Establishing Mathematical Model to Predict Ship Resistance Forces

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Abstract: Resistance forces of water affecting to the ship hull at every single time during ship motions change very complexly. For simulating the ship motion in 6 degrees of freedom on a bridge simulator, these forces need to be calculated. Previous studies showed that resistance forces were estimated by empirical or semi-empirical methods, basic hydrodynamic theory has not solved all components of resistance forces. Moreover, for simulating the ship motions at the initial design stage when experimental value is not available it is necessary to estimate resistance forces by theoretical method. Fully estimating damping forces by theoretical method is a practical challenge. This study aims to find out general equations to reasonably estimate all damping coefficients in 6 degrees of freedom for simulating ship motions on bridge simulators.

Keywords: Fluid Resistance, Damping Coefficients, Hydrodynamic Coefficient, Mathematical Modeling, Ship Simulation

1. Introduction

The motions of a ship in water are basically derived from Newton's deferential equations of motion. It can be basically presented in 6 degrees of freedom (6DOF) under the matrix equations [1]:

$$M\begin{bmatrix}\dot{u}\\\dot{v}\\\dot{w}\\\dot{p}\\\dot{q}\\\dot{r}\end{bmatrix} + C_{S,A}(v)\begin{bmatrix}u\\v\\w\\p\\q\\r\end{bmatrix} + D_n(v)\begin{bmatrix}u\\v\\w\\p\\q\\r\end{bmatrix} + g(\eta) = \begin{bmatrix}X\\Y\\Z\\K\\M\\N\end{bmatrix} \quad (1)$$

Where M is generalized mass matrix of the ship and added mass, $C_{S,A}(v)$ is Coriolis and centripetal matrix of the ship and added mass due to motion or rotation about the initial frame, $v = [u, v, w, p, q, r]^T$ is velocity matrix, $\ddot{x} = [\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}]^T$ is acceleration matrix. $g(\eta)$ is generalized gravitational/buoyancy forces and moments. $F = [X, Y, Z, K, M, N]^T$ is matrix of external forces and moments effecting to the ship.

The resistance forces affecting to the ship hull in 6DOF are very complexly. In general, they are described by a subdivision into linear and non-linear forces and can be expressed:

$$d(v_r) = D_l\begin{bmatrix}u\\v\\w\\p\\q\\r\end{bmatrix} + D_n(v)\begin{bmatrix}u\\v\\w\\p\\q\\r\end{bmatrix} = D(v)\begin{bmatrix}u\\v\\w\\p\\q\\r\end{bmatrix} \quad (2)$$

Where D_l, D_n (v) are linear damping and non-linear damping. In this paper 6 motion and rotation components are defined in the body-fixed reference frame with motion parameters defined as below:

***Table 1.** Parameters of 6DOF defined in the body-fixed reference frame.*

DOF	Description	Velocities	Forces
1	surge - motion in x direction	u	X
2	sway - motion in y direction	v	Y
3	heave - motion in z direction	w	Z
4	roll – rotation about x axis	p	K
5	pitch - rotation about y axis	q	M
6	yaw - rotation about z axis	r	N

The non-linear damping $D_n(v)$ is created by the effect of "viscous fluid". $D_n(v)$ is majority and is the most difficult to estimate even when the ship moves steady with stable speed [2]. It was previously estimated by empirical methods [3].

Because basic hydrodynamic theory has not solved all components of resistance forces. Therefore, to estimate empirical or semi-empirical formulas or simulation test are applied. There were several studies introducing various methods including empirical and theoretical for estimating the hull damping.

In theory, a simple set of equations is presented by Society of Naval Architects and Marine Engineers (SNAME) in 3 DOF including surge, sway and yaw [2]. Fedyaevsky and Sobolev introduced equations to calculate cross-flow Drag in sway and yaw [4]. Nils Salvesen, E. O. Tuck and Odd Faltisen suggested a method to calculate damping components in "Ship Motions and Sealoads" [5].

Meanwhile main empirical methods can be referred to some studies of Wagner Smitt, Norbin, Inoue, Clake [6], Lee [7], Kijima and Nakiri [8]. These methods only derive damping coefficients mainly for 3DOF including surge, swaye and yaw. It is obviously that these experimental methods cannot help in case of modeling the ship at the initial design stage.

As theoretical method, Clarke typically applied the slender body strip theory for a flat plate and introduced an equation set for estimation [9]. However, it could only derive the same above components without solving for a model with 6DOF. More details on various studies can be referred in the reviews of J. P. Hooft [3].

Recent studies have trended to improve the accuracy of previous methods or apply computational fluid dynamics (CFD) or numerical simulation to calculate resistances. K. Zelazny introduced a method to improve accuracy of ship resistance at preliminary stages of design [10]. Mucha et al. had a validation study on numerical prediction of resistance in shallow water based on the solution of the Reynolds-averaged Navier-Stokes (RANS) equations, a Rankine Panel method and a method based on slender-body [11]. The application of CFD can be typically referred to the study of Yasser M. Ahmed et al. [12]. These mentioned methods only calculate the total hull resistance at translation speeds.

For roll damping coefficients, it can be referred to study of Frederick Jaouen et al. [13] and the calculation of Yang Bo et al. by using numerical simulation based on CFD [14]. Burak Yildiz et al. introduced an URANS prediction of roll damping due to the effects of viscosity based on CFD [15] while Min Gu et al. presented a roll damping calculation based on numerical simulation on the RANS model in calm water [16]. In 2017, D. Sathyaseelan et al. introduced an efficient Legendre wavelet spectral method (LWSM) to ship roll motion model for investigating the nonlinear damping coefficients [17].

