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Exponential Mean and Ratio-Types Estimators of Population Mean Using Moments Under Calibration Approach

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Abstract: Calibration is a well-known technique for weight adjustment using various sets of constraints. This paper considers exponential ratio-type calibrated estimators for finite population mean using first three moments about the origin of the auxiliary variable in the calibration constraint under stratified random sampling. The exponential mean-type estimators for the second and third order moments are also suggested for the mentioned sampling scheme. When first three moments of the auxiliary variable are not known, then we use stratified double sampling scheme to estimate these moments. Thus, the result has been extended in the case of stratified double sampling and exponential mean-type and exponential ratio-type estimators have been developed using first three moments about the origin in the calibration constraints. The expression for mean squared error for the suggested estimators have been derived using the Taylor linearization method. For judging the performance of the proposed estimators, a simulation study has been carried out on two real datasets of MU284 population using R-software and their percentage root mean squared error (%RRMSE) and relative efficiency have been computed. The suggested estimators have been compared with the existing estimators given in the same setup and the new developed estimators are found to be more efficient than these estimators for the considered datasets.

Keywords: Auxiliary Information, Calibration Estimation, Stratified Random Sampling, Mean, Moments, Exponential Type Estimator

1. Introduction

The auxiliary information to improve estimates of the population parameters of the study variable is a common practice in sample surveys, specifically when there is a strong linear relationship between the study and auxiliary variables. Numerous authors have contributed in this aspect, including Cochran [4]. Singh and Vishwakarma [20] proposed modified exponential ratio and product estimators for finite population mean in double sampling. Onyeka [16] suggested a class of product-type exponential estimators of the population mean in simple random sampling scheme. They studied exponential estimators of population mean in post-stratified sampling using known value of some population parameters. Rashid et al. [17] proposed exponential estimators for population mean using transformed auxiliary variables. Kadilar [10] gave an exponential type estimator for the population mean in simple random sampling. Bhushan et al. [2] worked on the class of double sampling exponential ratio type estimators using auxiliary information on an attribute. Tailor et al. [23] recommended improved ratio and product-type exponential estimators for population mean in case of post-stratification.

The calibration estimation is a method of adjusting weights to estimate the population parameters of finite population with the help of auxiliary information, defined by Deville and Sarndal [5], following this, many researchers have contributed in development of calibration estimators using different calibration constraints for different population parameters under various sampling schemes (see Singh [19], Kim et al. [11], Singh and Arnab [21], Koyuncu and Kadilar [12, 13], Mouhamed et al. [15], Clement and Enang [3], Garg and Pachori [6-9], etc.).

The purpose of the current study is to suggest new calibration estimators for population mean under stratified random sampling and stratified double sampling by making use of first three moments about origin in the calibration constraints. Section 2 contains the already existing estimators. Three exponential-type estimators using first three moments about origin in the calibration constraint and two mean-type estimators using second and third moment about origin in the calibration constraint and defined in Section 3. These estimators have been compared with the existing estimators of Singh [22] that have first-order moment about origin in the constraint and Tracy et al. [24]. These suggested estimators have been extended under the stratified double sampling in Section 4. While Section 5 includes the simulation study on two real datasets of the MU284 population in which the %RRMSE and %RE are computed. The conclusion of the study is given in Section 6.

2. Notations in Calibration Estimator Under Stratified Sampling

Suppose a heterogeneous finite population U of size N is divided into L homogeneous strata of sizes $N_1, N_2, \ldots, N_L$ such that $N=\sum_{h=1}^{L} N_h$. A sample of size n_h is drawn using simple random sampling without replacement (SRSWOR) from the hth stratum such that $\sum_{h=1}^{L} n_h = n$, where n is the required sample size. Let us consider a study variable (Y) and an auxiliary variable (X) which are positively correlated with each other. Suppose y_{hi} and x_{hi} are the ith units of Y and X, respectively, in the hth stratum for i= 1, 2,..., n_h and h = 1, 2, …, L. $W_h=\frac{N_h}{N}$ and $f_h=\frac{n_h}{N}$ are the hth stratum weight and sample fraction, respectively.

To estimate a population parameter, say mean $\bar{Y}=\sum_{h=1}^{L} W_h \bar{Y}_h$, population mean $\bar{X}=\sum_{h=1}^{L} W_h \bar{X}_h$ of the auxiliary variable is presumed to be known. The calibration estimator for population means $\bar{Y}$ under stratified random sampling given by Singh [22] is:

$$\bar{y}_{m1}=\sum_{h=1}^{L} W_h \bar{y}_h+\hat{\beta}_{m1}\left(\bar{X}-\sum_{h=1}^{L} W_h \bar{x}_h\right) \tag{1}$$

where $\hat{\beta}_{m1}=\left[\dfrac{\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h\right)-\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)-\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)^2}\right]$

Similarly, the calibration estimator of the population means $\bar{Y}$ under the stratified random sampling defined by Tracy et al. [24] is given as:

$$\bar{y}_{tr}=\sum_{h=1}^{L} W_h \bar{y}_h+\hat{\beta}_{t1}\left(\sum_{h=1}^{L} W_h\left(\bar{X}_h-\bar{x}_h\right)\right)+\hat{\beta}_{t2}\left(\sum_{h=1}^{L} W_h\left(S_{hx}^2-s_{hx}^2\right)\right) \tag{2}$$

where

$$\hat{\beta}_{t1}=\left[\frac{\left(\sum_{h=1}^{L} W_h Q_h s_{hx}^4\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h\right)-\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h s_{hx}^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h s_{hx}^2\right)}{\left(\sum_{h=1}^{L} W_h Q_h s_{hx}^4\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)-\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h s_{hx}^2\right)^2}\right]$$

$$\hat{\beta}_{t2}=\left[\frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h s_{hx}^2\right)-\left(\sum_{h=1}^{L} W_h Q_h s_{hx}^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h s_{hx}^4\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)-\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h s_{hx}^2\right)^2}\right]$$

