

# Methods for determination of the optical constants of the substance BiTe-BiSe

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**Abstract:** On the basis of the interference fringes in the transparency and reflection spectra of the free film or a film on the substrate when the substrate and the film material does not absorb light, can be determined by one of the optical constants of the material at a blank principal refractive index, if the film thickness is known.

**Keywords:** Structure, Optical Constants, Absorption Coefficient, Surface

## 1. Introduction

The principal refractive index  $n$  and the ratio of the absorption  $\alpha$  are directly connected with the microscopic parameters of the crystal, so an optical method for studying the structure of the substance, it is first necessary to determine  $n$  and  $\alpha$  separately or  $n^2 - \alpha^2$  and  $2n\alpha$  in a wide range of wavelengths and at different temperatures of the crystal. Various methods for determining the optical constants can be divided into two groups: the first  $n$  and  $\alpha$  are computed on the basis of the reflection coefficient in the second - transparency.

At the boundary between two media with different refractive indices  $n_1 = 1$  and  $n_2 = n$  is a division of the incident wave to the reflected and refracted. The amplitudes of the incident  $-E$  and reflected waves  $-E'$  are associated with the angle of incidence  $\varphi$  and refraction  $\varphi''$  Fresnel formulas:

$$E'_s = -\frac{\sin(\varphi - \varphi'')}{\sin(\varphi + \varphi'')} E_s, \quad E'_p = -\frac{tg(\varphi - \varphi'')}{tg(\varphi + \varphi'')} E_p, \quad (1)$$

Where the indices  $p$  and  $s$  correspond to the plane of incidence and the plane of incidence and the plane perpendicular to it. In deriving these equations, it was assumed that the magnetic permeability of the medium  $\mu_1 = \mu_2 = 1$ . The Fresnel formulas are valid in the case of the complex refractive index as  $n - ik = \sin\varphi / \sin\varphi''$ , in which case the angle of refraction  $\varphi''$  is also complex value.

The reflection coefficients  $R_s = \left| \frac{E'_s}{E_s} \right|^2$  or  $R_p = \left| \frac{E'_p}{E_p} \right|^2$ , as seen from the Fresnel formulas depend in a complicated

manner on the angle of incidence, the orientation of the vector  $E$ , the optical constants  $n$  and  $\alpha$ . At normal incidence ( $\varphi = \varphi'' = 0$ ) the relationship of the reflection coefficient and the optical constants is simplified:

$$R = R_s = R_p = \frac{(n-1)^2 + \alpha^2}{(n+1)^2 + \alpha^2}, \quad (2)$$

However, both the definition of the constants  $n$  and  $\alpha$  of the equation is not possible.

## 2. Data Collection and Analysis

In the case of  $n^2 \gg \alpha^2$ , where the absorption coefficient is small compared to the main index of refraction, the eligible (2) takes the form

$$R = \left( \frac{n-1}{n+1} \right)^2; \quad (3)$$

It can be used to determine the principal refractive indices, taking as a basis of measurement is only one value of the reflection coefficient of the natural radiation at an angle of incidence close to normal. The ratio of  $n$   $n^2 \gg \alpha^2$  is performed in a rather wide range of wavelengths  $\lambda > \lambda_g$  (where  $\lambda_g = hc/E_g$ ,  $E_g$  - band gap for direct allowed interband transitions) for most semiconductor crystals, even with a high content of carriers charging them, so according to formula (3) can be determined by  $n$  reflecting natural light. At infrared wavelengths, where the use of semitransparent mirrors impossible angle of incidence is typically not zero, and  $5 - 7^\circ$ . However, the magnitude of the reflection coefficient in the range of angles of incidence from 0 to

10 – 15° is weakly dependent on the angle of incidence, so it is usually believed that when  $R$  at the  $7^\circ$  is equal to  $R$  at  $0^\circ$ .

In the wavelength region of weak absorption of the crystal refractive index can also be found from the angular dependence  $R_p$  or  $R_p/R_s$  based on the Brewster angle  $\varphi_b$  whereby  $R_p$  and  $R_p/R_s$  reaches a minimum value. In this case,  $n = \text{tg } \varphi_B$ .

If the area is completely no absorption, refractive index can be experimentally determined and at normal incidence ( $\varphi = 0$ ) based on the measurement of plane transparent sample test material.