However, the previous mentioned methods did only solve a single degree or limited degrees of freedom. It is also considered that a complex method can cause delays in computer calculation that does not satisfy the real-time run of ship simulation systems. This study aims to derive a detailed mathematical model of hull resistance consisting circulatory forces (lift, drag) and cross-flow drag in calm water $(D_n(v))$ in a simple and numerable method applicable for real-time simulation.

2. Method to Determine Resistance Forces

2.1. Preliminary Approach

If X (x), Y (x), Z (x) are local damping forces in 3 motions surge, sway and yaw of each local hull section (x), total damping matrix D(υ) in 6 DOF can be determined by integrating over the ship length.

x_{cp} (x), y_{cp} (x), z_{cp} (x) are longitudinal, transversal and vertical local center of pressure as a function of longitudinal position.

The study approach is to determine local components of the matrix $d(v_r)$ for developing mathematical model of ship maneuvering.

$$d(v_r) = \begin{bmatrix} \int_{-L}^{L} X(x)dx \\ \int_{-L}^{L} Y(x)dx \\ \int_{-L}^{L} Z(x)dx \\ -\int_{-L}^{L} z_{cp}(x).Y(x)dx \\ \int_{-L}^{L} z_{cp}(x).X(x)dx \\ \int_{-L}^{L} \left(x_{cp}(x).Y(x) - y_{cp}(x).X(x) \right) dx \end{bmatrix} \quad (3)$$

In this study lift, drag, cross-flow drag in every single motion are solved separately then they will be summed up to have the total damping value.

Abkowitz stated that the combination of acceleration and velocity parameters, representing interaction between viscous and inertial flow phenomena, are considered to be negligibly small as there is no theoretical or empirical justification for their inclusion [18].

The impact of waves is considered as external forces F and is suggested to solve separately.

2.2. Fundamental Theory

A general resistance force effecting on ship hull moving in free water surface can be derived according to fluid hydrodynamic theory:

$$F = \frac{1}{2}\rho U^2 S C_f \quad (4)$$

Where U is vessel's velocity, ρ is water density, S is wetted surface and C_f is hydrodynamic coefficient. Considering the forces impacting the hull at a local section i[th] which is apart from the center of gravity a distance x_i.

At a particular section i[th], the local longitudinal, transversal and vertical velocity $[v(x), u(x), w(x)]$ need to be adjusted with velocity due to surge, pitch and yaw rotation $[p, q, r]$:

$$v(x) = v + xr + zq \quad (5)$$

$$u(x) = u + yr - zp \quad (6)$$

$$w(x) = w + yp + xq \quad (7)$$

$$U(x) = \sqrt{u(x)^2 + v(x)^2 + w(x)^2} \quad (8)$$

Due to ship motion, the velocity of water impacting oppositely the ship velocity causes resistance against the ship hull. When the ship is moving and rotating in water with current, relative longitudinal velocity $u_r(x)$ and relative transversal velocity $v_r(x)$ of a local section are determined:

$$u_r(x) = u(x) - u_c = u(x) - V_c cos(\beta_c - \Psi) \qquad (9)$$

$$v_r(x) = v(x) - v_c = v(x) - V_c sin(\beta_c - \Psi) \qquad (10)$$

Where ψ is ship heading, $\beta_c(x)$ is current drift angle at the local section. In case the current speed and drift at each section along the ship length are different, a full calculation of current for every section is critical to ensure the accuracy of damping due to current impact.

In each motion the resistance forces can be divided into 2 components: Lift (L) and Drag (D).

According to hydrodynamic fluid theory the lift and drag are basically described.

$$L(x) = \frac{1}{2}\rho U(x)^2 S(x).C_L(x) \qquad (11)$$

$$D(x) = \frac{1}{2}\rho U(x)^2 S(x).C_D(x) \qquad (12)$$

Where CL (x), CD (x) is non-dimension hydrodynamic coefficient of lift and drag forces depending Reynolds number: $C_{D,L} = C_{D,L}(\beta, Re)$.
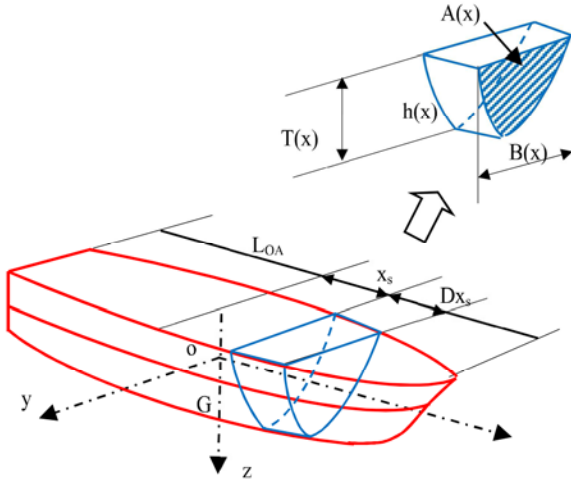

Figure 1. Describing a section of ship hull.

