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A Two-Connected Graph with Gallai's Property

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Abstract: The most famous examples of Hypo-Hamiltonian graph is the Petersen graph. Before the discovery of Hypo-traceable graphs, Tibor Gallai, in 1966, raised the question whether the graphs in which each vertex is missed by some longest path. This property will be called Gallai's property, various authors worked on that property. In 1969, Gallai's question was first replied through H. Walther[2], who introduced a planar graph on 25 vertices satisfying Gallai's criterion. Furthermore, H. Walther and H. Voss[3] and Tudor Zamfirescu introduced the graph with 12 vertices and it was guessed that order 12 is the smaller possibility of such a graphLater the question was modifies by Tudor Zamfirescu and asked that whether there exists graphs of Paths and Cycles, that is to say *i*-connected graphs (planar or non-planar respectively), such that each set of *j* points are disjoint from some longest paths or cycles., Several good examples answering Tudor Zamfirescu's questions were published. In this note a graphs is developed with the property that everyone vertex is missed by some longest cycle with connectivity 2, satisfying Gallai's property. The designed graphs can be useful in various fields of science and technology including computational geometry, networking, theoretical computer science and circuit designing.

Keywords: Hypo-Hamiltonian, Hypo-Traceable, Hamiltonian, Gallai's Property, Zamfirescu Criterion

1. Introduction

This A cycle that passes through each of the vertices only once and ends on the same vertex in graph G is called Hamiltonian cycle (Hamiltonian circuit). A path that also visit through every vertex once with no recurrences, and it does not have to start and end at the similar vertex in a graph G is said to be Hamiltonian path. A graph is said to be traceable if it has a Hamiltonian path and a graph is said to be Hamiltonian if it has a Hamiltonian cycle. A graph G is Hypo-Hamiltonian, if it is not Hamiltonian and deletion of one vertex at all from G results in Hamiltonian, a well-known counterexample of existence Hypo-Hamiltonian is Petersen graph.

The presence of Hypo-Hamiltonian graphs and earlier the modernization of the hypo traceable graphs, in 1966, Tibor Gallai (was a Hungarian mathematician. He worked in combinatory, especially in graph theory, and was a lifelong friend and collaborator of Paul Erdős. He was a Student of Dénes Kőnig and an advisor of László Lovász. He was a corresponding member of the Hungarian Academy of Sciences) [1] drew the attention towards the existence of the finite graph having property that everyone is missed by some longest path. Just later, in 1969, Gallai's question was first replied through H. Walther [2], who introduced a planar graph on 25 vertices satisfying Gallai's criterion. Furthermore, H. Walther and H. Voss [3] and Tudor Zamfirescu [4] introduced the graph with 12 vertices and it was guessed that order 12 is the smaller possibility of such a graph. In the case of planar graphs, such type of a graph with lowest number of vertices *i-e* with 17 vertices, was provided by W. Schmitz [5]. The first two-connected planar graphs where developed by Tudor Zamfirescu [6] with 82 nodes. The famous lowest illustration of such type of graphs nowadays has 26 nodes [7], conversely the lowest planar example up to now has order 32 [6].

In 1972, Tudor Zamfirescu [4] had developed idea related to the Gallai's property. Let $\mathrm{P}_\mathrm{i}^\mathrm{j} = \infty$ $(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} = \infty)$ if there does not exists any i −connected graph (planar graph) such that individually set of j points remains disjoint from some longest path condition $\mathrm{P}_\mathrm{i}^\mathrm{j} \neq \infty$ $(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} \neq \infty)$, let suppose that $\mathrm{P}_\mathrm{i}^\mathrm{j}$ $(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j})$ indicate the smallest number of vertices of an i −connected graph (planar graph) such that individually set of j vertices must be disjoint from some longest path. Analogously these cases are clearly C_i^j and $\bar{C}_i^j$ longest circuits as a replacement for longest path. To find the correct answers of the raised questions regarding such issues by Tudor Zamfirescu's work was carried out by W. Schmitz [5], H. Walther [8] and provided examples of $C_2^2 \leq 220$ and $\bar{C}_2^1 \leq 105$ [2], B. Grunbaum, [9], W. Hatzel [10], Tudor Zamfirescu [6], see also the studies [11, 12]. In 2019 furthermore two graphs with 18 & 22 vertices satisfying Gallai's property introduced by A. Naeem kalhoro & AD Jumani [13] also see the paper [14]. Also earlier some connected graphs are introduced on Gallai's property, Graphs of Paths and Cycles with 20 vertices graphs with similar properties by A. HJUNEJO, A. N KALHORO, I AHMED, I SOOMRO, R MUHAMMAD, I A JOKHIO, R Chohan, A D JUMANI. [15-17].

