

# Elastic Electron Scattering from $^{88}\text{Sr}$ and $^{89}\text{Y}$ Nuclei

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**Abstract:** The ground state proton momentum distributions and elastic charge form factors for  $^{88}\text{Sr}$  and  $^{89}\text{Y}$  nuclei have been derived and studied using the Coherent Density Fluctuation Model and formulated by means of the fluctuation function (weight function)  $|f(x)|^2$ . The fluctuation function has been connected to the charge density distribution of the nuclei and determined from the theory and experiment. The feature of the long-tail behavior at high momentum region of the PMD has been calculated by both the theoretical and experimental fluctuation functions. The calculated form factors  $F(q)$  of all nuclei under study are in good agreement with those of experimental data throughout all values of momentum transfer  $q$ .

**Keywords:** Proton Density Distributions, Elastic Electron Scattering form Factors, Momentum Distributions, fp Shell Nuclei, Coherent Density Fluctuation Model (CDFM)

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## 1. Introduction

The most accurate determination of the charge distributions in nuclei can be obtained from electron-nucleus scattering. The interest in charge densities results from the very important fact that they reflect the behavior of wave functions of protons in nuclei, where the charge density distribution is the sum of the proton wave functions squared. Charge density distributions for stable nuclei have been well studied by [1- 3]. For the case of the unstable exotic nuclei the corresponding charge distributions are planned to be studied by colliding electrons with these nuclei in storage rings (the GSI physics program [4] and the plan of RIKEN [5]. A number of interesting issue can be analyzed by the electron experiments. One of them is to study how the charge distribution evolves with increasing neutron number at fixed proton number or to what extent the neutron halo or skin may trigger sizable changes of the charge root-mean-squared radius and the diffuseness in the peripheral region of the charge distribution. The measurements of the charge form factor  $F_{ch}$  of various nuclei in a range of momentum transfers has stimulated extensive theoretical work for the calculation of this quantity. In [6] the calculations of the charge form factors of exotic nuclei were extended from light

(He, Li) to medium and heavy nuclei (Ni, Kr and Sn). For the He and Li isotopes the proton and neutron densities obtained in the LSSM method have been used, while for Ni, Kr and Sn isotopes the densities have been obtained in the deformed self-consistent mean-field Skyrme-Hartree-Fock (HF)+BCS method [7]. In [6] the charge form factors calculated not only within the PWBA but also in DWBA by the numerical solution of the Dirac equation [8] for electron scattering in the coulomb potential on the charge density of a given nucleus.

Another important characteristic of the nuclear ground state is the nucleon momentum distribution (NMD). In [9] the neutron and proton momentum distributions in some stable nuclei ( $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{56}\text{Fe}$  and  $^{208}\text{Pb}$ ) were calculated along with those of light neutron-rich isotopes of Li, Be, B and C using the natural -orbital representation (NOR) on the basis of the empirical data for  $n(k)$  in  $^4\text{He}$ . It is importance to study the NMD not only in stable but also in exotic nuclei. In [10] the NMD of even-even isotopes of Ni, Kr and Sn have been calculated in the framework of deformed self-consistent mean-field Skyrme (HF)+BCS method, as well as of theoretical correlation methods based on light-front dynamics and local density approximation. The isotopic sensitivities of the calculated neutron and proton momentum distributions are investigated together with the