Project the force L and D in the Descartes axis force component in 3 direction ox, oy, oz are obtained. The total forces in each direction are determined by integration over the ship length. Thus at each section, the force L and D are expressed:

$$
\begin{aligned}
X_{LD}(x) &= X_L(x) + X_D(x) \\
Y_{LD}(x) &= Y_L(x) + Y_D(x) \\
Z_{LD}(x) &= Z_L(x) + Z_D(x) \\
K_{LD}(x) &= -z_{cp}(x).Y_{LD}(x) \\
M_{LD}(x) &= z_{cp}(x).X_{LD}(x)dx \\
N_{LD}(x) &= x_{cp}(x).Y_{LD}(x) - y_{cp}(x).X_{LD}(x)
\end{aligned}
\qquad (13)
$$

The forces over the ship length are described:

$$
\begin{aligned}
X_{LD} &= \int_{-L}^{L} X_{LD}(x)dx \\
Y_{LD} &= \int_{-L}^{L} Y_{LD}(x)dx \\
Z_{LD} &= \int_{-L}^{L} Z_{LD}(x)dx \\
K_{LD} &= -\int_{-L}^{L} z_{cp}(x).Y_{LD}(x)dx \\
M_{LD} &= \int_{-L}^{L} z_{cp}(x).X_{LD}(x)dx \\
N_{LD} &= \int_{-L}^{L}\big(x_{cp}(x).Y_{LD}(x) - y_{cp}(x).X_{LD}(x)\big)dx
\end{aligned}
\qquad (14)
$$

2.3. Resistance Coefficients

The resistance forces are calculated separately in each single linear motion. The wet area S (x) is projected onto 2 directions: perpendicular to water flow direction that only creates the lift; parallel with the water flow direction that only creates the drag.

Based on "slender body theory" the hull of ship can be imagined behaving as a wing at an angle attack, the non-linear lift coefficient $C_{L\beta}$ is expressed as given in [2]. The drag coefficient $C_D(x)$ is expressed as given by Hoerner [19].

2.4. Deriving Velocity of Water Flow

The components of relative straight velocity of water at every single point on the hull wet area have a same value but opposite the velocity of this point:

$$
\begin{aligned}
u(x) &= -u \\
v(x) &= -v \\
w(x) &= -w
\end{aligned}
\qquad (15)
$$

To add the impact of rotation velocity, the angular velocities $[p, q, r]$ at each point are transferred into corresponding straight velocity. For a surface ships, resistance forces mainly impact to the lateral wet surfaces and the bottom wet surface.

2.4.1. Yaw Rotation Velocity (r)

a) Two sides

When the ship is yawing only half of lateral surface at the fore or aft is affected by water resistance. The velocities at each point at location (x, y) consist of longitudinal (u) and traversal (v) components:
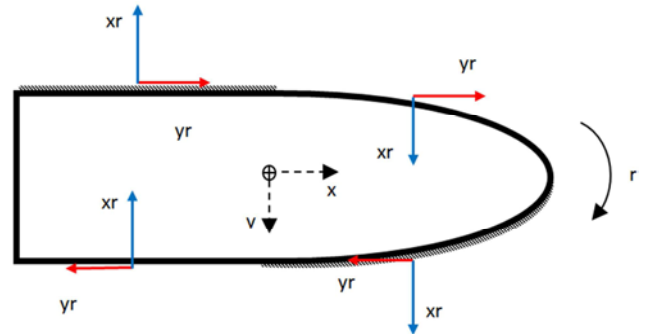

Figure 2. Longitudinal (u) and transversal (v) velocity due to yawing on the hull's lateral sides.

$$u_{rs}(x) = y_{cp}(x)r \tag{16}$$

$$v_{rs}(x) = x_{cp}(x)r \tag{17}$$

For ship with perpendicular side hull, $y_{cp}(x) = \pm\frac{B(x)}{2}$
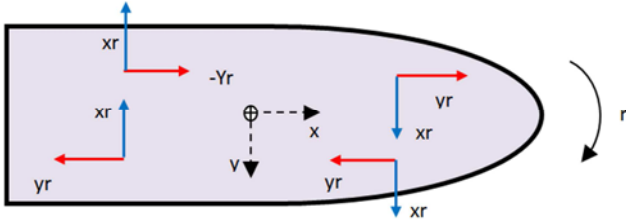
b) Bottom side

Figure 3. Longitudinal (u) and transversal (v) velocity due to yawing on the hull's bottom side.

The longitudinal velocity on the portside half and starboard side half of the bottom are opposite. The traversal velocity on the fore part and the aft part are also opposite.

$$u_{rb}(x) = \frac{y_{cp}(x)}{2}r \tag{18}$$

$$v_{rb}(x) = x_{cp}(x)r \tag{19}$$

The velocity on the bottom can be taken as average velocity at the distance of B (x)/4 apart from the fore-and-aft centre line.

2.4.2. Roll Rotation Velocity (p)

a) Two sides

The velocity on each side can be considered as the average velocity at half of the draft. The resistance force is only formed on the wet area positing the water flow.

$$v_{ps}(x) = z_{cp}(x)p \tag{20}$$

$$w_{ps}(x) = y_{cp}(x)p \tag{21}$$
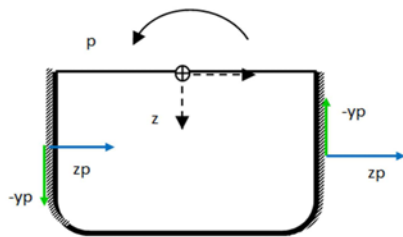
Where the $z_{cp}(x) = T(x)/2$

Figure 4. Transversal (v) and vertical (w) velocity due to rolling on the hull's lateral sides.

b) Bottom side

When rolling resistance force is only formed on the half of the bottom moving down (against the water flow).

$$v_{pb}(x) = z_{cp}(x)p \tag{22}$$

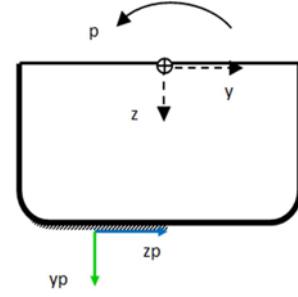$$w_{pb}(x) = y_{cp}(x)p \tag{23}$$

Figure 5. Transversal (v) and vertical (w) velocity due to rolling on the hull's bottom side.

