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Actuarial Analysis of Single Life Status and Multiple Life Statuses

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Abstract: Actuaries frequently employ probability models to analyse situations involving uncertainty. They are also not simply interested in modelling the future states of a subject but also model cash flows associated with future states. This study compared single life status and multiple life statuses using life functions. The expected time until death, annuity payments, insurance payable and premiums were estimated using age as a risk factor. The analysis also employed the De Moirve's law on mortality in estimating the rate of mortality. The analysis revealed that, the expected time until death for single life status and multiple life statuses are all increasing functions of age. It was realized also that, the premium for single life status was increasing with age and the same with multiple life statuses. But the premium for single life was higher than multiple life statuses. In the case of the multiple life statuses, it was revealed that, premium for joint life was higher than the last survivor and that a change in the interest rate or force of interest and the benefit did not changed the trend in premium payments.

Keywords: Single Life Status, Multiple Life Statuses, Annuity, Insurance and Premium

1. Introduction

Actuarial practice indorses that its scientific base is extensively applicable in life insurance. Therefore actuaries have established a large range of models and varieties of methods and techniques in order to carry out professed actuarial calculations. One of the most important reasons for actuarial modelling is to introduce reliable methods for the practical pricing of insurance contracts, i.e. for the calculation of premium, which the insured life should pay to the insurer, so that the latter will pay his or her next-of-kin the insured amount on the occurrence of the insured event. Another actuarial calculation is the valuation of an insurance contract, thus the determination of its value during the lifetime of the contract; insurance reserve, for which special requirements apply with regard to how the insurer can invest the assets backing it and which forms the base for assessing the creditworthiness of the insurer; its ability to meet its liabilities now and in the future. A traditional assumption in the theory of multiple life contingencies is that the remaining life times of the lives involved are mutually independent. Computational feasibility rather than practicality seems to be the main reason for making this assumption. Such effects may have a significant influence on present values related to multiple life actuarial functions.

[8] and [3] showed alternative ways of modelling dependence of times of death of coupled lives. They released a significant degree of positive correlation between lifetimes. This implies that, joint life annuities were under-priced while last survivor annuities are over-priced. [2] presented boundaries of single premiums for last survivor annuities. [7] and [6] studied bounds of single premiums. These studies showed the impact of dependency of two remaining lifetimes on the pricing of life insurance products on the lives concerned. Dependency, however, also affected the valuation of such contracts over time. The reserves were based on laws of mortality which apply to the policy valuation date. If the remaining lifetimes of a couple are dependent at the outset of a policy, then any of the two lives' survival probabilities may depend on the life status of the partner. Moreover, the joint

distribution of remaining lifetimes, given the survival of both partners to a certain date, is affected as well. [16] showed that a lot of well-known relationships between probabilities and single premiums in multiple life contingencies are not valid in case of dependent lifetimes. They established that, the validity of those relationships can be restored if the definition of individual survival probabilities allows for the life status of the partner.

Standard actuarial theory of multiple life insurance assume the independence of the future lifetimes of the insured lives. This may occur when a policy is issued to a married couple. Numerous clinical studies have showed that broken heart syndrome may cause an increase in the mortality rate after the death of a spouse [14] and [11]. Also in insurance and annuities, multiple life model play an important role and the application of multiple life actuarial models are common. The investment income from a fund can be paid to a group of beneficiaries as long as at least one of the group survives [1].

[10] extended the classical analysis of the endowment contract on a single life to multiple lives, covering the joint-life and the last survivorship status. The results indicated that, the independence assumption overestimated the joint-life net single and level premiums and underestimates the last survivor net single and level premiums. [15] investigated the condition under which multiple life models can be replaced by single life models, they proved that in a survivorship group, the force of mortality of the group must follow Gompertz law provided that, for the joint-life status of very two lives, one can find a single-life status whose time until-death's distribution is equal the joint-life status. Hence, the assumption that, the force of mortality follows Gompertz's law is the necessary and sufficient criterion to guarantee that every joint-life status survival pattern can be replaced by a single-life status in the group.

[4] used the Frechet-Hoeffding bounds and Norberg's Markov model in determining the effect of dependence of future lifetimes of couples with much emphases on the actuarial present values of widow's pension benefit. Their results showed an economically significant positive dependence between joint lives: in Norbergs model, the amount of premium were reduced approximately 10% compared to the standard model that assumes independence. Also, [5] showed that the effect of a possible dependence was rather moderate for classical multiple life contracts at about 5%.

Moreover, other models can be used to incorporate dependencies between life times, for example, the frailty models described by [13] or Markov models as described by [12]. [9] re-investigated joint mortality functions and the assertion that relates the joint life and last survivor random variables. They realized that the common assertion that the sum of the lifetime of joint life and last survivor were equal to the sum of the lifetimes of the single statuses was true and modified the definition of the statuses so that this common assertion holds. They used copula model to indicate that the life insurance premiums with spousal status classification are lower than those without the classification and that the percentage differences are higher for older spouses and for higher interest rates.

The purpose of this paper is investigate the pricing of insurance for single-life status and multiple-life statuses using age as a risk factor. This is to provide insurers with the concept of insurance pricing using age as a risk factor and to give the insured information on the nature premium or annuity payments to be made pertaining to age.

