

Design and estimate of the optimal parameters of adaptive control chart model using Markov chains technique

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Abstract: Present paper aims to plan and estimate the optimal parameters of an adaptive control chart model for monitoring the mean of a process using sample size and variable interval. Here, the $X_BARRA-VSSI$ chart has been chosen because of its two special features- firstly being an adaptive scheme with great potential for practical application, and secondly the chart only requires knowledge of the sample size and the time between sample selections after established the optimal parameters for the chart. To estimate the optimal parameters of chosen $\bar{X}-VSSI$ control chart model, the Markov chains technique has been applied. Two functions written in R language are presented in order to assist the user in planning a statistical project based on the $X_BARRA-VSSI$ adaptive scheme. Evaluating the effectiveness of the control chart of $\bar{X}-VSSI$ by means of Markov chains has been examined and the optimal parameters of the adaptive control chart model have been explored. In addition to this, a numerical example for application of the control chart model has also been illustrated and finally some conclusive observations with significant suggestions for its future scope are carried out.

Keywords: Sampling amplitude, hypothesis testing, statistical control of process (SCP), adaptive charts, Markov chains technique, standard normal cumulative function, statistical software, decision variable, transition matrix

1. Introduction

Control charts with adaptive schemes are tools used to monitor production processes and to signal the presence of special causes. However, the use of adaptive schemes is not common yet because these topics are rarely covered in textbooks and are not available in traditional software used for statistical analysis. More specifically, control charts are used to monitor the production process in order to signal deviations from the target value of a quality characteristic that one wants to monitor. Designing a control chart to use in practice, involves elaboration of a sampling plan by means of specification of sample size and time interval between removal of samples, and calculation of control limits. The

mechanism which involves determination of limits distance of chart control at centerline is closely related to statistical testing of hypothesis. Extending the control limits decreases the risk of monitored statistics to be located beyond the control limits, with the adjusted process of error type I. However, recently Leoni and Costa [10] proposed in their study that extending the boundaries increases the risk of monitored statistic being located within the control limits when the process is out of adjustment, known as error type II. In adaptive control charts, it is common to use Markov chains to evaluate the performance of chart according to the set of chosen parameters, for more details we refer Costa [6], Faraz and Saniga [9] and Zimmer *et al.* [33]. We remark here that the subjacent idea of dividing the variation interval of

monitored statistic on a finite set of status is used in order to assess the statistical properties. The transient statuses of the chain are located in the control region of chart and the absorbing status in the region established as out of control. The adaptive charts are not available in traditional statistical software packages, despite showing better performance than the charts with fixed parameters. Determination of adaptive parameters is not a trivial task, therefore, this paper proposes the use of a free software to plan and estimate the optimal parameters of an adaptive chart for \bar{X} with variable sample size and interval ($\bar{X}-VSSI$). The average number of samples until the moment in which the chart indicates the out of control condition (ARL) and the average time between the instant at which the process is changed and the time when the chart indicates the out of control condition (ATS) are performance measures used as reference to parameters choice.

Estimation of statistical parameters has acquired its prominent place in research community, specially, for statisticians, decision makers, scientists and management executives because of its wide applications in statistical projects, production industries and quality control problems. In this connection, some previous research workers are worth mentioning for their immense contribution. For example, we refer Costa [5, 6, 7], Park and Reynolds [26, 27], Maurya *et al.* [13, 14, 15, 21, 22]. At the same time, it is also relevant to focus here that the optimization techniques with parametric constraints in different frameworks of statistical, mathematical, management and engineering problems have been used by several researchers e.g. Arneja *et al.* [1], Maurya A.K. and Maurya V.N. [12], Maurya [17], Maurya *et al.* [21], Maurya *et al.* [24, 25]. It is also pertinent to remark here that detection of small or moderate deviations by traditional charts proposed by Shewhart [32] is slow, that is why several charts have been proposed. Some authors introduced the adaptive control charts which are called this way because they do not present all their fixed parameters. Construction of this kind of chart provides that at least one of its parameters may vary and it can be: the control limits, the sample size and the time interval in which a sample is collected. For instance, consider the chart of adaptive control in which vary the sample size and the time interval in which a sample is collected. In this scheme, according to information obtained by the most recent sample, one can modify the size and collect interval of next sample.