Where, $z_{cp}(x) = T(x)$; $y_{cp}(x) = \frac{B(x)}{4}$

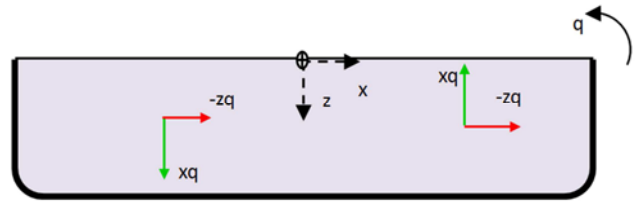### 2.4.3. Pitch Rotation Velocity (q)

a) Two sides

Figure 6. Longitudinal (u) and vertical (v) velocity due to pitching on the hull's lateral sides.

The longitudinal and vertical velocity on each side can be considered as the velocity at half of the draft.

$$u_{qs}(x) = z_{cp}(x)q \tag{24}$$

$$w_{qs}(x) = x_{cp}(x)q \tag{25}$$

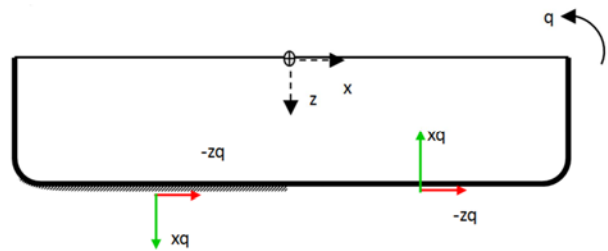Where, $z_{cp}(x) = \frac{T(x)}{2}$

b) Bottom side

Figure 7. Longitudinal (u) and vertical (v) velocity due to pitching on the hull's bottom sides.

The longitudinal and vertical velocity of point on the hull's bottom side is derived like on the lateral sides. However, the vertical resistance force is only formed on the fore or aft part which are moving down (against water flow).

$$v_{qb}(x) = z_{cp}(x)q \tag{26}$$

$$w_{qb}(x) = x_{cp}(x)q \tag{27}$$

Where, $z_{cp}(x) = T(x)$

2.5. Establishing Formulas Calculating Resistance Forces

2.5.1. Resistance Force Due to Longitudinal Water Flow

a) Lateral sides

The longitudinal relative motion of water flow on the hull's lateral side creates a drag (viscous damping) along the lateral projection wet area A_L and a lift against fore and aft projection wet area A_p. The forces are only formed on the side against the water flow. The longitudinal velocity ($u_{rs} = z_{cp}q = \pm T(x)q/2$) on the portside and starboard side are symmetric and exterminated.

Longitudinal resistance force (X): From the general formula (4), the drag and lift are described:

$$D(x) = -\frac{1}{2}\rho A_L(x)u^2(x)C_{D0}$$

$$L(x) = -\frac{1}{2}\rho A_p(x)u^2(x)C_{L\beta}$$

Where, $u(x) = u + \frac{T(x)}{2}q$

$$A_L(x) = 2T(x)dx$$

$$D(x) = -\rho C_{D0}u(x)^2 T(x)dx$$

$$L(x) = -\frac{1}{2}\rho C_{L\beta}u(x)^2 A_P(x)$$

The D (x), L (x) only impact on 1/2 area which is against the water flow.

$$u(x) \geq 0:$$

$$X_s = -\rho C_{D0}\int_{-L}^{L} u(x)|u(x)|T(x)dx$$
$$-\frac{1}{2}\rho C_{L\beta}\int_0^L u(x)|u(x)|A_P(x) \qquad (28)$$

$$u(x) < 0:$$

$$X_s = -\rho C_{D0}\int_{-L}^{L} u(x)|u(x)|T(x)dx$$
$$-\frac{1}{2}\rho C_{L\beta}\int_{-L}^0 u(x)|u(x)|A_P(x) \qquad (29)$$

Yawing resistance moment (N):

$$X(x) = -\frac{1}{2}\rho C_{D0}y_{cp}r|y_{cp}r|T(x)dx$$
$$-\frac{1}{2}\rho C_{L\beta}y_{cp}(x)r|y_{cp}(x)r|A_P(x) \qquad (30)$$

$$N(x) = X(x)y_{cp}(x)$$

$$N_{Xs} = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} r|r||y_{cp}(x)^3|T(x)dx$$
$$-\frac{1}{2}\rho C_{L\beta}\int_{-L}^L |r||y_{cp}(x)^3|A_P(x) \qquad (31)$$

Pitching resistance moment (M): The pitching moment M(x) is caused by the u (x).

$$u(x) = u + z_{cp}(x)q$$

$$M(x) = X(x) Z_{Xs}(x)$$

$$(x) \geq 0:$$

$$M_{Xs} = \rho C_{D0}\int_{-L}^{L} u(x)|u(x)|T(x)z_{cp}(x)dx -$$
$$\frac{1}{2}\rho C_{C\beta}\int_0^L u(x)|u(x)|z_{cp}(x)A_P(x) \qquad (32)$$

$$u(x) < 0:$$

$$M_{Xs} = -\rho C_{D0}\int_{-L}^{L} u(x)|u(x)|T(x)z_{cp}(x)dx -$$
$$\frac{1}{2}\rho C_{C\beta}\int_{-L}^0 u(x)|u(x)|z_{cp}(x)A_P(x) \qquad (33)$$

Where $Z_{Xs}(x) = z_{cp}(x)$ is moment M due to longitudinal resistance force.

b) Bottom side

Longitudinal force (X): The longitudinal movement of water along ship bottom mainly create viscous resistance on the bottom wet area A_b. The component $y_{cp}(x)/2$ are killed because they are opposite on port and starboard part.