2. Materials and Methods of Analysis

2.1. Data Source

This study employed ages of Ghanaians with reference from the mortality table of the Indian Institute of Actuaries.

2.2. Methods of Data Analysis

The mortality table is a tool which in a practical way represents a model of mortality and belongs to the basic mathematical toolbox in life insurance. Most often used in life insurance are complete mortality tables, which contain separate figures for each integer age$x \in \{0,1,2,\dots,\omega\}$, where ω is the assumed maximum age, which someone may attain.

2.2.1. Single Life Theory

De Moivre's law states that for all ages x such that $0 \leq x < \omega$, the expected number of survivors at age x and constant force of mortality are given by equation (1) and (2);

$$l_x = \omega - x \tag{1}$$

where, l_x is the expected number of survivors at age x, ω is the terminal age and x is the attained age.

$$\mu(x) = \frac{1}{\omega-x} \tag{2}$$

The survival function is given by;

$$S_x(t) = \frac{\omega-(x+t)}{\omega-x} \tag{3}$$

where ω is the terminal age.

The expected time until first death is given by;

$$e^o{}_x = \int_0^{\omega-x} S_{T(x)}(t)dt \tag{4}$$

where $S_{T(x)}(t)$ is the survival at time t.

The Actuarial Present Value (APV) is given by;

$$A_x = \frac{a_{\overline{\omega-x}|i}}{\omega-x} \tag{5}$$

where $a_{\overline{\omega-x}|} = \frac{1-(1+i)^{-n}}{i}$ is the annuity payment and i is the interest rate.

The annuity-insurance relation is given by;

$$\ddot{a}_x = \frac{1-A_x}{d} \tag{6}$$

where $d = \frac{i}{1+i}$ is the discounting factor and A_x is the APV.

The premium is given by;

$$P = \frac{bA_x}{\ddot{a}_x} \tag{7}$$

where b is the benefit, A_x is the APV and $\ddot{a}_x$ is the annuity.

For a continuous whole life insurance, the APV using a constant force model is given by;

$$\bar{A}_x = \frac{\mu}{\mu+\delta} \tag{8}$$

where μ is the constant force of mortality and δ is the force of interest. The annuity for a continuous whole life insurance is given by;

$$\bar{a}_x = \frac{1}{\mu+\delta} \tag{9}$$

where μ is the constant force of mortality and δ is the force of interest.

2.2.2. Multiple Life Theory

In multiple life theory consisting of two lives, the joint probability distribution of $(T(x), T(y))$ can be in two distinct forms; namely

a Joint Life Status

The status is said to fail at the first time of failure of one of the component lives or fails upon the death of one of the component lives. The waiting time T_1 until failure of the status is given by;

$$T_1 = \min\{T(x), T(y)\} \text{ or } T(x, y) = \min\{T(x), T(y)\}$$

The status survives t years from now if $T_1 > t$.

b Last Survivor Status

The status is said to fail at the last time of failure of the component lives or fails upon the death of the last component lives. The waiting time T_2 until failure of the status is given by;

$$T_2 = \max\{T(x), T(y)\} \text{ or } T(\overline{x, y}) = \max\{T(x), T(y)\}$$

This status fails within the next t years if $T_2 \leq t$ (both lives have died in t years). The status is surviving in t years if $T_2 > t$ (second death has not occurred by time t).

The Joint Life Status Force of Mortality Function with Independent Lives is given by;

$$\mu_{xy}(t) = \mu(x+t) + \mu(y+t) \tag{10}$$

where $\mu(x+t)$ is the force of mortality of a person aged $x+t$ and $\mu(y+t)$ is the force of mortality of a person aged $y+t$.

2.2.3. Expected Time Until Death for Multiple Life Statuses

The expected time until first death of the component lives is given by;

$$e^o{}_{xy} = \frac{1}{\mu_1+\mu_2} \tag{11}$$

where μ_1 is the constant force of mortality for (x) and μ_2 is the constant force of mortality for (y)

The expected time until death of the last survivor is given by;

$$e^o{}_{\overline{xy}} = e^o{}_x + e^o{}_y - e^o{}_{xy} \tag{12}$$

where $e^o{}_x$ and $e^o{}_y$ are the expected time until death for (x) and (y) respectively and $e^o{}_{xy}$ is the expected time until first death of the component lives.

2.2.4. Insurance for Multiple Life Statuses

For a continuous whole life insurance for joint lives, the APV using a constant force model is given by;

$$\bar{A}_{x:y} = \frac{\mu_1+\mu_2}{\mu_1+\mu_2+\delta} \tag{13}$$

where μ_1 is the constant force of mortality for (x) and μ_2 is the constant force of mortality for (y) and δ is the force of interest.

For a continuous whole life insurance for the last survivor, the APV using a constant force model is given by;

$$\bar{A}_{\overline{x:y}} = \bar{A}_x + \bar{A}_y - \bar{A}_{x:y} \tag{14}$$

2.2.5. The Annuities for Multiple Life Statuses

The Actuarial Present Value (APV) for Joint Life Annuity is given by;

$$\bar{a}_{x:y} = \frac{1}{\mu_1+\mu_2+\delta} \tag{15}$$

where μ_1 is the constant force of mortality for (x) and μ_2 is the constant force of mortality for (y) and δ is the force of interest.

