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How to Understand the Twin Paradox

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Abstract: The twin paradox is often misunderstood, both in textbook and science popularizations. This article is intended to help clarify misconceptions involving the famous thought experiment. Of special importance is how we define the inertial frame in special theory of relativity. Another common factor that is overlooked involves the difference between time dilation, which is a consequence of the Lorentz transformations and the concept of look back time, which is independent of that. The concept of the invariant space-time interval and Minkowski diagrams are used in making these issues clearer.

Keywords: Twin Paradox, Time Dilatation, Inertial Frame, Space-Time, Minkowski Diagrams, Special Theory of Relativity

1. Introduction

The concept of time dilation has fascinated many. By analysis of the twin paradox, in this article we will gain a more complete understanding of the principle behind this story concept. To understand this, it is helpful to understand the premises on which special theory of relativity is based. There have been discussions in the literature on this subject, Cf. the Refs. [1-3]. However, despite being otherwise good primers on the subject they, like most books fail to do the topic justice. In fact, they often mistake acceleration as necessary to explain the paradox. Only a few sources correct this fallacy, Cf. the Refs. [4-5].

2. The Postulates of Special Theory of Relativity

Recall that Einstein based his original paper "On the Electrodynamics of Moving Bodies", Cf. pp. 37-65 in the Ref. [6], on two simple principles. Principle 1: Two or more frames of reference in constant motion in relation to one another have the right to consider themselves as the observer at rest. Principle 2: The speed of light is a constant in all frames of reference, whether moving towards or away from it. The original principle is nothing new. It goes back to Galileo at the beginning of modern physics. The second was: for the speed of light to remain constant it meant that distance and time are no longer universal quantities. They are observer dependent. This much was suspected by others including Lorentz, who originally developed most of mathematics that is used in special theory of relativity, which is why they are called Lorentz transformations, Cf. pp. 11-34 in the Ref. [6]. Lorentz, however, believed in the ether as a medium in which light moved through. The ether is an absolute frame of frame. Different measurements of time would be found moving in relation to it, but the ether itself was the absolute frame of reference. Being at rest with the ether would give one "real" time. The speed of light was shown to be constant however. Einstein said we must accept the fact that the light moves at the same speed for all observers in the universe, and it is space and time that change. For all observers in the universe, the time of one inertial frame is much reality as that of another inertial frame. The term to remember in this is Inertial.

3. Origins of the Twin Paradox

The concept that time moves at a variable rate has fascinated people since Einstein first published the special theory of relativity in 1905. The Twin Paradox has been around in different forms for the last century, but its modern version comes from the papers of a professor named Herbert Dingle. Dr. Dingle was a professor who started writing to the journal Nature, claiming that he had found a flaw in the Einstein's special theory of relativity that would proved it could not be true. His argument went something like this: say there are two identical twins and one goes off in a rocket a speed near that of light. Since time passes slower for objects moving at near light speeds the travelling twin will take a round trip and return home younger than his Identical. So far so good (counter-intuitive as this claim may be it is a logical deduction from the axioms of special theory of relativity). In the contradiction comes in the principle of relativity itself. Why can not the moving twin take himself as the inertial frame or rest observer? If so then he will see the earth twin age slower and when he comes back he will claim his twin brother is younger! If only one of these situations is correct, but for both twins to say the other was younger leads to a contradiction. Dingle believed that this reduction ad absurdum proof of this showed that Einstein's special theory of relativity could not be true. Almost all introductory physics textbooks that have a chapter on special theory of relativity cover the twin paradox. The explanation usually given is that since the twin travelling on the rocket experienced accelerations it is not inertial, therefore the earth twin is correct, and when the other twin returns, he will be the younger. However the explanation is misleading. First it implies that General Relativity is required to solve it. That is not true. Second even without the acceleration, the journeys of the two twins are not mirror images of each other (see note 1. To explain it is interesting trip between the twins in a universal that follows only the laws of Newtonian Physics.)

