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Stochastic Modeling of a System with Maintenance and Replacement of Standby Subject to Inspection

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Abstract: The present paper develops a probabilistic model of a cold standby system considering the failure of unit in standby mode. Initially the model contains one unit in operation and another identical in cold standby mode. The unit in cold standby mode fails after passage of pre specified time and goes under inspection for feasibility check for maintenance or replacement, whereas the operative unit directly goes under repair at its failure. A single service facility available in the system handles the tasks of repair, inspection, maintenance or replacement. The replacement of unit in standby mode, at its failure, takes some time; that follows certain probability distribution. The theory of semi-Markov processes and regenerative point technique are used to develop and analyze the system model. For illustration, the results are obtained for a particular case.

Keywords: Cold Standby, Inspection, Maintenance, Replacement Time, Semi-Markov Process, Probability Distribution

1. Introduction

In case of single unit systems or configurations, the flawless working of the unit in operation is necessary to achieve desired goals. But no unit can work without failure forever. So, to deal with any emergency, spare units are needed to provide backup to the working unit. Though, introduction of spares requires money and space but by this practice the system performance in terms of availability, throughput, profit and reliability can be improved. Therefore, stochastic models of standby systems have been widely studied in literature [1-5]. Some studies focus on failure of service facility [6-9]. Moreover, some researches highlight the issue of switch failure [10-13]. But all these studies analyze the failure of operative unit only. They don't emphasize on state of unit in cold-standby mode.

As the cold-standby systems are described by operative as well as standby units. The best worth of standby unit is needed as the operative unit fails. Instantly, the standby unit switches in to operation and the system remains in up-state. Though there is no active load sharing with the standby unit in a cold-standby system but still there may be various factors that contribute to its deterioration such as temperature, moist, dust, corrosion, tropical changes etc. So presuming the standby unit as good as new, with the passage of time, is impractical. Despite of all these facts most of the past studies ignore this fact. An earlier work [14] highlights this issue however it studies mean time to system failure only and no-one else measure of system performance.

Keeping, the practical importance in view, a stochastic model for a two unit cold standby system with the possibility of standby failure is developed. Moreover, the present paper generalizes the work presented in [15]. The model consists of two identical units; one in operation and another as cold standby. When the operative unit fails it goes directly under repair whereas after completion of a pre-specified time the standby unit goes under inspection to check the feasibility for its maintenance (low intensity repair) or replacement. A single service facility takes care of all the repair, maintenance or replacement tasks. The failure times of operative as well as standby unit follow exponential distributions. However, the repair time, maintenance time and replacement time follow general distribution with distinct probability distribution functions. All the random variables, included here, are statistically independent. The repairs, maintenances and switches are perfect. The model is developed using semi-Markov process [16]. The model is analyzed at different regeneration epochs and the expressions for various performance measures are derived using regenerative point technique of renewal theory [17-18]. A particular case is considered for simulation and numerical illustration of results.

2. Notations

$E / \bar{E}$	: The set of regenerative/ non regenera-tive states
N_0	: The unit is operative and in normal Mode
C_s	: The unit is in cold-standby
μ	: Failure rate of cold-standby unit
λ	: Failure rate of operative unit
a / b	: Probability that maintenance feasible/ replacement feasible
F_{ui} / F_{UI}	: Failed unit under inspection /under inspection continuously from previous state
F_{ur} / F_{UR}	: Failed unit under repair / under repair continuously from previous state
F_{wr} / F_{WR}	: Failed unit waiting for repair/ waiting for repair continuously from previous State
F_{um} / F_{UM}	: Failed unit under maintenance / under maintenance continuously from previous state
F_{urp} / F_{URp}	: Failed unit under replacement / under replacement continuously from previous state
$h(t) / H(t)$	: pdf / cdf of inspection time
$g(t) / G(t)$	: pdf / cdf of repair time of unit
$f(t) / F(t)$	: pdf / cdf of replacement time of Unit
$m(t) / M(t)$	: pdf / cdf of maintenance time of Unit
$q_{ij}(t) / Q_{ij}(t)$	: pdf/ cdf of direct transition time from regenerative state S_i to regenerative State S_j Or failed state S_j without visiting any other regenerative state in $(0,t]$
$q_{ij.kr}(t) / Q_{ij.kr}(t)$	: pdf/ cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k , S_r once in $(0,t]$
$\mu_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	: Probability that server busy in the state S_i up to time t without making any transition to any other regenera-tive state or returning to the same state via one or more non-regenerative states
[s] / [c]	: Symbol for Laplace-Stietjes convolution/Laplace convolution
~ / *	: Symbol for Laplace- stietjes Transform(LST) /Laplace transform (LT)

Considering these symbols, the following are possible transition states of the system model (Fig. 1)

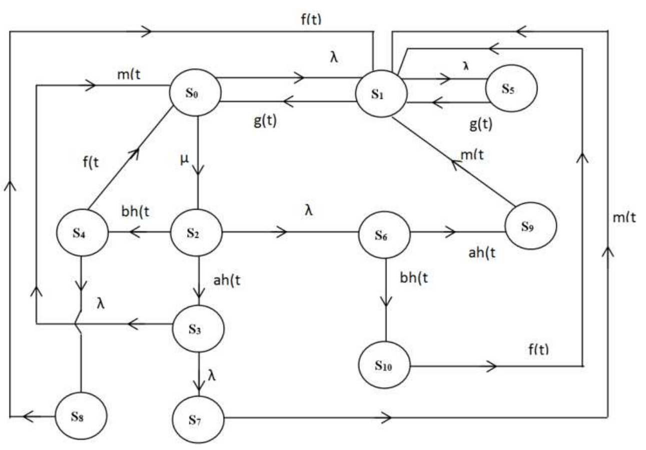

Figure 1. *System State Transition Diagram.*

The regenerative states (E):