2. Results and Discussions

The purpose of this work is to develop a 2-connected, non-planar graph with 12 vertices and a graph on Mobius strip satisfying Gallai's property.

Theorem 1. There is existing a non-planar graph G with 12 vertices and is a 2- connected satisfying Gallai's property.

Proof: One has only one planar graph for individually vertex v, there exist a longest cycle missing v. We use Figures 1 and 2.

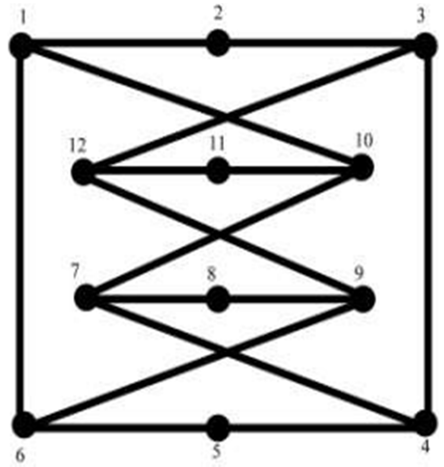

Figure 1. *Non-planar 12 vertices graph.*

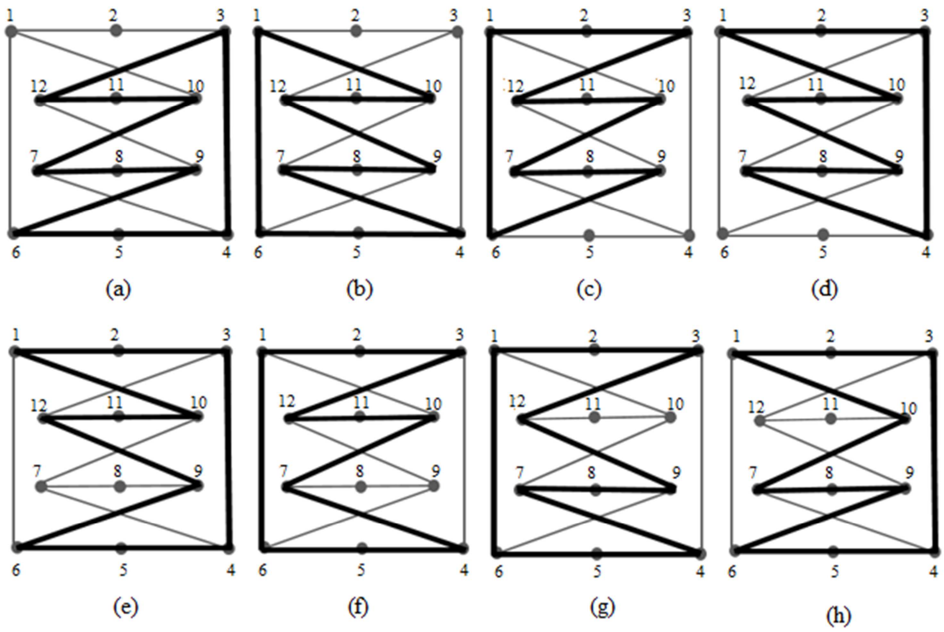

Figure 2. *Longest Cycles of non-planar 12-vertices graph.*

Consider the graph G of figure 1, With 12 vertices, let W be a longest cycle in G, the longest cycle of G have length $C(G) = 10$ avoiding v with $W \cap V = \emptyset$ empty intersection of all its longest Cycles. Figure 2, ('a' to 'h') shows longest Cycles and all vertices are missed by each of them, underlined vertices are as of 1 to 12, where each vertex is avoided by some longest Cycle. To confirm that all the vertices are avoiding individually by longest cycles, the results are shown in table 1.

Theorem 2. The graph Figure 1, created on Moebius band (or Mobius strip) satisfying the corresponding Gallai's property.

Figure 4, (from 'a' to 'h') shows longest Cycles and all vertices are missed by each of them, underlined vertices are as of 1 to 12, where each vertex is avoided by some longest Cycle. To confirm that all the vertices are avoiding individually by longest cycles.