4. Space Travel in a Newtonian Universe

In a universe that obeys the laws of classical physics there would be no universal speed limit on any object and time dilation and length contraction do not occur. What would a space trip to a nearby star and back be like in this universe? In this universe however light would still travel at 300,000 kilometers a second. Furthermore radio signals and all other electromagnetic waves travel at this speed in this thought experiment. Take two twins the same age. One Bob, stays at home. The other, Rob, takes a trip that would be 6 earth years away if we chose to measure distance in terms of time it would take a signal from there to reach the earth. In a classical universe distance and time are seen as invariants regardless of velocity. If Rob chooses to move at. 6c. in a universe with classical laws it would take him 10 years (time = distance/speed). He and his brother both agree on this in both frames of reference. Rob has agreed to send his brother a message when he makes it to the next star right before he turns back towards home. Since Rob trip will take ten years his clock will be ten years older once he reaches his turnaround point. Now Rob sends the radio message back to Bob. When Bob receives the message it will be sixteen years since his Brother Rob left? Why 16? Simply it takes 10 years to get the star system at. 60c plus 6 years for the signal to reach earth. Since the signal moves at the speed of light we that is a 6 years time trip in this universe. If the earth twin bob also agrees to keep sending out a radio signal from home that keeps give the amount of time that has past in his world what time will Rob read on the Bob's clock? Since it takes 6 years for a signal to travel so the clock will be 6 years behind. Since 10-6=4, Rob will read four years on the home clock. This is interesting. We see that even in a Newtonian universe trip is not symmetrical. Rob sees the home clock at 4 yeas during his mid trip where 10 years have passed. Bob, the home twin reads 10 years on his travelling brothers clock but 16 years will have passed on his clock when he receives it. If the reader is wondering why 16 years instead of 4 there is a simple explanation. We must wait for the time it takes for the brother to reach the next star system plus 6 years for the signal to travel. Remembering this is important. The stationary earth twin stays at a constant distance from the other star. They will be always be a 6 year delay between earth and there if signals are sent between them. The time the traveller Rob, sees at the halfway point is the duration of his trip minus the time it takes the signal to travel. All information is behind by that amount. In both cases the time of the observer's clock is greater than that of the other clock far away.

Rob turns back to go home and the trip back takes another 10 years. A total of 20 years have passed. Both Rob and Bob agree on this and they are both 20 years older than they were when the trip started. No paradox in a classical universe. No age difference either. The important point of this experiment is to compare the amount each sees the others clock when they away from each other. This will be important in understanding the twin paradox later and will help clarify some common errors.

5. Dingle's Fallacy

The above argument shows the main flaw in Dr. Herbert Dingle's argument. Some readers may already see where I am going with this. There is a only one inertial frame of reference, the stay at home twin (bob) viewpoint. Dingle tried to argue through the postulate of the theory of relativity that Rob, the rocket travelling twin, could use his spaceship as a inertial frame of reference. There is just one problem with that. Constant velocity or acceleration the rocket cannot be considered a inertial frame. The reader might ask why this is. The twin on earth, and all of the stars in our galaxy are at rest with reference to our Galaxy. The rocket travelling twin is not. That is why it is not a inertial frame. What would have with we take the rocket frame as at rest for the first part of the journey? As one the star moved to our rocket travelling twin our sun moved away by a equal amount. Yet they stay a fixed distance away from each other for the entire trip. That is why the rockets frame of reference cannot be considered a inertial. The fixed distance between the of objects in our galaxy by definition shows it is the true inertial frame of reference. The laws special theory of relativity and the Lorentz equations can only be used by the stay at home twin. There is another fallacy. This one is not Dingle's but one that many textbooks have perpetuated in order to explain the paradox itself. Many physics books say because the second ship experiences acceleration General Relativity is needed to solve the equations of for this problem. The entire problem can be solved using only special theory of relativity. As long as we have one inertial frame of reference we can use the Lorentz transformations to solve the paradox.

