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Human Fertility Behavior Through Birth Interval Models: Overview

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Abstract: Fertility is one of the responsible factors for the growth of human population. The demographers have given priority to understanding of the determinants of fertility through statistical techniques. Analytical models are suitable and appropriate tools and are widely used for better understanding of the phenomenon of the human fertility behavior. In other words, these models are useful in describing the action and interaction among various factors as well as for predicting the change in fertility behavior. The analytic models play an important role in estimation and interpretation of the fertility behaviors. In this paper, discuss the of birth intervals model based on realistic assumptions of human reproductive process, indirectly incorporating socio-cultural, bio-demographic factors, taboos and also use of contraceptive practices. In these derived models to describe the variation in the length of closed, forward, straddling and open birth interval with the realistic assumption that all the females are not exposed to the risk of conception immediately after the termination of post-partum amenorrhea (PPA) due to some factors or contraceptive practices. In these models, fecundability (λ) has been considered to be constant over the study period. The duration of time from the point of termination of PPA to the state of exposure has been taken as random variable which follows exponential distribution. The maximum likelihood estimation technique has been used for the estimation of parameter (λ) through different derived models.

Keywords: Fecundability, Birth Interval, Post Partum Amenorrhea (PPA), Foetal Wastage, Contraceptive Practices

1. Introduction

Fertility is one of the responsible factors of population change as well as the important positive force for the growth of human population. The demographers have given priority to understanding and define the human fertility behavior through statistical tools and techniques in proper manner. Fertility behavior is usually influenced by the interaction of a number of biological, socio-economic and demographic factors. It is now almost established that traditional socio-cultural practices affect fertility behavior in a complex manner. The social scientists and demographers have given priority to thorough study of the differentials and determinants of fertility through various mathematical and statistical modeling techniques. Within the physiological limits of human reproduction, it is estimated by a multiplicity of biosocial and demographic factors. The role of differentials in fertility has been reported by a number of researchers [1-3].

2. Some of the Main Biological Factors

Fecundability: It is defined as the probability that a non pregnant fecund woman will conceive in one unit of the time of the exposure to the risk of conception. The unit is taken as one month which is the length of a menstrual cycle.

Sterility: A female is said to be sterile if conception is impossible physiologically.

Foetal Wastage: A conception may not always result in a live birth. The outcome of the corresponding pregnancy may end in a spontaneous foetal death, an induced abortion and still birth.

Non Susceptible Period: This is the sum of the two parts; first, gestation period and second the interval after its termination and before the resumption of the ovulation, which is the known as post partum amenorrhea period.

There are two broad categories of the fertility model. First, the model which deals with the utilization of the data on point events like as conception, live births to women in a specified period of time. The second type of models utilizes the data on interval between the consecutive events. Both type of models have own usefulness as well as limitations. The present paper is associated with second type of model.

Mathematical models are very appropriate tools and are widely used for better understanding of the phenomenon of the complex process of human fertility behavior. These models are useful in describing inter relationship among various factors as well as for prediction of change in human fertility behavior. The importance of differentials in fertility has been reported by a number of researchers and demographers [1-3].

Probabilistic models are also suitable approach and are widely used by researcher for better understanding of the phenomenon of human fertility behavior due to involvement of probability. Probability models are useful in describing the relationship among various chance factors as well as for predicting the change in fecundability.

The stochastic models play an important and appropriate role in estimation and interpretation of the fertility parameters associated with time. Gini (1924) was the first in this area to initiate research in model construction, by introducing the concept of fecundability [4]. Sheps (1964) and Singh (1966) and others have given detailed discussions on the variables to be included in the model [5, 6]. Birth interval is a good index for current change in fertility behavior.

Various type of birth intervals discussed so far in the literature is:

First Birth Interval: The interval between marriages to first live birth. This interval gives the recent marital fertility performance.

Closed Birth Interval: The interval between two successive live births. This gives the actual fertility performance in between two successive birth as well as impact of PPA and temporary separation and impact of family planning.

Forward Birth Interval: The interval between survey date and the date of next live birth posterior to the survey date.