$S_0 = (N_o, C_s)$,
$S_1 = (F_{ur}, N_o)$,
$S_2 = (N_o, F_{ui})$,
$S_3 = (N_o, F_{um})$,
$S_4 = (N_o, F_{urp})$

The non-regenerative states ($\bar{E}$):

$S_5 = (F_{UR}, F_{wr})$,
$S_6 = (F_{wr}, F_{UI})$,
$S_7 = (F_{wr}, F_{UM})$,
$S_8 = (F_{wr}, F_{URp})$,
$S_9 = (F_{WR}, F_{um})$,
$S_{10} = (F_{WR}, F_{urp})$

3. Modeling of the System

3.1. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements:-

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)\,dt = \tilde{Q}_{ij}(0)$$
$$p_{01} = \lambda / (\lambda+\mu),\quad p_{02} = \mu / (\lambda+\mu),\quad p_{10} = g^*(\lambda),$$
$$p_{15} = 1 - g^*(\lambda),\quad p_{23} = ah^*(\lambda),\quad p_{24} = bh^*(\lambda),$$
$$p_{26} = 1 - h^*(\lambda),\quad p_{30} = m^*(\lambda),\quad p_{37} = 1 - m^*(\lambda),$$
$$p_{40} = f^*(\lambda),\quad p_{48} = 1 - f^*(\lambda),\quad p_{51} = g^*(0),\quad p_{69} = ah^*(0),$$
$$p_{6,10} = bh^*(0),\quad p_{71} = m^*(0),\quad p_{81} = f^*(0),\quad p_{91} = m^*(0),$$
$$p_{10,1} = f^*(0),\quad p_{11.5} = 1 - g^*(\lambda),\ p_{21.6,9} = [1 - h^*(\lambda)]a,$$
$$p_{21.6,10} = [1 - h^*(\lambda)]b,\quad p_{31.7} = [1 - m^*(\lambda)],\quad p_{41.8} = [1 - f^*(\lambda)]$$

It can be easily verified that

$$p_{01} + p_{02} = p_{10} + p_{15} = p_{23} + p_{24} + p_{26} = p_{30} + p_{37} = p_{40} + p_{48} = p_{51} = p_{69} + p_{6,10} = p_{71} = p_{81} = p_{91} = p_{10,1} = p_{10} + p_{11.5} = p_{23} + p_{24} + p_{21.6,9} + p_{21.6,10} = p_{30} + p_{31.7} = p_{40} + p_{41.8} = 1$$

The unconditional mean time taken by the system to transit to any regenerative state S_j when it is counted from epoch of entrance into that state S_i is given by;

$$m_{ij} = \int_0^\infty t\,d\{Q_{ij}(t)\} = -q_{ij}^{*\prime}(0)$$

And the mean sojourn time in the state S_i is given by; $\mu_i = E(t) = \int_0^\infty P(T > t)\,dt$, where T denotes the time to system failure.

We get,

$$\mu_0 = 1/(\lambda+\mu),\ \mu_1 = [1 - g^*(\lambda)]/\lambda,\quad \mu_2 = [1 - h^*(\lambda)]/\lambda,$$
$$\mu_3 = [1 - m^*(\lambda)]/\lambda,$$

$\mu_4=[1-f^*(\lambda)]/\lambda,\ \ \mu_5=-g^{*'}(0),\ \ \mu_6=-h^{*'}(0),$

$\mu_7=-m^{*'}(0),\ \ \mu_8=-f^{*'}(0),\ \ \mu_9=-m^{*'}(0),\ \ \mu_{10}=-f^{*'}(0),$

$\mu_1^{'}=\left[(1/\lambda)-g^{*'}(0)\right]\left[1-g^*(\lambda)\right],$

$\mu_2^{'}=\left[(1/\lambda)-h^{*'}(0)-am^{*'}(0)-bf^{*'}(0)\right]\left[1-h^*(\lambda)\right],$

$\mu_3^{'}=\left[(1/\lambda)-m^{*'}(0)\right]\left[1-m^*(\lambda)\right],$

$\mu_4^{'}=\left[(1/\lambda)-f^{*'}(0)\right]\left[1-f^*(\lambda)\right]$

Further

$\sum_j m_{ij}=\mu_i$

$m_{01}+m_{02}=\mu_0,\ m_{10}+m_{15}=\mu_1,\ \ m_{23}+m_{24}+m_{26}=\mu_2,$

$m_{30}+m_{37}=\mu_3,$

$m_{40}+m_{48}=\mu_4,\ \ m_{51}=\mu_5,\ \ m_{69}+m_{6,10}=\mu_6,\ \ m_{71}=\mu_7,$

$m_{81}=\mu_8,,\ \ m_{91}=\mu_9,\ m_{10,1}=\mu_{10},\ \ m_{10}+m_{11.5}=\mu_1^{'},$

