the Ionosphere increase with increase in magnetic field strength as well as with radiation and free convection parameters. It is observed that for; $\frac{(M^2 + G_r)^2}{f_{kn}(G_r - f_{kn})} \geq 0$, the plasma interactive state becomes more stable, otherwise some bit of oscillation is noticed. The stability is seen to depend on the magnetic (M^2) and thermal convection (G_r) parameters. Under this condition the signal propagation becomes less diffuse when the frequency of the signal is far greater than the plasma frequency, that is, $\omega \gg p$. The study aids our understanding of the effect of coupling frequency on the propagation of satellite signals through the ionosphere.

Keywords: Ambipolar Diffusion, Two-Component Plasma Flow, Planetary Ionosphere

1. Introduction

The study of the ionosphere and its characteristics are important for both practical and scientific purposes; significantly due to its influence on the propagation of electromagnetic waves. Its studies date back to the pioneer work of Guglielmo Marconi, who successfully transmitted radio signal across the transatlantic on the 12th of December, 1901 [1, 2]. Today, the understanding of the chemical and dynamical processes of the ionospheric and astrophysical plasma has found applications in the areas of condensed matter Physics, nuclear Physics, thin panel displays (plasma TV), particle accelerators and communication, just to mention but a few. The ionosphere, which is the ionized part of the Earth's upper atmosphere lies between the 60 and 1000 km, is known to be stratified into D, E, F1 and F2 layers respectively [3 - 5]. The solar extreme ultraviolet (EUV) and x-ray radiations as well as other cosmic radiations account for the ionization of the ionosphere [6 - 9], with the

D-layer being the least ionized.

Hence the D-layer, similar to the chromosphere of the solar atmosphere [10], is weakly ionized and as such a collision dominated region; collisions between electrons and ions, and neutral gas and the ionized species are frequent. The co-existence of the ionized species and the neutral gas explains the high recombination in the D-layer [11 - 14] as well as in the core of molecular clouds. Electrons which are very light moves faster and tend to separate from the more massive ions. This set up a charge imbalance, and the resulting field slows down the electrons and accelerates the ions, such that they both move together with the same velocity, drifting through the plasma as coupled particles, in the process colliding with other electrons, ions and the neutral gas. [1] and [15] show that the relative motion of ions and neutrals in a weakly ionized medium leads to the coupling of the ions and neutrals via mutual collision, and they drift together through the plasma resulting in ambipolar diffusion. According to [16], [17] and [18], ambipolar

diffusion is prevalent in the core of molecular clouds which is a key factor in star formation (especially in the transfer of mass and momentum [19]), and the transfer and dissipation of energy in the Sun's chromosphere [10].

Several researchers in the literature have extensively used numerical methods in the study of partially ionized plasma within the frame of single- or two-fluid models [20 - 26]. For example, [20] used semi-implicit scheme for two-fluid ambipolar diffusion to investigate instability in steady-state continuous shocks, while [21] described an explicit method for single-fluid ambipolar diffusion in the strong coupling limit. But, [27] investigated the heating through ambipolar diffusion in turbulent molecular clouds using a single-fluid approximation. Numerical schemes for the multi-fluid treatment of hall diffusion and ambipolar diffusion have been suggested by [28 - 30]. Whereas, [31] and [32] studied the properties of turbulence with ambipolar diffusion in a two-fluid approximation using three-dimensional (3D) numerical simulations model. Similarly, [26] presented a semi-implicit method for ambipolar diffusion using a two-fluid approximation. A fully explicit method for incorporating the single-fluid ambipolar diffusion into a multi-dimensional magnetohydrodynamic (MHD) code based on the total variation diminishing (TVD) scheme was described by [14]. While, [15] conducted a three-dimensional (3D) local sharing-box simulations to explore the effect of ambipolar diffusion on the non-linear evolution of magnetorotational instability (MRI) in the strong coupling limit. And just recently, [33] studied the dynamics of the interactions of neutrals and charged plasma particles in a single-fluid, ideal magnetohydrodynamic framework to describe the propagation of low-frequency waves in the partially ionized collision-dominated lower Earth's Ionosphere, and the ionosphere-magnetosphere coupling.

In all the above studies, the effect of temperature differentials with accompanied radiative heat transfer and free convection motion; which is paramount in astrophysical regions where high temperature are experienced was not considered. This is the crux of this present study.

The paper is organized in the following format: - In section 2, the physics and the mathematical formulation of the problem leading to equation of motion shall be presented, while section 3 has the method of analysis leading to the derivation of different wave-modes and the attendant effects on the electrons and ions density variation in the ionosphere. Section 4 on the other hand discusses the various results and section 5 presents the conclusion. The study will aid our understanding of the chemical and dynamical processes of the ionosphere. It would further shed a light on the physics and factors affecting navigational satellites and global positioning systems.