effect of pairing and nucleon-nucleon correlations. In the coherent density fluctuation model (CDFM), which is characterized by the work of Antonov et al. [11, 12], the local nucleon density distribution (NDD) and the nucleon momentum distribution (NMD) are simply linked and specified by an experimentally obtainable fluctuation function (weight function)  $|f(x)|^2$ . They [11, 12] studied the NMD of ( $^4\text{He}$  and  $^{16}\text{O}$ ),  $^{12}\text{C}$  and ( $^{39}\text{K}$ ,  $^{40}\text{Ca}$  and  $^{48}\text{Ca}$ ) nuclei employing weight functions  $|f(x)|^2$  specified by the two parameter Fermi (2PF) NDD [13], the data of Reuter et al. [14] and the model independent NDD [13], respectively. It is significant to remark that all above studies, employed the framework of the CDFM, proved a high momentum tail in the NMD. Elastic electron scattering from  $^{40}\text{Ca}$  nucleus was also studied in Ref. [11], where the calculated elastic differential cross sections ( $d\sigma/d\Omega$ ) were found to be in good agreement with those of 2PF [13]. AL-Rahmani [15] has been calculated the G. S. elastic charge form factors and proton momentum distribution for the upper region of the 2S-1d shell nuclei like ( $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$  and  $^{39}\text{K}$ ). At the same year, also, AL-Rahmani [15] has measured the nucleon momentum distributions and elastic electron scattering form factor of the ground state for some odd 2s-1d shell nuclei like ( $^{19}\text{F}$ ,  $^{25}\text{Mg}$ ,  $^{27}\text{Al}$  and  $^{29}\text{Si}$ ) by using the coherent density fluctuation model and expressed in terms of the fluctuation function (weight function)  $|f(x)|^2$ . In addition, through her works she found that the inclusion of the quadrupole form factors  $F_{C2}(q)$  in all nuclei under study which was described by the undeformed 2S-1d shell nuclei, was essential for obtained a notable accordance between the experimental and theoretical form factors. It is important to point out that all above calculations obtained in the framework of CDFM proved a high momentum tail in the PMD. Al-Rahmani and Faris [16] have calculated the EES and PMD of the ground state for Ge isotopes by using the Coherent Density Fluctuation Model and expressed in terms of the fluctuation function (weight function). Al-Rahmani and Hussien [17] have studied the (CDD) and elastic electron scattering form factors of some 2s-1d shell nuclei using the (PWBA) and demonstrated that

the inclusion of the higher 1f-2p shell in the calculation leads to produce a good result in comparison with those of the experimental data. Al-Rahmani and Kadhim [18] have calculated the (NMD) and elastic electron scattering form factors of the ground state for some 1f-2p shell nuclei for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei by using the Coherent Density Fluctuation Model and expressed in terms of the fluctuation function (weight function). In the present study, we employ the CDFM with weight functions originated in terms of theoretical CDD. We first try to derive a theoretical form for the CDD, applicable through out the lower region of the fp shell nuclei with  $Z \geq 38$ , based on the use of the single particle harmonic oscillator wave function and the occupation numbers of the states. The derived form of the CDD is employed in determining the theoretical weight function  $|f(x)|^2$  which is then used in the CDFM to study the proton momentum distribution (PMD) and elastic electron scattering charge form factors for  $^{88}\text{Sr}$  and  $^{89}\text{Y}$  nuclei.

## 2. Theory

The charge density distribution CDD of the shell nuclei can be evaluated by means of the radial part of the wave functions of a harmonic oscillator, since [16]

$$\rho_c(r) = \frac{1}{4\pi} \sum_{nl} \zeta_{nl} 2(2l+1) |R_{nl}(r)|^2 \quad (1)$$

where  $\rho_c(r)$  is the CDD of nuclei,  $\zeta_{nl}$  is the proton occupation probability of the state  $nl$  ( $\zeta_{nl} = 0$  or  $1$  for closed shell nuclei and  $0 < \zeta_{nl} < 1$  for open shell nuclei) and  $R_{nl}(r)$  is the radial part of the single-particle harmonic oscillator wave function. To derive an explicit form for the CDD of fp shell nuclei, we suppose that there is a core of filled 1s, 1p, 1d, 2s and 1f7/2 shells and the proton occupation numbers in 2p3/2, 1f5/2, 2p1/2 and 1g9/2 shells are take as free parameters. Using this assumption in eq. (1), we get

