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Magneto-Hydrodynamics (MHD) Bioconvection Nanofluid Slip Flow over a Stretching Sheet with Microorganism Concentration and Bioconvection Péclet Number Effects

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Abstract: This study analyzes the effect of slip parameter, microorganism concentration and bioconvection Péclet number on Magneto-hydrodynamics (MHD) bioconvection nanofluid flow over a stretching sheet. Similarity transformation is employed to convert the governing partial differential equations into coupled non-linear ordinary differential equations with appropriate boundary conditions. These equations are solved numerically using fourth order Runge Kutta-Fehlberg integration method along with a shooting technique. The dimensionless velocity, temperature, nanoparticle concentration and density of motile microorganisms were obtained together with the local skin friction, reduced Nusselt, Sherwood and motile microorganism density numbers. It was observed that nanoparticle concentration decreases with increase in the nanoparticle concentration slip but increases as magnetic field parameter increases. Also the velocity of the fluid decreases with increase in both velocity slip parameter ξ and magnetic field parameter M. It is also noticed that the temperature of the flow is continuously decreasing as the value of velocity slip parameter ξ, temperature slip parameter β and concentration slip parameter γ increase. Furthermore, as velocity and nanoparticle concentration slip parameters increase, the Nusselt number was observed to increase while the Sherwood number decreases. The skin friction coefficient also decreases as the values of velocity slip parameter increases. Finally we found that local microorganism transfer rate increases with greater values of bioconvection Lewis number Lb, microorganism concentration Ω and bioconvection Péclet number Pe. Comparisons between the previously published works and the present results reveal excellent agreement.

Keywords: Bioconvention Péclet, Slip Conditions, Nanofluid, MHD Flow and Heat Transfer

1. Introduction

Thermal conductivity plays a significant role in enhancing heat transfer. Fluids with small thermal conductivity such as oil, water and ethylene glycol are categorized as the poor heat transfer materials. There are several innovative techniques proposed for the heat transfer enhancement of fluids. One of such techniques is nanofluids. The term nanofluidswas coined by Choi [1] at the ASME Winter Annual Meeting, which refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles) with typical length on the order of 1-50 nm. Nanofluids are used to enhance the thermal conductivity of base fluids. They have several engineering and applications in cooling and process industries. Buongiorno and Hu [2] and Buongiorno [3] presented a study of convective transport in nanofluids. Also, Daungthongsuk and Wongwises [4] investigated the effects of thermophysical properties of nanofluids on the convective heat transfer. Kuznetsov and Nield [5] employed the Brinkman and Darcy models to study the onset of convection in a horizontal layer of a porous medium filled with a nanofluid. They also obtained similarity solution of natural convective boundary-layer flow of a nanofluid past a vertical plate. Rana and Bhargava [6] studied the steady boundary-layer flow and heat transfer for different types of nanofluids near a vertical plate with heat generation effects. Makinde and Aziz [7] reported the similarity solutions for the thermal boundary layer of a nanofluid past a stretching sheet with a convective boundary condition. Bioconvection is a phenomenon that occurs when microorganisms swim upward microorganisms like algae tend to concentrate in the upper portion of the fluid layer thus causing a top heavy density stratification that often becomes unstable. Kuznetsov [8] studied both non-oscillatory and oscillatory nanofluid biothermal convection in a horizontal layer of finite depth and analyzed the dependence of the thermal Rayleigh number on the nanoparticle Rayleigh number and the bioconvection Rayleigh number. Khan and Makinde [9] investigated MHD flow of nanofluids with heat and mass transfer along a vertical stretching sheet in the presence of motile gyrotactic microorganisms. Xuand Pop [10] obtained a more physically realistic result using a passively controlled nanofluid model by an analysis of bioconvection flow of nanofluids in a horizontal channel. In a recent paper, Khan *et al.* [11] investigated the effects of both Navier slip and magnetic field on boundary-layer flow of nanofluids containing gyrotactic microorganisms over a vertical plate. Their results show that the bioconvection parameters tend to reduce the local concentration of motile microorganisms.

In this study, our main objective is to report the effect of the bioconvection Lewis number Lb, microorganism concentration Ω and bioconvection Péclet number Pe on the rate of heat and mass transfer of a nanofluid over a stretching sheet with temperature and velocity slip, in which combined effects of magnetic field, nanoparticle, microorganism and thermal radiation are also considered.