2. Problem Formulations

In most regions of the ionosphere, particularly in the D-layer and some part of E-layer, ionization is seen to be weak. In this case the region is permeated with neutral gas

molecules along with ions and electrons. According to [15] in weakly ionized plasma, where ambipolar diffusion occurs as a result of the relative motion of the ions and neutrals, the inertia of the ionized species is usually negligible. It follows, therefore, that the ion velocity can be determined by the balance between the Lorentz force and ion-neutral gas collisional drag. It is this force balance along with other factors that determined and predict the motion of such an interacting system. If, therefore, V , B , P , g and ρ are respectively the velocity field vector, magnetic field vector, the plasma pressure, the gravitational field vector and the density of the plasma component species; while q_r is the radiation flux resulting from the interaction of the cosmic and x-rays from the solar body on the Earth's ionosphere; σ , the Stefan Boltzmann constant; α , the plasma optical property; T , the temperature of the medium and K_0 , χ_k , C_p , μ and e are the thermal conductivity, porosity, specific heat capacity at constant pressure, neutral gas viscosity and electron charge, f_{kn} , the coupling frequency between the ionized species and the neutral gas, whereas J is the current density, then the mathematical statements governing the flow of the two-component plasma model in the present of ambipolar diffusion are as follows;

$$\frac{\partial \rho_k}{\partial t} = Q - L - \nabla(\rho_k V_k) \quad (1)$$

$$\rho_k \frac{\partial V_k}{\partial t} = -\nabla P_k + \mu \nabla^2 V_k + \rho_k g + \frac{e \rho_k}{m_k} (E + V_k \times B) + \frac{\mu_r}{\chi_k} V_k \pm \rho_k f_{kn} (V_k - V_n) \quad (2)$$

$$\rho_k C_p \frac{\partial T_k}{\partial t} = k_0^2 \nabla^2 T_k - \nabla q_r \quad (3)$$

$$\nabla^2 q_r - 3 \alpha^2 q_r - 16 \sigma \alpha T^2 \nabla T = 0 \quad (4)$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B_1}{\partial t}, \quad (5)$$

$$\nabla \times B_1 = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} J, \quad (6)$$

$$\nabla \cdot E = 4\pi (e^+ \rho_i V_i + e^- \rho_e V_e), \quad (7)$$

$$\nabla \cdot B_1 = 0 \quad (8)$$

and

$$J = (e^+ \rho_i V_i + e^- \rho_e V_e) \quad (9)$$

where, $(k = i \text{ or } e)$ denote the ions and electron and n the neutral species respectively, while Q and L are the rate of production of ions from the neutral gas and the rate of loss of ions due to recombination. Here equations (1 to 4) express the flow variables of the two-component plasma model. Equation (1) is the continuity equation, which expresses the mass conservation of electrons, ions and neutral gas (atoms) in the ionosphere and $\nabla(\rho_k V_k)$ is the loss of ionization due to transport with V_k being the transport velocity. In the absence of Q and L , that is, the case in which $Q = 0$ and $L = 0$, the continuity equation assumes the form:

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k V_k) = 0 \quad (10)$$

On the other hand, equation (2) shows, the force balance of the plasma within the ionosphere, the term ∇P_k is the force due to the pressure gradient, whereas the present of $\rho_k g$, $\frac{e\rho_k}{m_k}(E + V_k \times B)$ and $\rho_k f_{kn}(V_k - V_n)$ $\{k \neq n\}$ terms indicate forces due to gravity, electromagnetic fields and collision (or friction) between the ions (i) and the neutrals (n) with f_{kn} being the collisional frequency.

Also, equation (3) is the energy conservation equation, which indicates the present of $k_0^2 \nabla^2 T_k$, the heat conduction term and ∇q_r , the heat absorption and transfer term. As we can readily see from equations (1) to (4), the flow variables are highly coupled and the equations expressing them are non-linear. Therefore, to tackle the problem analytically some useful assumptions are necessary. For example, in the intergalactic, interplanetary regions, stellar atmospheres and in some layers of the ionosphere the gases are seen to be rarefied [10], such that the corresponding optical property, α is far less than one (i.e., $\alpha \ll 1$). In this case, as was in the case of [34], the integro-differential equation in equation (4) becomes:

$$\nabla q_r = 4\sigma\alpha(T^4 - T_\infty^4)$$

where, T_∞ is the temperature at which the plasma gas is in a state of equilibrium. Also, if we further assume as in the case of [5, 35 – 38], that the temperature difference between adjacent layers of the plasma is not much compared to each other, thus;

$$T_{k,n} = T_\infty + \phi \quad (11)$$

where, ϕ is a small temperature correction factor, such that, $\theta(T) \gg \phi \gg \theta(T_\infty)$, then, the heat transfer flux vector equation reduced to;

$$\nabla q_r = 16\sigma\alpha T^3(\phi + T_\infty) \quad (12)$$

This equation shall be substituted into equation (3) and solved along with equations (1) and (2). Equations (5-9) are the Maxwell's equations; where, B_1 is the induced magnetic field vector and $B = B_0 + B_1$, with B_0 being the applied magnetic field. Eliminating B_1 by combining equations (5) and (6) we have the following form of equation:

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{4\pi}{c} \frac{\partial \mathbf{J}}{\partial t} \quad (13)$$

This brings the mathematical statement of the problem to conclusion.