The APV of the last survivor annuity is given by;

$$\bar{a}_{\overline{x:y}} = \bar{a}_x + \bar{a}_y - \bar{a}_{x:y} \tag{16}$$

where $\bar{a}_x$ and $\bar{a}_y$ are the annuities for a continuous whole life insurance of (x) and (y), $\bar{a}_{x:y}$ is APV for Joint Life Annuity.

2.2.6. Premium for Multiple Life Statuses

The premium for Joint Life is given by;

$$P = \frac{b\bar{A}_{x:y}}{\bar{a}_{x:y}} \tag{17}$$

where b is the benefit, $\bar{A}_{x:y}$ is the APV for the continuous whole life insurance for Joint life and $\bar{a}_{x:y}$ is the APV for Joint Life Annuity.

The premium for Last survivor is given by;

$$P = \frac{b\bar{A}_{\overline{x:y}}}{\bar{a}_{\overline{x:y}}} \tag{18}$$

where b is the benefit, $\bar{A}_{\overline{x:y}}$ is the APV for the continuous whole life insurance for the last survivor and $\bar{a}_{\overline{x:y}}$ is the APV of the last survivor annuity.

3. Results and Discussion

From figure 1, the mortality rate was seen as an increasing function of age. There was a smaller mortality rate from age 1 to age 49 but with an increasing mortality rate from age 50 upwards indicating that mortality rate is an increasing function of age. From age 80, the mortality rate raises to 1 in probability indicating that the chances of death is higher as one approaches the terminal age and as such requires higher premium payments. The expected time until death was seen as a decreasing function of age. At age 1, the expected time until death was almost the same as the terminal age. Also the expected time until death was almost zero at age 80 upwards since it was approaching the terminal age. The insurance payment gradually increases with age, thus from age 1 there was insurance payment which was lesser than payments at age 65 upwards. Also as ones age increases, the premium payments also increases. The annuity payments were seen as a decreasing function of age, meaning once the age goes up, the regular payments reduces. Thus the premium at age 1 to age 10 were quite lesser than premiums thereafter. It could be seen that from age 12 upwards the premium was increasing with age and that approaching the terminal age the premium was almost closer to the benefit. This indicates that, in pricing life insurance age plays a crucial role in determining the premium payments for the insurance coverage in that the higher ones age, the higher the premium and vice versa when other risk factors are held constant.

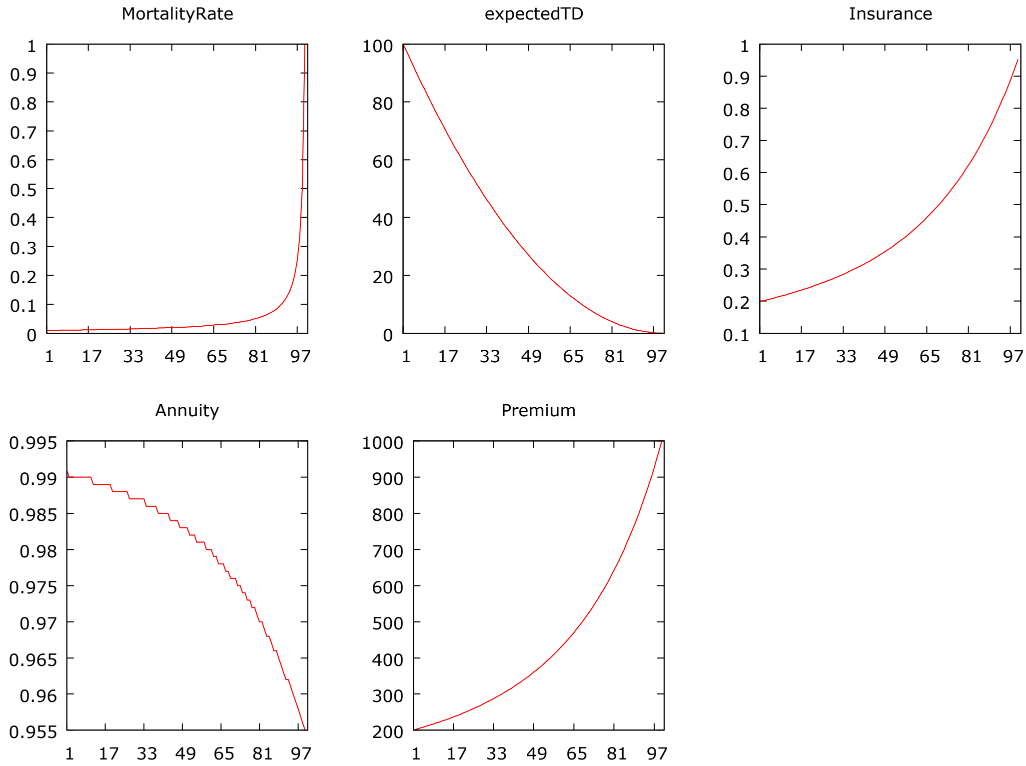

Figure 1. *A graph of Mortality, Expected time until death, insurance, annuity and premium for Single Life.*