2. Mathematical Formulation

Consider a two-dimensional steady state boundary layer flow of a nanofluid over stretching sheet with surface temperature T_w and concentration C_w. The stretching velocity of the sheet is $u_w = ax$, with a being a constant. Let the wall mass transfer be V_w, which will be determined later. The flow is assumed to be generated by stretching sheet issuing from a thin slit at the origin. The sheet is then stretched in such a way that the speed at any point on the sheet becomes proportional to the distance from the origin. The ambient temperature and concentration are T_∞ and C_∞ respectively. The flow is subjected to the combined effects of thermal radiation R and a transverse magnetic field of strength B_0, which is assumed to be applied in the positive y direction

It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Using scale analysis and following Buongiorno [12] and Kuznetsov [13], the governing equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho c_p}\left(\frac{\partial q_r}{\partial y}\right)+\tau\{D_b\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_t}{T_\infty}[\left(\frac{\partial T}{\partial y}\right)^2]+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho}u^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_b\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_t}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_C}{C_\omega-C_\infty}[\frac{\partial}{\partial y}(n\frac{\partial C}{\partial y})+\frac{\partial}{\partial x}(n\frac{\partial C}{\partial x})]=D_n\left(\frac{\partial^2 n}{\partial x^2}+\frac{\partial^2 n}{\partial y^2}+2\frac{\partial^2 n}{\partial x\partial y}\right) \tag{5}$$

In the above expressions u and v are the nanofluid velocity components, T is the temperature, n is the density of motile microorganisms, p ρ, α, B_0, D_b, D_t and D_n are the fluid pressure, the density of the base fluid, electrical conductivity, magnetic field, the Brownian diffusion, thermophoresis diffusion coefficient and the diffusivity of microorganisms respectively.

The given boundary conditions are

$$\text{At } y=0, u=u_w+L\frac{\partial u}{\partial y}, v=V_m, T=T_w+\delta\frac{\partial T}{\partial y}, C=C_w+\varepsilon\frac{\partial C}{\partial y} and n= n_w+\zeta\frac{\partial n}{\partial y} \tag{6}$$

$$n\to n_\infty \text{as y}\to\infty u\to U_\infty=0, T\to T_\infty, C\to C_\infty, n\to n_\infty \text{as y}\to\infty \tag{7}$$

Where $u_w = ax$, $T_w = T_\infty + g(\frac{x}{l})^2$,

$C_w = C_\infty + C(\frac{x}{l})^2$, $n_w = n_\infty + D(\frac{x}{l})^2$

Where L, δ , ε and ζ are the velocity, the thermal concentration and density slip factors respectively, and when $L = \delta = \varepsilon = \zeta = 0$, the no-slip condition is recovered, l reference length of a sheet. The above boundary condition is valid when $x \ll l$ which occurs very near to the slit. We introduce the following similarity functions:

$$\eta = y\sqrt{\frac{a}{v}} \tag{8}$$

$$\psi=\sqrt{av}xf(\eta)$$

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}, u=axf'(\eta), v=-\sqrt{av}f(\eta) \tag{9}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_{w-T_\infty}} \tag{10}$$

$$\phi(\eta)=\frac{C-C_\infty}{C_{w-C_\infty}}$$

Where ψ is the stream function, the continuity equation is satisfied

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \tag{11}$$

The radiative heat flux in the x-direction is considered negligible as compared to y-direction. The radiative heat flux q_r is given by

$$q_r = -4\frac{\sigma^*}{3K^*}\frac{\partial T^4}{\partial y^4} \tag{12}$$

Where σ^* and K^* are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature difference within the flow is sufficiently small such that the term T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a Taylor series about a free stream temperature T_∞ as follows:

$$T = T_\infty^4 + 4T_\infty^3(T - T_\infty^4) + 6T_\infty^2(T - T_\infty^4)^2 + ... \tag{13}$$

If we neglect higher-order terms in the above Eq. (13) beyond the first order in $(T - T_\infty)$, we have:

$$T^4 \cong 4T_\infty^3 - 3T_\infty^4 \tag{14}$$

Thus, substituting Eq. (14) into Eq. (12), we have:

$$q_r = -\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial T}{\partial y} \tag{15}$$

Using the boundary layer approximations with the similarity functions, the transformed governing equations and boundary conditions are then obtained as:

$$f''' + ff'' - (f'^2) - Mf' = 0 \tag{16}$$

$$(1+\frac{4}{3}R)\theta'' + P_r f\theta' + P_r Ec f''^2 + P_r M Ec f'^2 + Pr N_b\theta'\phi' + Pr N_t\theta'^2 = 0 \tag{17}$$

$$\phi'' + Lef\phi' + \frac{N_t}{N_b}\theta'' = 0 \tag{18}$$

$$\chi'' + L_b f\chi' - P_e[\emptyset''(\Omega+\chi) + \chi'\phi'] = 0 \tag{19}$$

$$f(0) = \lambda,\ f'(0) = 1 + \xi f''(0),\ \theta(0) = 1 + \beta\theta'(0),\ \phi(0) = 1 + \gamma\phi'(0),\ \chi(0) = 1 + \varphi\chi'(0) \tag{20}$$

$$f'(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0,\ \chi(\infty) = 0 \text{ as } \eta \to \infty \tag{21}$$