3. Method of Solutions and Analysis

In a typical plasma setting viscosity, coriolis and centripetal accelerations, and tidal waves are usually neglected for simplicity, thus equation (2) becomes;

$$\rho_k \frac{\partial V_k}{\partial t} = -\nabla P_k + \rho_k g + \frac{e\rho_k}{m_k}(E + V_k \times B) + \frac{\mu_r}{\chi_k} V_k \pm \rho_k f_{kn}(V_k - V_n) \quad (14)$$

such that, for the ions, we have;

$$\rho_i m_i \frac{\partial V_i}{\partial t} = -K_B T_i \nabla \rho_i + \rho_i m_i g + e\rho_i(E + V_i \times B) + \frac{\mu_r}{\chi_i} m_i V_i + \rho_i m_i f_{in}(V_i - V_n) \quad (15)$$

and for the electrons, we have;

$$\rho_e m_e \frac{\partial V_e}{\partial t} = -K_B T_e \nabla \rho_e - \rho_e m_e g - e\rho_e(E + V_e \times B) + \frac{\mu_r}{\chi_e} m_e V_e + \rho_e m_e f_{en}(V_e - V_n) \quad (16)$$

whereas, for the neutral gas, it becomes;

$$\rho_n m_n \frac{\partial V_n}{\partial t} = -K_B T_n \nabla \rho_n - \rho_n m_n g + \frac{\mu_r}{\chi_n} m_n V_n + \rho_n m_n f_{ni}(V_n - V_i) \quad (17)$$

Experimental observation [3] shows that the ion-neutral gas collision frequency f_{in} or f_{ni} and the electron-neutral gas collision frequency f_{en} can be given as;

$$f_{in} = 2.6 \times 10^{-15}(\rho_n + \rho_i)[M'_n]^{1/2} \quad (18)$$

and

$$f_{en} = 5.4 \times 10^{-16} \rho_n T_e^{1/2} \quad (19)$$

where, M'_n and T_e denote the mean neutral molecular mass and electron temperature.

Now, if ρ_n and ρ_i , V_n and V_i are respectively the density of neutral gas and ions as well as their velocities. In the absent of every other force the equilibrium motion of the plasma can be determined by the balance between the magnetic field (that is, the Lorentz force) and the ambipolar field which exists as a result of the relative motion between the ions and the neutral gas, thus;

$$\mathbf{J} \times \mathbf{B} = \gamma \rho_n \rho_i m_i (V_i - V_n) \quad (20)$$

where, $\gamma = \frac{\langle \sigma v \rangle}{m_n + m_i}$, with $\langle \sigma v \rangle$ being the momentum

transfer rate between the ions and the neutral gas, and m_i and m_n are the masses of the ions and neutral gas. This indicates, according to [15, 33] that the neutral gas gets hook up to the magnetic field through its interactions with the ions, such that the induction equations for the neutrals assumed the forms;

$$\rho_n \frac{\partial V_n}{\partial t} = \nabla \times (v_n \times B) - \frac{4\pi}{c} \nabla \times \left[\frac{v_A^2}{\gamma \rho_i} J_\perp \right] \quad (21)$$

where, $v_A = \frac{B}{\sqrt{(4\pi\rho_n)}}$ is the Alfven velocity while J_\perp is the component of the current density in the perpendicular direction. The term $\frac{4\pi}{c} \nabla \times \left[\frac{v_A^2}{\gamma \rho_i} J_\perp \right]$ accounts for the ambipolar diffusion, with the ambipolar diffusivity defined as $\eta_A = \frac{v_A^2}{\gamma \rho_i}$.

The effect of ambipolar diffusion on the propagation of plasma waves and satellite signals can be deduced by adopting a spectral analysis and solutions of the form;

$$f(x, y, t) = F(y) \exp[i(k \cdot x - \omega t)] \quad (22)$$

for all the flow variables presented in section 2. Where, $F(y)$ is the amplitude of the incident beam propagated in the vertical y-direction with the frequency, ω and $k \cdot x$ expresses

the phase.

Introducing the following neutrality condition as in [5, 39], thus:-

$$\rho = \rho_i + \rho_e$$

and

$$e^+ \rho_i + e^- \rho_e = 0$$

Such that the velocity of the centre of gravity which enhance the collective behaviour of the plasma becomes;

$$U = \frac{m_i \rho_i V_i + m_e \rho_e V_e}{m_i \rho_i + m_e \rho_e} \quad (23)$$

Eliminating V_n , the neutral wind velocity in equations (15) and (16), we have;

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + 2f_{in} \right) V_i = \frac{e^+}{m_i} \left(\frac{\partial}{\partial t} + f_{in} \right) [E + \frac{V_i \times B}{c}] + H_i \quad (24)$$

for the ions-neutral gas interaction and,

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + 2f_{en} \right) V_e = \frac{e^-}{m_e} \left(\frac{\partial}{\partial t} + f_{en} \right) [E + \frac{V_e \times B}{c}] + H_e \quad (25)$$

for the electron-neutral gas interaction.

where, $H_k = 2g f_{kn} - K_B f_{kn} \left(\frac{T_k}{m_k} \frac{\nabla \rho_k}{\rho_k} + \frac{T_n}{m_n} \frac{\nabla \rho_n}{\rho_n} \right)$;