Here, we consider the $\bar{X}-VSSI$ control chart model because it is a scheme with great potential for practical application, for the chart requires only knowledge of sample size and the time among samples selection after established the optimal parameters. The statistical properties of control chart are optimized considering the approach presented by Zimmer *et al.* [33] who used the Markov chain to establish the parameters keeping statistical risks of errors type I and type II under control. The Markov chain technique pays key role in statistical analysis and it is very much useful in solving optimization problems including queueing network theory. In this connection, Maurya [16, 17, 18, 19, 20, 23] and references therein are worth mentioning among others. The rest of the paper is organized as follows: section 2 presents the brief description of $\bar{X}-VSSI$ control chart. In section 3, it

is described the procedure to evaluate the performance of a $\bar{X}-VSSI$ chart using Markov chains. Section 4 is planned for the numerical illustration for application of the control chart model and some significant conclusions have been drawn in section 5. Some valuable recommendations and significant suggestions for future scope of the present study are also incorporated in section 5.

2. Description of $\bar{X}-VSSI$ Control Chart Model

Related literature shows that several noteworthy researchers contributed to design and analyze control charts in different framework. The rigorous study of literature reveals that Reynolds *et al.* [31] was the first contributor to consider the adaptive design of control chart by varying the time interval in which a sample is collected. Then there appeared a large number of papers for the purpose of varying the other control chart parameters, being proven that this technique generally increases the chart power in detection of special causes just modifying the quality characteristic average (variable) that is desired to monitor. In this connection for more details we refer Bai and Lee [2], Magalhães *et al.* [11] and Park and Reynolds [26, 27]. The $\bar{X}-VSSI$ control chart is adaptive with respect to the sample size and time interval at which a sample is collected. This chart was used by Costa [6], Park and Reynolds [27] and Prabhu *et al.* [28, 29] to monitor a process statistics.

In a control chart with sample size and interval variables (see Figure 1), the sample size and the time interval in which a sample is collected can vary according to the information provided by the most recent sample collected. In this chart type, random samples of different sizes are collected at variable intervals of length according to the function:

$$(n(i), h(i)) = \begin{cases} (n_2, h_1) & \text{if } w < Z_{i-1} < k \\ (n_1, h_2) & \text{if } -w < Z_{i-1} < w \\ (n_2, h_1) & \text{if } -k < Z_{i-1} < -w \end{cases} \quad (2.1)$$

where $i = 1, 2, \dots$, is the sample number; $n(i)$ is the size of the i^{th} sample ($n(i) = n_1 < n(i) = n_2$); $h(i)$ is the time performed to remove the i^{th} sample ($h(i) = h_1 > h(i) = h_2$); k and w are limits that define control regions; Z_i is control statistics calculated by:

$$Z_i = (\bar{x}_i - \mu_0) \cdot (\sigma_0 / \sqrt{n(i)})^{-1} \quad (2.2)$$

Where \bar{x}_i is the sample mean of the i^{th} subgroup; μ_0 and σ_0 are the mean and standard deviation of the process when in control.

The choice between the pairs $(n(i), t(i))$ depends on the position of the last point (Z_{i-1}) marked on the chart. For a chart $\bar{X}-VSSI$, one can divide the control region in three mutually exclusive and exhaustive regions, as follows (see

Figure 1):

- Region within the alarm limits: $I_1 = [-w, w]$
- Region between the limits for alarm and control: $I_2 = [-k, -w) \cup (w, k]$
- Region outside the control limits: $I_3 = [-\infty, -k) \cup (k, \infty]$

If the statistic Z_i falls within the region $I_1 = [-w, w]$, the control (or inspection) is relaxed using the pair (n_1, h_1) , otherwise if the current point Z_i lies within the region $I_2 = [-k, -w) \cup (w, k]$, the control will be tighter by using the pair (n_2, h_2) .