Before ending this part of the discussion there is one more point that should be brought up. This is a fact that seems not to have been mentioned in any of the articles or books the author of this paper has read. It is assumed at the onset that the two frames are mirror images of one another. The point from then on is to show why the the travelling cannot be a inertial frame of reference. However a closer look will show that even without any accelerations the two situations are not symmetrical. One takes the travelling twin and says that because he is seen moving at constant velocity in a linear direction by the stay at home twin, then the travelling twin will see the same thing from his point of view, that is constant linear movement of the earth bound twin moving away from him. However one subtle point has been overlooked in the literature. The stay at home twin is on earth, which is moving in a orbit around the sun. The movement of the Earth's orbit means that the stay at home twin will not see his brother's motion away from him as constant or linear. Neither twins are pure inertial frames. Notice how this fact reinforces the argument about the movement with reference to the galaxy. Inertial frames are usually defined in a relation to another system which is does have independent motion of it's own. Frames are usually only inertial relative to a given situation. Pure inertial frames only exist outside of the 'local' influence of gravity. This is seen only special cases such as the Hubble redshift of galaxies.

The understand how the concepts of this we have abandon our common sense assumptions about how the world works. Light speed of is a invariant and a constant. Invariant because all frames of reference measure it with the same speed. Constant because its speed in a vacuum is always 300,000 kilometers per second (sometimes slower certain mediums but never faster). This is the second postulate of special theory of relativity. It also states this is the maximum velocity any object can obtain in the universe. Strange things happen as one travels near the speed of light. Because the speed stays constant distance and time are no longer invariant quantities. In everyday life a train moves so many kilometers in so many seconds. But in reality distances between object change (not the object itself) and the time things occur is shorter. To understand the connection think of a world where the laws of nature would not allow a object to go faster than 100 kilometers per hour. If the train went 5000 kilometers in 50 minutes then it would be 100 kilometers per hour. But then another train accelerates faster it clocks in 500 kilometers in 5 minutes time. Note that though the values are different the ratio is the same. Space and time work like in near light speeds to prevent any object moving faster than light. The distance between objects appears to shrink. The shape of objects becomes distorted because of this. Likewise the rate of time slows down. It is this effect that leads to the change in ages of the two twins. The rate at which time and space diminish is the gamma factor. It is equal to the amount of time or distance measured at rest to multiplied by the square root of $1-(v/c)^2$. The v/c part is the velocity of the object divided by the velocity of light. For simplicities sake, we will let the unit system c=1 and velocity in percent of the speed of light in these equations. We can then ignore c so the equation is simply

$$y=(1-v^2)^{1/2} \tag{1}$$

6. Space Travel in Einstein's Universe

Look again at the the two twins Bob and Rob. Bob stays home just like before and Rob goes on a trip to the nearest star at sixty percent the speed of light or. 6. Bob is a rest we both agree on the velocity. From that we can calculate the differences in time and space between the two different observers. The rest frames of measurements of distance and time will be called t and s respectively. For Bob the distance is 6 light years. Remember this is Einstein's physics so distance and speed are measured in terms of light not the opposite. For home twin bob time is determined by the formula

$$T=d/v \tag{2}$$

as in classical physics. With 6 lights distance he would measure the time of a object moving at. 6 light years ten years to make the trip.

$$6/0.6=10 \tag{3}$$

Now though we have the travelling twin Rob. His measurements of distance and time will be given the variables t' and s' respectively. For his case we have to bring in the gamma factor y. Therefore

$$d'=d(1-v^2). \tag{4}$$

Likewise time is measured as

$$t'=d'/v \tag{5}$$

Both observers agree on the value of v. Solving for equation 4 we the distance $(1\text{-}0.6^2)^{1/2} = (1\text{-}0.36)^{1/2} = (0.64)^{1/2} = 0.8$. Since we know that we can solve for d'=60.8=4.8, we can calculate the time t' with this information t'=4.8/0.6=8. Here is the major difference between Einstein's and Newton's view. There are two different times measured. Time is frame dependent. Often t' is solved by multiplying t, the rest time by the gamma factor. In that case that it would be