Also, by multiplying equation (24) with $m_i \rho_i$ and equation (25) with $m_e \rho_e$ following the method adopted by [5, 39], and thereafter adding the results considering the case in which, $f_{in} = f_{en} = f_c$, and in conjunction with equations (23), we have;

$$\left(\frac{\partial}{\partial t} + 2f_c \right) \frac{\partial U}{\partial t} = \frac{1}{c} \left(\frac{\partial}{\partial t} + f_c \right) (J \times B) + [m_i \rho_i H_i + m_e \rho_e H_e] \quad (26)$$

Similarly, multiplying equation (24) with $\rho_i e^+$ and equation (25) with $\rho_e e^-$; and adding their results, and then noting that;

$$\frac{(e^+)^2 \rho_i V_i}{m_i} + \frac{(e^-)^2 \rho_e V_e}{m_e} = \left(\frac{e^+}{m_i} + \frac{e^-}{m_e} \right) J - \frac{e^+ e^-}{m_i m_e} (m_i \rho_i + m_e \rho_e) U$$

we have;

$$\begin{aligned} \left(\frac{\partial}{\partial t} + 2f_c \right) \frac{\partial J}{\partial t} &= \left[\frac{(e^+)^2 \rho_i}{m_i} + \frac{(e^-)^2 \rho_e}{m_e} \right] \left(\frac{\partial}{\partial t} + f_c \right) E + \frac{1}{c} \left(\frac{e^+}{m_i} + \frac{e^-}{m_e} \right) \frac{\partial}{\partial t} (J \times B) \\ &- \frac{e^+ e^-}{m_i m_e c} (m_i \rho_i + m_e \rho_e) \frac{\partial U}{\partial t} \times B + f_c \frac{1}{c} \left(\frac{e^+}{m_i} + \frac{e^-}{m_e} \right) (J \times B) \\ &- f_c \frac{e^+ e^-}{m_i m_e c} (m_i \rho_i + m_e \rho_e) (U \times B) + e^+ \rho_i H_i + e^- \rho_e H_e \end{aligned}$$

$$\left[\begin{array}{c} \left\{ \left(\omega^4 + 4f_c \omega^3 - \left(\frac{a_3}{c} + 4f_c^2 \right) \omega^2 - \left(\frac{a_5}{c} + \frac{a_3}{c} f_c \right) \omega \right) \right\} \{-a_1(\omega^3 - 3f_c \omega^2 - 4f_c^2 \omega)\} \\ \left\{ \frac{4\pi\omega}{c^2} \right\} \left\{ \frac{\omega^2}{c^2} - k^2 \right\} \end{array} \right] \begin{bmatrix} J \\ E \end{bmatrix} = \begin{bmatrix} 2f_c(e^+ \rho_i H_i + e^- \rho_e H_e) \\ 0 \end{bmatrix} \quad (30)$$

whose Slater determinant becomes;

$$\left\| \begin{array}{c} \left\{ \left(\omega^4 + 4f_c \omega^3 - \left(\frac{a_3}{c} + 4f_c^2 \right) \omega^2 - \left(\frac{a_5}{c} + \frac{a_3}{c} f_c \right) \omega \right) \right\} \{-a_1(\omega^3 - 3f_c \omega^2 - 4f_c^2 \omega)\} \\ \left\{ \frac{4\pi\omega}{c^2} \right\} \left\{ \frac{\omega^2}{c^2} - k^2 \right\} \end{array} \right\| = 0$$

and further analysis of the above equation yields the following frequency spectrum;

Now, taking a time derivative of the above equation and multiplying the result by $\left(\frac{\partial}{\partial t} + 2f_c \right)$, and in accordance with the relationship stated in equation (26), we arrived at the following;

$$\begin{aligned} \left[\frac{\partial}{\partial t} + 2f_c \right] \frac{\partial^2 J}{\partial t^2} &= a_1 \left(\frac{\partial}{\partial t} + 2f_c \right) \left(\frac{\partial}{\partial t} + f_c \right) \frac{\partial E}{\partial t} + a_2 \left(\frac{\partial}{\partial t} + 2f_c \right) \frac{\partial^2 J}{\partial t^2} \times B_0 \\ &- a_3 \left\{ \frac{\partial}{\partial t} \left[\frac{1}{c} \left(\frac{\partial}{\partial t} + f_c \right) (J \times B) \times B_0 + [m_i \rho_i H_i + m_e \rho_e H_e] \times B_0 \right\} \right. \\ &+ a_4 \left(\frac{\partial}{\partial t} + 2f_c \right) \frac{\partial J}{\partial t} \times B_0 - a_5 \left\{ \frac{1}{c} \left(\frac{\partial}{\partial t} + f_c \right) (J \times B) \times B_0 + [m_i \rho_i H_i \right. \\ &\quad \left. + m_e \rho_e H_e] \times B_0 \right\} \\ &\left. + \left(\frac{\partial}{\partial t} + 2f_c \right) [e^+ \rho_i H_i + e^- \rho_e H_e] \right] \quad (27) \end{aligned}$$

where,

$$a_1 = \left[\frac{(e^+)^2 \rho_i}{m_i} + \frac{(e^-)^2 \rho_e}{m_e} \right], a_2 = \frac{1}{c} \left(\frac{e^+}{m_i} + \frac{e^-}{m_e} \right),$$

$$a_3 = \frac{e^+ e^-}{m_i m_e c} (m_i \rho_i + m_e \rho_e), a_4 = f_c a_2, a_5 = f_c a_3$$

