

Empirical Bayes One-side Test for Inverse Exponential Model Based on Negative Associate Samples

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Abstract: By using the kernel-type density estimation and empirical distribution function in the case of identically distributed and negatively associated samples, the empirical Bayes one-sided test rules for the parameter of inverse exponential distribution are constructed based on negative associate sample under weighted linear loss function, and the asymptotically optimal property is obtained. It is shown that the convergence rates of the proposed empirical Bayes test rules can arbitrarily close to $O(n^{-1/2})$ under suitable conditions.

Keywords: Empirical Bayes Test, Negatively Associated Sample, Asymptotic Optimality, Weighted Linear Loss Function, Inverse Exponential Distribution

1. Introduction

Empirical Bayes method, first proposed by Robbins [1], is a very important method in statistical inference field, and it has many applications in the field of reliability, lifetime prediction, medical research, ect. [2-6]. Empirical Bayes hypothesis test problem is a hot topic, which attracts many scholars' attention.

For example, Karunamuni [7] discussed the optimal convergence rate of empirical Bayes tests for uniform distributions. Xu et al. [8] investigated the convergence rates of empirical Bayes tests of parameters in one-side truncated distribution families under an asymmetric loss function. Qian and Wei [9] studied the two-sided empirical Bayes test for a class of scale exponential model under a weighed loss function. Chen et al. [10] studied the two-sided empirical Bayes test problem for the continuous one-parameter exponential model with contaminated data by a kernel probability density method. Under Type-II censored samples. Wang et al. [11] studied the empirical Bayes test problem for two-parameter exponential distribution. However, in many situations, such as penetration theory, reliability analysis and multivariable analysis, random samples are probably not independent identical distribution but negatively associated (NA) samples [12, 13]. For instance, Wei [14] developed a

empirical Bayes test of parameter for exponential distribution in the case of NA samples using the linear weighted loss functions. Shi et al. [15] developed a two-sided empirical Bayes test for the parameter of the truncated distribution family in the case of NA samples. Wang and Shi [16] studied the Bayes test problem for a special exponential family under NA samples.

The definition of the NA sequence was first introduced by Esary in [17] as follows:

Definition 1. A sequence of random variables X_1, X_2, \dots, X_n is said to be a NA sample, if for every pair of disjoint non-empty subsets A_1 and A_2 of set $\{1, 2, \dots, n\}$,

$$\text{Cov}(f_1(X_i, i \in A_1), f_2(X_i, i \in A_2)) \leq 0$$

Where f_1 and f_2 are increasing or decreasing for every independent variable, and their covariance exists. Random variable sequence $\{X_j, j \in N\}$ are said to be NA, if for every natural number $n \geq 2$, X_1, X_2, \dots, X_n is negatively associated.

Assume X is a random variable drawn form inverse exponential distribution with the probability density function is

$$f(x|\theta) = \frac{\theta}{x^2} \exp\left(-\frac{\theta}{x}\right), x > 0 \tag{1}$$

Here $\theta > 0$ is the unknown shape parameter, $\Omega = \{x | x > 0\}$ is the sample space, and $\Theta = \left\{ \theta > 0 \mid \int_{\Omega} f(x|\theta) dx = 1 \right\}$ is the parameters space.

Remark 2. It is easily shown that if the random variable X is distributed with exponential distribution, then X^{-1} is distributed with inverse exponential distribution.

The inverse exponential distribution is one of the most important lifetime distributions [18], and the applications in life test, reliability and statistical inference attract many authors' attention. For example, Prakash [19] discussed the Bayes estimation of the inverse exponential model under symmetric and asymmetric loss functions. Rao [20] studied the reliability estimation for the stress-strength model, which the stress and strength are both distributed with inverse exponential distribution. Singh et al. [21] developed a two-stage group acceptance sampling plan for a generalized inverse exponential distribution under truncated life test.

In this paper, we will first construct the empirical Bayes test function for testing Eq. (2) in inverse exponential model. Then, we will prove that the proposed test is asymptotically optimal and its convergence rate will also be derived under certain suitable conditions. The remains of this paper is organized as follows: Section 2 will establish a Bayes decision rule for one-side hypothesis test problem. Then Section 3 will put forward a new empirical Bayes decision rule combining the kernel-type density function with empirical distribution function. Finally, Section 4 will give the conclusion of this paper.