If the test sample of a substance with a refractive index  $n$  surrounded by air ( $n_0 = 1$ ), the transparency of the sample  $t$  can be represented by the formula

$$t = \frac{1}{I_0} \sum_i I_i = (1 - R)^2(1 + R^2 + R^4 + \dots) = \frac{1-R}{1+R} \quad (4)$$

Thus, on the basis of transparency obtain reflectance  $R$ , and then taking (3), we find that the refractive index

$$n = \frac{1+\sqrt{R}}{1-\sqrt{R}} = \frac{1+[(1-t/(1+t))^{1/2}]}{1-[(1-t/(1+t))^{1/2}]} \quad (5)$$

We note in passing that the total reflection from the sample in this case (without interference and absorption)

$$R_{sum} = \frac{1}{I_0} \sum_j I_{j\text{ref}} = R\{1 + (1 - R)^2[1 + R^2 + R^4 + \dots]\} = \frac{2R}{1+R} = \frac{(n-1)^2}{n^2+1} \quad (6)$$

In full accordance with the law of conservation of energy

$$R_{sum} + t = 1.$$

In the field of high transparency material most accurate method for determining the refractive index of the prism is a method in which the sample is made in the form of prism, wherein the sample is made in the form of a prism with a refractive angle  $A$ . Parallely light beam with a wavelength  $\lambda$  after passing through the lens is deflected by an angle  $\delta$ , and the highest measurement accuracy is obtained for  $n$  equal to the angle of incidence entering the prism and the angle of refraction at the exit of the prism. The angle  $\delta$  in the case minimal and the method is sometimes called prism method smallest deviation and the refractive index is determined by

$$\text{the formula } n = \frac{\sin \frac{A+\delta_{min}}{2}}{\sin \frac{A}{2}}.$$

By this method refractive index can be measured to within 0.01% for the most common semiconductor chip [1].

In real semiconductor crystals wavelength region where absorption can be neglected in most cases is small, but the condition  $n^2 \gg \kappa^2$  is performed in a much greater range of wavelengths, so the determination of the refractive index on the basis of the magnitude of the reflection coefficient of unpolarized radiation (at an angle of incidence close to normal) by the formula (3) often provides a sufficiently reliable value of the refractive index  $n$  in a wide range of photon energies.

The absorption coefficient  $\kappa$  when this condition  $n^2 \gg \kappa^2$  can be found by measuring the transparency of the film of the

test material according to the formula  $t = \frac{(1-R)^2[1+(\lambda\alpha/\pi n)^2]}{(e^{\beta-R}e^{-\beta})^2+4R\sin^2(\xi+\zeta)}$ , where  $\beta = \alpha d/2$ ,  $d$ -thickness of the sample, the phase shift  $\xi = 2\pi nd/\lambda$  (at normal incidence of the interfering beams); angle  $\zeta$  is calculated according to the condition  $\text{tg } \zeta = \alpha\lambda \left\{ 2\pi \left[ n^2 + \frac{\lambda^2 \alpha^2}{16\pi^2} \right] \right\}^{-1}$ . Methods for determination of  $n$  and  $\kappa$  on the interference will be described below.

If the interference is not observed, the relationship between  $t$ ,  $R$  and  $\alpha$  is simplified:

$$t = \frac{(1-R)^2[1+(\lambda\alpha/\pi n)^2]}{e^{\alpha d-R}e^{-\alpha d}}. \quad (7)$$

Since  $n^2 \gg \kappa^2$ , ie,  $\left(\frac{\lambda\alpha}{4\pi n}\right) < 1$ , then change the transparency of  $(1-R)/(1+R)$  to 10% of the absorption coefficient can be calculated from the formula

$$t = \frac{(1-R)^2 e^{-\alpha d}}{1-R^2 e^{-2\alpha d}}. \quad (8)$$

Even this simplified formula requires cumbersome calculations, so it is usually for the determination of  $\alpha$  make tables or nomograms build  $\alpha d$  depending on the transparency of  $t$  for different reflection coefficients  $R$ .

When transparency  $t < 10\%$  to calculate the absorption coefficient formula (8) takes the form

$$\alpha = \frac{1}{d} \ln \frac{(1-R)^2}{t}. \quad (9)$$

In the calculation of the absorption coefficient films deposited on a substrate material which has a refractive index different from the refractive index material film and a vacuum, it is necessary to take into account reflection from the boundaries of the film substrate, a substrate-vacuum. In the simplest case, a strong absorption in the film material and negligible free from the substrate of the sample (9) to be applied formula

$$\alpha = \frac{1}{d} \ln \frac{(1-R_1)(1-R_2)(1-R_{12})}{t}, \quad (10)$$

where  $R_{12}, R_2, R_1$  – reflection coefficients borders film - substrate vacuum, the film-vacuum.