At this juncture, it is expedient to introduce a travelling wave solution of the form stated in equation (22), such that the expressions in equations (12) and (27) becomes;

$$k^2 E - k(k \cdot E) = \frac{\omega^2}{c^2} E + \frac{4\pi i \omega}{c^2} J \quad (28)$$

and,

$$\begin{aligned} (\omega^4 + i4f_c \omega^3 - 4f_c^2 \omega^2) J &= a_1 (i\omega^3 - 3f_c \omega^2 - 2if_c^3 \omega) \times E + \\ &a_2 (i\omega^3 - 2f_c \omega^2) (J \times B) \\ &- \frac{a_3}{c} (-\omega^2 - if_c \omega) (J \times B) \times B + a_4 (-\omega^2 - 2if_c \omega) (J \times B) - \frac{a_5}{c} (- \\ &\quad i\omega) (J \times B) \times B \\ &- \frac{a_5}{c} f_c (J \times B) \times B - a_3 [m_i \rho_i H_i + m_e \rho_e H_e] \times B + 2f_c [e^+ \rho_i H_i \\ &\quad + e^- \rho_e H_e] \end{aligned} \quad (29)$$

But, $(J \times B) \times B$ can be expressed as $J - (J \cdot B) \times B$, such that, equations (28) and (29) for the case in which the plasma is unbounded [that is, not frozen by the magnetic field (i.e., $B = 0$), a free plasma state, and $(k \cdot E) = 0$], which indicates propagations in the perpendicular direction, we arrived at the following matrix equation;

$$\omega^6 + b_4\omega^5 + b_3\omega^4 - b_2\omega^3 + b_1\omega^2 + b_0\omega = 0 \quad (31)$$

where,

$$b_0 = ck^2 (a_5 + a_3 f_c), b_1 = 4f_c$$

$$b_2 = 4\pi a_1 - \frac{a_3}{c} - 4f_c^2 - c^2 k^2, b_3 = 12\pi a_1 f_c + \frac{a_5}{c} + \frac{a_3}{c} f_c + 4f_c c^2 k^2,$$

$$\text{and } b_4 = ck^2 a_3 + 4f_c^2 c^2 k^2 - 16\pi a_1 f_c^2.$$

Substituting for a_1 , a_3 and a_5 , we have;

$$b_0 = 2k^2 \left[\frac{\rho_e e^+ e^-}{m_e} + \frac{\rho_e e^+ e^-}{m_i} \right], b_1 = 4f_c,$$

$$b_3 = 3f_c \omega_\rho^2 + \frac{2f_c}{c^2} \left[\frac{\rho_e e^+ e^-}{m_e} + \frac{\rho_e e^+ e^-}{m_i} \right] + 4f_c c^2 k^2,$$

$$b_2 = \omega_\rho^2 - \frac{1}{c^2} \left[\frac{\rho_e e^+ e^-}{m_e} + \frac{\rho_e e^+ e^-}{m_i} \right] - (4f_c^2 + c^2 k^2),$$

and

$$b_4 = k^2 \left[\frac{\rho_e e^+ e^-}{m_e} + \frac{\rho_e e^+ e^-}{m_i} \right] - 4f_c^2 \omega_\rho^2 + 4f_c^2 c^2 k^2.$$

where, $\omega_\rho^2 = 4\pi \left[\frac{(e^+)^2 \rho_i}{m_i} + \frac{(e^-)^2 \rho_e}{m_e} \right] \Rightarrow \omega_\rho^2 = \omega_{\rho_i}^2 + \omega_{\rho_e}^2$,
the plasma frequency

$$\omega_{\rho_i}^2 = \left[\frac{4\pi(e^+)^2 \rho_i}{m_i} \right], \text{ the ion frequency} \quad (32)$$

and,

$$\omega_{\rho_e}^2 = \left[\frac{4\pi(e^-)^2 \rho_e}{m_e} \right], \text{ the electron frequency} \quad (33)$$

If we consider the case in which the plasma is bounded by the magnetic field (i.e., $B_0 \neq 0$); it is possible to isolate the following wave modes and their frequencies [5, 39 – 40];

$$\Omega_i = \frac{1}{C} \left[\frac{e^+ B_0}{m_i} \right], \quad (\text{Ion cyclotron frequency})$$

$$\Omega_e = \frac{1}{C} \left[\frac{e^- B_0}{m_e} \right], \quad (\text{Electron cyclotron frequency})$$

$$\Omega_{mg} = \mu g B_0, \quad (\text{Magneto-gravity waves})$$

$$\Omega_g = g \left[\frac{\rho_e e^+}{m_i} + \frac{\rho_e e^-}{m_e} \right] = g(\Omega_{ig} + \Omega_{eg}) \quad (\text{Gravity waves})$$

$$\Omega_a = k_B (\Omega_{ig} + \Omega_{eg}) \quad (\text{Acoustic waves})$$

and,

$$\Omega_{mi} = k_B B_0 \quad (\text{Magneto-acoustic waves})$$

The above spectral analysis, considering equation (30); yields the current density j and the accompanied induced electric field E as:-

$$j = \frac{2f_c(\omega^2 - k^2 c^2)}{c^2 \Delta} [e^+ \rho_i H_i + e^- \rho_e H_e] \quad (34)$$

and

$$E = \frac{8\pi\omega f_c}{c^2 \Delta} [e^+ \rho_i H_i + e^- \rho_e H_e] \quad (35)$$