Where: $u(x) = u + T(x)q$

$$A_b(x) = B(x)dx$$

$$D(x) = -\frac{1}{2}\rho C_{D0}u(x)^2 B(x)dx$$

The component T (x)q only effected as on the fore or the aft part of the bottom area:

$$X_b = -\frac{1}{2}\rho C_{D0}\int_{-L}^L u(x)|u(x)|B(x)dx \qquad (34)$$

Yawing resistance moment (N): N (x) formed by the counter forces $\pm B(x)r/4$. Other forces components have the same direction and symmetric over longitudinal axis. Thus, they are exterminated.

$$N(x) = \frac{1}{2}\rho C_{D0}\left(\frac{B(x)}{4}r\right)^2 2\frac{B(x)}{4}\frac{B(x)}{2}dx$$

$$N_{Xb} = -\frac{1}{128}\rho C_{D0}\int_{-L}^L r|r|B(x)^4 dx \qquad (35)$$

Pitching resistance moment (M): The resistance moment M is caused by the longitudinal velocity $u(x) = u + T(x)q$.

$$M(x) = X(x) Z_{Xb}(x)$$

$$M_{Xb} = -\frac{1}{2}\rho C_{D0}\int_{-L}^L u(x)|u(x)|B(x)Z_{Xb}(x)dx \qquad (36)$$

Where, $Z_{Xb}(x) = T(x) - Zg$ the lever of the resistance moment on the bottom side.

2.5.2. Resistance Force Due to Transversal Water Flow

a) Lateral sides

The transversal relative motion of water flow on the hull's

lateral side creates a drag (viscous damping) along fore/aft projection wet area A_p and a lift against the lateral projection wet area A_L. The forces are only formed on the side against the water flow.

Transversal resistance force (Y):

$$D(x) = -\frac{1}{2}\rho A_P(x)v^2(x)C_{D0}$$

$$L(x) = -\frac{1}{2}\rho A_L(x)v^2(x)C_{C\beta}$$

Where: $v(x) = v + x_{cp}(x)r - z_{cp}(x)p$

$$A_L(x) = T(x)dx$$

$$D(x) = -\frac{1}{2}\rho C_{D0}v(x)^2 A_P(x)$$

$$L(x) = -\frac{1}{2}\rho C_{C\beta}v(x)^2 T(x)dx$$

$$v(x) < 0:$$

$$Y_s = -\frac{1}{2}\rho C_{D0}\int_{-L}^{0} v(x)|v(x)|A_P(x)$$

$$-\frac{1}{2}\rho C_{C\beta}\int_{-L}^{L} v(x)|v(x)|T(x)dx \qquad (37)$$

$$v(x) \geq 0:$$

$$Y_s = -\frac{1}{2}\rho C_{D0}\int_{0}^{L} v(x)|v(x)|A_P(x)$$

$$-\frac{1}{2}\rho C_{C\beta}\int_{-L}^{L} v(x)|v(x)|T(x)dx \qquad (38)$$

Yawing resistance moment (N): The moment N (x) is caused by $Y_s(x)$.

$$N(x) = Y(x)x_{cp}(x)$$

$$v(x) = v + x_{cp}(x)r - z_{cp}(x)p$$

$$N_{Xs} = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} v(x)|v(x)|x_{cp}(x)A_P(x)$$

$$-\frac{1}{2}\rho C_{C\beta}\int_{-L}^{L} v(x)|v(x)|T(x)x_{cp}(x)dx \qquad (39)$$

Rolling resistance moment (K): The rolling moment K (x) is caused by the velocity Y_s (x).

$$K(x) = Y(x)Z_{Ys}(x)$$

$$v(x) = v + x_{cp}(x)r - z_{cp}(x)p$$

$$K_{Ys} = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} v(x)|v(x)|Z_{Ys}(x)A_P(x)$$

$$-\frac{1}{2}\rho C_{C\beta}\int_{-L}^{L} v(x)|v(x)|T(x)Z_{Ys}(x)dx \qquad (40)$$

Where $Z_{Ys}(x) = T(x)/2 - Zg$ is the lever of moment K(x) caused by the resistance fornce Y(x).

b) Bottom sides
Transversal resistance force (Y):

$$D(x) = -\frac{1}{2}\rho A_b(x)v^2(x)C_{D0}$$

Where: $v(x) = v + x_{cp}(x)r - T(x)p$

$$A_b(x) = B(x)dx$$

$$D(x) = -\frac{1}{2}\rho C_{D0}v(x)^2 B(x)dx$$

$$Y_b = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} v(x)|v(x)|B(x)dx \qquad (41)$$

Yawing resistance moment (N): The moment N (x) is caused by the force $Y_b(x)$.

$$N(x) = Y(x)x_{cp}(x)$$

$$N_{Yb} = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} v(x)|v(x)|B(x)x_{cp}(x)dx \qquad (42)$$

Rolling resistance moment (K): The moment K (x) is caused by Y_b (x).

$$K(x) = Y(x)Z_{Ys}(x)$$

$$K_{Yb} = -\frac{1}{2}\rho C_{D0}\int_{-L}^{L} v(x)|v(x)|B(x)Z_{Yb}(x)dx \qquad (43)$$

Where $Z_{Yb}(x) = T(x)/2 - Zg$ is the lever of the moment K (x) caused by the force $Y_b(x)$.