Straddling Birth Interval: Any closed birth interval that straddles the survey date.

Open Birth Interval: The interval between the dates of birth of last child to the date of the survey. This provides the latest fertility performance.

The present paper associated with the overview of derived models for the closed birth interval, open birth interval, forward birth interval and straddling birth interval based on realistic assumptions to explain the better situation understanding. Model of open birth interval defined as the duration of time between the survey points to last live birth. One of the limitations of closed birth interval is that it is defined only for those females who are capable of proceeding from i^{th} to $(i+1)^{th}$ birth. Thus, this closed birth interval is not defined for those females who either become sterile after i^{th} birth or deliberately avoid further births. This limitation is absent in open birth interval. Closed birth interval can provide estimate of sterility. Open birth interval relates to an event of the recent past. Therefore, estimate of fecundability through open birth interval is more reliable.

To describe the variation in the length of open birth interval a number of models and procedures to estimate the parameters involve. Various researchers discussed the distribution of open birth interval. Srinivasan (1966) has shown that a comparison of the distribution of open birth interval at successive point of time in a population [7]. Srinivasan (1966), reported that open birth interval may be considered as an index of fertility [7]. Sheps et al. (1970) derived some general distribution for open birth interval [8]. Singh and Yadav (1977), Singh et al. (1979) and Pandey (1981) derived the distributions of open birth interval for the parity specific as well as regardless of parity [9-11]. Singh et al. (1982) derived parity specific probability density function for open birth interval to females of specified marital duration on the assumption that conception rate and sterility changes with parity and there is one to one correspondence between conception and a live birth [12]. Mishra (1983) derived some continuous time models for open birth intervals for fixed parity and marital duration as well as regardless of parity incorporating the foetal wastage [13]. Bhattacharya et al. (1988) have derived a probability model to describe the length of interval between successive live births by taking different parametric form of risk function and one to one correspondence between conception and a live birth [14]. Singh (1989) derived time dependent model for inter live birth interval with finite exposure period by taking into account intrauterine mortality and a distribution for the non-susceptible period [15]. Singh (1989) also derived the probability model of open birth interval taking fecundability to be time dependent and some chance to foetal westage [15]. Singh (1992) derived analytical models for human fertility behavior and their applications with the consideration of socio cultural factors [16]. Mturi (1997) studied the determinants of birth intervals among non contracepting Tanzanian women [17]. Rao (2006) studied correlates of inter-birth interval and their implications of optional birth spacing strategies in Mozambique [18]. Recently, Singh et al. (2011) discussed the demographic and socio-economic determinants of birth interval dynamics [19]. Yadav et al. (2013) estimate the parity progression ratios from open and closed birth interval data [20].

However, the aforesaid models assume that females are exposed to the risk of conception immediately after the termination of PPA which is not realistic in the real situation. Because of the socio cultural factors or contraceptive practices, females may not be exposed to the risk of conception immediately after the termination of PPA. A significant proportion of females are not exposed to the risk of conception immediately after termination of PPA due to socio-cultural or contraceptive practices. Singh (2013) presented research article on closed birth interval and estimate the fecundability with this realistic concepts [21]. Singh (2014) and Singh (2015) estimate the fecundability through forward birth interval and straddling birth interval on the same realistic assumptions [22-23].

3. Concepts of Derived Probability Distribution Models and Assumptions

Due to the socio cultural factors or contraceptive practices, some females not exposed to the risk of conception immediately after the termination of PPA. Probability models has been derived for the estimation of fecundability based on assumptions of human reproductive process, indirectly incorporating socio, bio-demographic factors, taboos and use of contraceptive practices. In these models, fecundability (λ) and the duration of time from the point of termination of PPA to the state of exposure as random variable (μ) which follows exponential distribution. The maximum likelihood estimation technique has been used for the estimation of parameters λ and μ through derived model [16].