Such that, the ratio j/E becomes;

$$\delta_\rho = \frac{j}{E} = \frac{\omega^2 - k^2 c^2}{4\pi\omega} \Rightarrow \text{Plasma conductivity}$$

[5, 39] show that $\omega^2 = \omega_p^2 + k^2 c^2$, simplifying the plasma conductivity to;

$$\delta_\rho = \frac{j}{E} = \frac{\omega_p^2}{4\pi\omega} \quad (36)$$

In the above analysis we have considered the situation in which the temperature of the plasma is constant, that is, assumed to be either the electron temperature, T_e and/or ion temperature, T_i . But in the case where the temperature varies with altitude (i.e., height of the ionosphere) then equation (2) along with equation (12) are considered such that after introducing the following scaling parameters:-

$$\Theta = \frac{T - T_0}{T_k - T_0}, \rho_k = \frac{\rho_k}{\rho_{0k}}, \omega = \frac{\omega}{\omega_k}, t = \omega_k t \text{ and } v = \frac{v}{c} \text{ in}$$

addition to a few algebraic steps, the ion-neutral collision and electron-neutral collision equations reduced to:

$$\begin{aligned} (k^2 - \frac{1}{4}\omega - M_1^2)V_i + \eta^2 V_n &= G_r \Theta_i \\ (k^2 - \frac{1}{4}\omega - \chi_1^2)V_n + \eta^2 V_i &= G_r \Theta_n \end{aligned} \quad (37)$$

for the ion-neutral collision and;

$$\begin{aligned} (k^2 - \frac{1}{4}\omega - M_1^2)V_e + \eta^2 V_n &= G_r \Theta_e \\ (k^2 - \frac{1}{4}\omega - \chi_1^2)V_n + \eta^2 V_e &= G_r \Theta_n \end{aligned} \quad (38)$$

for the electron-neutral collision.

And eliminating V_n , the neutral gas velocity, we have;

$$\omega_{in}^2 - h_1 \omega_{in} + h_0 = 0 \quad (39)$$

and

$$\omega_{en}^2 - h_1 \omega_{en} + h_0 = 0 \quad (40)$$

where, $h_1 = 2k^2 + (\chi_1^2 + M_1^2 + G_r)$, $h_0 = k^4 + (\chi_1^2 + M_1^2 + G_r)k^2 + (M_1^2 + G_r)\chi_1^2 + \eta^2 G_n - \eta^4$,

$M_1^2 = (\chi^2 + M^2 + \eta^2)$ and $\chi_1^2 = (\chi^2 + \eta^2)$, whereas, G_r is the thermal convection parameter, M^2 is the magnetic parameter, η^2 is the coupling parameter, and χ^2 is the porosity.

The amplitude of the velocity can be deduced from equation (2) for a steady state situation, thus;

$$[\rho_i m_i f_{in} - \frac{\mu}{\chi_c}] V_i = (\rho_i m_i f_{in}) V_n - K_B T_i \nabla \rho_i + \rho_i m_i g + e \rho_i (E + V_i \times B)$$

for the ions-neutral gas interaction and,

$$[\rho_e m_e f_{en} - \frac{\mu}{\chi_c}] V_e = (\rho_e m_e f_{en}) V_n - K_B T_e \nabla \rho_e + \rho_e m_e g - e \rho_e (E + V_e \times B)$$

for the electron-neutral interaction, such that;

$$V_i = \beta^* \rho_i m_i f_{in} V_n + \beta^* [\rho_i m_i g - k_B T_i \nabla \rho_i] + \beta^* e^+ \rho_i [E + V_i \times B]$$

$$V_e = \beta^* \rho_e m_e f_{en} V_n + \beta^* [\rho_e m_e g - k_B T_e \nabla \rho_e] + \beta^* e^- \rho_e [E + V_e \times B]$$

where, $\beta^* = \frac{1}{(\rho_k m_k f_{kn} - \frac{\mu_r}{\chi_c})}$

According to [3], the vector $[E + V \times B]$ can be expanded, thus;

$$\beta^* e^k \rho_k [E + V \times B] = \frac{1}{1 + (\eta_k^*)^2} \left[\frac{\eta_k^*}{B} E \right] + \left[\left(\frac{\eta_k^*}{B} \right)^2 - \left(\frac{\eta_k^*}{B} \right)^3 \right] \{ [(E + V \times B) \cdot B] \times B \}$$