Experimental procedure for measuring the absorption coefficient according BiTe-BiSe to the reflection and transmission usually consists of the following. Initially, only one plane is polished sample, wherein the condition  $\alpha d \gg 1$ , and measure the reflection coefficient  $R$  in all the possible range of wavelengths. The sample is then made thin enough ( $\alpha d \leq 1$  or  $\frac{1-R}{1+R} > t > 10\%$  in the most transparent), in order to be able to reliably measure the intensity passing there through beam in the wavelength region of interest. The absorption coefficient is determined by the formula (7) for  $t < 10\%$ .

To determine the absorption coefficient, it is possible merely to measure one only transparency, but it is necessary to prepare two samples of different thicknesses one from the same material. If  $\alpha d_1 > 1$  and  $\alpha d_2 > 1$ , the transparency of both the sample the absorption coefficient of the test material

can be calculated by the formula

$$\alpha = \frac{1}{d_2 - d_1} \ln \frac{t_1}{t_2} \approx \frac{1}{d_2 - d_1} \ln \frac{I_1}{I_2}, \quad (11)$$

Where  $I_1$  and  $I_2$  – intensity of the beam transmitted through the sample thickness  $d_1$  and  $d_2$  respectively. In that case, when the experimenter is the only example given composition from which the two samples prepared impossible because of its small size, it is possible to measure the opacity of the same sample, ground- it to a certain thickness  $d_1$  and  $d_2$ .

Sometimes the method of approximate determination of the absorption coefficient  $\alpha_j$  in variable  $j$ , where the reflection coefficient  $R = const$ , or changes only slightly compared with the change in transparency. Here

$$\alpha_j - \alpha_0 \approx \frac{1}{d} \ln \frac{t_0}{t_j}, \quad (12)$$

If  $\left(\frac{1-R_j}{1-R_0}\right)^2 \ll \frac{t_0}{t_j}$ . In the case where  $\alpha_j \gg \alpha_0$ , obtained

$$\alpha_j \approx \frac{1}{d} \ln \frac{t_0}{t_j}$$

This formula implies that the more precisely defines the absorption coefficient  $\alpha_j$ , the weaker the dependence  $R(j)$ .

If  $\alpha_j \geq \alpha_0$  or  $\alpha_j \leq \alpha_0$ , then (12) can be used to determine the difference  $\alpha_j - \alpha_0 = \Delta\alpha(j)$ , which allows us to calculate  $\alpha_j$ , if you have previously been measured absorption coefficient  $\alpha_0$  the initial conditions. This method is particularly suitable for measuring the dependence of  $\alpha(j)$  to the wavelength region where  $R \approx const$ . Then, instead of transparency  $t_0/t_j$  in the formula (12) is substituted intensity ratio  $I_0/I_j$  rays passing through the sample during the initial and intermediate conditions. Value  $\alpha_j$  is defined as the sum or difference  $\alpha_0$  and  $\Delta\alpha_j$ . The convenience of this method is associated with a fixed sample. In the study of the temperature dependence of absorption  $\alpha(T)$  is fixedly mounted on the thermal conductivity sample has a much more stable and reliable temperature-controlled sample than sliding on curtain.

The analysis of formula (9) to calculate the absorption coefficient  $\alpha$  and  $R = 35\%$  indicates that the change in transparency  $10^3$  times  $\alpha$  corresponds to a change only 6 times, so the absorption studies in a wide range of values requires a set of samples of the same material with different thickness.

For each thickness  $d$  of the maximum absorption coefficient  $\alpha_{max}$  determined from the minimum of transparency  $t_{min}$ ;  $\alpha_{min}$  value estimate for reasons of the maximum permissible error, the relative magnitude of which  $\delta\alpha = \Delta\alpha/\alpha$  for the formula (9) takes the form

$$\delta\alpha = \delta d + \left(\frac{2R}{1-R} \delta R + \delta t\right) \left[\ln \frac{(1-R)^2}{t}\right]^{-1} \quad (13)$$

Therefore, if it is supposed to measure  $\alpha$  with an error not exceeding the amount of error amounts  $(\delta R + \delta d + \delta t)$ , where  $R = 35\%$ .

$$\alpha_{min} = \frac{1}{d}, \quad (14)$$

which corresponds  $t_{max} = 16\%$ . The absorption coefficient  $\alpha < 1/d$  will be determined with less accuracy than  $\alpha_{min}$ . For example, the accuracy of  $\alpha = 0,2/d$  will  $\delta\alpha = \delta d + 5(\delta R + \delta t)$ , wherein  $e \approx 35\%$ , transparency, therefore the absorption coefficient of not calculated according to the formula (9), and formula (8).