Table 1, shows the estimates for Single Life Status. The constant force of mortality, from age 0 to 4 was constant at 0.010 rate per death, age 5 to 13 have 0.011 rate per death indicating a 0.001 increase from the previous cohort (age 0 to 4). Age 14 to 19 have 0.012 rate per death with age 20 to 25 having 0.013 rate per death. Also, age 26 to 31 have 0.014 rate per death. Age 32 to 35 have 0.015 rate per death with age 36 to 39 having 0.016 rate per death. A mortality rate difference of 0.001 existed for the cohorts (age 40 to 42, age 43 to 45, age 46 to 48, age 49 to 51, age 52 to 53, age 54 to 55, age 56 to 57 and age 58 to 59) respectively from the previous cohort (age 36 to 39) indicated that the mortality rate for each cohort was the same. From age 60 to 99, the mortality rate kept increasing until it was 1.000 at age 99. At the terminal age, there is an assumption that no person reaches that age and thus there will not be any mortality at that age. All these indicated that according to De Moivres Law, mortality is an increasing function of age. The expected time until death of each individual age was not the same, thus decreasing from time 100 years for age 0 to time 0 years for age 100. This indicates that, considering the age of an individual with all other perils held constant, the expected time until death was a decreasing function of age. The Actuarial Present Value (APV) for the individual lives was also not the same and thus shows an increasing function of age (from 0.198 for age 0 to 0.952 for age 99) since the pricing of life insurance takes into consideration the age of the individual. The annuity payments are decreasing function of age thus 0.991 for age 0 to 0.955 for age 99. For an individual to receive a benefit of GH¢ 1000 with an interest rate of 5%, then premium payment tends to be an increasing function of age, from GH¢ 200.373 for age 0 to GH¢ 997.625 for age 99. But these premium estimates will still be increasing with different interest rates and benefits when all other perils aside attain age are held constant. This shows that, no matter the interest and benefits, an insured for a life policy will have a high premium to pay when the age is high and vice versa. Also insurers of life products must consider the age insured to be able to apply the required premium payments.

Table 1. Estimates for Single Life Actuarial Functions.

Age	i = 5% and b = 1000				
x	$\mu(x)$	e^o_x	A_x	$\ddot{a}_x$	Premium (P)
0	0.010	100.000	0.198	0.991	200.373
1	0.010	98.010	0.200	0.990	202.338
2	0.010	96.040	0.202	0.990	204.340
3	0.010	94.090	0.204	0.990	206.379
4	0.010	92.160	0.206	0.990	208.457
5	0.011	90.250	0.208	0.990	210.574
6	0.011	88.360	0.211	0.990	212.731
7	0.011	86.490	0.213	0.990	214.930
8	0.011	84.640	0.215	0.990	217.172
9	0.011	82.810	0.217	0.990	219.457
10	0.011	81.000	0.219	0.990	221.787
11	0.011	79.210	0.222	0.989	224.164
12	0.011	77.440	0.224	0.989	226.588
13	0.011	75.690	0.227	0.989	229.060
14	0.012	73.960	0.229	0.989	231.583
15	0.012	72.250	0.232	0.989	234.156
16	0.012	70.560	0.234	0.989	236.783
17	0.012	68.890	0.237	0.989	239.464
18	0.012	67.240	0.239	0.989	242.200
19	0.012	65.610	0.242	0.988	244.994
20	0.013	64.000	0.245	0.988	247.847
21	0.013	62.410	0.248	0.988	250.760
22	0.013	60.840	0.251	0.988	253.736
23	0.013	59.290	0.254	0.988	256.775
24	0.013	57.760	0.257	0.988	259.881
25	0.013	56.250	0.260	0.988	263.054
26	0.014	54.760	0.263	0.987	266.297
27	0.014	53.290	0.266	0.987	269.612
28	0.014	51.840	0.269	0.987	273.001
29	0.014	50.410	0.273	0.987	276.465
30	0.014	49.000	0.276	0.987	280.008
31	0.014	47.610	0.280	0.987	283.632
32	0.015	46.240	0.283	0.987	287.339
33	0.015	44.890	0.287	0.986	291.131
34	0.015	43.560	0.291	0.986	295.011
35	0.015	42.250	0.295	0.986	298.983
36	0.016	40.960	0.299	0.986	303.047
37	0.016	39.690	0.303	0.986	307.208
38	0.016	38.440	0.307	0.985	311.469
39	0.016	37.210	0.311	0.985	315.832
40	0.017	36.000	0.315	0.985	320.300
41	0.017	34.810	0.320	0.985	324.877
42	0.017	33.640	0.324	0.985	329.567
43	0.018	32.490	0.329	0.984	334.373
44	0.018	31.360	0.334	0.984	339.297
45	0.018	30.250	0.339	0.984	344.346
46	0.019	29.160	0.344	0.984	349.521
47	0.019	28.090	0.349	0.983	354.828
48	0.019	27.040	0.354	0.983	360.270
49	0.020	26.010	0.360	0.983	365.852
50	0.020	25.000	0.365	0.983	371.579
51	0.020	24.010	0.371	0.982	377.455
52	0.021	23.040	0.377	0.982	383.485
53	0.021	22.090	0.383	0.982	389.674
54	0.022	21.160	0.389	0.981	396.027
55	0.022	20.250	0.395	0.981	402.551
56	0.023	19.360	0.401	0.981	409.250
57	0.023	18.490	0.408	0.981	416.130
58	0.024	17.640	0.415	0.980	423.198
59	0.024	16.810	0.422	0.980	430.460
60	0.025	16.000	0.429	0.980	437.923
61	0.026	15.210	0.436	0.979	445.593
62	0.026	14.440	0.444	0.979	453.477
63	0.027	13.690	0.452	0.978	461.584
64	0.028	12.960	0.460	0.978	469.920
65	0.029	12.250	0.468	0.978	478.494
66	0.029	11.560	0.476	0.977	487.314
67	0.030	10.890	0.485	0.977	496.388
68	0.031	10.240	0.494	0.976	505.726
69	0.032	9.610	0.503	0.976	515.337
70	0.033	9.000	0.512	0.976	525.231
71	0.034	8.410	0.522	0.975	535.418
72	0.036	7.840	0.532	0.975	545.908
73	0.037	7.290	0.542	0.974	556.712
74	0.038	6.760	0.553	0.974	567.842
75	0.040	6.250	0.564	0.973	579.310
76	0.042	5.760	0.575	0.973	591.127
77	0.043	5.290	0.586	0.972	603.308
78	0.045	4.840	0.598	0.972	615.865
79	0.048	4.410	0.611	0.971	628.812
80	0.050	4.000	0.623	0.970	642.165
81	0.053	3.610	0.636	0.970	655.937
82	0.056	3.240	0.649	0.969	670.146
83	0.059	2.890	0.663	0.968	684.807
84	0.063	2.560	0.677	0.968	699.937
85	0.067	2.250	0.692	0.967	715.556
86	0.071	1.960	0.707	0.966	731.681
87	0.077	1.690	0.723	0.966	748.332
88	0.083	1.440	0.739	0.965	765.529
89	0.091	1.210	0.755	0.964	783.295
90	0.100	1.000	0.772	0.963	801.650
91	0.111	0.810	0.790	0.962	820.619
92	0.125	0.640	0.808	0.962	840.226
93	0.143	0.490	0.827	0.961	860.497
94	0.167	0.360	0.846	0.96	881.457
95	0.200	0.250	0.866	0.959	903.134
96	0.250	0.160	0.886	0.958	925.559
97	0.333	0.090	0.908	0.957	948.761
98	0.500	0.040	0.930	0.956	972.771
99	1.000	0.010	0.952	0.955	997.625
100		0.000			