3. Proposed Calibration Estimator Under Stratified Sampling

Bahl and Tuteja [1] suggested an exponential ratio-type estimator of a finite population mean given as:

$$\bar{y}_{bt} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{3}$$

The traditional stratified estimator of population mean in stratified random sampling is given as:

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{4}$$

Following Bahl and Tuteja [1], the exponential ratio-type estimator in stratified random sampling defined by Malik et al. [14] is given as:

$$\bar{y}_{st.bt} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right) \tag{5}$$

This paper puts forward new exponential ratio-type calibration estimators of a population mean in stratified random sampling using first three moments about origin along with their mean-type estimators. In general, r^{th} population moment about origin (or, r^{th} raw moment) in the h^{th} stratum is given as:

$$\mu'_{hr} = \frac{1}{N_h} \sum_{i=1}^{N_h} X^r_{hi}; r = 1, 2 \, and \, 3 \tag{6}$$

The r^{th} sample moment about origin in the h^{th} stratum is given as:

$$m'_{hr} = \frac{1}{n_h} \sum_{i=1}^{n_h} x^r_{hi}; \; r = 1, 2 \, and \, 3 \tag{7}$$

We propose three exponential ratio-type calibrated estimators under stratified random sampling given as follows:

3.1. Calibration Estimator Using First Moment About Origin

Using first moment about origin, we propose first exponential type calibrated estimator as:

$$\bar{y}_{ce.1} = \sum_{h=1}^{L} \Omega_{h1} \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right) \tag{8}$$

where the calibration weights Ω_{h1}; (h = 1, 2, …, L) are selected in order to minimize the Chi-square type distance function defined as:

$$\sum_{h=1}^{L} \frac{(\Omega_{h1} - W_h)^2}{W_h Q_h} \tag{9}$$

subject to the following calibration constraints:

$$\sum_{h=1}^{L} \Omega_{h1} = \sum_{h=1}^{L} W_h \tag{10}$$

$$\sum_{h=1}^{L} \Omega_{h1} m'_{h1} = \sum_{h=1}^{L} W_h \mu'_{h1} \tag{11}$$

The Lagrange function is given as:

$$L_1 = \sum_{h=1}^{L} \frac{(\Omega_{h1} - W_h)^2}{Q_h W_h} - 2\lambda_1 (\sum_{h=1}^{L} \Omega_{h1} - \sum_{h=1}^{L} W_h) - 2\lambda_2 (\sum_{h=1}^{L} \Omega_{h1} m'_{h1} - \sum_{h=1}^{L} W_h \mu'_{h1}) \tag{12}$$

where λ_1 and λ_2 are the Lagrange's multipliers. To determine the optimum value of Ω_{h1}, we differentiate the Lagrange function given in equations (12) with respect to Ω_{h1} and equate it to zero. Thus, the calibration weights can be obtained as:

$$\Omega_{h1} = W_h + W_h Q_h(\lambda_1 + \lambda_2 m'_{h1}) \tag{13}$$

On substituting this value from equation (13) to (10) and (11), we get the calibrated weights given as:

$$\Omega_{h1} = W_h + W_h Q_h \left[\frac{-(\sum_{h=1}^{L} W_h(\mu'_{h1} - m'_{h1}))(\sum_{h=1}^{L} W_h Q_h m'_{h1})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h1})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})^2} \right] + W_h Q_h m'_{h1} \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h(\mu'_{h1} - m'_{h1}))}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h1})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})^2} \right] \tag{14}$$

After substituting the optimum calibrated weights in equation (14), we obtain the proposed estimator:

$$\bar{y}_{ce.1} = \sum_{h=1}^{L} W_h \bar{y}_{bt.h} + \hat{\beta}_{ce.1} \sum_{h=1}^{L} W_h(\mu'_{h1} - m'_{h1}) \tag{15}$$

where,

$$\hat{\beta}_{ce.1} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h1} \bar{y}_{bt.h}) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})(\sum_{h=1}^{L} W_h Q_h \bar{y}_{bt.h})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h1})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})^2} \right] \text{ and } \bar{y}_{bt.h} = \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right)$$

3.2. Calibration Estimator Using Second Moment About Origin

The exponential type calibrated estimator with second moment about origin is given as:

$$\bar{y}_{ce.2} = \sum_{h=1}^{L} \Omega_{h2} \bar{y}_{bt.h} \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right) \tag{16}$$

where Ω_{h2} are the calibrated weights chosen in such a way in order to minimize the Chi-square type distance function defined as:

$$\sum_{h=1}^{L} \frac{(\Omega_{h2} - W_h)^2}{W_h Q_h} \tag{17}$$

subject to the following calibration constraints:

$$\sum_{h=1}^{L} \Omega_{h2} = \sum_{h=1}^{L} W_h \tag{18}$$

$$\sum_{h=1}^{L} \Omega_{h2} m'_{h2} = \sum_{h=1}^{L} W_h \mu'_{h2} \tag{19}$$

The Lagrange function for the same is:

$$L_2 = \sum_{h=1}^{L} \frac{(\Omega_{h2} - W_h)^2}{Q_h W_h} - 2\lambda_1(\sum_{h=1}^{L} \Omega_{h2} - \sum_{h=1}^{L} W_h) - 2\lambda_2(\sum_{h=1}^{L} \Omega_{h2} m'_{h2} - \sum_{h=1}^{L} W_h \mu'_{h2}) \tag{20}$$

where λ_1 and λ_2 are the Lagrange's multipliers. Thus, the calibration weight is attained as:

$$\Omega_{h2} = W_h + W_h Q_h(\lambda_1 + \lambda_2 m'_{h2}) \tag{21}$$

On substituting the value from equation (21) to calibration constraints given in equations (18) and (19), the calibrated weight is given as follows:

$$\Omega_{h2} = W_h + W_h Q_h \left[\frac{-(\sum_{h=1}^{L} W_h(\mu'_{h2} - m'_{h2}))(\sum_{h=1}^{L} W_h Q_h m'_{h2})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right] + W_h Q_h m'_{h2} \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h(\mu'_{h2} - m'_{h2}))}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right] \tag{22}$$

On substituting the optimum calibrated weight value from equation (22), the proposed estimator is given as:

$$\bar{y}_{ce.2} = \sum_{h=1}^{L} W_h \bar{y}_{bt.h} + \hat{\beta}_{ce.2} \sum_{h=1}^{L} W_h(\mu'_{h2} - m'_{h2}) \tag{23}$$

where, $\hat{\beta}_{ce.2} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h2} \bar{y}_{bt.h}) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})(\sum_{h=1}^{L} W_h Q_h \bar{y}_{bt.h})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right]$

and, $\bar{y}_{bt.h} = \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right)$

Similarly, the mean-type calibrated estimator under stratified sampling using second moment about origin is defined as:

$$\bar{y}_{m2} = \sum_{h=1}^{L} \Omega_{h2} \bar{y}_h \tag{24}$$

where Ω_{h2} are the calibrated weights obtained by minimizing the Chi-square type distance measure $\sum_{h=1}^{L} \frac{(\Omega_{h2} - W_h)^2}{W_h Q_h}$, subject to two calibration constraints given as:

$$\sum_{h=1}^{L} \Omega_{h2} = \sum_{h=1}^{L} W_h \tag{25}$$

$$\sum_{h=1}^{L} \Omega_{h2} m'_{h2} = \sum_{h=1}^{L} W_h \mu'_{h2} \tag{26}$$

Minimization of Chi-square type distance measure subject to second-order moment calibration constraints, the calibrated estimator is given as:

$$\bar{y}_{m2} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{m2} \sum_{h=1}^{L} W_h(\mu'_{h2} - m'_{h2}) \tag{27}$$

where $\hat{\beta}_{m2} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h2} \bar{y}_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})(\sum_{h=1}^{L} W_h Q_h \bar{y}_h)}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right]$

3.3. Calibration Estimator Using Third Moment About Origin

On using third order moment about origin, the exponential ratio-type calibrated estimator becomes:

$$\bar{y}_{ce.3} = \sum_{h=1}^{L} \Omega_{h3} \bar{y}_{bt.h} \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right) \tag{28}$$

minimizing Chi-square type distance function defined as:

$$\sum_{h=1}^{L}\frac{(\Omega_{h3}-W_h)^2}{W_hQ_h} \tag{29}$$

subject to the following calibration constraints:

$$\sum_{h=1}^{L}\Omega_{h3}=\sum_{h=1}^{L}W_h \tag{30}$$

$$\sum_{h=1}^{L}\Omega_{h3}m'_{h3}=\sum_{h=1}^{L}W_h\mu'_{h3} \tag{31}$$

The Lagrange function is given as:

$$L_3=\sum_{h=1}^{L}\frac{(\Omega_{h3}-W_h)^2}{Q_hW_h}-2\lambda_1(\sum_{h=1}^{L}\Omega_{h3}-\sum_{h=1}^{L}W_h)-2\lambda_2(\sum_{h=1}^{L}\Omega_{h3}m'_{h3}-\sum_{h=1}^{L}W_h\mu'_{h3}) \tag{32}$$

where λ_1 and λ_2 are the Lagrange's multipliers. Therefore, the calibration weights are given as follows:

$$\Omega_{h3}=W_h+W_hQ_h(\lambda_1+\lambda_2m'_{h3}) \tag{33}$$

After substituting the value of Ω_{h3} from equation (33) to equations (30) and (31), the calibrated weights are obtained as:

$$\Omega_{h3}=W_h+W_hQ_h\left[\frac{-(\sum_{h=1}^{L}W_h(\mu'_{h3}-m'_{h3}))(\sum_{h=1}^{L}W_hQ_hm'_{h3})}{(\sum_{h=1}^{L}W_hQ_hm'^2_{h3})(\sum_{h=1}^{L}W_hQ_h)-(\sum_{h=1}^{L}W_hQ_hm'_{h3})^2}\right]+W_hQ_hm'_{h2}\left[\frac{(\sum_{h=1}^{L}W_hQ_h)(\sum_{h=1}^{L}W_h(\mu'_{h3}-m'_{h3}))}{(\sum_{h=1}^{L}W_hQ_hm'^2_{h3})(\sum_{h=1}^{L}W_hQ_h)-(\sum_{h=1}^{L}W_hQ_hm'_{h3})^2}\right] \tag{34}$$