Where the primes denote differentiation with respect to η and the parameter appearing in eqs. (16-19) are defined as:

$$Pr = \frac{v}{\alpha},\ Le = \frac{v}{D_b},\ Lb = \frac{v}{D_m}\ Pe = \frac{bw_c}{D_m},\ \Omega = \frac{n_\infty}{n_w - n_\infty},\ N_b = \frac{\tau(C_w - C_\infty)D_b}{v},$$

$$N_t = \frac{\tau(T_w - T_\infty)D_t}{vT_\infty},\ Ec = \frac{(u_w)^2}{c_p(T_w - T_\infty)},\ R = \frac{4T_\infty^3\sigma^*}{3K^*K},\ M = \frac{\sigma B_0^2}{\rho a},\ \xi = L\sqrt{\frac{a}{v}},\ \beta = \delta\sqrt{\frac{a}{v}},$$

$$\gamma = \varepsilon\sqrt{\frac{a}{v}},\ \varphi = \zeta\sqrt{\frac{a}{v}} \tag{22}$$

In Eqs. (16-20), Pr, Le, Lb, Ω, Nb, Nt, Pe, Ec, R, M, ξ, β, γ and φ represent Prandtl number, Traditional Lewis number, Bioconvection Lewis number, Bioconvection concentration difference parameter, Brownian motion parameter, thermophoresis parameter, Bioconvection Péclet number, Eckert number, thermal radiation, Magnetic parameter, Velocity slip parameter, Thermal slip parameter, Concentration slip parameter and Density slip parameter respectively. The quantities of practical interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu_x, the local Sherwood number Sh_x and the local density number of the motile microorganisms Nn_x which are defined as:

$$C_f = \frac{\tau_w}{\rho_f u_0^2},\ Nu_x = \frac{q_w x}{k(T_w - T_\infty)},\ Sh_x = \frac{q_m x}{D_b(T_w - T_\infty)},\ Nn_x = \frac{xq_n}{D_n(n_w - n_\infty)} \tag{23}$$

Where the heat flux q_w and mass flux h_m are given as:

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},\ h_m = -D_b\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{24}$$

Using Eqs. (21) and (22)

$$C_{fx} = (Re_x)^{-\frac{1}{2}} C_f = -f''(0),\ Nu_x = (Re_x)^{-\frac{1}{2}} Nu = -\theta'(0),\ Sh_x = (Re_x)^{-\frac{1}{2}} Sh = -\phi'(0),\ Nn_x = (Re_x)^{-\frac{1}{2}} Nn = -\chi'(0)$$

3. Numerical Solution

The governing equations with the associated boundary conditions Eqs. (16)- (19) are numerically solved using fourth order Runge-Kutta method based shooting technique with Matlab package. Firstly, they were converted into first order linear differential equations using shooting technique and Runge-Kutta method to solve first order differential equations. We assumed the unspecified initial conditions for unknown variables, the transformed first order differential equations are integrated numerically as an initial valued problem until the given boundary conditions are satisfied.

Equations (16)-(19) are reduced to systems of first order differential equations

4. Results and Discussion

This section deals with graphical and numerical results of all the governing parameters arises in the governing flow problem. Eqs. (16-19) subject to the boundary conditions, Eqs. (20) and (21), were solved numerically using a fourth-fifth order Runge-Kutta Fehlberg method to solve the boundary value problems numerically. The validity of this study is generally attested in tables 1-2 to by its agreement with other generally accepted and published works in the same line of study. The present work's validity was tested by comparing it with the works of Wubshet and Bandari. [14] (MHD boundary layer flow and heat transfer of a nanofluid past a permeable strecting sheet with velocity and thermal slip), Andersson [15] (Slip flow past a stretching surface) and Hayat et al, [16] (MHD flow and heat transfer over permeable stretching sheet with slip conditions). The result from this research was found to be in excellent agreement with theirs comparison of Results for the skin friction $f''(0)$ and reduced Nusselt number $-\theta'(0)$. Numerical values of local Nusselt number, skin friction coefficient, local Sherwood number and motile microorganism of local density are presented in Tables 3-4 against all the pertinent parameters.

Table 1. *Comparison of skin friction coefficient $-f''(0)$ for different values of ξ when $\lambda = \varphi = 0$.*

ξ	M	Anderson [15]	Haya et al [16]	Wubshet and Bandari [14]	Present Result
10		0.0812	0.081249		0.0812
0.0		1.0000	1.0000		0.9998
	0.0			1.2808	1.2799
	0.5			1.5000	1.5000
	1.0			1.6861	1.6861
	1.5			1.8508	1.8508
	2.0			2.0000	2.0000

Table 2. *Comparison of local Nusselt number $-\theta'(0)$ for different values of Le and Ntwhen $\lambda = M = R = 0.5, \xi = \beta = \gamma = \varphi = 1$, $Pr = 1$, $Ec = Nb = 0.2$.*