Also from figure 2, the mortality rate was seen as increasing function of age for the joint life status. From age 1 to 49, the mortality rate was constant until age 50 when it kept increasing all through to the terminal age. The expected time until death for both joint life (ExpectedTDJL) and last survivor (ExpectedTDLS) are seen as a decreasing function of age. But the expected time until death for the last survivor status was higher than the joint life status, and that the expected time until death does not hit zero for the last survivor as it does for the joint life. The insurance payable for both the joint life (InsuranceJL) and last survivor (InsuranceLS) were all increasing function of age but the joint life has a higher insurance payment than the last survivor. The annuity payment was seen as a decreasing function of age with the last survivor (AnnuityLS) having a higher annuity payment than the joint life (AnnuityJL). Nevertheless, the premium payment was seen an increasing function of age. The premium for joint life (PremiumJL) was higher than that of the last survivor (PremiumLS). This indicates that, in considering two lives for life policy, the ages of the two matters in that the age of either life can easily influence the premium payment especially in the case of joint life statuses.

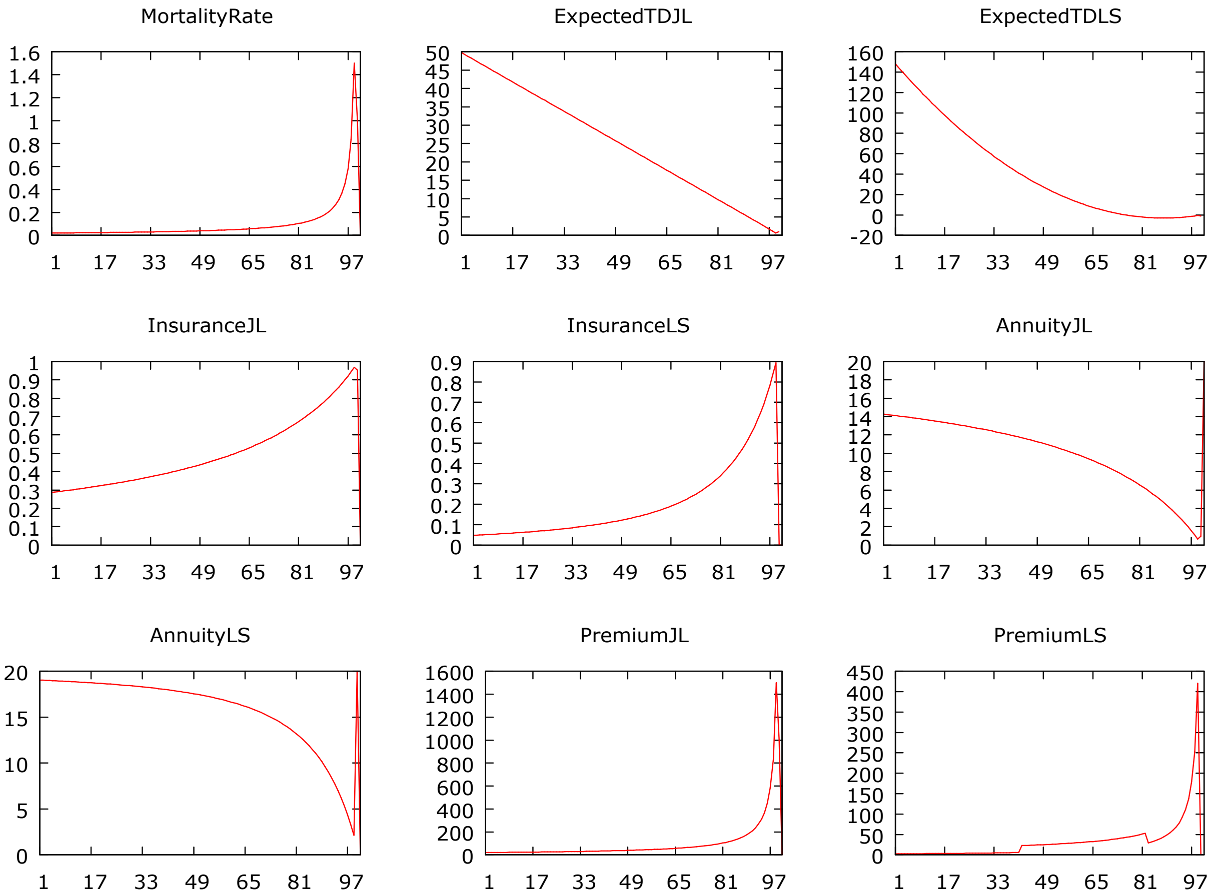

Figure 2. *A graph of Mortality Rate, Expected time until death (Joint Life and Last Survivor), Insurance (Joint life and Last survivor), Annuity (Joint life and Last survivor) and Premium (Joint life and Last survivor).*

Table 2. *Estimates for Multiple Life Actuarial Functions.*

Age	**δ = 5% and benefit = 1000**								
x	μ_{xy}	e^{o}_{xy}	$e^{o}_{\overline{xy}}$	$\bar{A}_{xy}$	$\bar{A}_{\overline{xy}}$	$\bar{a}_{xy}$	$\bar{a}_{\overline{xy}}$	**Premium (P) xy**	**Premium (P)** $\overline{xy}$