After replacing value of the optimum calibrated weight in equation (34), the proposed estimator becomes:

$$\bar{y}_{ce.3}=\sum_{h=1}^{L}W_h\bar{y}_{bt.h}+\hat{\beta}_{ce.3}\sum_{h=1}^{L}W_h(\mu'_{h3}-m'_{h3}) \tag{35}$$

where, $\hat{\beta}_{ce.3}=\left[\frac{(\sum_{h=1}^{L}W_hQ_h)(\sum_{h=1}^{L}W_hQ_hm'_{h3}\bar{y}_h)-(\sum_{h=1}^{L}W_hQ_hm'_{h3})(\sum_{h=1}^{L}W_hQ_h\bar{y}_h)}{(\sum_{h=1}^{L}W_hQ_hm'^2_{h3})(\sum_{h=1}^{L}W_hQ_h)-(\sum_{h=1}^{L}W_hQ_hm'_{h3})^2}\right]$

$\mu'_{h1}=\frac{1}{N_h}\sum_{i=1}^{N_h}X_{hi}$, $\mu'_{h2}=\frac{1}{N_h}\sum_{i=1}^{N_h}X^2_{hi}$, $\mu'_{h3}=\frac{1}{N}\sum_{i=1}^{N_h}X^3_{hi}$ are the population moments about origin in the h^th stratum.

$m'_{h1}=\frac{1}{n_h}\sum_{i=1}^{n_h}x_{hi}$, $m'_{h2}=\frac{1}{n_h}\sum_{i=1}^{n_h}x^2_{hi}$, $m'_{h3}=\frac{1}{n_h}\sum_{i=1}^{n_h}x^3_{hi}$ are the sample moments about origin in the h^th stratum.

Also, the mean-type calibrated estimator using third moment about origin under stratified sampling is suggested as:

$$\bar{y}_{m3}=\sum\nolimits_{h=1}^{L}\Omega_{h3}\bar{y}_h \tag{36}$$

where Ω_{h3} are the calibrated weights obtained by minimizing the Chi-square type distance measure $\sum_{h=1}^{L}\frac{(\Omega_{h3}-W_h)^2}{W_hQ_h}$, subject to two calibration constraints:

$$\sum_{h=1}^{L}\Omega_{h3} = \sum_{h=1}^{L}W_h \tag{37}$$

$$\sum_{h=1}^{L}\Omega_{h3}m'_{h3} = \sum_{h=1}^{L}W_h\mu'_{h3} \tag{38}$$

Minimization of Chi-square type distance measure subject to second-order moment calibration constraints, the calibrated estimator is given as:

$$\bar{y}_{m3} = \sum_{h=1}^{L}W_h\bar{y}_h + \hat{\beta}_{m3}\sum_{h=1}^{L}W_h(\mu'_{h3} - m'_{h3}) \tag{39}$$

where $\hat{\beta}_{m3} = \left[\dfrac{(\sum_{h=1}^{L}W_hQ_h)(\sum_{h=1}^{L}W_hQ_hm'_{h3}\bar{y}_h) - (\sum_{h=1}^{L}W_hQ_hm'_{h3})(\sum_{h=1}^{L}W_hQ_h\bar{y}_h)}{(\sum_{h=1}^{L}W_hQ_hm'^2_{h3})(\sum_{h=1}^{L}W_hQ_h) - (\sum_{h=1}^{L}W_hQ_hm'_{h3})^2}\right]$

3.4. Expressions for Mean Squared Error (MSE)

The mean-type calibration estimators defined in equations (27) and (39) for k = 2 and 3 can be rewritten by first order Taylor expansion as:

$$\bar{y}_{m.k} = \bar{Y} + \sum_{h=1}^{L}W_h\left(\bar{y}_h - \bar{Y}_h\right) - \left\{\frac{\sum_{h=1}^{L}W_h\bar{Y}_h\mu'_{xkh} - \left(\sum_{h=1}^{L}W_h\mu'_{xkh}\right)\left(\sum_{h=1}^{L}W_h\bar{Y}_h\right)}{\sum_{h=1}^{L}W_h\mu'_{xkh} - \left(\sum_{h=1}^{L}W_h\mu'_{xkh}\right)^2}\right\} \times \left(\sum_{h=1}^{L}W_h\left(\mu'_{xkh} - m'_{xkh}\right)\right)$$

The mean squared errors of the estimators k = 2 and 3 up to second order of approximation are given as:

$$MSE\left(\bar{y}_{mk}\right) = E\left[\bar{y}_{mk} - \bar{Y}\right]^2 = \sum_{h=1}^{L}W_h^2 f'_h\left[\bar{Y}_h^2C_{yh}^2 + \beta_{mk}^2\mu'^2_{xkh}C_{xkh}^2 - 2\beta_{mk}\rho_h\bar{Y}_h\mu'_{xkh}C_{yh}C_{xkh}\right]$$

where $f_h' = \left(\dfrac{1}{n_h} - \dfrac{1}{N_h}\right)$

$$C_{yh}^2 = \frac{1}{(N_h - 1)}\sum_{i=1}^{N_h}\left(Y_{hi} - \bar{Y}_h\right)^2$$

$$C_{xkh}^2 = \frac{1}{\mu'^2_{xkh}(N_h - 1)}\sum_{i=1}^{N_h}\left(X_{hi}^k - \mu'_{xkh}\right)^2$$

$$\beta_{mk} = \frac{\sum_{h=1}^{L}W_h\bar{Y}_h\mu'_{xkh} - \left(\sum_{h=1}^{L}W_h\mu'_{xkh}\right)\left(\sum_{h=1}^{L}W_h\bar{Y}_h\right)}{\left(\sum_{h=1}^{L}W_h\mu'^2_{xkh}\right) - \left(\sum_{h=1}^{L}W_h\mu'_{xkh}\right)^2}$$