1. The female has led married life throughout the period of observation.
2. Let h be the constant duration of non- susceptibility associated with each live birth comprised of gestation and the period of PPA.
3. The duration of non- susceptibility after the termination of PPA which is caused by some social factors or use of contraceptive practices be a non negative random variable. Let the female after termination of her PPA will enter into susceptible state in a small interval $(t, t+\Delta t)$ is $\mu\ \Delta t + 0\ \Delta t$; $\mu > 0$, $\Delta t > 0$ and $t > 0$
4. Let the female who is susceptible to conception at time t will conceive in a small interval $(t, t+\Delta t)$ is $\lambda\ \Delta t + 0\ \Delta t$; $\lambda > 0$, $\Delta t > 0$ and $t > 0$
5. Let each conception results in to a live birth.

3.1. Closed Birth Interval Model

Closed birth interval or interval between two successive live births. The main importance of the closed birth interval is due to inclusion of amenorrhoiec period, temporary separation due to social taboos or use of contraceptives under the above assumptions. The interval between i^{th} and $(i+1)^{th}$ birth say T_i ($i \geq 1$) in the absence of risk of foetal wastage is the sum of four components.

a. z: the duration of PPA
b. y: duration of non- susceptible period caused by some social factors or use of contraception just after the termination of PPA.
c. x: waiting time from the date of entrance into the susceptible period to the first conception.
d. g: the gestation period.

Therefore, $T_i = z + y + x + g$

Under the above assumption the probability density function and the corresponding distribution function of the interval between i^{th} and $(i+1)^{th}$ birth say $f_i(t)$ and $F_i(t)$, will be as

$$f_i(t) = [\mu\lambda/(\mu-\lambda)]\ [e^{-\lambda(t-h)} - e^{-\mu(t-h)}] \text{ if } i \geq 1 \text{ and } t > h$$

$$F_i(t)= [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}] \text{ if } i \geq 1 \text{ and } t > h$$

Now the probability density function and probability distribution function i^{th} and $(i+1)^{th}$ birth who have given their $(i+1)^{th}$ birth during the first T years after i^{th} birth will be given as

$$f^*(t) = f(t)/F(t) \text{ if } h < t < T$$

According to above condition the probability density $f^*(t)$ and probability distribution function of present model of closed birth interval defined as

$$f^*(t) = [\mu\lambda/(\mu-\lambda)]\ [e^{-\lambda(t-h)} - e^{-\mu(t-h)}] / [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}] \text{ if } h < t < T$$

$$F^*(t) = [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}] / [1- \mu/(\mu-\lambda)\ e^{-\lambda(T-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(T-h)}] \text{ if } h < t \leq T$$

3.2. Forward Birth Interval Model

Forward Birth Interval is based on renewal theory, it is well known that the limiting forms of the probability density functions of backward recurrence time and forward recurrence time which are similar to open birth interval are identical on the assumption that the renewal densities do not change over time. Obviously, if renewal density change after some time (say survey point), the distribution of forward birth interval and open birth interval will not be identical, Pandey (1981) has derived the forward birth interval under the assumption that a family planning programme has been introduced in the population at the survey point and has obtained the expressions for mean and variance for different situations [11].

If the marital duration of the female is large enough the probability density function of forward birth interval which are similar to open birth interval regardless of parity is given as;

$$W(t) = [1- F(t)] / \mu^*$$

Where, F(t) and μ^* are the distribution function and mean length of the waiting time between two consecutive live births respectively. The expressions for F(t) and μ^* under the above mentioned real assumptions [16, 22];

$$F(t) = 1- [\{1/(\mu-\lambda)\}\{\mu\ e^{-\lambda(t-h)} - \lambda e^{-\mu(t-h)}] \text{ if } t > h$$

F(t) = 0 otherwise

$$\mu^* = h + 1/\mu + 1/\lambda$$

Substituting the value of F(t) and μ^*. Modified probability density function of forward birth interval

$$W(t) = 1/ [h + 1/\mu + 1/\lambda];\ 0 < t < h$$

$$= [\{1/(\mu-\lambda)\}\ \{\mu\ e^{-\lambda(t-h)} - \lambda\ e^{-\mu(t-h)}]/ [h + 1/\mu + 1/\lambda];\ t > h$$

and the corresponding distribution function is

$$W(t) = t / [h + 1/\mu + 1/\lambda];\ 0 < t \leq h = [h + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\}\ e^{-\lambda(t-h)} - \{\lambda/(\mu-\lambda)\}\ e^{-\mu(t-h)]} / (h + 1/\mu + 1/\lambda);\ t > h$$