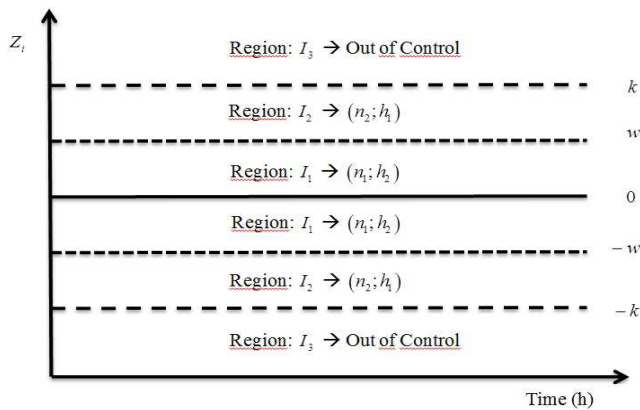


Figure 1. Regions of a chart control with size of sample and range

3. Statistical Performance of $\bar{X}-VSSI$ Control Chart Model

The statistical performance of a control chart can be evaluated by calculating the ARL and ATS statistics. Depending on the process operation conditions, one has the ARL when the process is in control (ARL_0), that is, the expected number of samples between two successive false alarms and the ARL for process out of control (ARL), which represents the expected number of samples between the occurrence of special cause which alters the monitored parameter and signal triggered by the chart. Similarly, one has the ATS when the process is in control (ATS_0), representing the average time between two successive false alarms and ATS for process out of control (ATS), representing the expected time between the occurrence of special cause and the signal triggered by the chart.

3.1. Markov Chains Technique for ARL and ATS Computation

It is possible to calculate the ARL and ATS statistics using Markov chains. One observes the expected number of transitions before the monitored statistic lies in the absorbing state of the chain. Here, we note that the Markov chain technique proposed in Zimmer *et al.* [33] has been used in our present study to assess the ARL in control and out of control, ARL_0 and ARL , respectively. Each transition probability is calculated as the probability of the statistic falls within one of the regions of the control range (I_1, I_2, I_3). In

this chain, there are two transient states and one absorbing state that correspond to the process out of control.

The state transition matrix of chain that represents the operation of process in control, P_0 , can be divided into four sub-matrices:

$$P_0 = \begin{bmatrix} Q_0 & R_0 \\ 0 & I \end{bmatrix} \quad (3.1)$$

where sub-matrix Q_0 is the transition matrix between transient states; R_0 is the transition matrix from transient states to the absorbing state; 0 is the matrix that states the impossibility of going from an absorbing state to a transient state, and I is the identity matrix.

In a Markov chain, the element (i, j) of the matrix $[I - Q_0]^{-1}$ represents the average number of visits to the j transient state before reaching the absorbing state, given that the process started at the i^{th} state. Each control transition probability is calculated as the probability of a point of monitored statistic falls within one of the regions of the control range. Therefore, the average number of samples between two successive false alarms is calculated by following matrix equation (3.2):

$$ARL_0 = \{b\}^T [I - Q_0]^{-1} \{1\} \quad (3.2)$$

where $\{b\}^T$ is a vector with initial probabilities; I is the identity matrix; $\{1\}$ is a unit vector and Q_0 is a transition matrix obtained by following matrix equation (3.3):

$$Q_0 = \begin{bmatrix} \Phi(w) - \Phi(-w) & 2[\Phi(k) - \Phi(w)] \\ \Phi(w) - \Phi(-w) & 2[\Phi(k) - \Phi(w)] \end{bmatrix} \quad (3.3)$$

where $\Phi(\cdot)$ denotes the standard normal cumulative function; K and w are the limits that define the region of the chart control.

The average time that the chart can produce a false alarm is given by following matrix equation (3.4):

$$ATS_0 = \{b\}^T [I - Q_0]^{-1} \{h\} \quad (3.4)$$

where $\{h\}$ is a vector with the sampling intervals.

Moreover, the transition matrix of the process running out of control is given by the following matrix equation (3.5):

$$P_\delta = \begin{bmatrix} Q_\delta & R_\delta \\ 0 & I \end{bmatrix} \quad (3.5)$$

In order to calculate the performance measures ARL_δ and ATS_δ , the following transition matrix equations (3.6)-(3.7) are used:

$$ARL_\delta = \{b\}^T [I - Q_\delta]^{-1} \{1\} \quad (3.6)$$

$$ATS_\delta = \{b\}^T [I - Q_\delta]^{-1} \{h\} \quad (3.7)$$

being the transition matrix given by:

$$Q_{\delta} = \begin{bmatrix} Q_{\delta 11} & Q_{\delta 12} \\ Q_{\delta 21} & Q_{\delta 22} \end{bmatrix} \quad (3.8)$$

where:

$$\begin{aligned} Q_{\delta 11} &= \Phi(w - \delta\sqrt{n_1}) - \Phi(-w - \delta\sqrt{n_1}); \\ Q_{\delta 21} &= \Phi(w - \delta\sqrt{n_2}) - \Phi(-w - \delta\sqrt{n_2}); \\ Q_{\delta 12} &= [\Phi(k - \delta\sqrt{n_1}) - \Phi(w - \delta\sqrt{n_1})] + [\Phi(-k - \delta\sqrt{n_1}) - \Phi(-w - \delta\sqrt{n_1})]; \\ Q_{\delta 22} &= [\Phi(k - \delta\sqrt{n_2}) - \Phi(w - \delta\sqrt{n_2})] + [\Phi(-k - \delta\sqrt{n_2}) - \Phi(-w - \delta\sqrt{n_2})]. \end{aligned}$$