Le	Nt	Wubshet and Bandari [14]	Present Result
5	0.2	0.3980	0.3974
10		0.3993	0.3990
15		0.4001	0.3999
20		0.4005	0.4005
	0.1	0.3947	0.3943
	0.3	0.3915	0.3911
	0.4	0.3884	0.3880
	0.5	0.4002	0.4000

Table 3. *Computation showing the skin friction coefficient $-f''(0)$, local Nusselt $-\theta'(0)$, local Sherwood number $-\emptyset'(0)$ and local microorganisms density number $-\chi'(0)$ when $M = 1$, $R = 0.5$, $Pr = 10$, $Ec = Nt = Nb = Pe = 0.2$, $\Omega = Lb = 0.1$ for different values of ξ, β, γ and φ.*

ξ	β	γ	φ	$-f''(0)$	$-\theta'(0)$	$-\phi9(0)$	$-\chi''(0)$
0.0	0.5	0.5	0.5	1.6861	0.7595	1.1331	0.2186
0.1				1.4062	0.8370	1.0647	0.2384
0.2				1.2116	0.8835	1.0178	0.2498
0.3				1.0673	0.9337	0.9833	0.2568
0.5	0.0	0.5	0.5		1.6539	0.7417	0.3414
	0.2				1.3027	0.8338	0.3073
	0.4				1.0467	0.9061	0.2773
	0.6		0.5		0.8644	0.9997	0.2535
		0.0			0.7888	2.5663	0.1014
		0.1			0.8545	1.9016	0.1000
		0.2			0.8927	1.5108	0.0993
		0.3			0.9176	1.2534	0.0989
0.5	0.5	0.5	0.0				0.3025
			0.1				0.2941
			0.2				0.2861
			0.3				0.2786

Table 3 presents the variation of the skin friction coefficient, local Nusselt number, local Sherwood number and local microorganism density number in relation to different slip parameters. It is observed that as velocity slip increases both local Nusselt number and local microorganism density number increase while skin coefficient and local Sherwood number decrease. This concludes that increment in velocity slip leads to increase in rate of heat transfer as well as microorganism density but decrease in mass rate.

The effect of thermal slip parameter on the physical quantities, we observe that as temperature slip increase the rate of heat transfer and microorganism density are decreased.

The effects of nanoparticle concentration slip parameter also observed in table 3. It was noticed that mass transfer rate and microorganism density decrease as nanoparticle concentration slip increases, but heat transfer rate increases.

Table 3 also illustrated the effect of motile microorganism density slip parameter on various physical quantities. It was found that wall motile microorganisms flux decreased with higher motile microorganism density slip effect.

Table 4 presents the variation of the local microorganism density number - $\chi''(0)$ in relation to the bioconvection Lewis number Lb, microorganism concentration Ω and bioconvectionPéclet number Pe. On observing this table, as the values Lb, Ω and Pe increase, the local microorganism density number increase. Therefore for higher value of value Lb, Ω and Pe the microorganism speed will be reduced and the diffusivity of the microorganisms will be reduced. This will leads to reduced microorganisms' density in the boundary layer and results in increase mass transfer rate.

Table 4. *Computation showing the local microorganisms density number -* $\chi''(0)$ *when* $M = 1$, $R = 0.5$, $\xi = \beta = \gamma = \alpha_1 = 0.5$, *=Pe = 0.2, Nt =Nb = Ec = 0.2 for different values of Ec,Nt,Nb and Pr=10.*

Lb	Pe	Ω	- $\chi''(0)$
0.1	0.2	0.2	0.2646
0.2			0.3105
0.3			0.3584
0.4			0.4053
	0.1		0.1924
	0.2		0.2646
	0.3		0.3313
	0.4		0.3931
		0.0	0.2502
		0.05	0.2574
		0.1	0.2646
		0.15	0.2718
		0.2	0.2790

Table 5. *Computation showing the local Nusselt -*$\theta'(0)$*, local Sherwood number -*$\phi'(0)$ *and local microorganisms density number -* $\chi'(0)$*when* $\xi = \beta = \gamma = \varphi = 0.5$, *Pr = 10, Ec =Nt = Nb =Pe = 0.2,* Ω *= Lb =0.1 for different values of Le, M and R.*

Le	M	R	$-\theta'(0)$	-ϕ9(0)	- $\chi''(0)$
4.0	1.0	0.5	0.9510	0.7987	0.2662
6.0			0.9479	1.0408	0.2637
8.0			0.9518	1.1962	0.2628
10			0.9575	1.3059	0.2626
12			0.9636	1.3880	0.2627
5.0	0.0		1.0297	0.9420	0.2874
	0.5		0.9813	0.9393	0.2735
	1.0		0.9480	0.9350	0.2646
	1.5		0.9232	0.9302	0.2583
	2.0		0.9038	0.9255	0.2536
	1.0	0.0	1.0627	0.8981	0.2815
		0.1	1.0362	0.9064	0.2778
		0.2	1.0117	0.9142	0.2743
		0.3	0.9890	0.9216	0.2709
		0.4	0.9678	0.9285	0.2677

Table 5 presents the variation of the local Nusselt number, local Sherwood number and local microorganism density number in relation to magnetic field, Lewis number and thermal radiation. On observing this table, as the values of Lewis number increase, the values of local Nusselt number and local Sherwood number increase, but opposite effect is observed in local microorganism density number. In addition, as the values of magnetic field parameters increase, both the values of the local Nusselt number, local Sherwood number and local microorganism density number decrease. However, increase in the thermal radiation increases local Sherwood number but reduces rate of heat transfer and local microorganism density number.