For $\eta^* \ll 1$, that $O(\eta^{*2}) \rightarrow 0$, which is the case for a weakly ionized plasma, as is in the D-layer of the ionosphere. In this case the velocity amplitude becomes;

$$V_k = \beta_k^* \rho_k m_k f_{kn} V_n + \beta_k^* [\rho_k m_k g - k_B T_k \nabla \rho_k] + \eta_k^* \frac{E}{B}$$

such that

$$V_k = \beta_k^* \rho_k m_k f_{kn} V_n + \beta_k^* \rho_k k_B T_k \left[\frac{\rho_k m_k g}{k_B T_k} - \frac{\nabla \rho_k}{\rho_k} \right] + \eta_k^* \frac{E}{B}$$

and further simplification yields;

$$V_k - \beta_k^* \rho_k m_k f_{kn} V_n = \beta_k^* \rho_k k_B T_k \left[\frac{g}{H_k} - \frac{\nabla \rho_k}{\rho_k} \right]$$

In the absence of porosity, that is, the permeability of the medium is zero, the results agree with those of [3], thus;

$$V_k = V_n + G_k + \eta_k \frac{E}{B}$$

and the ambipolar velocity for the ions and electron becomes;

$$V_i - V_n = G_i + \eta_i \frac{E}{B} \quad (41)$$

and

$$V_e - V_n = G_e + \eta_e \frac{E}{B} \quad (42)$$

where, $G_k = D_k \left[\frac{g}{H_k} - \frac{\nabla \rho_k}{\rho_k} \right]$, $D_k = \frac{k_B T_k}{m_k f_{kn}}$ is the diffusion coefficient of the ions or electrons as the case may be and $H_k = \frac{k_B T_k}{m_k}$ is the scale height of the respective species and $\eta_k = \frac{\Omega_k}{m_k f_{kn}}$ with Ω_k being the gyro-frequency of the respective species.

4. Results and Discussion

The plasma medium of the Earth's planetary ionosphere is highly coupled due to the interactions of plasma species as they move with their respective thermal velocities [15, 33, 41]. The electrons have higher thermal velocity therefore; tend to move further away from the comparatively massive ions and in the process set up a polarization current as a result of charge imbalance which drags the ions along with the electrons. This results in ambipolar diffusion as the coupled species move through the plasma interacting with the neutral gas, and electric and magnetic fields in the terrestrial environment [3, 15]. The velocity of these coupled species as a function of collision frequency is shown in figure 1. The profile shows that the electron diffusion velocity decreases as the coupling frequency increases.

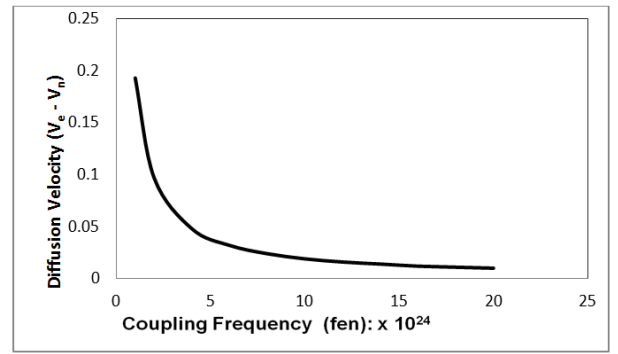


Figure 1. Variation of Diffusion Velocity with Coupling Frequency.

The increase in coupling frequency reduces the diffusivity of plasma medium; therefore, ambipolar diffusion of the coupled species is dampened [42], due to high collision. But, when the plasma temperature is increased the diffusivity is enhanced, due to the fact that the plasma species attain higher thermal velocities. The study further shows that these phenomena affect the propagation of electromagnetic signal via the ionosphere as a result of the interactions of the coupled species in the plasma medium. Furthermore, the coupled plasma species is seen to oscillate in response to the external field as the electromagnetic signal traverse the ionosphere. The interaction of the signal with the coupled species reduces the velocity amplitude of the signal, leading to loss of signal strength with time. The result of this is either the signal is totally lost (absorbed), refracted or partially transmitted, depending on the frequency of propagation in relation to the plasma frequency. [5] Shows that for any signal frequency (ω) much larger than the plasma frequency (that is, $\omega \gg \omega_p$) the propagated signal suffer less diffusion.

In this case the effect of ambipolar diffusion would not be significant. On the other hand for a case in which $\omega \ll \omega_p$, ambipolar diffusion effect is highly noticeable, leading to reduction or loss of signal strength [33]. This is mostly the case in the D-region of the ionosphere, where ionization is weak. Figure 2 illustrates the behaviour of the diffusion velocity with changes in the E/B ratio for large and small coupling frequency ranges.

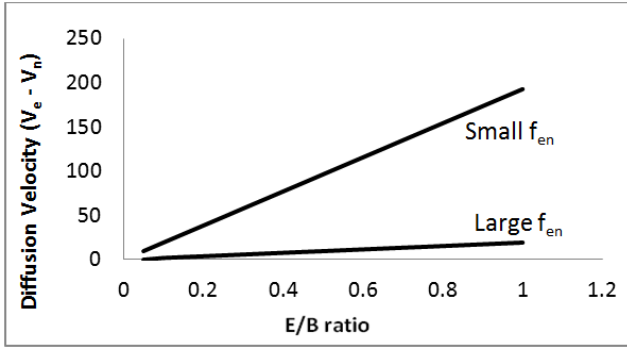


Figure 2. Variation of Diffusion Velocity with Electric-Magnetic Field Ratio.