The exponential-type calibration estimators defined in equations (15), (23) and (35) for k = 1, 2 and 3 can be rewritten by first order Taylor expansion as:

$$\bar{y}_{ce.k} = \bar{Y} + \sum_{h=1}^{L} W_h \left(\bar{y}_h - \bar{Y}_h\right) - \left\{ \frac{\sum_{h=1}^{L} W_h \bar{Y}_h \mu'_{xkh} - \left(\sum_{h=1}^{L} W_h \mu'_{xkh}\right)\left(\sum_{h=1}^{L} W_h \bar{Y}_h\right)}{\sum_{h=1}^{L} W_h \mu'_{xkh} - \left(\sum_{h=1}^{L} W_h \mu'_{xkh}\right)^2} \right\} \times \left(\sum_{h=1}^{L} W_h \left(\mu'_{xkh} - m'_{xkh}\right)\right)$$

The mean squared errors of the estimators for k = 1, 2 and 3 up to second order of approximation are given as:

$$MSE\left(\bar{y}_{ce.k}\right) = E\left[\bar{y}_{ce.k} - \bar{Y}\right]^2 = \sum_{h=1}^{L} W_h^2 f'_h \left[\bar{Y}_h^2 \left(C_{yh}^2 - \frac{C_{xkh}}{2} \right)^2 + \beta_{ce.k}^2 \mu'^2_{xkh} C_{xkh}^2 - \beta_{ce.k} \bar{Y}_h \mu'_{xkh} \left(2\rho_h C_{yh} C_{xkh} - C_{xkh}^2 \right) \right]$$

where $\beta_{ce.k} = \dfrac{\sum_{h=1}^{L} W_h \bar{Y}_{st.bt} \mu'_{xkh} - \left(\sum_{h=1}^{L} W_h \mu'_{xkh}\right)\left(\sum_{h=1}^{L} W_h \bar{Y}_{st.bt}\right)}{\left(\sum_{h=1}^{L} W_h \mu'^2_{xkh}\right) - \left(\sum_{h=1}^{L} W_h \mu'_{xkh}\right)^2}$

For different values of Q_h, we can obtain different forms of the suggested calibration estimators.

4. Proposed Calibration Estimator in Stratified Double Sampling

The resulting outcome for stratified sampling is extended in case of stratified double sampling, for which a preliminary sample of size m_h units as a first phase sample is drawn by using SRSWOR, and a subsample of n_h units is drawn from the preliminary sample of size m_h units by SRSWOR. Let $\bar{x}_h^* = \frac{1}{m_h} \sum_{i=1}^{m_h} x_{hi}$ be the first phase sample mean and $\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$, the second phase sample mean of auxiliary variable and study variable, respectively.

4.1. Calibration Estimator Using First Moment About Origin

For first-order moment about origin, the exponential-type calibrated estimator under stratified double sampling is given as:

$$\bar{y}_{ce.d1} = \sum_{h=1}^{L} W_h \bar{y}_{bt.h} + \hat{\beta}_{ce.d1} \sum_{h=1}^{L} W_h (m_{h1}^{*\prime} - m'_{h1}) \tag{40}$$

where $\hat{\beta}_{ce.d1} = \left[\dfrac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h1} \bar{y}_{bt.h}) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})(\sum_{h=1}^{L} W_h Q_h \bar{y}_{bt.h})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h1})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h1})^2} \right]$

and $\bar{y}_{bt.h} = \bar{y}_h \exp\left(\dfrac{\bar{x}_h^* - \bar{x}_h}{\bar{x}_h^* + \bar{x}_h} \right)$

4.2. Calibration Estimator Using Second Moment About Origin

The exponential type calibrated estimator considering second-order moment about origin becomes:

$$\bar{y}_{ce.d2} = \sum_{h=1}^{L} W_h \bar{y}_{bt.h} + \hat{\beta}_{ce.d2} \sum_{h=1}^{L} W_h (m_{h2}^{*\prime} - m'_{h2}) \tag{41}$$

where $\hat{\beta}_{ce.d2} = \left[\dfrac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h2} \bar{y}_{bt.h}) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})(\sum_{h=1}^{L} W_h Q_h \bar{y}_{bt.h})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right]$

Likewise, after minimization of Chi-square type distance measure depending on the second-order moment calibration constraints, the mean-type calibration estimator under stratified double sampling is specified as:

$$\bar{y}_{m.d2} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{m.d2} \sum_{h=1}^{L} W_h (m_{h2}^{*\prime} - m'_{h2}) \tag{42}$$

where $\hat{\beta}_{m.d2} = \left[\dfrac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h2} \bar{y}_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})(\sum_{h=1}^{L} W_h Q_h \bar{y}_h)}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h2})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h2})^2} \right]$

4.3. Calibration Estimator Using Third Moment About Origin

The exponential ratio-type calibrated estimator using third order moment about origin becomes:

$$\bar{y}_{ce.d3} = \sum_{h=1}^{L} W_h \bar{y}_{bt.h} + \hat{\beta}_{ce.d3} \sum_{h=1}^{L} W_h (m_{h3}^{*\prime} - m'_{h3}) \tag{43}$$

where $\hat{\beta}_{ce.d3} = \left[\dfrac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h3} \bar{y}_{bt.h}) - (\sum_{h=1}^{L} W_h Q_h m'_{h3})(\sum_{h=1}^{L} W_h Q_h \bar{y}_{bt.h})}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h3})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h3})^2} \right]$