2.5.3. Resistance Force Due to Vertical Water Flow

a) Lateral sides
The vertical relative motion of water flow on the hull's lateral side creates a drag (viscous damping) along the lateral projection wet area A_L. The lift force is small and cen be ignored.

Transversal resistance force (Z): Resistance force due to rolling $y_p = p/2$ on each lateral side can be assumed as zero due to their same in value and opposite direction. The resistance forces of the other components due to pitching are determined:

$$D(x) = -\frac{1}{2}\rho A_L(x)w^2(x)C_{D0}$$

Where: $w(x) = w - x_{cp}(x)q$

$$A_L(x) = 2T(x)dx$$

$$Z(x) = -\rho C_{D0}w(x)^2 T(x)dx$$

$$Z_s = +\rho C_{D0}\int_{-L}^{L} w(x)|w(x)|T(x)dx \qquad (44)$$

Rolling resistance moment (K): The moment K (x) is mainly caused by $y_{cp}(x)p$. The vertical resistance forces on the port side and starboard side is opposite. Therefore, no moment K (x) is formed.

$$K(x) = -\frac{1}{2}\rho C_{D0}\big(y_{cp}(x)p\big)^2 T(x)y_{cp}(x)$$

$$K_{Zs} = -\frac{1}{2}\rho C_{D0} \int_{-L}^{L} p|p|T(x)\left|y_{cp}(x)^3\right|dx \qquad (45)$$

Pitching resistance moment (M): The moment M (x) on each lateral side is caused by the force Z (x).

$$M(x) = -Z(x)x_{cp}(x)$$

$$M_{Zs} = -\rho C_{D0} \int_{-L}^{L} w(x)|w(x)|T(x)x_{cp}(x)dx \qquad (46)$$

b) Bottom side

The vertical movement of water against ship bottom mainly create a lift on the bottom wet area A_b. The viscous resistance component is small and can be ignored.

Vertical resistance (Z): The resistance force due to rolling p/2 only appears on the fore or aft botom area where impacted by the vetical water flow.

$$L(x) = -\frac{1}{2}\rho A_b(x)w^2(x)C_{L\beta}$$

Where: $w(x) = w - x_{cp}(x)q$; $A_b(x) = \frac{1}{2}B(x)dx$

Port area: $w_1(x) = w - \frac{B(x)}{4}p - x_{cp}(x)q$

Starboard area: $w_2(x) = w + \frac{B(x)}{4}p - x_{cp}(x)q$

$$Z(x) = -\frac{1}{2}\rho C_{L\beta}w_1(x)^2\frac{B(x)}{2}dx - \frac{1}{2}\rho C_{L\beta}w_2(x)^2\frac{B(x)}{2}dx$$

$Z(x)$ is always negative and only appear when $w_1(x) > 0$ or $w_2(x) > 0$:

$$Z_{Zb} = -\frac{1}{4}\rho C_{L\beta} \int_{-L}^{L}(w_1(x)^2 + w_2(x)^2)B(x)dx \qquad (47)$$

Rolling resistance moment (K): The moment K (x) is mainly due to the force $B(x)p/4$ forming on the fore bottom side. The vertical resistance due to pitching on the bottom side does not creat K (x).

$$K(x) = Z(x)\frac{B(x)}{4}$$

$$A_b(x) = \frac{B(x)}{2}dx$$

$$K_{Zb} = -\frac{1}{2}\rho C_{C\beta} \int_{-L}^{L} p|p|\left(\frac{B(x)}{4}\right)^2\frac{B(x)}{2}\frac{B(x)}{4}dx$$

$$K_{Zb} = -\frac{1}{256}\rho C_{C\beta} \int_{-L}^{L} p|p|B(x)^3 \qquad (48)$$

Pitching resistance moment M:
The moment M (x) due to Z (x) apprear on the bottom side at position where $w_1(x) > 0$ or $w_2(x) > 0$.

$$M(x) = Z(x)x$$

$$Z_{Zb} = \frac{1}{4}\rho C_{C\beta} \int_{-L}^{L}(w_1(x)^2 + w_2(x)^2)x_{cp}dx \qquad (49)$$

2.5.4. Cross-Flow Drag

Cross-flow drag principle assumes that the sway flow of water crossing the ship hull creates viscous drag called cross-flow drag.

Since this study for modelling ship fitted with non-conventional propellers, the transversal speed or the $|\beta_c - \Psi|$ is remarkable and can reach 90°. Where Ψ is ship heading. Therefore, the cross-flow drag need to be taken in account.

The Cross-flow drag is basically expressed basing on the formula (4) [20]:

$$Y_{Dc} = \frac{1}{2}\rho \int_0^L U_r(x)^2 T(x)C_{DC}(x)dx \qquad (50)$$

Where $C_{DC}(x)$ is cross-flow coefficient and T (x) is draft at local section i^{th}. $C_{DC}(x)$ can be predicted as:

$$C_{DC} = C_{DC0}\lfloor \sin\beta(x)\rfloor \sin\beta(x) \qquad (51)$$

$$Y_{Dc} = \frac{1}{2}\rho \int_0^L U_r(x)^2 C_{Dc0}\sin\beta(x)\lfloor \sin\beta(x)\rfloor T(x)dx$$

$$Y_{Dc} = \frac{1}{2}\rho \int_0^L C_{Dc0}T(x)(v_r + xr)\lfloor v_r + xr\rfloor dx \qquad (52)$$