The distribution derived above model implicitly continuance of observations for a long time after T which may not always feasible. Thus, if the study is terminated after time T_1 from T, the modified probability density function and corresponding distribution function will be derived as,

$w_{T1}^*(t) = w(t) / W(T1)$

$W(T_1) = \int[1- F(t)] / \mu^* dt$ Range 0 to T_1

$= [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(T_1-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(T_1-h)}] / (h + 1/\mu + 1/\lambda); t > h$

$w_{T1}*(t) = 1 / [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(T_1-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(T_1-h)}]; 0 < t \leq h$

$= [\{\mu/(\mu-\lambda)\} e^{-\lambda(t-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(t-h)}] / [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(T_1-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(T_1-h)}]$

If t> h and distribution function

$W_{T1}^* (t) = t / [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(T_1-h)} - \{\lambda/(\mu-\lambda)\} e^{\mu(T_1-h)}]; 0 < t \leq h$

$= [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(t-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(t-h)]} / [h_v + 1/\mu + 1/\lambda - \{\mu/(\mu-\lambda)\} e^{-\lambda(T_1-h)} - \{\lambda/(\mu-\lambda)\} e^{-\mu(T_1-h)]}$ if $t > h$

3.3. Straddling Birth Interval Model

Due to the socio cultural factors or contraceptive practices, some females not exposed to the risk of conception immediately after the termination of PPA. Probability model on successive live births has been derived for the estimation of fecundability based on assumptions of human reproductive process, indirectly incorporating socio, bio-demographic factors, taboos and use of contraceptive practices. In this model, fecundability (λ) and the duration of time from the point of termination of PPA to the state of exposure as random variable (μ) which follows exponential distribution. The maximum likelihood estimation technique has been used for the estimation of parameters λ and μ through derived model [16, 23].

T units at a distant time point, after Return Marriage (R. M.) or effective marriage, a sample is drawn and observed until the next birth occurs and the intervals straddling the date of survey are obtained. Further, it is assumed that all the females survive until the delivery of the next live birth. In this paper derived the probability model of straddling birth interval to females of larger marital duration under the above realistic assumptions. Female of marital duration T where T is distant time point since R. M. is observed. Then the probability density function of the straddling birth interval regardless of parity v (t) will be given as

$$v(t) = t\, f(t) / \mu^*$$

where:

f(t) = Probability density function of the waiting between two consecutive live births.

$f(t) = [\mu\lambda/(\mu-\lambda) \{e^{-\lambda(t-h)} - e^{-\mu(t-h)} \}]$

$\mu^* = [h + 1/\lambda + 1/\mu]$

$v(t) = t [\mu\lambda/(\mu-\lambda)] [e^{-\lambda(t-h)} - e^{-\mu(t-h)}] / [h + 1/\lambda + 1/\mu]; t > h$

and the corresponding distribution function,

$$V(t) = [h + 1/\lambda + 1/\mu - 1/(\mu- \lambda)\{ \mu(t + 1/\lambda) e^{-\lambda(t-h)} - \lambda (t + 1/\mu) e^{-\mu(t-h)}\}]/ [h + 1/\lambda + 1/\mu]; t>h$$

Consider the situation when the females are observed for a period, say T_1, starting from T. In this case all the females can not have a birth on or before the period of observation. Now, if we consider the straddling birth interval of those females who give their births during the period of observation T_1, the modified probability density function (p. d. f.) and corresponding distribution function will be,

$$v^*_{T1}(t) = \{t.\ f(t)\}/ F^*(T_1); t \leq T_1 = \{T_1.\ f(t)\}/ F^*(T_1); t > T_1$$