Correspondingly, by using third-order moment about origin, the mean-type calibrated estimator for stratified double sampling is defined as:

$$\bar{y}_{m.d3} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{m.d3} \sum_{h=1}^{L} W_h (m_{h3}^{*\prime} - m'_{h3}) \tag{44}$$

where $\hat{\beta}_{m.d3} = \left[\dfrac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h m'_{h3} \bar{y}_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h3})(\sum_{h=1}^{L} W_h Q_h \bar{y}_h)}{(\sum_{h=1}^{L} W_h Q_h m'^2_{h3})(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m'_{h3})^2} \right]$

4.4. Expressions for Mean Squared Error (MSE)

The mean squared errors of the estimators defined in equations (42) and (44) for k = 2 and 3 up to second order of approximation for mean-type calibration estimators are given as:

$$MSE\left(\bar{y}_{m.dk}\right) = E\left[\bar{y}_{m.dk} - \bar{Y}\right]^2 = \sum_{h=1}^{L} W_h^2 \left[f'_h \bar{Y}_h^2 C_{yh}^2 + f''_h \left\{ \beta_{m.dk}^2 \mu'^2_{xkh} C_{xkh}^2 - 2\beta_{d.mk} \rho_h \bar{Y}_h \mu'_{xkh} C_{yh} C_{xkh} \right\} \right]$$

where $C_{xkh}^2 = \dfrac{1}{\mu'^2_{xkh}(N_h - 1)} \sum_{i=1}^{N_h} \left(X_{hi}^k - \mu'_{xkh} \right)^2$

The mean squared errors for the exponential-type calibration estimators defined in equations (40), (41) and (43) for k = 1, 2 and 3 up to second order of approximation are given as:

$$MSE\left(\bar{y}_{ce.dk}\right) = E\left[\bar{y}_{ce.dk} - \bar{Y}\right]^2$$
$$= \sum_{h=1}^{L} W_h^2 \left[f'_h \bar{Y}_h^2 C_{yh}^2 + f''_h \beta_{ce.dk}^2 \mu'^2_{xkh} C_{xkh}^2 + f''_h \left(\frac{1}{4} C_{xkh}^2 - \rho_h C_{yh} C_{xkh} \right) - 2 f''_h \beta_{ce.dk} \rho_h \bar{Y}_h \mu'_{xkh} \left(\rho_h C_{yh} C_{xkh} - \frac{C_{xkh}^2}{2} \right) \right]$$

5. Simulation Study

To study performance of the proposed calibrated estimators, two combinations of a real population MU284 given in Appendix B of Sarndal et al. [18] are considered. It comprises 284 units, divided into eight strata of varying sizes. The description of both datasets used here are as follows:

Real Data I:

X: SS82, Number of Social-Democratic seats in the municipal council;

Y: REV84, Real state value according to 1984 assessment (in millions).

Table 1. *Descriptive Statistics for Real Data I.*

X		Y	
Min.	8.00	Min.	347
1st Quartile	17.00	1st Quartile	1146
Median	21.00	Median	1854
Mean	22.19	Mean	3078
3rd Quartile	27.00	3rd Quartile	3345
Max.	46.00	Max.	59877

Real Data II:

X: S82, Total number of seats in the municipal council.

Y: REV84, Real state value according to 1984 assessment (in millions).

Table 2. *Descriptive Statistics for Real Data II.*

X		Y	
Min.	31.00	Min.	347
1st Quartile	41.00	1st Quartile	1146
Median	49.00	Median	1854
Mean	47.64	Mean	3078
3rd Quartile	51.00	3rd Quartile	3345
Max.	101.00	Max.	59877

The random samples of varying sizes are drawn by proportional allocation using simple random sampling without replacement (SRSWOR) from each stratum. A simulated study is done by generating 25,000 samples in R-software. The performance of the estimators is measured in terms of percentage relative root mean square error (%RRMSE) and Percentage Relative Efficiency (%RE) defined as:

(i) In the case of Stratified Sampling

$$\%RRMSE(\bar{y}_{\alpha}) = \sqrt{\frac{1}{25000}\sum_{i=1}^{25000}\left(\frac{(\bar{y}_{i\alpha} - \bar{Y})}{\bar{Y}}\right)^2} \times 100 \ ; \ \alpha = tr, m1, m2, m3, ce.1, ce.2, ce.3$$

$$\%RE(\bar{y}_{\alpha}) = \left(\frac{\bar{y}_{tr}}{\bar{y}_{\alpha}}\right) \times 100; \ \ \alpha = m1, m2, m3, ce.1, ce.2, ce.3$$

(ii) In the case of Stratified Double Sampling

$$\%RRMSE(\bar{y}_{\alpha.d}) = \sqrt{\frac{1}{25000}\sum_{i=1}^{25000}\left(\frac{(\bar{y}_{i\alpha.d} - \bar{Y})}{\bar{Y}}\right)^2} \times 100 \ ;$$

$$\alpha.d = tr.d, m.d1, m.d2, m.d3, ce.d1, ce.d2, ce.d3$$

$$\%RE(\bar{y}_{\alpha.d}) = \left(\frac{\bar{y}_{tr.d}}{\bar{y}_{\alpha.d}}\right) \times 100; \ \ \alpha.d = m.d1, m.d2, m.d3, ce.1, ce.2, ce.3$$

The results so obtained are given in Table 3, Table 4, Table 5 and Table 6.