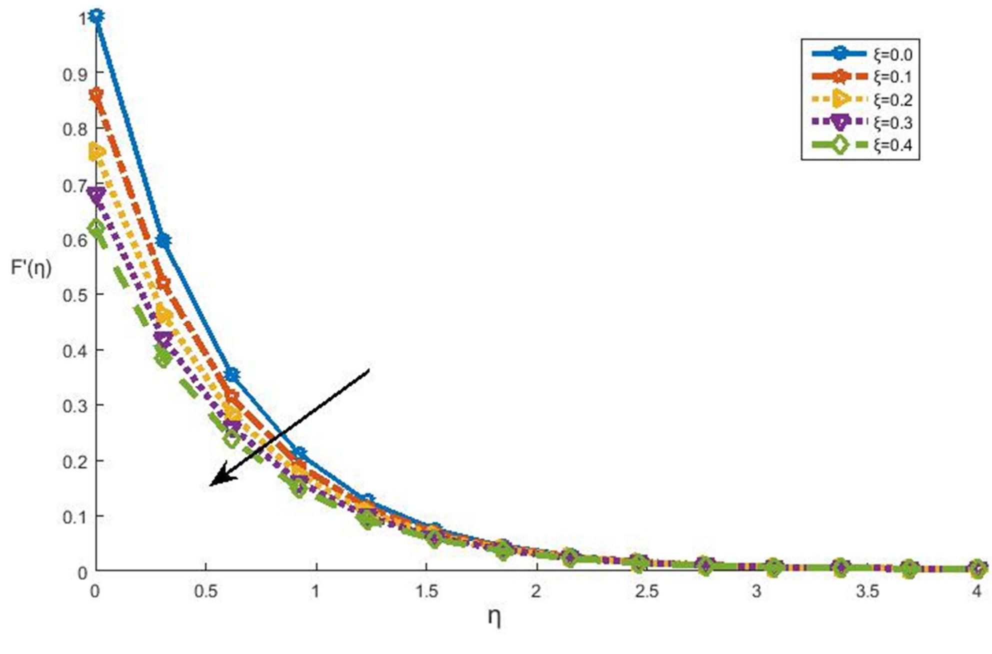

Figure 1. *Effect of velocity slip parameter* ξ *on velocity distribution when m = 0.*

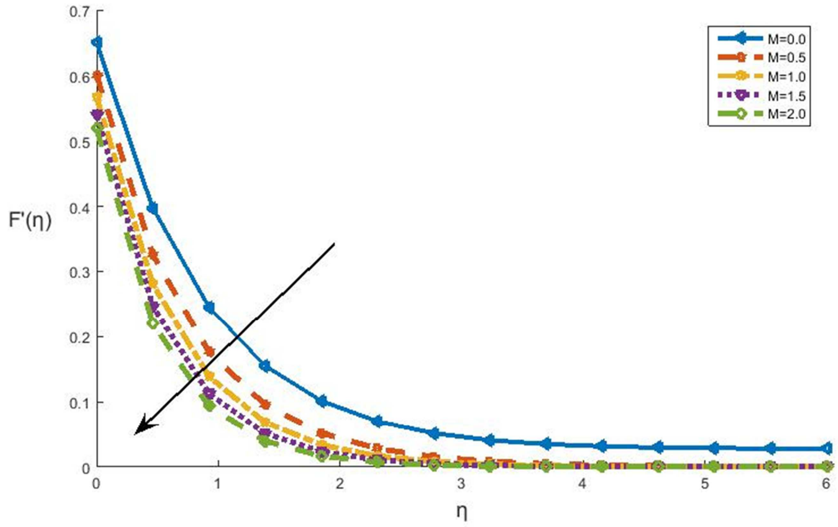

Figure 2. *Effect magnetic field parameter M on velocity distribution when R=0.5, Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1, φ =β= =0.5, Le=5, Pe=0.2Pe=0.2 and Pr=10.*