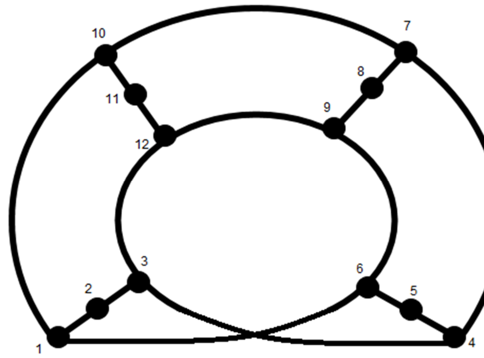

Figure 3. *Moebius band.*

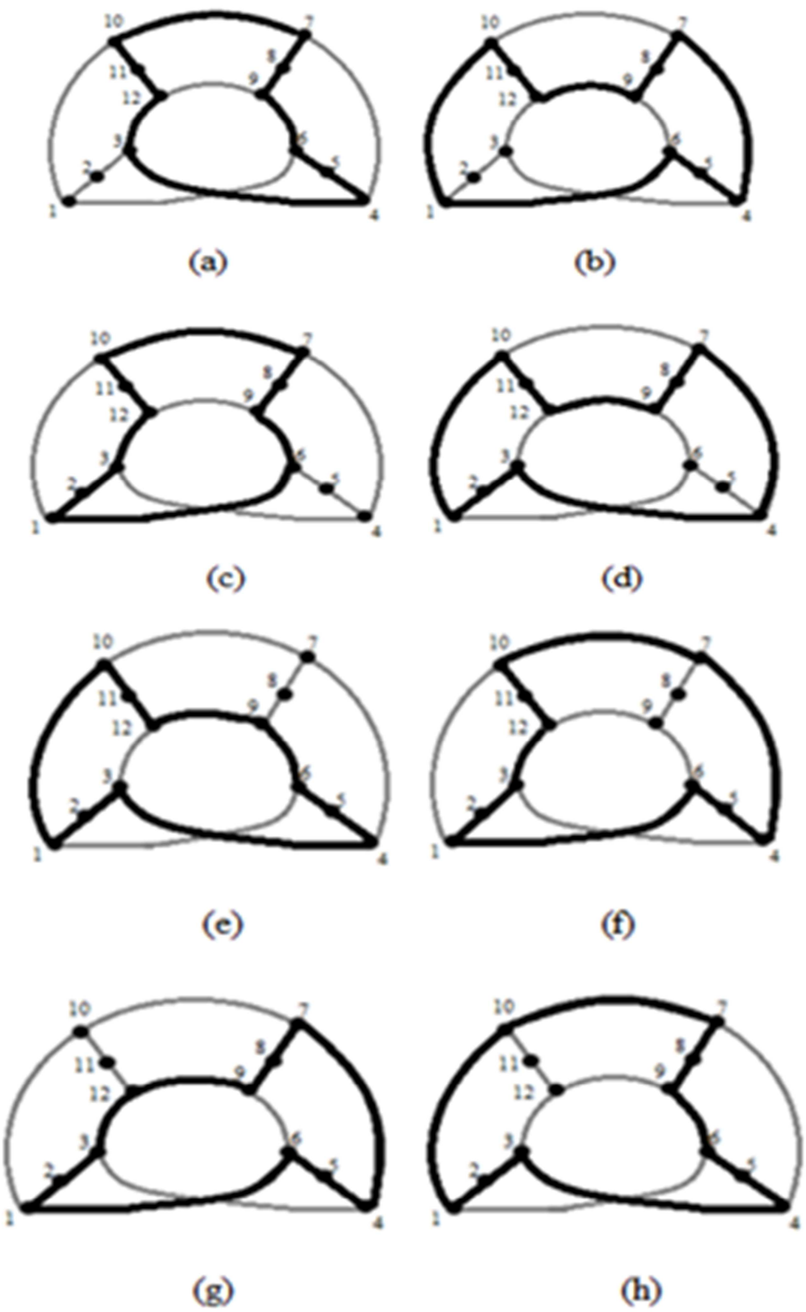

Figure 4. *Longest Cycles.*

To show that completely vertices avoided as each of the largest cycle, in table 1.

Table 1. *Results for every vertices missed by longest cycles.*

Cycles	Largest Cycle	Vertices missed
(a)	12, 11, 10, 7, 8, 9, 6, 5, 4, 3, 12	1, 2
(b)	10, 11, 12, 9, 8, 7, 4, 5, 6, 1, 10	2, 3
(c)	2, 3, 12, 11, 10, 7, 8, 9, 6, 1, 2	4, 5
(d)	7, 8, 9, 12, 11, 10, 1, 2, 3, 4, 7	5, 6
(e)	2, 1, 10, 11, 12, 9, 6, 5, 4, 3, 2	7, 8
(f)	2, 3, 12, 11, 10, 7, 4, 5, 6, 1, 2	8, 9
(g)	2, 3, 12, 9, 8, 7, 4, 5, 6, 1, 2	10, 11
(h)	2, 1, 10, 7, 8, 9, 6, 5, 4, 3, 2	11, 12

3. Conclusion

The above results shows that we have developed non-planar 2-connected graph in which each vertices is missed by some longest cycle. Results shown in table 1. Confirm our claim that there exists a 2-connected non-planar graph with 12 vertices satisfying Gallai's property.

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