Thus, if in the test area is changed by the absorption coefficient  $\alpha = 10cm^{-1}$  to  $\alpha = 10^4cm^{-1}$ , needed in the worst case for sample thicknesses  $d_1$  and  $d_2 > d_1$ .  $d_1$  thickness is determined from the formula (8), where the value is substituted  $\alpha_{max} = 10^4cm^{-1}$  and  $t_{min}$ . For example, when  $t_{min} = 0,01\%$  and  $R = 35\%$ ,  $d_1 = 8,3 mkm$ . Thickness  $d_1$  is determined from the formula (8), where the value is substituted  $\alpha_{max} = 10^4cm^{-1}$  and  $t_{min}$ . For example, when  $t_{min} = 0,01\%$  and  $R = 35\%$ ,  $d_1 = 8,3 mkm$ . Thickness  $d_2$  is calculated on the basis of (14) and the value  $\alpha_{min}$ ; for  $\alpha_{min} = 10cm^{-1}$ ,  $d_2 = 1 mm$ . This pattern allows to study the absorption region of 10 to  $84sm^{-1}$ , and the first sample from 240 to  $10^4cm^{-1}$ . To cover the interval  $84 - 240 cm^{-1}$  is required third sample with a thickness  $d_3 = 200 mkm$ .

When measuring small absorption coefficients of the condition of minimum error of measurement of the absorption coefficient  $\alpha d = 1$  requires the use of thick specimens. In modern solid-state lasers are used materials with an absorption coefficient reaching  $10^{-5}cm^{-1}$ .

From formula (13) implies that the measurement error of the absorption coefficient  $\delta\alpha$  increases with decreasing  $\delta d$  hyperbolically, and the relative error  $\delta I \approx \delta I_0 \approx 1\%$  for samples having  $\alpha d \approx 10^{-2}$ ,  $\delta\alpha \approx 140\delta I \approx 140\%$ . This limits the measurement values of  $\alpha d \approx 0,05$ , and due to a significant distortion of the light beam in the samples longer than 5cm real measurable quantity is  $\alpha \geq 10^{-2}cm^{-1}$ .

To measure less than the value of the absorption coefficient proposed a number of methods [2-5]. In [3] describes how to install, allowing to measure in a monochromatic laser beam with a wavelength of 1.06 micron absorption coefficient up to  $10^{-3}cm^{-1}$  of the active elements of a neodymium glass at length sample of 80 mm with a relative error of not more than 30% at the limit of sensitivity. In this setting, use a relative method, however measured values are not  $I_0$  and  $I$ , and combinations of these variables

$$A_0 = I_0 - I \text{ and } B_0 = I_0 + I. \quad (15)$$

The calculation formula for the absorption coefficient  $\alpha$ , obtained from [6] in this case is

$$\alpha = \frac{1}{d} \ln \left[ (1-R)^2 \frac{B_0 + A_0}{B_0 - A_0} \right] \quad (16)$$

When evaluating a random relative error in determining the absorption coefficient of variation must be considered a value of  $\beta = I_0/I_0'$ , which characterizes the degree of compensation of the light beams in the measuring channel in the absence of sample  $I_0'$  and in the channel comparison  $I_0$ , which is due to the finite accuracy of the compensation will be somewhat differ from one randomly. Then, instead of (15) should be more correct to write