Table 3. *Percentage Relative Root Mean Square Error (%RRMSE) under Stratified Random Sampling.*

Q_h	Sample Size (n)	$\bar{y}_{tr}$	$\bar{y}_{m1}$	$\bar{y}_{ce.1}$	$\bar{y}_{m2}$	$\bar{y}_{ce.2}$	$\bar{y}_{m3}$	$\bar{y}_{ce.3}$
Real Data I								
	30	24.99	23.25	21.53	22.71	21.11	22.56	20.96
1	35	23.30	22.18	20.50	21.61	20.04	21.32	19.76
	40	20.53	20.38	19.04	19.93	18.70	19.86	18.65
	30	24.47	23.18	21.53	22.57	21.06	22.41	20.91
$\frac{1}{\bar{x}_h}$	35	22.86	22.17	20.53	21.51	20.01	21.18	19.70
	40	20.29	20.38	19.08	19.84	18.67	19.72	18.58
Real Data II								
	30	18.83	18.46	17.38	16.52	15.86	15.37	15.01
1	35	16.82	17.41	16.36	15.44	14.82	14.25	13.94
	40	15.62	16.20	15.40	14.54	14.05	13.42	13.17
	30	18.60	18.30	17.21	16.39	15.73	15.20	14.85
$\frac{1}{\bar{x}_h}$	35	16.59	17.32	16.25	15.36	14.72	14.11	13.81
	40	15.43	16.10	15.28	14.46	13.96	13.31	13.06

__Table 4.__ Percentage Relative Efficiency (%RE) under Stratified Random Sampling.

Real Data I								
Q_h	**Sample Size (n)**	$\bar{y}_{tr}$	$\bar{y}_{m1}$	$\bar{y}_{ce.1}$	$\bar{y}_{m2}$	$\bar{y}_{ce.2}$	$\bar{y}_{m3}$	$\bar{y}_{ce.3}$
	30	100.00	107.48	116.07	110.04	118.38	110.77	119.23
1	35	100.00	105.05	113.66	107.82	116.27	109.29	117.91
	40	100.00	100.74	107.83	103.01	109.79	103.37	110.08
	30	100.00	105.57	113.66	108.42	116.19	109.19	117.03
$\frac{1}{\bar{x}_h}$	35	100.00	103.11	111.35	106.28	114.24	107.93	116.04
	40	100.00	99.56	106.34	102.27	108.68	102.89	109.20
Real Data II								
	30	100.00	102.00	108.34	113.98	118.73	122.51	125.45
1	35	100.00	96.61	102.81	108.94	113.50	118.04	120.66
	40	100.00	96.42	101.43	107.43	111.17	116.39	118.60
	30	100.00	101.64	108.08	113.48	118.25	122.37	125.25
$\frac{1}{\bar{x}_h}$	35	100.00	95.79	102.09	108.01	112.70	117.58	120.13
	40	100.00	95.84	100.98	106.71	110.53	115.93	118.15

__Table 5.__ Percentage Relative Root Mean Square Error (%RRMSE) under Stratified Double Sampling.

Real Data I								
Q_h	**Sample Size (m; n)**	$\bar{y}_{tr.d}$	$\bar{y}_{m.d1}$	$\bar{y}_{ce.d1}$	$\bar{y}_{m.d2}$	$\bar{y}_{ce.d2}$	$\bar{y}_{m.d3}$	$\bar{y}_{ce.d3}$
	90; 30	25.95	24.33	22.97	23.90	22.62	23.87	22.59
1	100; 30	24.51	23.80	22.45	23.49	22.22	23.47	22.19
	90; 35	25.11	22.80	21.53	22.48	21.28	22.43	21.23
	100; 35	23.55	22.43	21.14	22.02	20.81	21.93	20.74
	90; 30	25.51	24.29	22.95	23.78	22.55	23.73	22.50
$\frac{1}{\bar{x}_h}$	100; 30	24.03	23.74	22.43	23.36	22.14	23.31	22.10
	90; 35	24.66	22.79	21.53	22.40	21.23	22.32	21.16
	100; 35	23.14	22.45	21.17	21.97	20.79	21.84	20.68
Real Data II								
	90; 30	20.45	20.52	19.57	18.95	18.25	17.88	17.35
1	100; 30	20.09	20.10	19.17	18.50	17.83	17.43	16.95
	90; 35	18.68	19.18	18.33	17.78	17.14	16.84	16.37
	100; 35	18.33	19.05	18.14	17.51	16.86	16.47	16.01
	90; 30	20.36	20.47	19.50	18.96	18.23	17.87	17.33
$\frac{1}{\bar{x}_h}$	100; 30	19.96	20.03	19.08	18.49	17.80	17.41	16.92
	90; 35	18.63	19.18	18.29	17.81	17.15	16.85	16.36
	100; 35	18.25	19.03	18.09	17.53	16.85	16.47	15.99