C_{Dc0} can be estimated as [19] by assuming the ship as ellipsoid. Thus, KDc and NDc are calculated basing on formula (3):

$$K_{Dc} = -\int_0^L Y_{Dc}z_{cp}dx \qquad (53)$$

$$N_{Dc} = \int_0^L Y_{Dc}x_{cp}dx \qquad (54)$$

2.5.5. Total Damping Forces and Moments

The total damping force and moment matrix is eventually derived :

$$
\begin{aligned}
X_T &= X_s + X_b \\
Y_T &= Y_s + Y_b + Y_{DC} \\
Z_T &= Z_s + Y_b \\
K_T &= K_s + K_b + K_{DC} \\
M_T &= M_s + M_b \\
N_T &= N_s + N_b + N_{DC}
\end{aligned} \qquad (55)
$$

2.6. Computer Calculation and Simulation

For assessing the estimation results, some ship models are used. This paper presents the container ship Triple-E ship model with general particulars: L = 399m, B = 59m, T = 16m, Displacement = 257,343MT, V = 251,067 m^3.

The equations of damping coefficients are computerised with Matlab. For this purpose, the ship is divided longitudinally into 20 sections. Lewis transformation method is used for mapping the ship [21]. The value of added masses are calculated based on theoretical method as the previous study in [22].

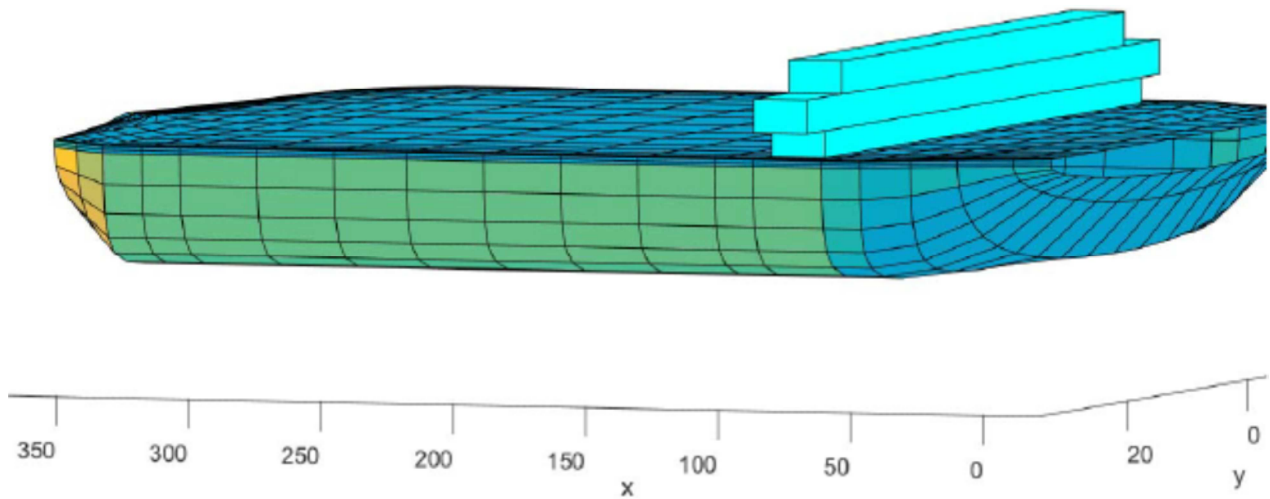**Figure 8.** *Overall view of the container ship Triple-E.*

Figure 9. *Body plan presented using Lewis forms in Matlab.*

Typical results of calculation of damping coefficients at various ship speeds (u, v) and yaw rates (rate of turn) in Matlab are listed in the Table 2 and 3.

Table 2. *Damping coefficients at various single motion parameter (straight velocity or angular velocity).*

u (m/s)	5	0	0	0	0	0
v (m/s)	0	5	0	0	0	0
w (m/s)	0	0	1	0	0	0
p (rad/s)	0	0	0	10	0	0
q (rad/s)	0	0	0	0	10	0
r (rad/s)	0	0	0	0	0	10
C_X	-0.0024	0	0	0	0	0
C_Y	0	-2.0327	-2.0327	-2.0327	-2.0327	-2.9366
C_Z	0	0	0	0	0	0
C_K	0	-0.0902	-0.0902	-0.0902	-0.0902	-0.1303
C_M	-0.0002	0	0	0	0	0
C_N	0	1.7647	1.7647	1.7647	1.7647	2.5704

Table 3. *Damping coefficients with combination of various motion parameters (straight velocity and angular velocity).*

u (m/s)	1	2	3	4	5	10
v (m/s)	1	2	3	4	5	10
w (m/s)	1	2	3	4	5	5
p (rad/s)	1	2	3	4	5	5
q (rad/s)	1	2	3	4	5	5
r (rad/s)	1	2	3	4	5	10
C_X	-0.0013	-0.0012	-0.0012	-0.0012	-0.0012	-0.0012
C_Y	-1.2264	-1.2264	-1.2264	-1.2264	-1.2264	-1.2289
C_Z	-2.3626	-2.3626	-2.3626	-2.3626	-2.3626	-0.5907
C_K	-0.0544	-0.0544	-0.0544	-0.0544	-0.0544	-0.0545
C_M	1.9707	1.9707	1.9707	1.9707	1.9707	0.4926
C_N	1.0697	1.0697	1.0697	1.0697	1.0697	1.0719