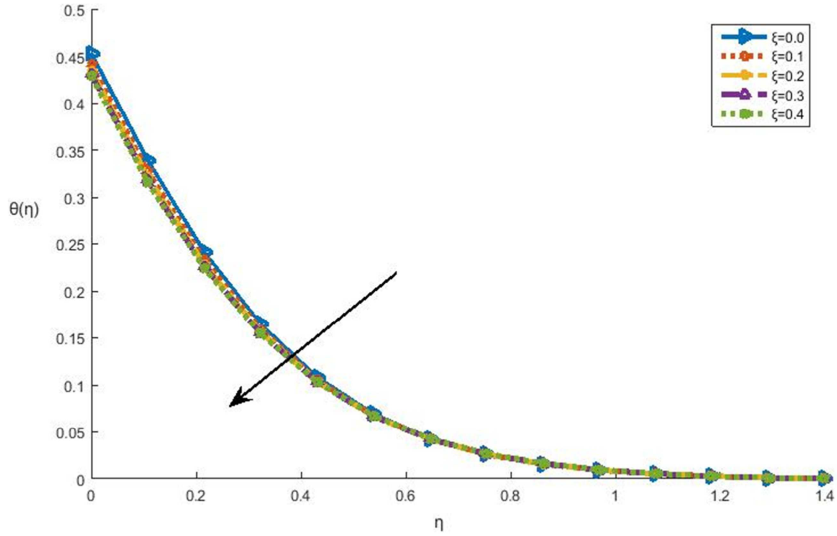

Figure 3. *Effect of velocity slip parameter ξ on temperature distributionwhen Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,φ =β= =0.5, Le=5, Pe=0.2 and Pr=10.*

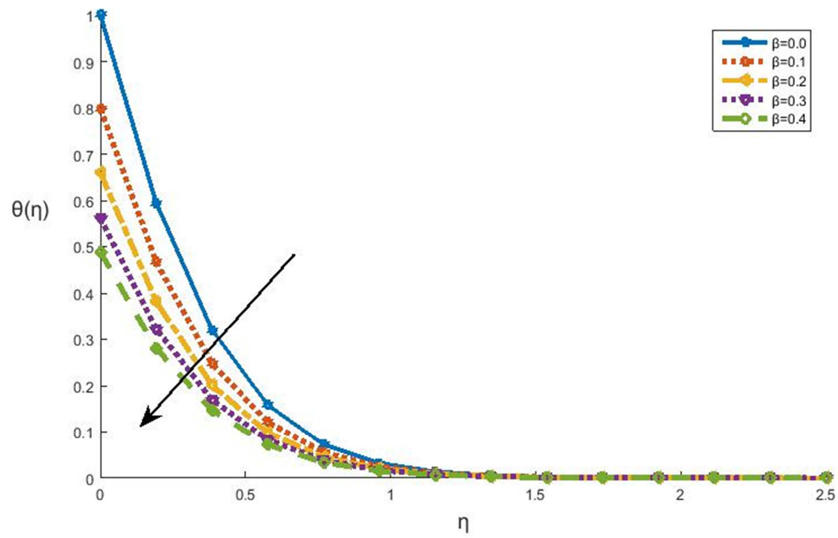

Figure 4. *Effect of temperature slip β on temperature distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,ξ =φ =Υ=05, Le=5, Pe=0.2 and Pr=10.*

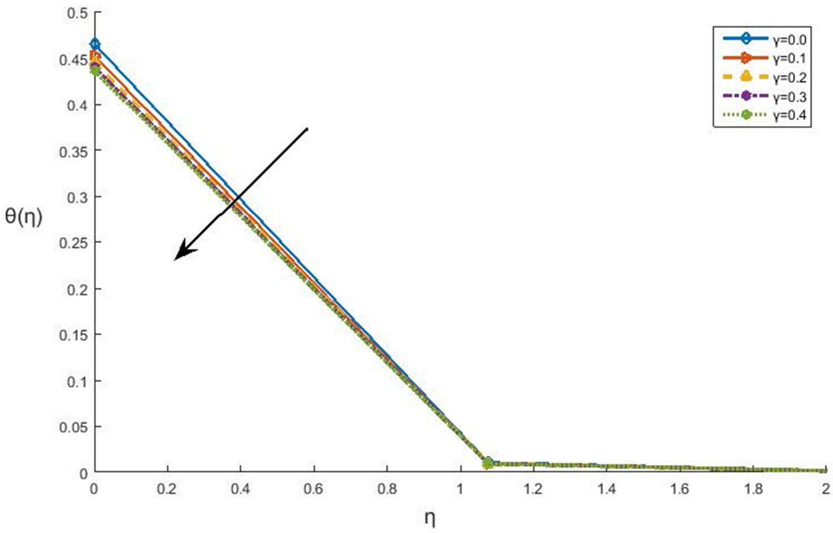

Figure 5. *Effect of nanoparticle concentration slip parameter γ on temperature distribution whenNt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,ξ =β=φ =0.5, Le=5, Pe=0.2 and Pr=10.*