$m_{23}+m_{24}+m_{21.6,9}+m_{21.6,10}=\mu_2^{'},\ \ m_{30}+m_{31.7}=\mu_3^{'},$

$m_{40}+m_{41.8}=\mu_4^{'}$

3.2. Reliability and MTSF

Let $\varphi_i(t)$ be the cdf of the passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$:

$$\begin{aligned}
\varphi_0(t)&=Q_{01}(t)[s]\varphi_1(t)+Q_{02}(t)[s]\varphi_2(t)\\
\varphi_1(t)&=Q_{10}(t)[s]\varphi_0(t)+Q_{15}(t)\\
\varphi_2(t)&=Q_{23}(t)[s]\varphi_3(t)+Q_{24}(t)[s]\varphi_4(t)+Q_{26}(t)\\
\varphi_3(t)&=Q_{30}(t)[s]\varphi_0(t)+Q_{37}(t)\\
\varphi_4(t)&=Q_{40}(t)[s]\varphi_0(t)+Q_{48}(t)
\end{aligned}\tag{1}$$

Taking LST of above relations (1), we get the following matrix form.

$$\begin{bmatrix}\tilde\varphi_0\\ \tilde\varphi_1\\ \tilde\varphi_2\\ \tilde\varphi_3\\ \tilde\varphi_4\end{bmatrix}=\begin{bmatrix}1 & -\tilde Q_{01} & -\tilde Q_{02} & 0 & 0\\ -\tilde Q_{01} & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & -\tilde Q_{23} & -\tilde Q_{24}\\ -\tilde Q_{30} & 0 & 0 & 1 & 0\\ -\tilde Q_{40} & 0 & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}0\\ \tilde Q_{15}\\ \tilde Q_{26}\\ \tilde Q_{37}\\ \tilde Q_{48}\end{bmatrix}$$

Here it should be noted that the argument s is omitted for brevity. Solving for $\tilde\varphi_0(s)$, we get

$$\tilde\varphi_0(s)=\frac{\tilde Q_{01}\tilde Q_{15}+\tilde Q_{02}[\tilde Q_{26}+\tilde Q_{23}\tilde Q_{37}+\tilde Q_{24}\tilde Q_{48}]}{1-\tilde Q_{01}\tilde Q_{10}-\tilde Q_{02}[\tilde Q_{23}\tilde Q_{30}+\tilde Q_{24}\tilde Q_{40}]}\tag{2}$$

The reliability of system model can be obtained by using Inverse Laplace transform of as below i.e.

$$R(t)=L^{-1}\left[\frac{1-\tilde\varphi_0(s)}{s}\right]$$

The mean time to system failure (MTSF) is given by

$$\begin{aligned}
\text{MTSF}&=\lim_{s\to0}\frac{1-\tilde\varphi_0(s)}{s}\\
&=\frac{\mu_0+p_{01}\mu_1+p_{02}[\mu_2+p_{23}\mu_3+p_{24}\mu_4]}{1-p_{01}p_{10}-p_{02}[p_{23}p_{30}+p_{24}p_{40}]}
\end{aligned}\tag{3}$$

4. Cost-Benefit Analysis

4.1. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are as follows:

$$\begin{aligned}
A_0(t)&=M_0(t)+q_{01}(t)[c]A_1(t)+q_{02}(t)[c]A_2(t)\\
A_1(t)&=M_1(t)+q_{10}(t)[c]A_0(t)+q_{11.5}(t)[c]A_1(t)\\
A_2(t)&=M_2(t)+q_{23}(t)[c]A_3(t)+q_{24}(t)[c]A_4(t)\\
&\quad+\{q_{21.6,9}(t)+q_{24.6,10}(t)\}[c]A_1(t)\\
A_3(t)&=M_3(t)+q_{30}(t)[c]A_0(t)+q_{31.7}(t)[c]A_1(t)\\
A_4(t)&=M_4(t)+q_{40}(t)[c]A_0(t)+q_{41.8}(t)[c]A_1(t)
\end{aligned}\tag{4}$$

$M_i(t)$ be the probability that system is up initially in state $S_i\in E$ is up at time t without visiting any other regenerative state, we have

$$M_0(t)=e^{-(\lambda+\mu)t},\ \ M_1(t)=e^{-\lambda t}\bar G(t),\ \ M_2(t)=e^{-\lambda t}\bar H(t),$$
$$M_3(t)=e^{-\lambda t}\bar M(t),\ \ M_4(t)=e^{-\lambda t}\bar F(t),$$

Taking LST of above relation (4) and putting them in matrix form, we get

$$\begin{bmatrix}A_0^*\\ A_1^*\\ A_2^*\\ A_3^*\\ A_4^*\end{bmatrix}=\begin{bmatrix}1 & -q_{01}^* & -q_{02}^* & 0 & 0\\ -q_{10}^* & [1-q_{11.5}^*] & 0 & 0 & 0\\ 0 & -[q_{21.6,9}^*+q_{21.6,10}^*] & 1 & -q_{23}^* & -q_{24}^*\\ -q_{30}^* & -q_{31.7}^* & 0 & 1 & 0\\ -q_{40}^* & -q_{41.8}^* & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}M_0^*\\ M_1^*\\ M_2^*\\ M_3^*\\ M_4^*\end{bmatrix}$$

Solve the matrix form for $A_0^*(s)$, we get the steady state availability as

$$\begin{aligned}
A_0(\infty)&=\lim_{s\to0}sA_0^*(s)\\
&=\frac{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1+p_{02}p_{10}[\mu_2+p_{23}\mu_3+p_{24}\mu_4]}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]}
\end{aligned}\tag{5}$$