$$A = I_0[\beta - (1 - R)^2 e^{-ad}], B = I_0[\beta + (1 - R)^2 e^{-ad}],$$

To calculate the random relative error of measurement of the absorption coefficient will expression  $\delta\alpha = D^{-1} \sqrt{C_1^2 sh^2 D + C_2^2}$ , where  $D = ad$ ;  $C_1^2 = \delta A^2 + \delta B^2$ ;  $C_2^2 = \delta B^2$ . Analysis function  $\delta\alpha(D)$  shows that it has at least  $sh D(Dch D - sh D) = C_2^2/C_1^2$ . Optimal  $ad$ , where the accuracy of determining the maximum  $\alpha$ . When  $\frac{C_2}{C_1} \approx 0,1$   $D_{min} \approx 0,45$ , and  $\delta \approx C_1 \approx \sqrt{\delta A^2 + \delta B^2}$ . If set the value of  $D = 0,01$ , in which the degree of compensation is 0.1%, then with the same respect  $C_2/C_1 \approx 0,1$  value  $\delta\alpha \approx 10C_1$ . When  $\delta A \approx \delta B \approx 1\%$  value  $\delta\alpha \approx 20\%$ .

In [4] to measure even smaller values of the absorption coefficient, the method of integrating cavities. The essence of this method is that the receiver does not detect the radiation transmitted through the sample, and the light scattered in the integrating cavity model. By setting [4] were able to measure the absorption coefficient of  $4 \cdot 10^{-3} cm^{-1}$  with an accuracy of 20% for a sample of  $35 cm^3$ . It is concluded that the possibility of measuring the absorption coefficient up to  $5 \cdot 10^{-5} cm^{-1}$  in a sample of no more than  $50 cm^3$ .

### 3. Results and Discussion

When passing of light through the semiconductor layers coplanar with a thickness comparable to the wavelength of light, the interference fringes occur, and for normal incidence mathematically dependent on the wavelength transparency, refractive index and thickness is expressed instead of (4) the formula

$$t = \frac{(1-R_{12})^2}{1+R_{12}^2-2R_{12}cos\delta}, \tag{17}$$

where  $\delta = \frac{4\pi}{\lambda} nd$  and  $R_{12} = \left(\frac{n-1}{n+1}\right)^2$ .

Reflection from the film can be found from the energy conservation law

$$R = 1 - t. \tag{18}$$

It follows from (17) that in the spectra of transparency at wavelengths

$$\lambda_{max} = \frac{4nd}{m}, \quad m = 2, 4, 6 \dots \tag{19}$$

Maxima are observed, and at wavelengths

$$\lambda_{min} = \frac{4nd}{m}, \quad m = 1, 3, 5 \dots$$

Minimum is observed.

In the reflection spectra of thin films free conditions take the form of maxima

$$2nd = m \frac{\lambda_{max}}{2}, \quad m = 1, 3, 5, \dots, \tag{20}$$

and the minimum

$$2nd = m \frac{\lambda_{max}}{2}, \quad m = 2, 4, 6, \dots$$

If the index  $n(\lambda)$  depends on the wavelength, then the interference fringes can not be determined  $n_\lambda = const$ , then based on the wavelength  $\lambda_m$  and  $\lambda_{m-1}$ , corresponding to neighboring extreme in the spectrum of transparency can be determined by the product of  $nd$  the equation  $2nd = m\lambda_m = (m - 1)\lambda_{m-1}$ , where

$$nd = \frac{\lambda_m \lambda_{m-1}}{2(\lambda_{m-1} - \lambda_m)}. \tag{21}$$

With the known thickness of the layer (21) is calculated refractive index

$$n = \frac{\lambda_m \lambda_{m-1}}{2d(\lambda_{m-1} - \lambda_m)} \tag{22}$$

In the case where the nonlinear refractive index, and weakly depends on the wavelength determined by this method in some middle range of wavelengths  $\lambda_{m-1} - \lambda_m$  refractive index. If the refractive index is linearly dependent on the wavelength:

$$n(\lambda) = n(0) + \alpha\lambda,$$

then the formula (22) calculating a refractive index  $n(0)$ , corresponding to  $\lambda \rightarrow 0$ .

From formulas (17) and (18) with (19) and (20) it follows that the maximum transparency of the free film  $t_{max} = 1$  and a maximum reflection from the film  $R_{max} = \frac{4R}{(1+R)^2} = \left(\frac{n^2-1}{n^2+1}\right)^2$ , while  $t_{min} = \left(\frac{1-R}{1+R}\right)^2 = \left(\frac{2n}{n^2+1}\right)^2$ . Transparency in the minimum and maximum can be determined by the formula

$$\frac{t_{min}}{t_{max}} = t_{min} = \left(\frac{1-R}{1+R}\right)^2 = \left(\frac{2n}{n^2+1}\right)^2 = t_0^2 \tag{23}$$

where  $t_0$  - transparency film, according to (14) in the wavelength range  $\lambda \gg 4nd$ , when the interference does not occur.