$$\rho_c(r) = \frac{1}{4\pi} \left\{ 2|R_{1s}(r)|^2 + 6|R_{1p}(r)|^2 + 10|R_{1d}(r)|^2 + 2|R_{2s}(r)|^2 + 8|R_{1f}(r)|^2 \right. \\ \left. + \mu_{2p3/2} |R_{2p3/2}(r)|^2 + \mu_{1f5/2} |R_{1f5/2}(r)|^2 + \mu_{2p1/2} |R_{2p1/2}(r)|^2 + \mu_{1g9/2} |R_{1g9/2}(r)|^2 \right\} \quad (2)$$

where  $\mu_{2p3/2}, \mu_{1f5/2}, \mu_{2p1/2}, \mu_{1g9/2}$  parameters are assumed as a free parameter to be adjusted to obtain agreement with the experimental (CDD). After introducing the form of  $R_{nl}(r)$  with a harmonic oscillator size parameter  $b$  in Eq.(2), an analytical form for the ground state CDD of the 2p-1g shell nuclei is expressed as

$$\rho_c(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \left\{ 5 + \frac{5}{3} (\mu_{2p3/2} + \mu_{2p1/2}) \left(\frac{r}{b}\right)^2 + \left[ 4 - \frac{4}{3} (\mu_{2p3/2} + \mu_{2p1/2}) \right] \left(\frac{r}{b}\right)^4 \right. \\ \left. + \left[ \frac{64}{105} + \frac{28}{105} (\mu_{2p3/2} + \mu_{2p1/2}) + \frac{8}{105} (\mu_{1f5/2}) \right] \left(\frac{r}{b}\right)^6 + \left[ \frac{16}{945} (\mu_{1g9/2}) \right] \left(\frac{r}{b}\right)^8 \right\} \quad (3)$$

The mean square charge radius (MSR) can be determined according to the following equation

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_c(r) r^4 dr, \quad (4)$$

Where the normalization condition of the  $\rho_c(r)$  is given by

$$Z = 4\pi \int_0^\infty \rho_c(r) r^2 dr, \quad (5)$$

And the corresponding MSR is

$$\langle r^2 \rangle = \frac{b^2}{Z} \left\{ \frac{192}{2} + \frac{9}{2} (\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) + \frac{9}{2} (\mu_{1f_{5/2}}) + \frac{11}{2} (\mu_{1g_{9/2}}) \right\} \quad (6)$$

The PMD,  $n(k)$ , for the  $1f-2p$  shell nuclei is studied using two distinct methods. In the first method, it is determined by the shell model using the single-particle harmonic oscillator wave functions in momentum representation and expressed as

$$n(k) = \frac{b^3 e^{-b^2 k^2}}{\pi^{3/2}} \left[ \begin{aligned} &5 + \left\{ \frac{5}{3} (\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) \right\} (bk)^2 + \left\{ 4 - \frac{4}{3} (\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) \right\} (bk)^4 \\ &+ \left\{ \frac{64}{105} + \frac{4}{105} (\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) + \frac{8}{105} (\mu_{1f_{5/2}}) \right\} (bk)^6 \\ &+ \left\{ \frac{16}{945} (\mu_{1g_{9/2}}) \right\} (bk)^8 \end{aligned} \right] \quad (7)$$

whereas in the second method, the  $n(k)$  is determined by the CDFM, where the mixed density is given by [11, 12]

$$\rho(\vec{r}, \vec{r}') = \int_0^\infty |f(x)|^2 \rho_x(\vec{r}, \vec{r}') dx, \quad (8)$$

since

$$\rho_x(\vec{r}, \vec{r}') = 3\rho_0(x) \frac{j_1(k_F(x)|\vec{r} - \vec{r}'|)}{k_F(x)|\vec{r} - \vec{r}'|} \times \theta\left(\bar{x} - \frac{|\vec{r} + \vec{r}'|}{2}\right), \quad (9)$$

is the density matrix for  $Z$  protons uniformly distributed in the sphere with radius  $x$  and density  $\rho_0(x) = 3Z/4\pi x^3$ . The Fermi momentum is defined as [11, 12]

$$k_F(x) = \left( \frac{3\pi^2}{2} \rho_0(x) \right)^{1/3} \equiv \frac{V}{x}; \quad V = \left( \frac{9\pi Z}{8} \right)^{1/3} \quad (10)$$

and the step function  $\theta$ , in Eq. (9), is defined by

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \quad (11)$$

According to the density matrix definition of Eq.(9), one-particle density  $\rho(r)$  is given by its diagonal element as [11, 12]

$$\rho_c(r) = \rho_c(r, r')|_{r=r'} = \int_0^\infty |f(x)|^2 \rho_x(r) dx, \quad (12)$$