The vector with initial probabilities $\{b\}^T$ is defined according to the initial conditions of operation in the process:

$$\{b\}^T = \left\{ \frac{\Phi(w) - \Phi(-w)}{\Phi(k) - \Phi(-k)}, \frac{2[\Phi(k) - \Phi(w)]}{\Phi(k) - \Phi(-k)} \right\} \quad (3.9)$$

We also remark here that the steady state conditions are used in our present study for sake of simplicity in view of computation. It is therefore, we assumed that the process starts in control and at some future instant; the steady state model occurs in a special case that causes a shift at the target value of monitored statistic.

3.2. Optimal Statistical Project for the \bar{X} -VSSI Control Chart

Planning a control chart can be formalized as an optimization problem in which the decision variables are the parameters of the chart. Figure 2 illustrates the objective function and constraints that define the best set of parameters of the \bar{X} -VSSI chart.

Objective Function :

$$\min ATS(n_1, n_2, h_1, h_2, w, k | \delta)$$

Subject to :

$$ATS(n_1, n_2, h_1, h_2, w, k | \delta = 0) = ATS_0;$$

$$E(n) = n_0;$$

$$E(h) = h_0;$$

$$0 < w < k;$$

$$h_{min} \leq h_1 \leq h_2 \leq h_{max};$$

$$n_{min} \leq n_1 \leq n_2 \leq h_1 \cdot r_{insp}$$

Figure 2. Objective function and constraints for the parameters of the \bar{X} -VSSI control chart

In Figure 2, n_1 and n_2 are the sample sizes; h_1 and h_2 are the time intervals between samples collection; w and k are control limits of the chart; δ is the displacement degree of occurred in the average of the process; ATS_0 is the mean time between two successive false alarms; n_0 is the expected value of the collected sample size with the process in control; h_0 is the expected time to collect a sample with the process in

control and r_{insp} is the quantity of parts (a piece, a component, etc.) which can be inspected per time unit considered in h_0 .

In order to illustrate that the optimization problem is reduced to find the pair (n_1, n_2) that minimizes the objective function, consider without generality loss that $E(h) = h_0 = 1$ the time unit (for example: 1 hour, 0.5 hour and etc.) and $ARL_0 = 370.4$. Thus, $AT S_0 = ARL_0 = 370.4$ and $k = 3$.

The expected value of the sample size with the process in control, $E(n) = n_0$, is given by following equation (3.10):

$$E(n) = n_0 = \frac{\Phi(w) - \Phi(-w)}{\Phi(k) - \Phi(-k)} n_1 + \frac{2[\Phi(k) - \Phi(w)]}{\Phi(k) - \Phi(-k)} n_2 \quad (3.10)$$

A pair of samples (n_1, n_2) is selected; since $(n_1, n_2), n_0$ and k are known, w can be inferred directly from the expression (3.10).

The shortest range of the optimal sampling (h_1) is given by:

$$h_1 = \frac{n_2}{r_{insp}} \quad (3.11)$$

where r_{insp} is the amount of parts (a piece, a component, etc.) which can be inspected per unit of considered time $E(h) = h_0$. For example, if $r_{insp} = 60$ given that $h_0 = 1$ hour, it is assumed that it is possible to inspect 60 parts every hour. For more details, we refer Celano [3, 4].

Once defined h_0, h_1, w and k, h_2 is obtained by means of the expected time to collect a sample:

$$E(h) = h_0 = \frac{\Phi(w) - \Phi(-w)}{\Phi(k) - \Phi(-k)} h_1 + \frac{2[\Phi(k) - \Phi(w)]}{\Phi(k) - \Phi(-k)} h_2 \quad (3.12)$$

The optimization problem is finally reduced to finding the pair (n_1, n_2) which minimizes the objective function. The next section presents an application example of how to plan an optimal statistical project that shows which values for the pair (n_1, n_2) should be used. For this, it has been used the R software [30] to obtain the optimal parameters of a \bar{X} -VSSI chart.