For a system of film - substrate spectrum transparency laws remain the same as for the free film, if  $n_1 < n_2 > n_3$  and  $k_1 = k_2 = k_3 = 0$ . The film and the substrate are completely transparent. If  $n_1 < n_2 < n_3$ , when the substrate material is optically denser material film, the film material, the maxima and minima in the spectra of transparency shifted  $\lambda/2$  with respect to the case,  $n_1 < n_2 > n_3$ , due to the difference of phase discontinuities at the interfaces media. Thus, on the basis of the interference fringes in the transparency and reflection spectra of the free film or a film on the substrate when the substrate and the film material does not absorb light, can be determined by one of the optical constants of the material at a blank principal refractive index, if the film thickness is known.

When the film thickness ( $d \gg \lambda$ ), then the transparency and reflection of its refractive index associated with the formulas (4) and (6), of which the refractive index can be determination for each wavelength.

In the case of the absorbing layer and the substrate for the

light incident from the environment  $n_1$  on film, the relationship of reflection from the film and the optical constants is [7]:

$$R_1 = \frac{(g_1^2 + h_1^2)e^{Y_2} + (g_2^2 + h_2^2)e^{-Y_2} + A \cos \delta_2 + B \sin \delta_2}{e^{Y_2} + (g_1^2 + h_1^2)(g_2^2 + h_2^2)e^{-Y_2} + C \cos \delta_2 + D \sin \delta_2}, \quad (24)$$

Where

$$\begin{aligned} A &= 2(g_1 g_2 + h_1 h_2), & B &= 2(g_1 h_2 - g_2 h_1), \\ C &= 2(g_1 g_2 - h_1 h_2), & D &= 2(g_1 h_2 + g_2 h_1), \\ g_1 &= \frac{n_1^2 - n_2^2 - k_2^2}{(n_1 + n_2)^2 + k_2^2}, & h_1 &= \frac{2n_1 k_2^2}{(n_1 + n_2)^2 + k_2^2}, \\ g_2 &= \frac{n_2^2 - n_3^2 + k_2^2 - k_3^2}{(n_2 + n_3)^2 + (k_2 + k_3)^2}, & h_2 &= \frac{2(n_2 k_3 - n_3 k_2)}{2(n_2 + n_3)^2 + (k_2 + k_3)^2}, \\ \gamma_2 &= \frac{4\pi k_2 d_2}{\lambda}, & \delta_2 &= \frac{4\pi}{\lambda} n_2 d_2. \end{aligned}$$

Note that in contrast to the transparent film and the substrate, for which the reflection of light incident from the air  $n_1$ , is represented by the light incident from the substrate side  $n_3$ , for absorbing film and the substrate, these values reflect different, and for light incident on the absorbing film absorbing medium, in contrast to the formula (24), the reflection can be expressed as [7]

$$R_3 = \frac{(g_2^2 + h_2^2)e^{Y_2} + (g_1^2 + h_1^2)e^{-Y_2} + A \cos \delta_2 + B \sin \delta_2}{e^{Y_2} + (g_1^2 + h_1^2)(g_2^2 + h_2^2)e^{-Y_2} + C \cos \delta_2 + D \sin \delta_2}, \quad (25)$$

Thus, when  $n_3 \neq n_1$  and  $R_3 \neq R_1$ ; obviously when  $n_3 = n_1$   $R_3 = R_1$ .

Transparency of the film - substrate in this case of absorbing media, as before, can be found from the relation  $t = 1 - R$ , which follows from the law of conservation of energy.

As can be seen from the expressions (24) and (25), the relationship between the optical constants  $n_2$  and  $k_2$  film on an absorbing substrate is very complicated.

The expression for the transparency of the system for normal incidence of extreme points is [8]

$$t_{ext} = 16n_2^2 n_3 \times \left[ (n_2 + 1)^3 (n_3^2 + n_2) e^{Y_2} - (n_2 - 1)^3 (n_3^2 - n_2) e^{-Y_2} + (-1)^m 2(n_2^2 - 1)(n_3^2 - n_2^2) \right]^{-1} \quad (26)$$

Where  $m$  - order of interference, which is determined from the formula

$$\delta = \frac{4\pi}{\lambda} (n_2 - ik_2) d_2 \approx m\pi, \quad (27)$$

as  $k_2 \ll 1$ . If  $k_2 = 0$ , then (27) has acquired the appearance of  $\delta = \delta_2 = \frac{4\pi}{\lambda} n_2 d_2 = m\pi$ . For absorbing film BiTe-BiSe extremes in the spectrum of transparency and reflection of the greater offset relative extrema at  $k_2 = 0$ , than more  $k_2$  [9-10].