Typical results of calculation of damping coefficients at various ship speeds (u, v) and yaw rates (rate of turn) are also displayed in curves by Matlab. The results indicate that the damping coefficients are calculated reasonably.
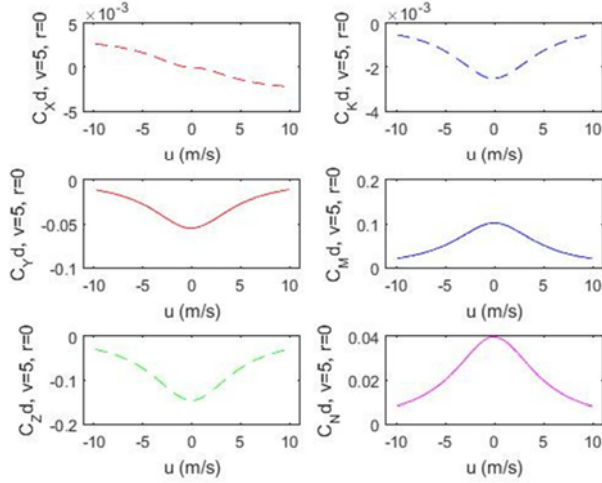
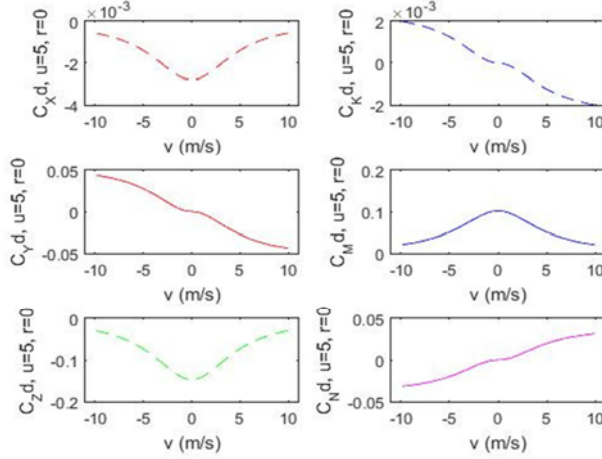*Figure 10. Curves of damping coefficients over u, (v=5knots, r=0 deg/m).*

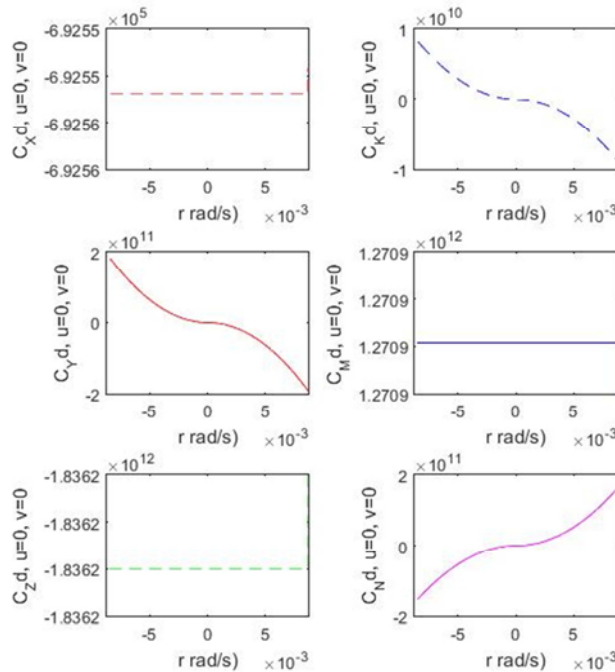*Figure 11. Curves of damping coefficients over v, (u=5knots, r=0 deg/m).*

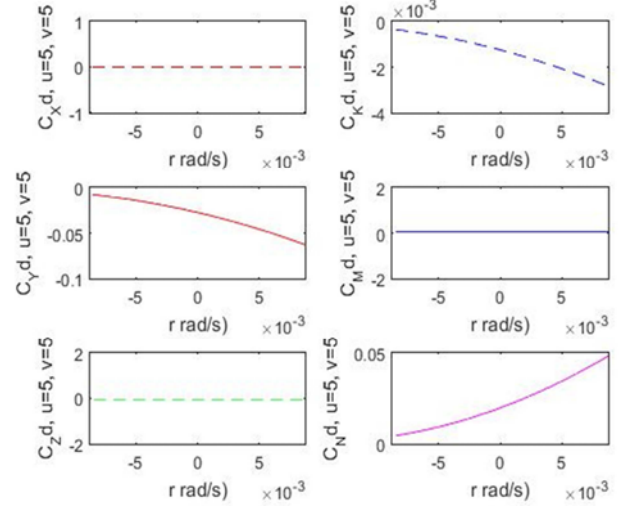*Figure 12. Curves of damping coefficients over r, (u=0 knot, v=0 knot).*

Figure 13. Curves of damping coefficients over r, (u=5 knots, v=5 knots).

The plotting curves of damping coefficients C_X, C_Y, C_Z, C_K, C_M, C_N of lift, drag, cross-flow drag and total damping over u, v or r separately. Value u, v or r increase from minus to plus on horizontal axis. The curves show that the coefficient values are logically. The reasonability of the damping coefficients is assessed basing on the results of simulating on computer by Matlab.

3. Conclusion

The suggested equations can be used for calculating damping coefficients of ship in 6 DOF. Mapping transformation of ship hull combined with the suggested equations can estimate hydrodynamic coefficients of ship's hull to simulate the motion of the ship with reasonable behaviour.

With mapping transformation and suggested equations to calculate resistance coefficients, a mathematical modelling of a ship can be made relatively fast. This method can also reduce a quantity of complex hydrodynamic data to be transferred into the computer. Therefore, it can reduce time for calculation and data transaction in real-time ship simulators.

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