4. Numerical Illustration for Application of the Model

In this section, it is proposed two functions (see Appendix A) developed for use in R software environment that evaluate the performance of the \bar{X} -VSSI control chart and solve the optimization problem shown in Figure 2.

The first function, called *VSSI*, evaluates the performance of the control chart calculating the *ATS* when supplied by the user: $n_1, n_2, n_0, \delta, h_0$ and r_{insp} .

The second function, *VSSI objective function for optimum*, solves the optimization problem shown in Figure 2. Here, it is necessary to provide: $n_0, \delta, h_0, r_{insp}$ and a value

for n max which is referred to the largest size of admissible sample to collect.

```
> VSSI(n1=2,n2=8,n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60)
  ATSD    n1    n2    h1    h2    delta
93.5959  2.0000  8.0000  8.0000 112.0000  1.0000
  ATSO    ARL0    k    w    n0    h0    r_insp
22223.8980 370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
```

Figure 3. Parameters obtained with the function VSSI ($n1=2$, $n2=8$, $n0_FSR=5$, $\delta=1$, $ARL0=370.3983$, $h0=60$, $r_insp=60$).

In order to illustrate the use of functions, consider the example presented in Costa *et al.* [8]. A packaging line has an average value of milk 1000 ml and standard deviation estimated to be 4.32 ml. Monitoring is performed in the process average by inspecting samples of size $n_0 = 5$ at each time unit. Suppose that this unit is equal to $h_0=1$ hour. In this example, the parameters planned for the control chart are fixed, meaning thereby the sample size, the sampling interval and limits do not change after estimated. To use the \bar{X} -VSSI control chart in the example shown, it is necessary to calculate the control limits (w and k) and the sampling scheme (n_1, h_1) and (n_2, h_2) . Choosing $(n_1=2; n_2=8)$, keeping $n_0=5$; $\delta=1, 0$; $ARL0=370.3983$; $h_0=1$ hour (60 min.) and $r_insp=60$, the VSSI function provides the parameters shown in Figure 3.

In the example demonstrated in our present study, $\delta=1.0$ means that the process average went from $\mu_0=1000$

(in control) to $\mu_1 = \mu_0 + \delta \cdot \sigma_0 = 1000 + 1 \cdot 4.32 = 1004.32$ (out of control).

Consider the case in which $\delta = 2.0$. Figure 4 illustrates the results obtained with the VSSI function. It is observed that the ATS is lower ($ATS_{-2} < ATS_{-1}$), because, when major shifts in the process mean occur, the performance of the chart is better.

```
> VSSI(n1=2,n2=8,n0_FSR=5,delta=2,ARL0=370.3983,h0=60,r_insp=60)
  ATSD    n1    n2    h1    h2    delta
63.1409  2.0000  8.0000  8.0000 112.0000  2.0000
  ATSO    ARL0    k    w    n0    h0    r_insp
22223.8980 370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
```

Figure 4. Parameters obtained with the function VSSI ($n1=2, n2=8, n0_FSR=5, \delta=2, ARL0=370.3983, h0=60, r_insp=60$).

However, an optimal scheme to monitor this process is what performs best, i.e. the lowest ATS. By means of the VSSI Optimum function, one can obtain the parameters that minimize the ATS. Figure 5 shows the best schemes for the cases shown in Figures 3 and 4.

In this case, the user who wants to control the average value of a process considering the possibility of a displacement presented here, just build the \bar{X} -VSSI control chart with the parameters shown in Figure 5. Additional \bar{X} -VSSI charts can be constructed easily by modifying the input values of VSSI and VSSI Optimum functions.

```
> VSSI.otimo(n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
  ATSD    n1    n2    h1    h2    delta    ATSO    ARL0    k    w    n0    h0    r_insp
91.5370  3.0000  7.0000  7.0000 113.0000  1.0000 22223.8980 370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
> VSSI.otimo(n0_FSR=5,delta=2,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
  ATSD    n1    n2    h1    h2    delta    ATSO    ARL0    k    w    n0    h0    r_insp
60.6035  4.0000  6.0000  6.0000 114.0000  2.0000 22223.8980 370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
```

Figure 5. Parameters obtained with the function VSSI Optimum ($n0_FSR$, δ , $ARL0$, $h0$, r_insp , $nmax$).