In Eq. (14),  $\rho_x(r)$  and  $|f(x)|^2$  have the following forms [11, 12]

$$\rho_x(r) = \rho_0(x) \theta(x - |\vec{r}|) \quad (13)$$

$$|f(x)|^2 = \frac{-1}{\rho_0(x)} \frac{d\rho_c(r)}{dr} \Big|_{r=x}. \quad (14)$$

The weight function  $|f(x)|^2$  of Eq. (14), determined in terms of the ground state  $\rho_c(r)$ , satisfies the following normalization condition [11, 12]

$$\int_0^\infty |f(x)|^2 dx = 1, \quad (15)$$

and holds only for monotonically decreasing  $\rho_c(r)$ , i.e.

$$\frac{d\rho_c(r)}{dr} < 0.$$

On the basis of Eq. (12), the PMD,  $n(k)$ , is given by [11, 12]

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k) dx, \quad (16)$$

where

$$n_x(k) = \frac{4}{3} \pi x^3 \theta(k_F(x) - |\vec{k}|), \quad (17)$$

is the Fermi-momentum distribution of the system with density  $\rho_0(x)$ . By means of Eqs. (14), (16) and (17), an explicit form for the PMD is expressed in terms of  $\rho_c(r)$  as

$$n^{CDFM}(k; [\rho_c]) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{Z} \times \left[ 6 \int_0^{V/k} \rho_c(x) x^5 dx - \left(\frac{V}{k}\right)^6 \rho_c\left(\frac{V}{k}\right) \right], \quad (18)$$

with normalization condition

$$Z = \int n^{CDFM}(k) \frac{d^3k}{(2\pi)^3}. \quad (19)$$

The elastic monopole charge form factors  $F_{C0}(q)$  of the target nucleus are also expressed in the CDFM as [11, 12]

$$F_{C0}(q) = \frac{1}{Z} \int_0^\infty |f(x)|^2 F(q, x) dx, \quad (20)$$

where the form factor of uniform charge density distribution is given by

$$F(q, x) = \frac{3Z}{(qx)^2} \left[ \frac{\sin(qx)}{(qx)} - \cos(qx) \right], \quad (21)$$

Inclusion of the correction due to the finite nucleon size  $f_{fs}(q)$  and the center of mass correction  $f_{cm}(q)$  in the calculations requires multiplying the form factor of Eq. (20) by these corrections. Here,  $f_{fs}(q)$  is considered as free nucleon form factor which is assumed to be the same for

protons and neutrons [19]

$$f_{fs}(q) = \exp\left[-\frac{0.43q^2}{4}\right], \quad (22)$$

The correction  $f_{cm}(q)$  removes the spurious state arising from the motion of the center of mass when shell model wave function is used and is given by [19]

$$f_{cm}(q) = \exp\left[\frac{q^2 b^2}{4A}\right], \quad (23)$$

Multiplying the right hand side of Eq. (20) by these corrections yields:

$$F_{C0}(q) = \frac{1}{Z} \int_0^\infty |f(x)|^2 F(q, x) dx f_{fs}(q) f_{cm}(q). \quad (24)$$