2. Bayes Test

Given the inverse exponential distribution with probability density function (1), the hypothesis needed to be tested is

$$H_0 : \theta \leq \theta_0 \leftrightarrow H_1 : \theta > \theta_0 \tag{2}$$

Here θ_0 is a given constant. This section will construct a new Bayes test rule for hypothesis test problem (2) under the following weighted linear loss functions:

$$L_0(\theta, d_0) = \frac{\theta - \theta_0}{\theta} I(\theta > \theta_0) \tag{3}$$

$$L_1(\theta, d_1) = \frac{\theta_0 - \theta}{\theta} I(\theta \leq \theta_0) \tag{4}$$

Here $d = \{d_0, d_1\}$ is the action space, d_0 indicates accepting H_0 , d_1 indicates refusing H_0 .

Remark 1. The weighted linear loss function has many advantages: it is invariant and makes the expression of Bayes test function more concise and makes the empirical Bayes

test function easy to construct.

Suppose that X has conditional probability density in Eq. (1) and parameter θ has the unknown prior distribution function $G(\theta)$, $g(\theta)$ is the corresponding prior probability density function. Then the marginal probability density function of X is:

$$f_G(x) = \int_{\Theta} f(x|\theta) dG(\theta) \tag{5}$$

Let $\delta(x) = P(\text{accept } H_0 | X = x)$ be the randomized decision rule. The Bayes risk function of $\delta(x)$ is

$$R(\delta(x), G(\theta)) = \int_{\Theta} \int_{\Omega} [L_0(\theta, d_0) f(x|\theta) \delta(x) + L_1(\theta, d_1) f(x|\theta) (1 - \delta(x))] dx dG(\theta)$$

Then

$$R(\delta(x), G(\theta)) = \int_{\Omega} \beta(x) \delta(x) dx + C_G \tag{6}$$

Where $C_G = \int_{\Theta} L_1(\theta, d_1) dG(\theta)$, and

$$\beta(x) = \int_{\Theta} \frac{\theta - \theta_0}{\theta} f(x|\theta) dG(\theta) \tag{7}$$

According to equation (5), we can get

$$\begin{aligned} \beta(x) &= \int_{\Theta} f(x|\theta) dG(\theta) - \theta_0 \int_{\Theta} \frac{1}{\theta} f(x|\theta) dG(\theta) \\ &= f_G(x) - \theta_0 \int_{\Theta} \frac{1}{\theta} f(x|\theta) dG(\theta) \end{aligned}$$

Define

$$P_G(x) = \int_{\Theta} e^{-\frac{\theta}{x^2}} dG(\theta), \tag{8}$$

then

$$\beta(x) = f_G(x) - x^{-2} \theta_0 P_G(x) \tag{9}$$

A test is said to be a Bayes test with respect to $G(\theta)$ if $R_G = \inf_{\delta} R(\delta(x), G(\theta))$.

Define

$$\delta_G(x) = \begin{cases} 1, & \beta(x) \leq 0 \\ 0, & \beta(x) > 0 \end{cases} \tag{10}$$

It is easy to show that $\delta_G(x)$ is a Bayes test with respect to $G(\theta)$.

The Bayes risk function is:

$$R_G = \int_{\Omega} \beta(x) \delta_G(x) dx + C_G \tag{11}$$

as

$$P_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i < x) \tag{13}$$

Remark 1. If the prior distribution $G(\theta)$ is known, $\delta(x)$ equals to $\delta_G(x)$, we can get R_G . Unfortunately the $G(\theta)$ is unknown, we need to use the empirical Bayes approach.

Therefore, the estimator of $\beta(x)$ is

$$\beta_n(x) = f_n(x) - x^{-2} \theta_0 P_n(x) \tag{14}$$

3. Empirical Bayes Test Function

Finally, the empirical Bayes test function could be defined as follows:

This section we will construct the empirical Bayes test function. Suppose that $X_1, X_2, \dots, X_n, X_{n+1}$ are identically distributed and weakly stationary NA sequence. The same probability density function is $f_G(x)$. X_1, X_2, \dots, X_n are the historical sample, X_{n+1} is the current sample. In this paper, we assume

$$\delta_n(x) = \begin{cases} 1, & \beta_n(x) \leq 0 \\ 0, & \beta_n(x) > 0 \end{cases} \tag{15}$$

(A1) $f_G(x) \in C_{s,\alpha}, x \in R^1$, where $C_{s,\alpha}$ denotes a family of probability density function with s order derivatives and its absolute value not exceeding α , α is a positive integer. $s \geq 2$ is a positive integer.