0	0.020	49.749	148.261	0.287	0.048	14.265	19.040	20.101	2.520
1	0.020	49.249	144.801	0.289	0.049	14.224	19.025	20.305	2.562
2	0.021	48.749	141.381	0.291	0.050	14.182	19.010	20.513	2.605
3	0.021	48.249	138.001	0.293	0.050	14.139	18.994	20.726	2.649
4	0.021	47.749	134.661	0.295	0.051	14.096	18.978	20.943	2.694
5	0.021	47.249	131.361	0.297	0.052	14.052	18.961	21.165	2.740
6	0.021	46.749	128.101	0.300	0.053	14.007	18.944	21.391	2.787
7	0.022	46.249	124.881	0.302	0.054	13.962	18.927	21.622	2.836
8	0.022	45.749	121.701	0.304	0.055	13.916	18.909	21.859	2.886
9	0.022	45.249	118.561	0.307	0.055	13.870	18.890	22.100	2.937
10	0.022	44.749	115.461	0.309	0.056	13.822	18.872	22.347	2.990
11	0.023	44.249	112.401	0.311	0.057	13.774	18.852	22.600	3.044
12	0.023	43.749	109.381	0.314	0.058	13.725	18.833	22.858	3.099
13	0.023	43.249	106.401	0.316	0.059	13.676	18.812	23.122	3.157
14	0.023	42.749	103.461	0.319	0.060	13.625	18.792	23.393	3.215
15	0.024	42.249	100.561	0.321	0.061	13.574	18.770	23.669	3.276
16	0.024	41.749	97.701	0.324	0.063	13.522	18.748	23.953	3.338
17	0.024	41.248	94.882	0.327	0.064	13.469	18.726	24.243	3.403
18	0.025	40.748	92.102	0.329	0.065	13.415	18.703	24.541	3.469
19	0.025	40.248	89.362	0.332	0.066	13.361	18.679	24.846	3.537
20	0.025	39.748	86.662	0.335	0.067	13.305	18.654	25.158	3.607
21	0.025	39.248	84.002	0.338	0.069	13.249	18.629	25.479	3.679
22	0.026	38.748	81.382	0.340	0.070	13.191	18.603	25.808	3.754
23	0.026	38.248	78.802	0.343	0.071	13.133	18.577	26.145	3.831
24	0.026	37.748	76.262	0.346	0.073	13.073	18.549	26.491	3.910
25	0.027	37.248	73.762	0.349	0.074	13.013	18.521	26.847	3.992
26	0.027	36.748	71.302	0.352	0.075	12.951	18.492	27.212	4.077
27	0.028	36.248	68.882	0.356	0.077	12.889	18.462	27.588	4.164
28	0.028	35.748	66.502	0.359	0.078	12.825	18.432	27.973	4.254