$$t'=t(1-v^2) \tag{6}$$

which is t'=10.8=8. Either way we get 8 years for the travelling twin, so we know the formulas match. It is interesting to look at the point of view of this for each twin. For Bob at home watching it looks as if the rate of his brother's clock as slowed down. For travelling Rob's point of view the distance between space itself has contracted. But either way it comes to 8 years. Now let’s go the half way point. Twin Rob stops his spaceship and before he turns around he sends a message back to Bob telling him the showing the amount of time that has passed on his journey. When Bob receives he will say that time for Rob has slowed down but he will see no change in the distance between them. Once again it must be noted both twins agree on the amount of time that has passed for the travelling twin. The next question is when will Bob receive the message? Since from his point of view was a ten year journey we must add that to the time it takes a signal of light to reach back to earth. That is 6 years. So the total time is 10 years for the journey plus 6 years to get the signal which equals 16 years. So far we have the travelling twin has had 8 years elapse between the time he started his journey and the time he made it to the next solar system. He sends his signal to l back to earth. At a six light year distance away the signal would be reach earth 16 years after the twin brother had left home. Note 16 years. is twice that of 8. The t' factor equation of the Lorentz equation $t'=t(1-v^2)^{1/2}$ tells us the travelling twin will measure 8 years have passed. The inverse of the equation of t for the home time is

$$t=t'(1-v^2)^{-1/2} \tag{7}$$

which in this case equals 10 But 16 years have passed. Where did the extra 6 years come from? The six comes from the distance light must travel to reach Earth. The original form assumes that you are dealing with a frame the moving and the rest object are interchangeable. The Twin on earth is at rest with our galaxy. The travelling twin is not. Corrections have to be made. The time at which at the signal is received is given by f where the full equation would be

$$t=t'(1-v^2)^{-1/2}+ L \tag{8}$$

where is the distance in light years from earth to the destination travelled for a inertial observer $f = t'(1-v^2)^{-1/2} + 6 = 16$. This gives us the time at which the traveller will get the signal. Now it is time for Rob to turn back and go home. For Rob this is another 8 years in time. Simple mathematics tells us 8+8=16 and therefore the entire journey is 16 years time for Rob, the traveller. The amount of time Rob experienced on his journey is dependent on how much or little distance the ship travels do the length contraction factor. It is in direct proportion actually. "Length contraction is time dilation for another frame of reference" says Jon Ogborn, the Ref. [1]. Finally Rob is home. The two brothers are both on earth and in the same frame of reference now. Here if we algebraically invert the formula for the traveller time t' we could get the answer to the amount of time that has passed for earth twin Bob.

Since $t'=t(1-v^2)^{1/2}$, we know the denominator is 0.8 and t' is equal to 16 at the end of the trip we get t=16/0.8=20 years There is no contradiction the traveller Rob ages more than his stay at home brother Bob. Furthermore they both agree it was 8 years away and 8 years back. One thing of interest here is the time of we can use the simple form of the Lorentz transformations to find the time for the home clock. We cannot use this formula when the travelling twin is away it his turning around point. There we must the distance inertially in light years must be added to find the time in which the home traveller would receive the signal. The Lorentz equations will only give you the right time for the home twin at the end of the journey when they are both at rest with reference to each other. This is important and we will see why.