4.2. Busy Period Analysis for Server

4.2.1. Due to Inspection

Let $B_i^I(t)$ be the probability that the server is busy in inspection of the unit due to cold-standby failure at an instant 't' given that the system entered state S_i at time t=0. The recursive relations for $B_i^I(t)$ are as follows:

$$\begin{aligned}
B_0^I(t) &= q_{01}(t)[c]B_1^I(t) + q_{02}(t)[c]B_2^I(t) \\
B_1^I(t) &= q_{10}(t)[c]B_0^I(t) + q_{11.5}(t)[c]B_1^I(t) \\
B_2^I(t) &= W_2^I(t) + q_{23}(t)[c]B_3^I(t) + q_{24}(t)[c]B_4^I(t) \\
&\quad + \{q_{21.6,9}(t) + q_{21.6,10}(t)\}[c]B_1^I(t) \\
B_3^I(t) &= q_{30}(t)[c]B_0^I(t) + q_{31.7}(t)[c]B_1^I(t) \\
B_4^I(t) &= q_{40}(t)[c]B_0^I(t) + q_{41.8}(t)[c]B_1^I(t)
\end{aligned} \tag{6}$$

$W_i^I(t)$ be the probability that the server is busy in state S_i due to inspection for failure of cold-standby unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2^I(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t}[c]1)\bar{H}(t)$$

Taking LST of above relation (6), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{*I} \\ B_1^{*I} \\ B_2^{*I} \\ B_3^{*I} \\ B_4^{*I} \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 \\ -q_{10}^* & [1-q_{11.5}^*] & 0 & 0 & 0 \\ 0 & -[q_{21.6,9}^* + q_{21.6,10}^*] & 1 & -q_{23}^* & -q_{24}^* \\ -q_{30}^* & -q_{31.7}^* & 0 & 1 & 0 \\ -q_{40}^* & -q_{41.8}^* & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ W_2^{*I} \\ 0 \\ 0 \end{bmatrix}$$

Solve above matrix form for $B_0^{*I}(s)$, we get the time for which server is busy due to inspection as:

$$\begin{aligned}
B_0^I(\infty) &= \lim_{s\to 0} sB_0^{*I}(s) \\
&= \frac{W_2^{*I}(0)p_{02}p_{10}}{p_{10}\mu_0 + [1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1' + p_{02}p_{10}[\mu_2' + p_{23}\mu_3' + p_{24}\mu_4']}
\end{aligned} \tag{7}$$

4.2.2. Due to Maintenance

Let $B_i^M(t)$ be the probability that the server is busy in maintenance of the unit due to failure of cold standby unit at an instant 't' given that the system entered state S_i at time t=0. The recursive relations for $B_i^M(t)$ are as follows:

$$\begin{aligned}
B_0^M(t) &= q_{01}(t)[c]B_1^M(t) + q_{02}(t)[c]B_2^M(t) \\
B_1^M(t) &= q_{10}(t)[c]B_0^M(t) + q_{11.5}(t)[c]B_1^M(t) \\
B_2^M(t) &= q_{23}(t)[c]B_3^M(t) + q_{24}(t)[c]B_4^M(t) + \\
&\quad \{q_{21.6,9}(t) + q_{21.6,10}(t)\}[c]B_1^M(t) \\
B_3^M(t) &= W_3^M(t) + q_{30}(t)[c]B_0^M(t) + q_{31.7}(t)[c]B_1^M(t) \\
B_4^M(t) &= q_{40}(t)[c]B_0^M(t) + q_{41.8}(t)[c]B_1^M(t)
\end{aligned} \tag{8}$$

$W_i^M(t)$ be the probability that the server is busy in state S_i due to maintenance of unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3^M(t) = e^{-\lambda t}\bar{M}(t) + (\lambda e^{-\lambda t}[c]1)\bar{M}(t)$$

Taking LST of above relation (8), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{*M} \\ B_1^{*M} \\ B_2^{*M} \\ B_3^{*M} \\ B_4^{*M} \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 \\ -q_{10}^* & [1-q_{11.5}^*] & 0 & 0 & 0 \\ 0 & -[q_{21.6,9}^* + q_{21.6,10}^*] & 1 & -q_{23}^* & -q_{24}^* \\ -q_{30}^* & -q_{31.7}^* & 0 & 1 & 0 \\ -q_{40}^* & -q_{41.8}^* & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ W_3^{*M} \\ 0 \end{bmatrix}$$

Solve above matrix form for $B_0^{*M}(s)$, we get the time for which server is busy due to maintenance as given below.

$$\begin{aligned}
B_0^M(\infty) &= \lim_{s\to 0} sB_0^{*M}(s) \\
&= \frac{W_3^{*M}(0)p_{02}p_{10}p_{23}}{p_{10}\mu_0 + [1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1' + p_{02}p_{10}[\mu_2' + p_{23}\mu_3' + p_{24}\mu_4']}
\end{aligned} \tag{9}$$