Assume E_n represents the mean value of the joint distribution of X_1, X_2, \dots, X_n , hence the comprehensive risk of $\delta_n(x)$ is

(A2) Suppose $s \geq 2$ is an arbitrarily certain natural number, $K_r(x) (r=0,1,\dots,s-1)$ is a bounded function of Borel measurable function. When outside of the interval $(0,1)$, their value are zero, and they satisfy the following conditions:

$$R(\delta_n(x), G(\theta)) = \int_{\Omega} \beta(x) E_n[\delta_n(x)] dx + C_G \tag{16}$$

$$\frac{1}{t!} \int_0^1 y^t K_r(y) dy = \begin{cases} 1, & t=r \\ 0, & t \neq r, t=0,1,2,\dots,s-1 \end{cases}$$

Definition 1. If $\lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) = R_G$, then the random variable sequence $\{\delta_n(x)\}$ is called asymptotic optimality empirical Bayes test functions.

Definition 2. If $R(\delta_n(x), G(\theta)) - R_G = O(n^{-q}), q > 0$,

Then we say the convergence rate of the test function sequence $\{\delta_n(x)\}$ is $O(n^{-q})$.

Define the Kernel density estimation of $f_G(x)$ as:

By Definitions 1 and 2, we conclude that the optimal evaluation of the empirical Bayes test functions depends on the degree of its risk approximation Bayes risk.

$$f_n(x) = \frac{1}{nb_n} \sum_{j=1}^n K_r\left(\frac{x - X_j}{b_n}\right) \tag{12}$$

Where $\{b_n\}$ is positive integer sequence, and $\lim_{n \rightarrow \infty} b_n = 0$.

In this paper, we let c, c_1, c_2, \dots denote different positive constants in different cases even in the same expression. In order to prove Theorems 1 and 2, which will be illustrated in Section 4, we need to present the following lemmas:

(A3) The covariance of $\{X_n | n \geq 2\}$ satisfies condition:

Lemma 1[22] Suppose R_G and $R(\delta_n(x), G(\theta))$ definite in equation (11) and equation (16) separately, then

$$\sum_{j=1}^{\infty} |cov(X_i, X_j)| \leq c < \infty$$

$$\begin{aligned} 0 &\leq R(\delta_n(x), G) - R_G \\ &\leq \int_{\Omega} |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) dx \end{aligned} \tag{18}$$

By equations (1) and (8), we can get

Lemma 2 [23] Suppose $f_n(x)$ definite as (13), where X_1, X_2, \dots are identically distributed and weakly stationary NA sequence. Assume both (A1)- (A3) hold, for $\forall x \in \Omega$

$$\begin{aligned} E[I(X_i < x)] &= \int_{-\infty}^x \left[\int_{\Theta} \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dG(\theta) \right] dx \\ &= \int_{\Theta} \left[\int_{-\infty}^x \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dx \right] dG(\theta) \\ &= \int_{\Theta} \left[\int_0^x \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dx \right] dG(\theta) \\ &= \int_{\Theta} [e^{-\frac{\theta}{x}}]_0^x dG(\theta) \\ &= P_G(x) \end{aligned}$$

(1) If $f_G(x)$ is continuous for x , when $\lim_{n \rightarrow \infty} b_n = 0$ and $\lim_{n \rightarrow \infty} nb_n = \infty$ hold, then

$$\lim_{n \rightarrow \infty} E_n |f_n(x) - f_G(x)|^2 = 0 \tag{19}$$

(2) If $f_G(x) \in C_{s,\alpha}$, let $b_n = n^{-\frac{1}{4+2s}}$, for $0 < \lambda \leq 1$, we have

Hence, the unbiased estimator of $P_G(x)$ can be constructed

$$E_n |f_n(x) - f_G(x)|^{2\lambda} \leq c \cdot n^{-\frac{\lambda s}{s+2}} \quad (20)$$