__Table 6.__ Percentage Relative Efficiency (%RE) under Stratified Double Sampling.

Real Data I								
Q_h	**Sample Size (m; n)**	$\bar{y}_{tr.d}$	$\bar{y}_{m.d1}$	$\bar{y}_{ce.d1}$	$\bar{y}_{m.d2}$	$\bar{y}_{ce.d2}$	$\bar{y}_{m.d3}$	$\bar{y}_{ce.d3}$
	90; 30	100.00	106.66	112.97	108.58	114.72	108.71	114.87
1	100; 30	100.00	102.98	109.18	104.34	110.31	104.43	110.46
	90; 35	100.00	110.13	116.63	111.70	118.00	111.95	118.28
	100; 35	100.00	104.99	111.40	106.95	113.17	107.39	113.55
	90; 30	100.00	105.02	111.15	107.28	113.13	107.50	113.38
$\frac{1}{\bar{x}_h}$	100; 30	100.00	101.22	107.13	102.87	108.54	103.09	108.73
	90; 35	100.00	108.21	114.54	110.09	116.16	110.48	116.54
	100; 35	100.00	103.07	109.31	105.33	111.30	105.95	111.90
Real Data II								
	90; 30	100.00	99.66	104.50	107.92	112.05	114.37	117.87
1	100; 30	100.00	99.95	104.80	108.59	112.68	115.26	118.53
	90; 35	100.00	97.39	101.91	105.06	108.98	110.93	114.11
	100; 35	100.00	96.22	101.05	104.68	108.72	111.29	114.49
	90; 30	100.00	99.46	104.41	107.38	111.68	113.93	117.48
$\frac{1}{\bar{x}_h}$	100; 30	100.00	99.65	104.61	107.95	112.13	114.65	117.97
	90; 35	100.00	97.13	101.86	104.60	108.63	110.56	113.88
	100; 35	100.00	95.90	100.88	104.11	108.31	110.81	114.13

6. Conclusion

Three exponential-type ratio calibration estimators using first three moments about the origin of the auxiliary variable as well as two mean-type estimators using second and third moment about the origin have been suggested in this study under stratified random sampling and stratified double sampling. Tables 1 and Table 3 depict the percentage relative root mean squared error (%RRMSE) while Table 2 and Table 4 comprise the percentage relative efficiency (%RE) of the estimators given by Singh [22], Tracy et al. [24], and three suggested exponential-type estimators and two mean-type estimators for different values of sample size and Q_h. Table 1 and Table 3 shows the %RRMSE of the existing and the proposed estimators which depict that as the values of sample size increases, the values of %RRMSE decrease and hence the percentage relative efficiency (%RE) of the suggested estimators increases, specified in Table 2 and Table 4.

From the results given in Tables 1 to 4, it can be inferred that the efficiency of the suggested calibration estimators is higher than the estimators given by Singh [22] and Tracy et al. [24]. Hence, it can be concluded that the proposed calibration estimators are more efficient than the other estimators for estimating the population means based on a simulation study conducted on two real datasets under both sampling schemes.

Appendix

Beside comparing the proposed estimators with the existing estimators, the comparative study has also been done among the mean-type and the exponential-type estimators by calculating the percentage relative gain (%RG) of exponential-type estimators with respect to their mean-type estimators, as:

Table A1. *Percentage Relative Gain (%RG) of Exponential-type Estimators with respect to their Mean Estimators for Stratified Sampling* $\%RG(\bar{y}_{ce.\alpha})=\left(\frac{RRMSE(\bar{y}_{m\alpha})-RRMSE(\bar{y}_{ce.\alpha})}{RRMSE(\bar{y}_{m\alpha})}\right)\times 100;\ \alpha=1,2,3$.

Q_h	Real Data I Sample Size (n)	%RG ($\bar{y}_1$)	%RG ($\bar{y}_2$)	%RG ($\bar{y}_3$)	Real Data II %RG ($\bar{y}_1$)	%RG ($\bar{y}_2$)	%RG ($\bar{y}_3$)
1	30	7.40	7.05	7.09	5.85	4.00	2.34
	35	7.57	7.27	7.32	6.03	4.02	2.18
	40	6.58	6.17	6.09	4.94	3.37	1.86
$\frac{1}{\bar{x}_h}$	30	7.12	6.69	6.69	5.96	4.03	2.30
	35	7.40	6.97	6.99	6.18	4.17	2.13
	40	6.38	5.90	5.78	5.09	3.46	1.88

Table A2. *Percentage Relative Gain (%RG) of Exponential-type Estimators with respect to their Mean Estimators under Stratified Double Sampling* $\%RG(\bar{y}_{ce.d\alpha})=\left(\frac{RRMSE(\bar{y}_{m.d\alpha})-RRMSE(\bar{y}_{ce.d\alpha})}{RRMSE(\bar{y}_{m.d\alpha})}\right)\times 100;\ \alpha=1,2,3$ ..

Q_h	Real Data I Sample Size (m; n)	%RG ($\bar{y}_{d.1}$)	%RG ($\bar{y}_{d.2}$)	%RG ($\bar{y}_{d.3}$)	Real Data II %RG ($\bar{y}_{d.1}$)	%RG ($\bar{y}_{d.2}$)	%RG ($\bar{y}_{d.3}$)
1	90; 30	5.59	5.36	5.36	4.63	3.69	2.96
	100; 30	5.67	5.41	5.45	4.63	3.62	2.75
	90; 35	5.57	5.34	5.35	4.43	3.60	2.79
	100; 35	5.75	5.50	5.43	4.78	3.71	2.79
$\frac{1}{\bar{x}_h}$	90; 30	5.52	5.17	5.18	4.74	3.85	3.02
	100; 30	5.52	5.22	5.19	4.74	3.73	2.81
	90; 35	5.53	5.22	5.20	4.64	3.71	2.91
	100; 35	5.70	5.37	5.31	4.94	3.88	2.91

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