The curve for the small coupling frequency indicates a much larger velocity growth rate than that for large coupling frequency as the E/B ratio increases. Further analysis shows that when the E/B ratio is much greater than one (i.e. $E/B \gg 1$, which implies that $B \rightarrow 0$, or that B is along E direction, or that the magnetic field component is negligible compared to the electric field); the ionized species oscillates along the direction of the electric field and thereby leading to increase in the diffusion velocity in the vertical direction. But, when the ratio is far less than one (i.e., $E/B \ll 1$) the magnetic field component become much larger than the electric field; therefore, the species moves along the electric field in the perpendicular direction of the magnetic field, the Lorentz forces act on them setting up a torque such that they gyrate in response to the magnetic flux. This results in a reduction of diffusion rate. The flow of plasma across magnetic field lines depends on the ratio of the coupling frequency, f_{kn} , to the gyro-frequency, Ω_k [43]. The analysis indicates that for $f_{kn} > \Omega_k$, the collision prevents the plasma from gyrating along the magnetic field lines, instead they move in the electric field direction. But, for $f_{kn} < \Omega_k$, the plasma species gyrates along the magnetic field lines. The variation of coupling frequency, f_{kn} as the electron density increases for different wave numbers at constant magnetic strength ($M^2 = 0.5$) is shown in figure 3.

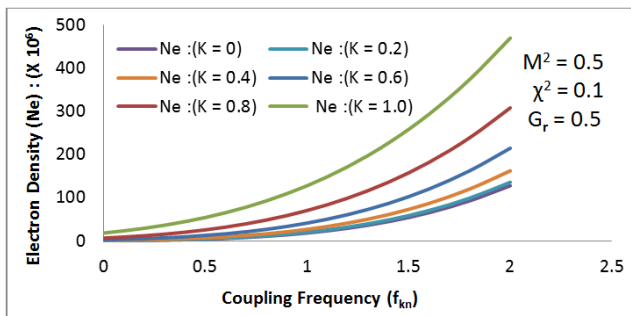


Figure 3. Variation of Electron Density against Coupling Frequency.

Similarly, the profiles presented in figures (4) and (5) indicate that the plasma conductivity (δ_p) increases with electron density as well as with the height of the ionosphere. At the peak electron density which is at about 450 km up into the ionosphere, the conductivity attains maximum value and tappers off as the ionization density reduces. These findings are in agreement with those of [3, 43].

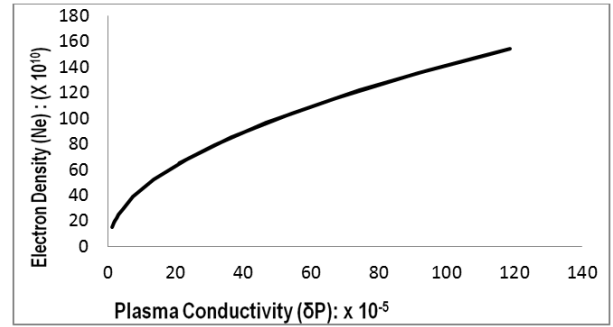


Figure 4. Variation of Plasma Conductivity with Electron Density.

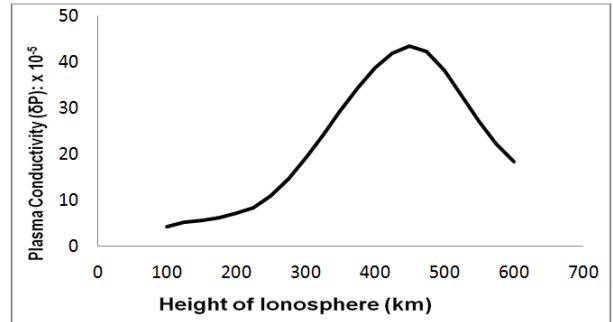


Figure 5. Variation of Plasma Conductivity with Height of Ionosphere.

On the whole, the general analysis shows that when $\frac{(M^2 + G_r)^2}{f_{kn}(G_r - f_{kn})} \geq 0$, the plasma interactive state becomes more stable, otherwise some bit of oscillation is noticed. The stability is seen to depend among others on the magnetic field strength (M^2) and thermal convection (G_r). Under this condition the signal propagation becomes less diffuse when the frequency is far greater than the plasma frequency, that is, $\omega \gg \omega_p$.

The overall analysis presented in this work aids our understanding of the physics of signal and satellite propagation in the present of ambipolar diffusion.

5. Conclusion

Ambipolar diffusion, which results from the interactions in a weakly ionized plasma, has been found to have a profound effect on the interactive state and the flow structure of ionospheric plasma. The stability condition observed in the study, aside from shedding a light on the physics and ionospheric plasma interactions, will serve as a veritable tool in mitigating factors affecting signal propagation in navigational satellites and global positioning systems.

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