5. Conclusive Observations and Policy Recommendations

Using the Markov chains technique the effectiveness of the control chart of \bar{X} -VSSI has been examined in this paper. In order to explore the optimal parameters that minimize the ATS, two functions written in the language for R environmental were created with the purpose of solving the optimization problem that involves minimizing the ATS and presentation of the best parameters to be used in creation and use of the \bar{X} -VSSI control chart. Based on results explored herein and numerical example illustrated, some significant conclusions are drawn as following:

- Adaptive schemes are more efficient than the known schemes of control charts with fixed parameters. However, the use of adaptive schemes for control charts is not common in practice, since the traditional statistical software present no routines for these types of charts.
- With the programs presented here, the user has a tool in which it is able to plan the use of \bar{X} -VSSI control chart

to monitor the average value of a desired quality characteristic.

- Steady state analysis of \bar{X} -VSSI control chart model has been done to estimate its optimal parameters.
- It is observed that the ATS is lower ($ATS_{-2} < ATS_{-1}$), because, when major shifts in the process mean occur, the performance of the chart is better.

In addition to this, some following valuable suggestions for future scope of the present study are hereby recommended:

- The present research study can be extended for transient state conditions.
- Instead of the Markov chains technique, some other conventional optimization techniques may be used for the same \bar{X} -VSSI control chart model or its versions.
- It is suggested that future works present, with the support of the R software, how to plan statistical projects for control charts with adaptive schemes for other statistics such as standard deviation and sampling amplitude.

Appendix A

Source code to evaluate the performance and choosing an optimal statistical project for the control chart \bar{X} - VSSI in the R environment.

It is presented below two functions called *VSSI* and *VSSI.optimum*. To use them, just copy them into the R environment and follow the application example.

Function: VSSI

Function Thatevaluates the \bar{X} - VSSI Chart Development by Means of Markov Chain

```
rm(list=ls(all=TRUE))
VSSI <- function(n1,n2,n0_FSR,delta,ARL0,h0,r_insp) {
k0 <- qnorm(1-(1/(2*ARL0)))
time<- h0
h0 <- 1
b_vector<- matrix(c(1,0), nrow=1, ncol=2) #vector {b}
fi_k0 <- pnorm(k0)
w0<- qnorm((fi_k0*(n2-n0_FSR)/(n2-n1)+0.5*(n0_FSR-
n1)/(n2-n1)))
h1 <- n2/r_insp
h2 <- h0*(pnorm(k0)-pnorm(-k0))/(pnorm(w0)-pnorm(-w0))-
h1*(2*(pnorm(k0)-pnorm(w0))/(pnorm(w0)-pnorm(-w0))
# In control-TransitionProbabilities
p0_o_o <- pnorm(w0)-pnorm(-w0)
p0_o_ab <- 2*(pnorm(k0)-pnorm(w0))
p0_ab_o <- p0_o_o
p0_ab_ab <- p0_o_ab
# steady state probabilities
p1 <- p0_ab_o/(p0_o_ab+p0_ab_o)
p2 <- p0_o_ab/(p0_o_ab+p0_ab_o)
# TransitionMatrix
P <- matrix(c(p0_o_o, p0_o_ab, 1-p0_o_o-p0_o_ab,
p0_ab_o, p0_ab_ab, 1-p0_ab_o-p0_ab_ab, 0, 0, 1), nrow =
3, ncol=3, byrow=TRUE,dimnames = list(c("O", "A or B",
"OOC"),c("O", "A or B", "OOC")))
# Fundamental matrixofMarkov
Qo <- matrix(c(p0_o_o, p0_o_ab, p0_ab_o, p0_ab_ab),
nrow=2, ncol=2, byrow=TRUE,
dimnames = list(c("O", "A or B"), c("O", "A or B")))
Id <- matrix(c(1,0,0,1), nrow=2, ncol=2, byrow=TRUE)
# unit vector
one<- matrix(c(1,1), nrow=2, ncol=1)
# [(I-Q0)^-1]
Id_Qo<- solve (Id-Qo)
# vector {n}
n_vector <- matrix(c(n1,n2), nrow=2, ncol=1)
#Out ofcontrol-TransitionProbability
pd_o_o<- pnorm(w0-delta*sqrt(n1))-pnorm(-w0-
delta*sqrt(n1))
pd_o_ab <- pnorm(k0-delta*sqrt(n1))-pnorm(w0-
delta*sqrt(n1))+pnorm(-w0-delta*sqrt(n1))-pnorm(-k0-
delta*sqrt(n1))
pd_ab_o <- pnorm(w0-delta*sqrt(n2))-pnorm(-w0-
```

```
delta*sqrt(n2))
pd_ab_ab <- pnorm(k0-delta*sqrt(n2))-pnorm(w0-
delta*sqrt(n2))+pnorm(-w0-delta*sqrt(n2))-pnorm(-k0-
delta*sqrt(n2))
#matrixofstatesttransition
Pd <- matrix(c(pd_o_o, pd_o_ab, 1-pd_o_o-pd_o_ab,
pd_ab_o, pd_ab_ab, 1-pd_ab_o-pd_ab_ab, 0, 0, 1), nrow =
3, ncol=3, byrow=TRUE,
dimnames = list(c("O", "A or B", "OOC"), c("O", "A or
B", "OOC")))
# fundamental matrixofMarkov
Qd <- matrix(c(pd_o_o, pd_o_ab, pd_ab_o, pd_ab_ab),
nrow=2, ncol=2, byrow=TRUE,
```