Lemma 3 Suppose $P_G(x)$ and $P_n(x)$ definite in equation (8) and equation (13) separately, X_1, X_2, \dots are identically distributed and weakly stationary NA sequence, for $0 < \lambda \leq 1$, we have

$$E_n |P_n(x) - P_G(x)|^{2\lambda} \leq n^{-\lambda}. \quad (21)$$

Proof. According to equation (14), we have

$$\begin{aligned} & E(P_n(x) - P_G(x))^{2\lambda} \\ &= E\{[P_n(x) - P_G(x)]^2\}^\lambda \\ &\leq [Var(P_n(x))]^\lambda \end{aligned}$$

Where

$$\begin{aligned} Var(P_n(x)) &= E\left\{\frac{1}{n} \sum_{i=1}^n [I(X_i < x) - P_G(x)]\right\}^2 \\ &= \frac{1}{n^2} \sum_{i=1}^n Var[I(X_i < x)] \\ &\quad + \frac{2}{n^2} \sum_{1 \leq i < j \leq n} Cov(I(X_i < x), I(X_j < x)) \\ &= Q_1 + Q_2 \end{aligned}$$

Because $I(X_i < x)$ is a decreasing about X_i , and X_1, X_2, \dots are identically distributed and weakly stationary NA sequence.

Then

$$\frac{2}{n^2} \sum_{1 \leq i < j \leq n} Cov(I(X_i < x), I(X_j < x)) \leq 0$$

Thus we have

$$\begin{aligned} Var(P_n(x)) &= Q_1 + Q_2 \\ &\leq Q_1 \\ &= \frac{1}{n^2} \sum_{i=1}^n Var[I(X_i < x)] \\ &= \frac{1}{n} Var[I(X_1 < x)] \\ &= \frac{1}{n} \{E[I^2(X_1 < x)] - [E(I(X_1 < x))]^2\} \\ &\leq \frac{1}{n} \end{aligned}$$

Hence, we have

$$\begin{aligned} & E_n |P_n(x) - P_G(x)|^{2\lambda} \\ &\leq \{E_n [P_n(x) - P_G(x)]^2\}^\lambda \\ &\leq n^{-\lambda} \end{aligned}$$

for any $0 < \lambda \leq 1$. The proof of Lemma 3 is completed.

4. Asymptotic Optimality of Empirical Bayes Test

Theorem 1. Suppose $\delta_n(x)$ be the empirical Bayes test function defined in equation (16), X_1, X_2, \dots are identically distributed and weakly stationary NA sequence, when (A1)-(A3) hold, if

$$(1) \{b_n\} \text{ is a positive sequence, and satisfy } \lim_{n \rightarrow \infty} b_n = 0, \lim_{n \rightarrow \infty} nb_n = \infty$$

$$(2) \int_{\theta} \frac{1}{\theta} dG(\theta) < \infty$$

$$(3) f_G(x) \text{ is continuous about } x$$

$$\text{Then } \lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) = R_G$$

$$\text{Proof. Let } Q_n(x) = |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|),$$

$$\text{Then } Q_n(x) \leq |\beta(x)|.$$

Apply (5) and Fubini theorem, we have

$$\begin{aligned} \int_{\Omega} |\beta(x)| dx &\leq \int_{\Omega} f_G(x) dx + |\theta_0| \int_{\Omega} \int_{\theta} \frac{1}{\theta} f(x|\theta) dG(\theta) dx \\ &\leq 1 + |\theta_0| \int_{\theta} \int_{\Omega} f(x|\theta) dx dG(\theta) \\ &= 1 + |\theta_0| \int_{\theta} \frac{1}{\theta} dG(\theta) < \infty \end{aligned}$$

Then by Lemma 1 and dominated convergence theorem, we have

$$0 \leq \lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) - R_G \leq \int_{\Omega} \lim_{n \rightarrow \infty} Q_n(x) dx \quad (22)$$

According to formulas (8) and (13), then by Markov and Jensen inequality, we have

$$\begin{aligned} Q_n(x) &\leq E_n |\beta_n(x) - \beta(x)| \\ &\leq E_n |f_n(x) - f_G(x)| \\ &\quad + |\theta_0 x^{-2}| E_n |P_n(x) - P_G(x)| \\ &\leq [E_n |f_n(x) - f_G(x)|^2]^{\frac{1}{2}} \\ &\quad + |\theta_0 x^{-2}| [E_n |P_n(x) - P_G(x)|^2]^{\frac{1}{2}} \end{aligned}$$