Where

$$F^*(T_1) = [h + 1/\lambda + 1/\mu - \{\mu\, e^{-(T1-h)} / \lambda(\mu-\lambda)\}+ \{\lambda\, e^{-(T1-h)} / \mu (\mu-\lambda)\}]$$

On the substituions of the value of $F^*(T_1)$ and f (t)

$v^*_{T1}(t) = t. [\mu\lambda/(\mu-\lambda) \{e^{-\lambda(t-h)} - e^{-\mu(t-h)} \}] / [h + 1/\lambda + 1/\mu - \{\mu\, e^{-(T1-h)} / \lambda(\mu-\lambda)\}+ \{\lambda\, e^{-(T1-h)} / \mu (\mu-\lambda)\}]; h < t \leq T_1$

$T_1. [\mu\lambda/(\mu-\lambda) \{e^{-\lambda(t-h)} - e^{-\mu(t-h)} \}] / [h + 1/\lambda + 1/\mu - \{\mu\, e^{-(T1-h)} / \lambda(\mu-\lambda)\}+ \{\lambda\, e^{-(T1-h)} / \mu (\mu-\lambda)\}]; t > T_1$

and the corresponding distribution function as

$$V^*_{T1}(t) = H_{11}(t) / F^* (T_1) ; h < t \leq T_1 = H_{12}(t) / F^* (T_1); t > T_1$$

Where;

$H_{11} = [h + 1/\lambda + 1/\mu - 1/(\mu- \lambda)\{ \mu(t + 1/\lambda) e^{-\lambda(t-h)} - \lambda (t + 1/\mu) e^{-\mu(t-h)}\}]$

$H_{12} = [h + 1/\lambda + 1/\mu - \mu/(\mu- \lambda)\{1/\lambda\, e^{-\lambda(T1-h)} + T.\ e^{-\lambda(t-h)}\} + \{\lambda/(\mu- \lambda)\{1/\mu\, e^{-\lambda(T1-h)} + T.\ e^{-\lambda(t-h)}\}$

If it is assumed that h takes value $h_1 < h_2 < \ldots\ldots.< h_q$ with respective proportions of females $b_1, b_2, \ldots\ldots\ldots, b_q$. The probability density function and probability distribution function of the derived model extend to,

$$v^{**}_{T1} = \sum b_v\, v(t/ h=h_v);\ v = 1, 2, \ldots\ldots q$$

$$V^{**}_{T1} = \sum b_v\, V(t/ h=h_v);\ v = 1, 2, \ldots\ldots q$$

3.4. Open Birth Interval Model

Suppose a homogeneous group of females of larger marital duration, says T, is sampled and their open birth intervals are recorded. The distribution of the open birth interval for such females is derived under the above realistic assumptions.

If the marital duration T of the female is large enough the probability density function of the open birth interval regardless of parity;

$$u(t) = [1 - F(t)] / \mu^*$$

Where:

F(t) and μ^* are the distribution function and mean length of the waiting between two consecutive live births respectively. The expressions for F(t) and μ^*

$F(t) = [1- 1/(\mu-\lambda) \{\mu e^{-\lambda(t-h)} - \lambda e^{-\mu(t-h)}\}]$ if $t > h$ = 0 otherwise

$$\mu^* = [h + 1/\lambda + 1/\mu]$$

Substitution of the value of F(t) and μ^*

$$u(t) = 1/(h + 1/\lambda + 1/\mu);\ 0 < t \le h$$

$$= [1- 1/(\mu-\lambda) \{\mu e^{-\lambda(t-h)} - \lambda e^{-\mu(t-h)}\}]/(h + 1/\lambda + 1/\mu);\ t > h$$

The distribution function corresponding to U(t) is

$U(t) = t / [h + 1/\lambda + 1/\mu];\ 0 < t \le h$

$= [h + 1/\lambda + 1/\mu - \{\mu/(\mu-\lambda)\}e^{-\lambda(t-h)} - \{\lambda/(\mu-\lambda)\}e^{-\mu(t-h)}] / [h + 1/\lambda + 1/\mu];\ t > h$