4.2.3. Due to Replacement

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacing the standby unit at its failure at an instant 't' given that the system entered regenerative state S_i at time t=0. The recursive relations for $B_i^{Rp}(t)$ are as follows:

$$\begin{aligned}
B_0^{Rp}(t) &= q_{01}(t)[c]B_1^{Rp}(t) + q_{02}(t)[c]B_2^{Rp}(t) \\
B_1^{Rp}(t) &= q_{10}(t)[c]B_0^{Rp}(t) + q_{11.5}(t)[c]B_1^{Rp}(t) \\
B_2^{Rp}(t) &= q_{23}(t)[c]B_3^{Rp}(t) + q_{24}(t)[c]B_4^{Rp}(t) + \{q_{21.6,9}(t) \\
&\quad + q_{21.6,10}(t)\}[c]B_1^{Rp}(t) \\
B_3^{Rp}(t) &= q_{30}(t)[c]B_0^{Rp}(t) + q_{31.7}(t)[c]B_1^{Rp}(t) \\
B_4^{Rp}(t) &= W_4^{Rp}(t) + q_{40}(t)[c]B_0^{Rp}(t) + q_{41.8}(t)[c]B_1^{Rp}(t)
\end{aligned} \tag{10}$$

$W_i^{Rp}(t)$ be the probability that the server is busy in state S_i due to replacing of unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4^{Rp}(t) = e^{-\lambda t}\bar{F}(t) + (\lambda e^{-\lambda t}[c]1)\bar{F}(t)$$

Taking LST of above relation (10), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{*Rp} \\ B_1^{*Rp} \\ B_2^{*Rp} \\ B_3^{*Rp} \\ B_4^{*Rp} \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 \\ -q_{10}^* & [1-q_{11.5}^*] & 0 & 0 & 0 \\ 0 & -[q_{21.6,9}^* + q_{21.6,10}^*] & 1 & -q_{23}^* & -q_{24}^* \\ -q_{30}^* & -q_{31.7}^* & 0 & 1 & 0 \\ -q_{40}^* & -q_{41.8}^* & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ W_4^{*Rp} \end{bmatrix}$$

Solve the matrix form for $B_0^{*Rp}(s)$, we get the time for which server is busy due to replacement is given by

$$B_0^{Rp}(\infty)=\lim_{s\to 0} sB_0^{*Rp}(s)$$
$$=\frac{W_1^{*Rp}(0)p_{02}p_{10}p_{24}}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \tag{11}$$

4.2.4. Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to failure at an instant 't' given that the system entered regenerative state S_i at time t=0. The recursive relations for $B_i^R(t)$ are as follows:

$$\begin{aligned}
B_0^R(t) &= q_{01}(t)[c]B_1^R(t)+q_{02}(t)[c]B_2^R(t)\\
B_1^R(t) &= W_1^R(t)+q_{10}(t)[c]B_0^R(t)+q_{11.5}(t)[c]B_1^R(t)\\
B_2^R(t) &= q_{23}(t)[c]B_3^R(t)+q_{24}(t)[c]B_4^R(t)+\{q_{21.6,9}(t)+\\
&\quad q_{21.6,10}(t)\}[c]B_1^R(t)\\
B_3^R(t) &= q_{30}(t)[c]B_0^R(t)+q_{31.7}(t)[c]B_1^R(t)\\
B_4^R(t) &= q_{40}(t)[c]B_0^R(t)+q_{41.8}(t)[c]B_1^R(t)
\end{aligned} \tag{12}$$

$W_i^R(t)$ be the probability that the server is busy in state S_i due to repairing of unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1^R(t)=e^{-\lambda t}\overline{G}(t)+(\lambda e^{-\lambda t}[c]1)\overline{G}(t)$$

Taking LST of above relation (12), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{*R}\\ B_1^{*R}\\ B_2^{*R}\\ B_3^{*R}\\ B_4^{*R}\end{bmatrix}=\begin{bmatrix} 1 & -q_{01}^{*} & -q_{02}^{*} & 0 & 0\\ -q_{10}^{*} & [1-q_{11.5}^{*}] & 0 & 0 & 0\\ 0 & -[q_{21.6,9}^{*}+q_{21.6,10}^{*}] & 1 & -q_{23}^{*} & -q_{24}^{*}\\ -q_{30}^{*} & -q_{31.7}^{*} & 0 & 1 & 0\\ -q_{40}^{*} & -q_{41.8}^{*} & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ W_4^{*R}\end{bmatrix}$$

Solve above matrix form for $B_0^{*R}(s)$, we get the time for which server is busy due to repair as below:

$$B_0^R(\infty)=\lim_{s\to 0} sB_0^{*R}(s)$$
$$=\frac{W_1^{*R}(0)[1-p_{02}\{p_{23}p_{30}+p_{24}p_{40}\}]}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \tag{13}$$

4.3. Expected Number of Inspections of the Standby Unit

Let $I_i(t)$ be the expected number of inspection of the failed unit by the server in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $I_i(t)$ are given as:

$$\begin{aligned}
I_0(t) &= Q_{01}(t)[s]I_1(t)+Q_{02}(t)[s][1+I_2(t)]\\
I_1(t) &= Q_{10}(t)[s]I_0(t)+Q_{11.5}(t)[s]I_1(t)\\
I_2(t) &= Q_{23}(t)[s]I_3(t)+Q_{24}(t)[s]I_4(t)+\{Q_{21.6,9}(t)+\\
&\quad Q_{21.6,10}(t)\}[s]I_1(t)\\
I_3(t) &= Q_{30}(t)[s]I_0(t)+Q_{31.7}(t)[s]I_1(t)\\
I_4(t) &= Q_{40}(t)[s]I_0(t)+Q_{41.8}(t)[s]I_1(t)
\end{aligned} \tag{14}$$