Appendix B

Source code to evaluate the performance and choosing an optimal statistical project for the \bar{X} - VSSI control chart in the R environment (Continued).

```
dimnames = list(c("O", "A or B"), c("O", "A or B")))
Id_Qd <- solve(Id-Qd) # [(I-Qd)^-1]
h_vector <- matrix(c(h2,h1), nrow=2, ncol=1) # vetor {h}
p1 <- (pnorm(w0)-pnorm(-w0))/(pnorm(k0)-pnorm(-k0))
p2 <- (2*(pnorm(k0)-pnorm(w0))/(pnorm(k0)-pnorm(-k0))
b_vector<- matrix(c(p1,p2), nrow=1, ncol=2) #vector {b} SS
ATS <- b_vector %*% Id_Qd %*% h_vector #b*[(I-Q0)^-
1]*h
#transformation for user time unit in minutes
h0 <- h0 * time; h1 <- h1 * time; h2 <- h2 * time; ATS <-
ATS * time; ATS0 <- ARL0*h0
parameters<-
c(ATS,n1,n2,h1,h2,delta,ATS0,ARL0,k0,w0,n0_FSR,h0,r_in
sp)
names(parameters)
c("ATSd","n1","n2","h1","h2","delta","ATS0","ARL0","k","
w","n0","h0","r_insp")
parameters<- round(parameters,4)
parameters}
```

Example of VSSI Function Application

```
# n1-sample size 1
# n2 - sample size 2
# n0_FSR-expected value (average) for the sample
size(process in control)
# delta-dislocation grade in the process average
# ARL0 – the expected number of samples between two
successive false alarm
# h0c –expected value (average) for the time interval to
collect a sample (process in control).
NOTE: launching the ho value in minutes.
# r_insp - pieces quantity which can be checked per
considered time unit in h0.
```

```
VSSI(n1=2,n2=8,n0_FSR=5,delta=1,ARL0=370.3983,h0=60
```

```
,r_insp=60)
```

Function: VSSI. Optimum

function that chooses the optimal parameters of the \bar{X} – VSSI chart

```
VSSI.optimum<-
function(n0_FSR,delta,ARL0,h0,r_insp,nmax){
  LI=n0_FSR-1 ; LS=n0_FSR+1
  n1opt=1 ; n2opt=2*n0_FSR-n1opt
  ATSopt=VSSI(n1opt,n2opt,n0_FSR,delta,ARL0,h0,r_insp)[1]
}
for (n1 in 1:LI) {
  for (n2 in LS:nmax) {
    x1 <- 0.5*n1+0.5*n2
    x2 <- n0_FSR
    if (identical(all.equal(x1, x2), TRUE)) {
      result <- VSSI(n1,n2,n0_FSR,delta,ARL0,h0,r_insp)[1]
      if (result < ATSopt) {
        n1opt=n1 ; n2opt=n2
        ATSopt=as.numeric(VSSI(n1,n2,n0_FSR,delta,ARL0,h0,r_insp)[1])
      }
    }
  }
}
print(VSSI(n1opt,n2opt,n0_FSR,delta,ARL0,h0,r_insp)) }
```

Application Example of VSSI. Optimum Function

nmax – maximum permissible value for the sample size
#Note: This function depends on the previous one. To use the function VSSI.optimumcopy also VSSI in the desktop of the R software.

```
VSSI.optimum(n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
```

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