In the distribution derived so far it has been assumed that non susceptible period h associated with a live birth is constant for all the females. But in practice it is empirically observed that the duration of PPA varies from female to female while it may be assumed to be a constant. If it is assumed that h takes value $h_1 < h_2 < \ldots\ldots. < h_q$ with respective proportions of females $b_1, b_2, \ldots\ldots\ldots, b_q$. The probability density function and probability distribution function extend to

Probability density function and corresponding extended to

$$u^*(t) = \sum b_v\, u\, (t/h = h_v);\ v = 1, 2, \ldots., q$$

$$U^*\,(t) = \sum b_v\, U\, (t/h = h_v);\ v = 1, 2, \ldots., q$$

The parameter λ and μ through method of maximum likelihood estimate (MLE) with the help of large sample data are obtained [16].

4. Application

The applications of the derived models on real observed data taken from Demographic Survey of Varanasi Rural, India. As a close approximation in the estimates for the present surveyed population we have taken four point observed values of PPA eq. 3 months, 6 months, 12 months and 18 months with respective proportion of females b_1= 0.25, b_2=0.35, b_3=0.32 and b_4=0.08, such that $\sum b_v = 1$. Further, gestation period g is taken as 9 months (h_1=1.00, h_2=1.25, h_3=1.75, and h_4=2.25). The remaining two parameters of the model λ and μ are estimated through Maximum Likelihood Estimates [16, 22-24].

5. Results

The estimated values of λ and μ through closed birth interval, forward birth interval, straddling birth interval and open birth interval presented in Table-1. The variances of estimated λ and μ and co-variance in between estimated values of λ and μ are presented in the Table-1. These estimates of λ & μ are almost same. These above mentioned estimates have considered and based on the rate of entrance into the state of exposure after the termination of PPA. Thus, the derived probability models explain the fertility behavior of observed data satisfactorily well. Therefore, conclude that the derived model describes the real situations and provides the better estimates of human fertility behavior. The present estimated values also explain the real impact of family planning programmes and their effectiveness in community. The present model can explain fertility behavior in better manner in comparison to conventional methods especially for rural population.

Table 1. *Estimated values of parameters (λ and μ), variances and co-variances.*

Parameters	**Estimated Values**			
	Closed Birth Interval	**Forward Birth Interval**	**Straddling Birth Interval**	**Open Birth Interval**
λ	1.3783	1.1051	1.0558	1.0507
μ	2.7450	2.8410	5.7511	2.9810
V(λ)	0.1336	0.0670	0.0017	0.0028
V(μ)	1.4430	0.7900	0.3027	0.9206
Cov(λ, μ)	0.4198	-0.0260	-0.0142	0.0026

6. Summary and Conclusion

Human fertility behavior is responsible for population growth and it is also for infant mortality. These factors are highly responsible for development of nation and puts extra pressure on economy of nation. Cognizant of these inherent problems, researcher, scientist and demographers have given high priority to a thorough understanding of the differential and determinants of fertility through mathematical and statistical methodologies.

The estimate of λ obtained by Mishra, 1983 were quite low as compared to estimates of Western countries probably due to the various social and cultural factors affecting human fertility in the rural parts of India viz., the frequent visits to females to their parents in their early marital life, practices of prolonged lactation, joint family system and various other social taboos [13]. The estimate of λ is quite high compared to estimates obtained previous estimate λ = 0.79 and in another research study λ = 0.78 [13, 26-27]. The previous studies as mentioned above have not considered the rate of entrance into the state of exposure after the termination of PPA. Srivastava (1989), Bhardwaj (1989) reported same pattern of fecundability [28-29]. However, the present derived model can explain and gives the better understanding of change in complex human fertility behavior. Derived model also helps to assessing the real impact of family planning programmes and their effectiveness. Therefore, derived models are explained the human fertility behavior in better manner and provides the estimate of the parameters with insignificant variance.

The high estimate of λ is may be due to improvement in the standard of living and also as an impact of urbanization. Further, with passage of time probably role of social customs and rituals has declined in last decades. Therefore, these models explained in better manner and provide the estimates of the parameters and derived models are more useful for rural and sub-urban localities.

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