It is important to point out that all physical quantities studied above in the framework of the CDFM such as  $n(k)$  and  $F_{C0}(q)$ , are expressed in terms of the weight function  $|f(x)|^2$ . In the previous work [11, 12], the weight function was obtained from the NDD of the 2PF, extracted by analyzing elastic electron-nuclei scattering experiments. In the present work, the theoretical weight function  $|f(x)|^2$  is expressed, by introducing the derived CDD of Eq. (3) into Eq. (14), as

$$|f(x)|^2 = \frac{8\pi x^4}{3Zb^2} \rho_c(x) - \frac{16x^4 e^{-x^2/b^2}}{3Zb^5 \pi^{1/2}} \left\{ \begin{aligned} &\frac{5}{6}(\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) + \left[4 - \frac{4}{3}(\mu_{2p_{3/2}} + \mu_{2p_{1/2}})\right] \left(\frac{x}{b}\right)^2 \\ &+ \frac{3}{2} \left[ \frac{64}{105} + \frac{28}{105}(\mu_{2p_{3/2}} + \mu_{2p_{1/2}}) + \frac{8}{105}(\mu_{1f_{5/2}}) \right] \left(\frac{x}{b}\right)^4 \\ &+ \frac{32}{945}(\mu_{1g_{9/2}}) \left(\frac{x}{b}\right)^6 \end{aligned} \right\} \quad (25)$$

### 3. Results and Discussion

The proton momentum distributions  $n(k)$  and elastic form factors,  $F(q)$ , for  $^{88}\text{Sr}$  and  $^{89}\text{Y}$  nuclei are studied by means of the CFM. The PMD of Eq. (18) is calculated in term of the CDD obtained firstly from theoretical consideration, as in Eq. (3) and secondly from the fit to the electron-nuclei scattering experiments, such as 2PF [13]. The harmonic oscillator size parameters  $b$  are chosen in such a way so as to imitate the experimental root mean square (rms) charge radii of nuclei. In table 1, we display the experimental values of the

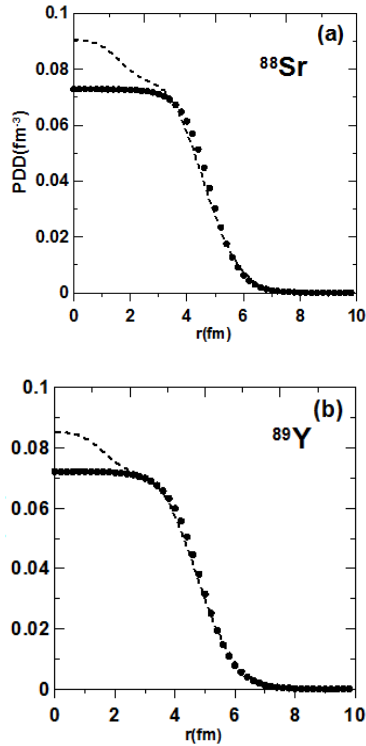
parameters  $c, z$  of 2PF distribution as well as the corresponding value of the central densities  $\rho_{\text{exp}}^{2PF}(0)$ , the root mean square charge radii  $\langle r^2 \rangle_{\text{exp}}^{1/2}$  for  $^{88}\text{Sr}$  and  $^{89}\text{Y}$  nuclei, in table 2, we present the chosen parameter of  $b$  (which reproduce the observed rms of  $^{88}\text{Sr}$  and  $^{89}\text{Y}$  nuclei) and the chosen occupation numbers of  $\mu_{2p_{3/2}}, \mu_{1f_{5/2}}, \mu_{2p_{1/2}}, \mu_{1g_{9/2}}$  required in Eq. (3) for evaluating the PDD and  $\langle r^2 \rangle_{\text{cal}}^{1/2}$

**Table 1.** The value of various parameters employed to PDD for ( $^{88}\text{Sr}$  and  $^{89}\text{Y}$ ) nuclei.

Nucleus	Type of PDD [14]	$P_{\text{exp}}(0)(\text{fm}^{-3})$ [14]	$\langle r^2 \rangle_{\text{exp}}^{1/2}(\text{fm})$ [14]	C [14]	Z [14]
$^{88}\text{Sr}$	2PF	0.09029581	4.17(2)	4.83(1)	0.496(11)
$^{89}\text{Y}$	2PF	0.08508776	4.27(2)	4.86(1)	0.542(11)

**Table 2.** The chosen parameters of  $b$  with chosen occupation numbers of  $\mu_{2p_{3/2}}, \mu_{1f_{5/2}}, \mu_{2p_{1/2}}, \mu_{1g_{9/2}}$  used in Eq. (3) to calculate the PDD and  $\langle r^2 \rangle^{1/2}_{\text{cal}}$ .