Then by lemma 2 and lemma 3, for $\forall x \in \Omega$, we have

$$0 \leq \lim_{n \rightarrow \infty} Q_n(x) \leq [\lim_{n \rightarrow \infty} E_n |f_n(x) - f_G(x)|^2]^{\frac{1}{2}} + |\theta_0 x^{-2} [\lim_{n \rightarrow \infty} E_n |P_n(x) - P_G(x)|^2]^{\frac{1}{2}}| = 0$$

Combining (19) with (20), the theorem is proved.

Theorem 2 Suppose $\delta_n(x)$ be the empirical Bayes test function defined in equation (16), X_1, X_2, \dots are identically distributed and weakly stationary NA sequence. Assume (A1) -(A3) hold, $f_G(x) \in C_{s,\alpha}$, $0 < \lambda < 1$ and $s \geq 2$ is the positive integer, and it satisfies the following conditions:

$$(B1) \int_{\Omega} |\beta(x)|^{1-\lambda} dx < \infty$$

$$(B2) \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} dx < \infty$$

Then with the choice of $b_n = n^{-\frac{1}{2s+1}}$, we have

$$R(\delta_n, G) - R_G = O(n^{-\frac{\lambda s}{2s+1}})$$

Proof. First, we can easily get the following result:

$$\begin{aligned} & P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) \\ &= \int_{|\beta_n(x) - \beta(x)| \geq |\beta(x)|} f(x | \theta) dG(\theta) \\ &\leq \int_{|\beta_n(x) - \beta(x)| \geq |\beta(x)|} \frac{|\beta_n(x) - \beta(x)|^{\lambda}}{|\beta(x)|^{\lambda}} f(x | \theta) dG(\theta) \\ &\leq \int_{\Omega} \frac{|\beta_n(x) - \beta(x)|^{\lambda}}{|\beta(x)|^{\lambda}} f(x | \theta) dG(\theta) \\ &= |\beta(x)|^{-\lambda} E |\beta_n(x) - \beta(x)|^{\lambda} \end{aligned}$$

Apply lemma 1 and Markov inequality, we conclude that

$$\begin{aligned} 0 &\leq R(\delta_n, G) - R_G \\ &\leq \int_{\Omega} |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) dx \\ &\leq \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |\beta_n(x) - \beta_G(x)|^{\lambda} dx \\ &\leq c_1 \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |f_n(x) - f_G(x)|^{\lambda} dx \\ &\quad + c_2 \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} E_n |P_n(x) - P_G(x)|^{\lambda} dx \\ &= A_n + B_n \end{aligned}$$

Where $A_n = c_1 \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |f_n(x) - f_G(x)|^{\lambda} dx$,

$$B_n = c_2 \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} E_n |P_n(x) - P_G(x)|^{\lambda} dx$$

By Lemma 2 and Lemma 3 and condition (B1), we have

$$A_n \leq c_1 n^{-\frac{\lambda s}{2s+1}} \int_{\Omega} |\beta(x)|^{1-\lambda} |\theta_0|^{\lambda} dx \leq c_3 n^{-\frac{\lambda s}{2s+1}} \tag{23}$$

$$B_n \leq c_2 n^{-\frac{\lambda}{2}} \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} dx \leq c_4 n^{-\frac{\lambda}{2}} \tag{24}$$

Then $0 \leq R(\delta_n, G) - R_G \leq c_3 n^{-\frac{\lambda s}{2s+1}} + c_4 n^{-\frac{\lambda}{2}}$

Finally it shows that

$$R(\delta_n, G) - R_G = O(n^{-\frac{\lambda s}{2s+1}})$$

Remark 2. From Theorems 1 to 2, we know that the new established empirical Bayes test function is asymptotically optimal and its convergence rates $O(n^{-\frac{\lambda s}{2s+1}})$ can arbitrarily close to $O(n^{-1/2})$ under suitable conditions. For positive associate (PA) random variables sequences, we can also use the similar method to construct the empirical Bayes test function and prove the asymptotic performance of the empirical Bayes test function.

Conflict of Interest

The author has declared that no conflict of interests exists.

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