Taking LST of above relation (14), we get the relations in following matrix form.

$$\begin{bmatrix} \widetilde{I}_0\\ \widetilde{I}_1\\ \widetilde{I}_2\\ \widetilde{I}_3\\ \widetilde{I}_4\end{bmatrix}=\begin{bmatrix} 1 & -\widetilde{Q}_{01} & -\widetilde{Q}_{02} & 0 & 0\\ -\widetilde{Q}_{10} & [1-\widetilde{Q}_{11.5}] & 0 & 0 & 0\\ 0 & -[\widetilde{Q}_{21.6,9}+\widetilde{Q}_{21.6,10}] & 1 & -\widetilde{Q}_{23} & -\widetilde{Q}_{24}\\ -\widetilde{Q}_{30} & -\widetilde{Q}_{31.7} & 0 & 1 & 0\\ -\widetilde{Q}_{40} & -\widetilde{Q}_{41.8} & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix} \widetilde{Q}_{02}\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

Solve the matrix form for $\widetilde{I}_0(s)$, we get the expected number of inspections per unit time by the server as follows:

$$I_0(\infty)=\lim_{s\to 0} s\widetilde{I}_0(s)$$
$$=\frac{p_{02}p_{10}}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \tag{15}$$

4.4. Expected Number of Repairs of the Unit

Let $R_i(t)$ be the expected number of repairs of the failed unit by the server in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $R_i(t)$ are given as:

$$\begin{aligned}
R_0(t) &= Q_{01}(t)[s]R_1(t)+Q_{02}(t)[s]R_2(t)\\
R_1(t) &= Q_{10}(t)[s][1+R_0(t)]+Q_{11.5}(t)[s][1+R_1(t)]\\
R_2(t) &= Q_{23}(t)[s]R_3(t)+Q_{24}(t)[s]R_4(t)+\{Q_{21.6,9}(t)+\\
&\quad Q_{21.6,10}(t)\}[s]R_1(t)\\
R_3(t) &= Q_{30}(t)[s]R_0(t)+Q_{31.7}(t)[s]R_1(t)\\
R_4(t) &= Q_{40}(t)[s]R_0(t)+Q_{41.8}(t)[s]R_1(t)
\end{aligned} \tag{16}$$

Taking LST of above relation (16), we get the relations in following matrix form.

$$\begin{bmatrix} \widetilde{R}_0\\ \widetilde{R}_1\\ \widetilde{R}_2\\ \widetilde{R}_3\\ \widetilde{R}_4\end{bmatrix}=\begin{bmatrix} 1 & -\widetilde{Q}_{01} & -\widetilde{Q}_{02} & 0 & 0\\ -\widetilde{Q}_{10} & [1-\widetilde{Q}_{11.5}] & 0 & 0 & 0\\ 0 & -[\widetilde{Q}_{21.6,9}+\widetilde{Q}_{21.6,10}] & 1 & -\widetilde{Q}_{23} & -\widetilde{Q}_{24}\\ -\widetilde{Q}_{30} & -\widetilde{Q}_{31.7} & 0 & 1 & 0\\ -\widetilde{Q}_{40} & -\widetilde{Q}_{41.8} & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix} 0\\ [\widetilde{Q}_{10}+\widetilde{Q}_{11.5}]\\ 0\\ 0\\ 0\end{bmatrix}$$

Solve the matrix form for $\widetilde{R}_0(s)$, we get the expected number of repairs per unit time by the server as given below:

$$R_0(\infty)=\lim_{s\to 0} s\tilde{R}_0(s)$$
$$=\frac{1-p_{02}\{p_{23}p_{30}+p_{24}p_{40}\}}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \quad (17)$$

4.5. Expected Number of Maintenances of the Standby Unit

Let $M_i(t)$ be the expected number of maintenances of the failed unit by the server in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $M_i(t)$ are given as:

$$\begin{aligned}
M_0(t)&=Q_{01}(t)[s]M_1(t)+Q_{02}(t)[s]M_2(t)\\
M_1(t)&=Q_{10}(t)[s]M_0(t)+Q_{11.5}(t)[s]M_1(t)\\
M_2(t)&=Q_{23}(t)[s]M_3(t)+Q_{24}(t)[s]M_4(t)+\\
&\quad Q_{21.6,9}(t)[s][1+M_1(t)]+Q_{21.6,10}(t)[s]M_1(t)\\
M_3(t)&=Q_{30}(t)[s][1+M_0(t)]+Q_{31.7}(t)[s][1+M_1(t)]\\
M_4(t)&=Q_{40}(t)[s]M_0(t)+Q_{41.8}(t)[s]M_1(t)
\end{aligned} \quad (18)$$

Taking LST of above relation (18), we get the relations in following matrix form.

$$\begin{bmatrix}\tilde{M}_0\\ \tilde{M}_1\\ \tilde{M}_2\\ \tilde{M}_3\\ \tilde{M}_4\end{bmatrix}=\begin{bmatrix}1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 & 0\\ -\tilde{Q}_{10} & [1-\tilde{Q}_{11.5}] & 0 & 0 & 0\\ 0 & -[\tilde{Q}_{21.6,9}+\tilde{Q}_{21.6,10}] & 1 & -\tilde{Q}_{23} & -\tilde{Q}_{24}\\ -\tilde{Q}_{30} & -\tilde{Q}_{31.7} & 0 & 1 & 0\\ -\tilde{Q}_{40} & -\tilde{Q}_{41.8} & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}0\\ 0\\ \tilde{Q}_{21.6,9}\\ [\tilde{Q}_{30}+\tilde{Q}_{31.7}]\\ 0\end{bmatrix}$$