It follows from (26) it follows that for  $\alpha_2 = 0$  in the

transparency of the system extremum for  $m$  even,

$$t_{ext} = \frac{2n_3}{n_3^2 + 1} = t_0 \quad (28)$$

If  $m$  is odd,

$$t_{ext}^0 = \frac{4n_2^2 n_3}{(n_2^2 + 1)(n_3^2 + n_2^2)} \quad (29)$$

On the basis of formula (29) can be calculated and constructed a nomogram, which allow to define  $n_2$  of transparency in the odd extremum and  $n_3$  transparency of one substrate, as the envelope of the family of curves corresponds  $n_2 = n_3$ .

The ratio of extreme values (28) and (29)

$$\kappa = \frac{t_{ext}^0(m - odd)}{t_{ext}^0(m - even)} = \frac{2n_2^2(n_3^2 + 1)}{(n_2^2 + 1)(n_3^2 + n_2^2)}$$

May be less or greater than unity depending on the ratio between  $n_2$  and  $n_3$ . When  $n_2 > n_3$   $\kappa < 1$ , and for  $n_2 < n_3$   $\kappa > 1$ . In other words, for  $n_4 = n_1 < n_2 < n_3$  even  $m$  corresponds to the minimum of transparency, while for  $n_4 = n_1 > n_2 > n_3$  even  $m$  corresponds to the maximum. This is connected with boundary conditions: at normal incidence to the interface with optical media paid more phase of the reflected wave at  $\pi$ , and the reflection from the interface with the less dense medium phase loss occurs.

## 4. Conclusion

Since the formula (26) includes two unknowns quantities  $n_2$  and  $k_2$  ( $\gamma_2 = \frac{4\pi}{\lambda} k_2 d_2$ ), then the method of determining  $n_2$  and  $k_2$  on the interference fringes is reduced to the method of successive approximations. First, find a first approximation  $n_2$  substituting the value of transparency in the odd extremum in the formula (26). Interference order  $m$  is calculated from the ratio of  $2nd = \lambda_m m = \lambda_{m-1}(m - 1)$ , which is believed  $n(\lambda) = const$ . If the film is not free, the pre-determined  $n_3$  of a transparent substrate according to the formula (28), then a first approximation  $n_2$  values into the formula (26) for even  $m$  and determining a first approximation of the value  $\gamma_2$ . Values  $n_3$ ,  $n_2$  and  $\gamma_2$  into the formula

$$t_{ext} = t_{ext}^0 \left[ 1 - A\gamma_2 + (A^2 - \frac{B}{2})\gamma_2^2 \right], \quad (30)$$

where

$$A = \frac{(n_3^2 + n_2)(n_2 + 1)^3 + (n_3^2 - n_2)(n_2 - 1)^3}{4(n_2^2 + 1)(n_3^2 + n_2^2)},$$

$$B = \frac{(n_3^2 + n_2)(n_2 + 1)^3 - (n_3^2 - n_2)(n_2 - 1)^3}{4(n_2^2 + 1)(n_3^2 + n_2^2)},$$

$t_{eksper}$  - the experimental value of transparency in the odd extremes.

Equation (30) is obtained from (26) for odd  $m$  decomposition  $e \pm \gamma_2$  in a row to second order in  $\gamma_2$ .

Calculated transparency in the odd extremum  $t_{ext}^0$  again

substituted into the formula (29) and determine the second approximation  $\gamma_2$  etc. This cycle is repeated as long as the following approximate values  $n_2$  and  $\gamma_2$  will be a little different from the previous one.

When  $n_2$  and  $\gamma_2$  finally found,  $k_2$  calculated using (24) and (27), by the formula

$$k_2 = \frac{\lambda}{4\pi d_2} \gamma_2 = \frac{n_2}{m\pi} \gamma_2,$$

Where the order of interference  $m$  is determined from (19) according to the values of wavelength for the two neighbouring extreme. For non-absorbing layer  $n_2$  are based on  $t_{ext}^0$  ( $m$ - odd), and immediately get the final value.

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