Age	$\delta = 5\%$ and benefit $= 1000$								
x	μ_{xy}	e^{o}_{xy}	$e^{o}_{\overline{xy}}$	$\bar{A}_{xy}$	$\bar{A}_{\overline{xy}}$	$\bar{a}_{xy}$	$\bar{a}_{\overline{xy}}$	Premium (P) xy	Premium (P) $\overline{xy}$
29	0.028	35.248	64.162	0.362	0.080	12.760	18.400	28.370	4.348
30	0.029	34.748	61.862	0.365	0.082	12.694	18.367	28.778	4.444
31	0.029	34.248	59.602	0.369	0.083	12.626	18.334	29.199	4.544
32	0.030	33.748	57.382	0.372	0.085	12.558	18.299	29.631	4.648
33	0.030	33.248	55.202	0.376	0.087	12.488	18.263	30.077	4.755
34	0.031	32.748	53.062	0.379	0.089	12.417	18.226	30.536	4.866
35	0.031	32.248	50.962	0.383	0.091	12.344	18.188	31.010	4.981
36	0.031	31.748	48.902	0.386	0.093	12.270	18.149	31.498	5.101
37	0.032	31.248	46.882	0.390	0.095	12.195	18.108	32.002	5.225
38	0.033	30.748	44.902	0.394	0.097	12.118	18.066	32.522	5.353
39	0.033	30.248	42.962	0.398	0.099	12.039	18.022	33.060	5.487
40	0.034	29.748	41.062	0.402	0.101	11.959	17.977	33.616	5.626
41	0.034	29.248	39.202	0.406	0.103	11.878	17.931	34.191	22.649
42	0.035	28.748	37.382	0.410	0.106	11.795	17.882	34.785	22.943
43	0.035	28.248	35.602	0.415	0.108	11.709	17.833	35.401	23.245
44	0.036	27.748	33.862	0.419	0.111	11.623	17.781	36.039	23.557
45	0.037	27.248	32.162	0.423	0.114	11.534	17.727	36.700	23.879
46	0.037	26.748	30.502	0.428	0.116	11.443	17.672	37.386	24.210
47	0.038	26.248	28.882	0.432	0.119	11.351	17.614	38.099	24.552
48	0.039	25.748	27.302	0.437	0.122	11.256	17.554	38.839	24.905
49	0.040	25.248	25.762	0.442	0.125	11.160	17.492	39.608	25.269
50	0.040	24.747	24.263	0.447	0.129	11.061	17.428	40.408	25.646
51	0.041	24.247	22.803	0.452	0.132	10.960	17.361	41.241	26.036
52	0.042	23.747	21.383	0.457	0.135	10.857	17.291	42.110	26.440
53	0.043	23.247	20.003	0.462	0.139	10.751	17.218	43.016	26.858
54	0.044	22.747	18.663	0.468	0.143	10.643	17.143	43.961	27.292
55	0.045	22.247	17.363	0.473	0.147	10.532	17.064	44.949	27.742
56	0.046	21.747	16.103	0.479	0.151	10.419	16.982	45.983	28.210
57	0.047	21.247	14.883	0.485	0.155	10.302	16.897	47.065	28.697
58	0.048	20.747	13.703	0.491	0.160	10.183	16.808	48.200	29.203
59	0.049	20.247	12.563	0.497	0.164	10.061	16.715	49.390	29.730
60	0.051	19.747	11.463	0.503	0.169	9.936	16.617	50.641	30.281
61	0.052	19.247	10.403	0.510	0.174	9.808	16.516	51.957	30.855
62	0.053	18.747	9.383	0.516	0.180	9.677	16.409	53.343	31.456
63	0.055	18.247	8.403	0.523	0.185	9.542	16.298	54.805	32.085
64	0.056	17.746	7.464	0.530	0.191	9.403	16.181	56.349	32.744
65	0.058	17.246	6.564	0.537	0.197	9.261	16.059	57.983	33.437
66	0.060	16.746	5.704	0.544	0.203	9.115	15.931	59.715	34.165
67	0.062	16.246	4.884	0.552	0.210	8.964	15.796	61.553	34.931
68	0.064	15.746	4.104	0.560	0.217	8.810	15.655	63.508	35.740
69	0.066	15.246	3.364	0.567	0.225	8.651	15.506	65.591	36.596
70	0.068	14.746	2.664	0.576	0.233	8.488	15.349	67.816	37.502
71	0.070	14.246	2.004	0.584	0.241	8.320	15.184	70.197	38.463
72	0.073	13.745	1.385	0.593	0.250	8.147	15.009	72.751	39.487
73	0.075	13.245	0.805	0.602	0.259	7.968	14.825	75.499	40.578
74	0.078	12.745	0.265	0.611	0.268	7.784	14.631	78.462	41.745
75	0.082	12.245	0.235	0.620	0.279	7.595	14.425	81.667	42.998
76	0.085	11.745	0.295	0.630	0.290	7.399	14.207	85.145	44.345
77	0.089	11.244	0.114	0.640	0.301	7.198	13.976	88.933	45.800
78	0.093	10.744	0.494	0.651	0.313	6.989	13.731	93.074	47.378
79	0.098	10.244	0.834	0.661	0.327	6.774	13.470	97.619	49.095
80	0.103	9.744	0.134	0.672	0.340	6.552	13.192	102.632	50.972
81	0.108	9.243	0.393	0.684	0.355	6.322	12.896	108.187	53.035
82	0.114	8.743	0.613	0.696	0.371	6.083	12.579	114.379	29.495
83	0.121	8.242	0.792	0.708	0.388	5.837	12.241	121.324	31.692
84	0.129	7.742	0.932	0.721	0.406	5.581	11.879	129.167	34.183
85	0.138	7.241	0.031	0.734	0.425	5.316	11.490	138.095	37.030
86	0.148	6.741	0.091	0.748	0.446	5.042	11.073	148.352	40.314
87	0.160	6.240	0.110	0.762	0.469	4.756	10.623	160.256	44.138
88	0.174	5.739	0.089	0.777	0.493	4.459	10.137	174.242	48.645
89	0.191	5.238	0.028	0.792	0.519	4.151	9.612	190.909	54.031
90	0.211	4.737	0.927	0.809	0.548	3.830	9.044	211.111	60.573
91	0.236	4.235	0.785	0.825	0.579	3.495	8.426	236.111	68.680
92	0.268	3.733	0.603	0.843	0.612	3.146	7.753	267.857	78.976
93	0.310	3.231	0.381	0.861	0.649	2.781	7.019	309.524	92.468
94	0.367	2.727	0.117	0.880	0.689	2.400	6.215	366.667	110.891
95	0.450	2.222	0.812	0.900	0.733	2.000	5.333	450.000	137.500
96	0.583	1.714	0.464	0.921	0.782	1.579	4.363	583.333	179.196
97	0.833	1.200	0.070	0.943	0.835	1.132	3.295	833.333	253.508
98	1.500	1.667	0.617	0.968	0.894	0.645	2.125	1500.000	420.499
99	1.000	1.000	0.990	0.952	0.000	0.952	20.000	1000.000	0.000
100	0.000			0.000	0.000	20.000	0.000	0.000	