Nucleus	B	$\langle r^2 \rangle^{1/2}_{\text{cal}}$	Occupation			
			$\mu_{2p_{3/2}}$	$\mu_{2p_{1/2}}$	$\mu_{1f_{5/2}}$	$\mu_{1g_{9/2}}$
$^{88}\text{Sr}$	2.15	4.170471	2.283	0.164	5.709	1.394
$^{89}\text{Y}$	2.193	4.268564	2.283	0.614	5.709	2.394

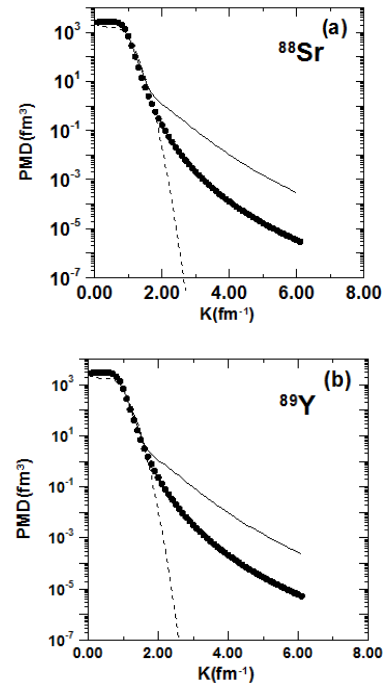


**Figure 1.** Proton density distribution as a function of  $r(\text{fm})$  for (a)  $^{88}\text{Sr}$  and (b)  $^{89}\text{Y}$  nuclei. The dashed curve is the calculated CDD of Eq. (3) when the higher shell of  $2p_{3/2}, 2p_{1/2}, 1f_{5/2}, 1g_{9/2}$ , are included. The solid circles are those fitted to the experimental data of Two Parameter Fermi (2PF) PDD [14].

In Figure 1, we explore the dependence of the CDD (in  $\text{fm}^{-3}$ ) on  $r$  (in  $\text{fm}$ ) for  $^{88}\text{Sr}$  [Figure 1(a)], and  $^{89}\text{Y}$  [Figure 1(b)] nuclei. The dotted symbols correspond to the experimental data [14]. The dashed curve corresponds to the calculated PDD when the higher shell of  $2p_{3/2}, 2p_{1/2}, 1f_{5/2}, 1g_{9/2}$ , are included in calculation of Eq. (3) with It is obvious that the dashed curves are in poor agreement with the experimental data, mainly for small  $r$ . These nuclei may need introducing other higher orbital to overcome this problem.

In Figure 2, we demonstrate the dependence of the  $n(k)$  (in  $\text{fm}^3$ ) on  $k$  (in  $\text{fm}^{-1}$ ) for  $^{88}\text{Sr}$  [Figure 2(a)] and  $^{89}\text{Y}$  [Figure 2(b)] nuclei. The long-dashed curves correspond to the PMD's of Eq. (7) evaluated by the shell model utilizing the single particle harmonic oscillator wave functions in the momentum space. The dotted symbols and solid curves correspond to the PMD's obtained by the CDFM of Eq. (18) utilizing the experimental and theoretical PDD, respectively. It is clear that the manner of the long-dashed distributions