7. Space-Time Intervals and Minkowski Diagrams.

When different frames of reference are used invariants are often searched for. The speed of light is a invariant quantity for example. It will be measured the same for all observers no matter what the frame of reference. Though Einstein's special theory of relativity was discovered in 1905, it did not reach its full and most complete form until 1908, when Minkowski published his famous paper “Space and Time”, Cf. pp. 75-96 in the Ref. [6]. Minkowski was mathematician who had taught Einstein when he was a undergraduate in college. Minkowski noticed something about special theory of relativity and the Lorentz transformations that nobody had noticed before that. There is a second constant quantity that is invariant aside from the speed of light. The space-time interval. The concept of the space-time interval goes back to Euclidean Geometry. The ancient geometer's noticed something interesting. Take a two dimensional plane. It has a x axis and a y axis. There are two separate points that form a straight line. One is 4 meters to the north, the other 3 meters to the left. Then take another two dimensional axis. Its origin at 0 just like the first but it is rotated 20 degrees to the right. These same two points now both are a different length from the origin when measured on this coordinate system. However take the distance between them as x^2+y^2 which is 9+16=25 and the square root which is 5. Using the Pythagorean theorem we have found the length between the two points which is 5. If we squared to the distance of the same points on the other axis and added them it would again come out to a distance of 5. The distance is a invariant in the geometry of Euclid. Minkowski had the insight to see that if space and time are plotted as coordinates and like points in a x and y Cartesian graph they also could form a invariant interval. Each inertial frame of reference would use a different time and space coordinate grid. With the vertical axis representing time and the horizontal axis representing space. Each point on the graph would be considered a event. The event is something that happens at a place and at a time. A star going supernova is a event. We can have a certain event on one coordinate system have a certain location in time and space but have different values for the locations in time and space in a different coordinate system for a separate inertial frame. But like the distance between two points in plane geometry a space time diagram also has a invariant quantity, it is called the space-time interval. In the geometry of Euclid we add the square of the distances between two points such as 3^2+4^2. With space and time it is somewhat different. Instead what we do is subtract the quantity from the time quantity so that

$$t^2-s^2=\text{Interval}^2 \tag{9}$$

Like the distance between two points in Euclidean geometry the space time interval is a constant for all frames of reference. The interval itself has a greater reality than that of either the space or time coordinate itself. This is often called space-time or space-time continuum in many science fiction stories. Lets apply the equation to the problem above. We said that the time for the ship at. 6c would be 10 years from his home twin's point of view. The distance is a separation of 6 light years. The square of the interval is the time $10^2 = 100$ minus the distance $6^2 = 36$, that is the squared interval is 64, and he space-time interval is 8. Does this number look familiar? The space time interval is the same as the time t' for the travelling twins clock. (Note that we cannot look for a space-time interval for time and distance of the ship because it has been shown that is not a inertial frame. It would not come out to the same number) Lorentz called t' the local time of the object. Today the term used is Proper Time. This not coincidence it follows logically from the equation. Remember that $t'=t(1-v^2)^{1/2}$, and the basic formula v=d/t. In our the above equation, the distance is represented by s. Solving for s (or d) we get s=dt. Using the interval, we get $t^2-s^2=t^2-(vt)^2$, solving we get $t^2-v^2t^2$ we can factor out t^2 to get

$$t^2(1-v^2)=\text{interval}^2 \tag{10}$$

That means the interval itself equals $t(1-v^2)^{1/2}$. This is the same formula for t' in the Lorentz equations. The concept of t' being the interval is consistent with what was found above. The interval is a invariant. It is constant in all frames of reference. In the above example both observers agree that the time for the travelling twin's clock aged 8 years both away and back to earth. it is important to note this is the time measured on the clock not the time when the home twin gets the signal. Both the traveller and the stay at home twin agree he aged 8 years going from earth to the next solar system. They both agree that he aged another 8 years and a total of 16 years passed for our travelling twin. Therefore t' in our example is the invariant space time interval. It is the proper time or the travellers time of the rocket ship. The fact that these the space time interval and the ship time t' are the same makes sense at a conceptual level. Look at the example above. Both twins agree on the amount of time that passed for the home twin for both parts of the journey (this is not the time passed when the traveller's clock is read from earth but how much is seen to have passed). The amount of time that passes on the travelling twins clock is a invariant. All frames of reference agree on it. It is also interesting to look at the home twins path along a Minkowski diagram. He was a rest throughout the journey. No distance was travelled. Using the interval formula we can see the that t^2-0 is the proper time for any observer totally at rest. But notice that since spatial distance is zero this means this is the largest the value of the proper time will be any frame of reference. If we graphed it in space time it would be a longer that of the travelling twin. Ironically the longest path on space-time is a path at rest! This is called the principle of maximal ageing and it is yet another way one can prove the stay at home twin ages will age more than his travelling brother.

8. Conclusion

One final interesting fact: Though space and time measurements of events may differ in all frames of reference the events happen in the same order. In other words, cause and effect are also invariant in space-time. Event A always precedes event B. If tachyons exist, then this relationship would not hold true. But that is a article for another time.

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