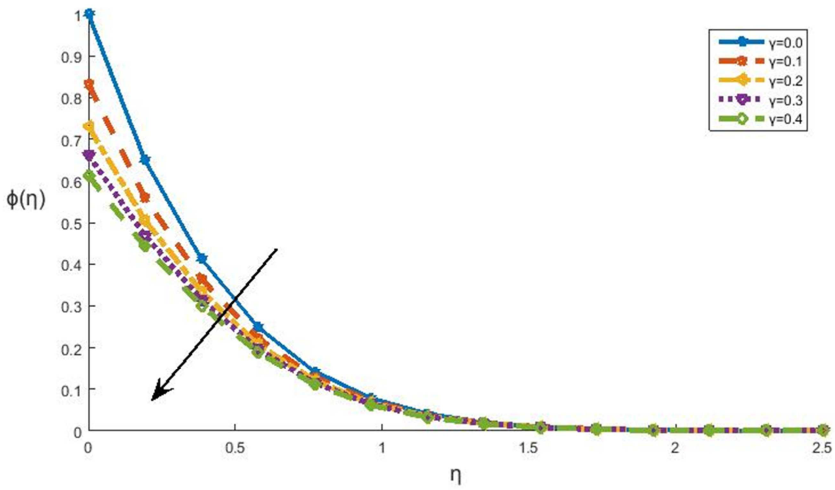

Figure 6. *Effect of nanoparticle concentration slip γon nanoparticle concentration distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,ξ =β = φ=0.5, Le=5, Pe=0.2 and Pr=1.*

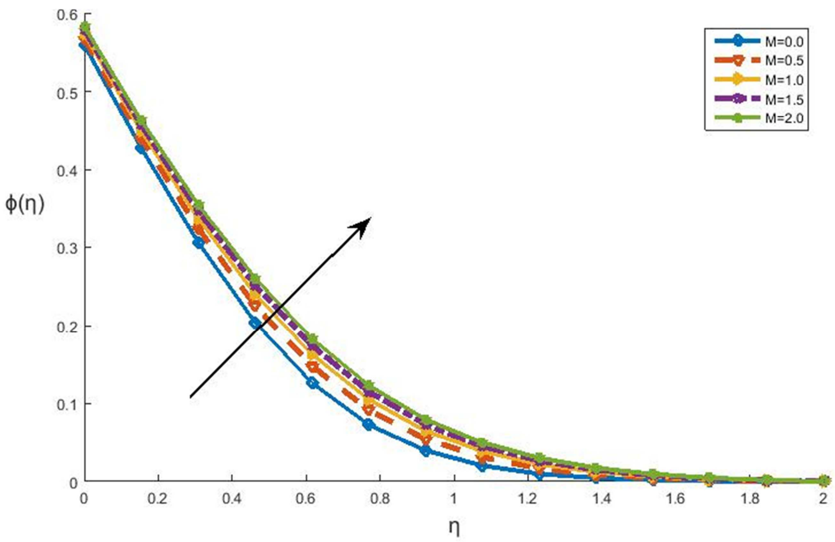

Figure 7. *Effect of magnetic field M on nanoparticle concentration distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1,ξ =β=γ= φ =0.5, Pe=0.2 and Pr=10.*

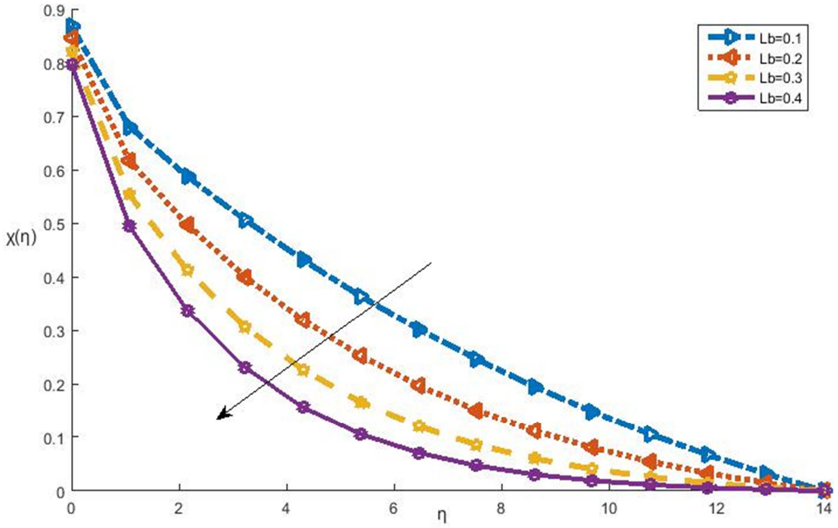

Figure 8. *Effect of bioconvection Lewis number Lb on motile microorganism density distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,ξ=β=φ=γ =0.5, Pe=0.2, Le=5 and Pr=10.*