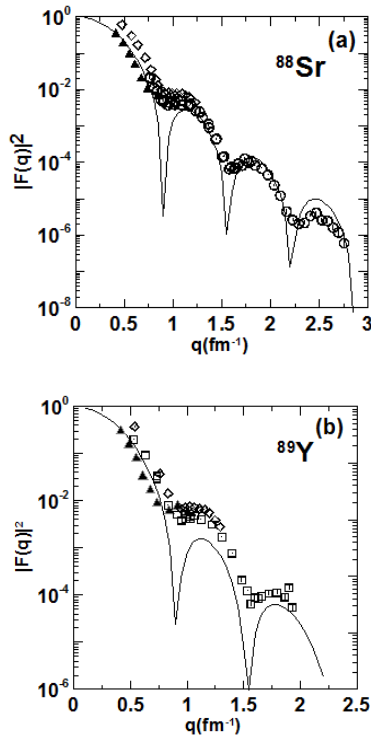
obtained by the shell model is in dissimilarity with the distributions imitated by the CDFM. The major property of the long-dashed distributions is the steep slope mode, when  $k$  increases. This behavior is in disagreement with the studies [11, 12, 20- 22] and it is recognized to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into consideration the significant effects of the short range dynamical correlation functions. Therefore, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the PMD [20, 21]. It is noted that the general structure of the dotted and solid distributions at the region of high momentum components is almost the same for  $^{88}\text{Sr}$ , and  $^{89}\text{Y}$  nuclei, where these distributions have the property of long-tail manner at momentum region  $k \geq 1.8 \text{ fm}^{-1}$ . The property of long-tail manner obtained by the CDFM, which is in agreement with the studies [11, 12, 20- 22], is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of Eq. (12), though their fluctuation functions  $|f(x)|^2$  are small.



**Figure 2.** Proton Momentum distribution as a function of  $k(\text{fm}^{-1})$  for (a)  $^{88}\text{Sr}$  and (b)  $^{89}\text{Y}$  nuclei. The solid curves and dotted symbols are the calculated PMD obtained in terms of the CDFM of Eq. (18) using the theoretical PDD of Eq. (3) and the experimental data of ref. [14], respectively. The long-dashed curves are the calculated PMD of Eq. (7) obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum representation.

The elastic electron scattering charge form factors from the considered nuclei are calculated in the framework of the CDFM through introducing the theoretical weight functions  $|f_c(x)|^2$  of Eq. (25) into Eq. (24). In Figure 3, we present the dependence of the form factors  $F(q)$  on the momentum

transfer  $q$  (in  $\text{fm}^{-1}$ ) for  $^{88}\text{Sr}$  [Figure 3(a)] and  $^{89}\text{Y}$  [Figure 3(b)] nuclei. It shows that the calculation  $F(q)$  (solid curve) is in agreement with the experiment  $F(q)$  (filled triangle [23], rhomboid [24], circle [25], square [26]) the first, second diffraction minima are located at ( $q \approx 0.9 \text{ fm}^{-1}$ ), ( $q \approx 1.55 \text{ fm}^{-1}$ ), respectively, while the third diffraction minimum experimental data of  $^{88}\text{Sr}$  nucleus was shifted slightly by about  $0.2 \text{ fm}^{-1}$  from the calculated one demonstrated by the solid curve which is located at  $q=2.2 \text{ fm}^{-1}$ .



**Figure 3.** Proton Form Factors as a function of  $q$  ( $\text{fm}^{-1}$ ) for (a)  $^{88}\text{Sr}$  and (b)  $^{89}\text{Y}$  nuclei. The solid curve is the calculated  $F(q)$  of the Eq. (24). The experimental data (filled triangle [23], rhomboid [24], circle [25], square [26])

## 4. Conclusions

The PMD and elastic charge form factors  $F(q)$ , which are evaluated by the *CDFM*, are formulated via the weight function  $|f(x)|^2$ . The weight function, which is related with the local density  $\rho_c(r)$ , is obtained from experiment and from theory. The property of the long-tail behavior of the PMD, which is in agreement with the other studies [11, 12, 21, 22], is achieved by both theoretical and experimental weight functions and is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of Eq. (12), though their weight functions are small. It is found that the theoretical CDD of Eq. (3) employed in the determination of theoretical weight function of Eq. (25) is capable to reproduce information about the PMD and elastic charge form factors as do those of the experimental data.

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