Solve the above matrix form for $\tilde{M}_0(s)$, we get the expected number of maintenances per unit time by the server as given below:

$$M_0(\infty)=\lim_{s\to 0} s\tilde{M}_0(s)$$
$$=\frac{p_{02}p_{10}[p_{23}+p_{21.6,9}]}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \quad (19)$$

4.6. Expected Number of Replacements of Standby Unit

Let $R_i^C(t)$ be the expected number of replacements of the unit failed in cold-standby by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^C(t)$ are as follows:

$$\begin{aligned}
R_0^C(t)&=Q_{01}(t)[s]R_1^C(t)+Q_{02}(t)[s]R_2^C(t)\\
R_1^C(t)&=Q_{10}(t)[s]R_0^C(t)+Q_{11.5}(t)[s]R_1^C(t)\\
R_2^C(t)&=Q_{23}(t)[s]R_3^C(t)+Q_{24}(t)[s]R_4^C(t)+Q_{21.6,9}(t)[s]\\
&\quad R_1^C(t)+Q_{21.6,10}(t)[s][1+R_1^C(t)]\\
R_3^C(t)&=Q_{30}(t)[s]R_0^C(t)+Q_{31.7}(t)[s]R_1^C(t)\\
R_4^C(t)&=Q_{40}(t)[s][1+R_0^C(t)]+Q_{41.8}(t)[s][1+R_1^C(t)]
\end{aligned} \quad (20)$$

Taking LST of above relation (20), we get the relations in following matrix form.

$$\begin{bmatrix}\tilde{R}_0^C\\ \tilde{R}_1^C\\ \tilde{R}_2^C\\ \tilde{R}_3^C\\ \tilde{R}_4^C\end{bmatrix}=\begin{bmatrix}1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 & 0\\ -\tilde{Q}_{10} & [1-\tilde{Q}_{11.5}] & 0 & 0 & 0\\ 0 & -[\tilde{Q}_{21.6,9}+\tilde{Q}_{21.6,10}] & 1 & -\tilde{Q}_{23} & -\tilde{Q}_{24}\\ -\tilde{Q}_{30} & -\tilde{Q}_{31.7} & 0 & 1 & 0\\ -\tilde{Q}_{40} & -\tilde{Q}_{41.8} & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}0\\ 0\\ \tilde{Q}_{21.6,10}\\ 0\\ [\tilde{Q}_{40}+\tilde{Q}_{41.8}]\end{bmatrix}$$

Solve above matrix form for $\tilde{R}_0^C(s)$, we get the expected number of replacements per unit time to cold-standby failure is given by

$$R_0^C(\infty)=\lim_{s\to 0} s\tilde{R}_0^C(s)$$
$$=\frac{p_{02}p_{10}[p_{24}+p_{21.6,10}]}{p_{10}\mu_0+[1-p_{02}(p_{23}p_{30}+p_{24}p_{40})]\mu_1^{'}+p_{02}p_{10}[\mu_2^{'}+p_{23}\mu_3^{'}+p_{24}\mu_4^{'}]} \quad (21)$$

4.7. Profit Analysis

System Profit=Total System Revenue- Total System Cost

So the Profit incurred to the system model in (0,t] is given as below:

$$P(t)=K_0A_0(t)-\sum_{i=1}^{8}C_iX_i(t) \quad (22)$$

As t→∞, we obtain the profit attained in steady state i.e.

$$P_0=\lim_{t\to\infty}P(t)=K_0A_0-\lim_{t\to\infty}\sum_{i=1}^{8}C_iX_i(t)$$

$$\text{Where } X_i(\infty)=\begin{cases}B_0^I; i=1\\ B_0^M; i=2\\ B_0^{Rp}; i=3\\ B_0^R; i=4\\ I_0; i=5\\ R_0; i=6\\ M_0; i=7\\ R_0^C; i=8\end{cases}$$

Where

K_0 = Revenue per unit up-time of the system

C_1 =Cost per unit time for which server is busy in inspection of cold-standby unit

C_2 = Cost per unit time for which server is busy due to maintenance

C_3 = Cost per unit time for which server is busy due to replacement

C_4 = Cost per unit time for which server is busy due to repair

C_5 = Cost per unit inspection of the unit

C_6 = Cost per unit repair of the unit

C_7 = Cost per unit maintenance of the unit

C_8 = Cost per unit replacement of the unit

5. Example (Exponential Case)

For numerical illustration let us suppose that the different random variables follow exponential distribution with different probability density function given as $h(t)=\alpha e^{-\alpha t}$, $g(t)=\beta e^{-\beta t}$, $m(t)=\theta e^{-\theta t}$, $f(t)=\gamma e^{-\gamma t}$. For the sake of convenience we pretend the following values for different parameters as: a=0.4, b=0.6, α=0.1, β=0.5, θ=0.3, μ=0.4, γ=3.5, $\lambda=0.001$, $K_0=50000$, $K_1=100$, $K_2=1000$, $K_3=35000$, $K_4=2000$, $K_5=180$, $K_6=3000$, $K_7=230$, $K_8=6000$

For this, we obtained the values for different measures of system performance as follows:

MTSF=1219.5
Availability=0.9916
Busy period due to inspection=0.713
Busy period due to maintenance=0.094076
Busy period due to replacement=0.012096
Busy period due to repair=0.001983
Expected number of inspections of standby=0.0712627
Expected number of repairs=0.000992
Expected number of maintenances of standby=0.028505
Expected number of replacements of standby=0.042758
System profit=49087.59

6. Simulation Study

For simulation, we consider the numerical values of different parameters as assumed above. The simulation results are presented in Table1 and Fig. 2. Table 1 shows the trends of system performance measures w.r.t. λ for different possible combinations of other parameters. Evidently, it can be observed that

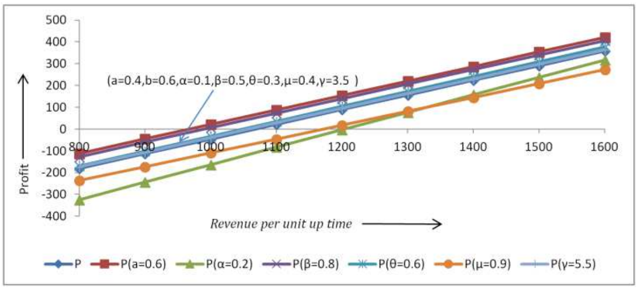

Figure 2. *System profit for different values of parameters vs. revenue per unit up time.*

Table 1. *Effect of α, β, θ, μ & γ on system performance w.r.t. λ (a=0.4,b=0.6,α=0.1,β=0.5,θ=0.3,μ=0.4,γ=3.5).*

Performance Index	λ		α=0.2	β=0.8	θ=0.6	μ=0.9	γ=5.5
	0.01	123.9	140.3	123.9	125.3	110.6	124.0
	0.02	63.0	71.1	63.1	63.7	55.8	63.1
MTSF	0.03	42.7	48.0	42.8	43.2	37.5	42.7
	0.04	32.5	36.4	32.6	32.9	28.4	32.6
	0.05	26.4	29.5	26.5	26.7	22.9	26.5
	0.01	0.9236	0.9614	0.9232	0.9264	0.9152	0.9236
	0.02	0.8616	0.9271	0.8606	0.8666	0.8465	0.8618
Availability	0.03	0.8105	0.8966	0.8085	0.8171	0.7897	0.8107
	0.04	0.7676	0.8692	0.7647	0.7755	0.7421	0.7678
	0.05	0.7310	0.8444	0.7272	0.7400	0.7015	0.7313
	0.01	45648.96	47324.66	45643.68	45810.51	45179.00	45664.72
Profit	0.02	42514.14	45571.91	42482.24	42778.55	41708.79	42531.75
	0.03	39921.89	44005.31	39853.49	40265.66	38839.96	39941.05
	0.04	37743.54	42595.99	37634.97	38149.39	36429.12	37764.03
	0.05	35887.95	41320.65	35738.96	36342.88	34375.02	35909.58

(i) All indices decline with increasing values of λ. This implies that the system performance declines with increasing rate of failure of operative unit.

(ii) As the value of α increase from α=0.1 to α=0.2 the mean time to system failure, availability and profit increase rapidly, though the rate of increase is less with higher values of λ. This indicates that system performance improves with increase in rate of inspection of unit in standby mode.

(iii) A slight increase in the values of performance indices is observed as β increase from β =0.5 to β =0.8. So increasing the repair rate of operative unit at its failure results improved system performance.

(iv) The rising trend remains with value of θ changes from θ=0.3 to θ=0.6 and γ=3.5 to 5.5 respectively. After inspection maintenance of standby unit enhances system performance.

(v) Simultaneously, it can be observed that the values of all performance measures decline with increase in the value of μ. As μ increase from μ=0.4 to μ=0.9 a sharp declining trend of MTSF, availability and profit can be observed. This trend reflects that increase in the rate with which the cold standby fails reduce system performance.

The Fig. 2 gives the cutoff points of profit for different levels of other parameters. As usual, irrespective of other parameters, the system profit increases with K_0. It is evident from the graph that the system remains profitable (keeping other parameters fix) if

$K_0 \geq 980,\ a = 0.6$

$K_0 \geq 990,\ \beta = 0.8$

$K_0 \geq 1050,\ \gamma = 5.5$

$K_0 \geq 1190,\ \mu = 0.9$

$K_0 \geq 1200,\ \alpha = 0.2$

The effect of μ on system profit is more than that of α if, $K_0 \leq 1300$. The system is equally profitable for given values of μ and α if $1300 \leq K_0 \leq 1350$. However, as $K_0 \geq 1350$, the system profit due to effect of α crosses the same due to effect of μ. Further for given values of θ, β and γ the rate of increase of profit w.r.t K_0 is almost same.

7. Discussion

The paper investigates a two unit cold standby system with the possibility of failure of the cold standby. It explores laws of probability theory to develop the stochastic model. The expressions for various system performance measures are derived. The numerical illustration of the study through an example and a simulation study reveals its practical importance. The results thus obtained indicate that a two unit cold standby system can be made more reliable and profitable by deploying a skilled service facility with higher inspection rate, repair rate and maintenance rate. Though the failures cannot be completely avoided but certain preventive measures can be taken to reduce the impact, such as periodic inspections, preventive maintenances etc.

This paper may provide suggestive guidelines for the professionals working in various safety and /or profit making installations such as space-satellite organizations, power generation companies, defense manufacturing and robotics etc.

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