Table 2, shows the estimates for the joint life and last survivor statuses. For multiple life statuses in this case two lives were considered at a time. The ages were grouped as follows: ages 0 and 1, ages 1 and 2, ages 2 and 3,…, ages 99 and 100. The mortality for joint life started from 0.020 for ages 0 and 1 and same for ages 1 and 2 but increases from there to 1.000 for ages 99 and 100. This indicated that, the mortality for joint life was an increasing function of age. The expected time until first death of the component lives (joint life status) was a decreasing function of age, from 50 years for ages 0 and 1, 49 years for ages 1 and 2 to 1 year for ages 99 and 100. This indicated that the expected time until death of the first component lives which can be either of the two depends on the age of any of the two when all other perils are held constant. The expected time until death of the last component lives (last survivor status) was also a decreasing function of age, that was from 148 years for ages 0 and 1, 145 years for ages 1 and 2 to approximately 1 year for ages 99 and 100. The expected time until death for last component lives has much surviving time than the expected time until death of the first component lives because in the joint life case, it fails only on the first death of one of the component lives. In this case the probability of death was higher than that of the last survivor where both component lives has to die for the status to fail and by the time the two components might have died the time survived would also have increased. The insurance payable immediately on the death of the first component lives (joint life) was an increasing function of age. Thus, from 0.287 for ages 0 and 1, 0.289 for ages 1 and 2 to 0.952 for ages 99 and 100. The insurance payable immediately on the death of the last component lives was also an increasing function of age, that is from 0.048 for ages 0 and 1, 0.049 for ages 1 and 2 to 0.894 for ages 98 and 99. This indicates that a unit insurance payable immediately on the death of first component lives was greater than that of the last survivor. This was because of the variation in the expected time until death of the joint life been much smaller than the last survivor and as such insurance payments on lives with shorter time until death paying more than those with more time until death. The annuity payments until the first death of the component lives was a decreasing function of age, that is from 14.265 for ages 0 and 1, 14.224 for ages 1 and 2 to 0.952 for ages 98 and 99. But age 100 had a higher annuity payment of 20.000 than all the component lives because it was considered to the terminal age. Also the annuity payments until the death of the last component lives was a decreasing function of age, that is from 19.040 for ages 0 and 1, 19.025 for ages 1 and 2 to 2.125 for ages 98 and 99 but ages 99 and 100 had annuity payments of 20.000 since there was an inclusion of the terminal age. In the same way, the annuity payments for the joint life was smaller than the last survivor because the annuity was paid until the first death of one of the component lives and since the expected time until first death was also smaller compared with last survivor. It was also realized that, annuity for the last survivor was continually paid until the last death of the component lives and in this case will make the payments accrual more than that of the joint life. For one to receive a benefit of GH¢ 1000 at 5% force of interest, the premium to be paid for both the joint life and last survivor was an increasing function of age and that the premium for the joint life was greater than the last survivor since the expected time for the last survivor was greater than the joint life when all other perils are held constant. Again the benefit and force of interest when changed will yield an increasing premium with age when all other perils are held constant.

4. Conclusion

This paper compared single life status and multiple life statuses using multiple and single life functions. The analysis revealed that, using age as the main risk evaluation, for single life status and multiple life statuses, their expected time until death are decreasing functions of age. It was realized also that, the premiums for both single life status and multiple life statuses were increasing with age, however the premium for single life was higher than multiple life statuses. In the case of the multiple life statuses, it was revealed that, premium for joint life was greater than the last survivor and that, a change in the interest rate or force of interest and the benefit did not change the trend in premium payments. Also no matter the interest and benefits, an insured for a life policy will have a high premium to pay when the age is high and vice versa and insurers of life products must consider the age of the insured to be able to apply the required premium payments. In considering two lives for life policy coverage, the ages of the two matters in that the age of either life can easily influence the premium payment especially in the case of joint life status in which the premium payments are higher than the last survivor status. Therefore, in pricing these insurances, the age of one is essential in determining the premium.

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