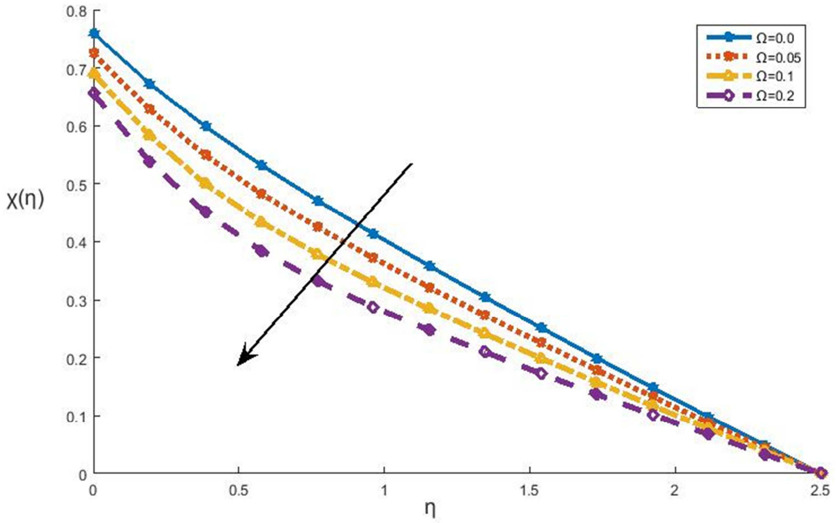

Figure 9. *Effect of microorganism concentration* Ω *on motile microorganism density distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=*Ω*=0.1, M=1,ξ=β=φ=γ =0.5, Pe=0.2, Le=5 and Pr=10.*

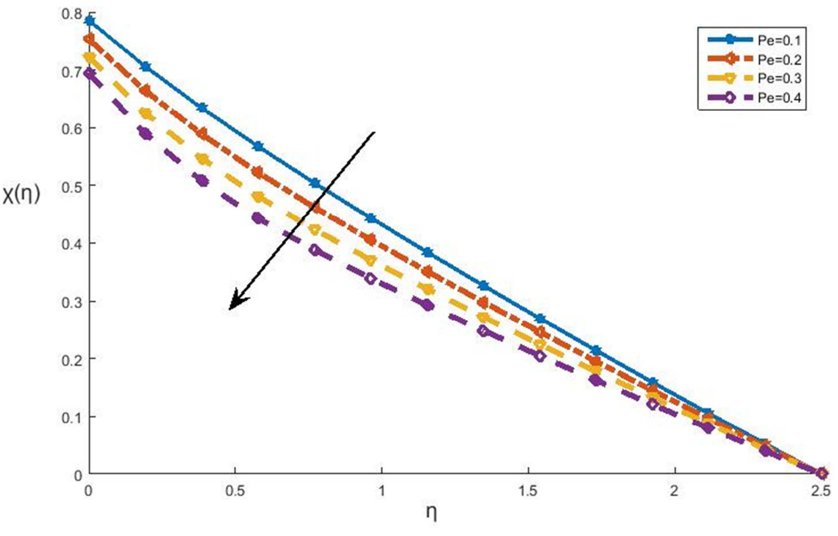

Figure 10. *Effect of bioconvectionpécletPe on motile microorganism density distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=*Ω*=0.1, M=1,ξ=β=φ=γ =0.5, Le=5 andPr=10.*

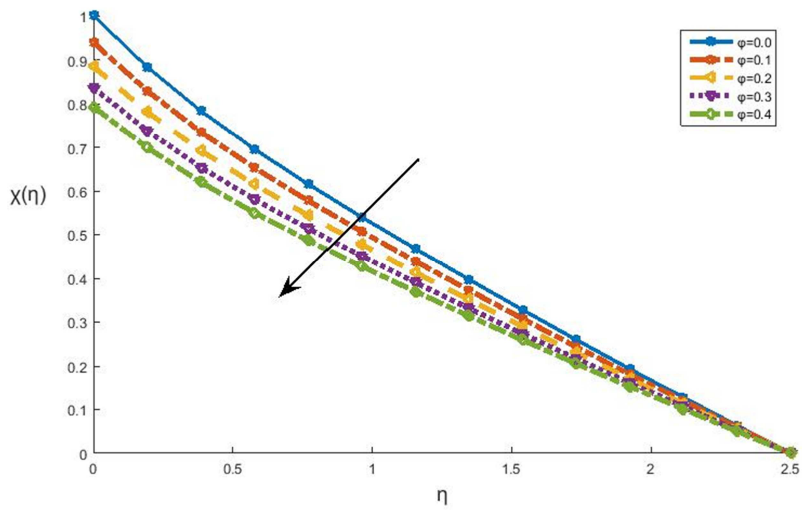

Figure 11. *Effect of microorganism density slip φ on motile microorganism density distribution when Nt=Nb=Ec=0.2, R=0.5, Lb=Ω=0.1, M=1,ξ=β=φ=γ =0.5, Pe=0.2, Le=5 and Pr=10.*

Figures1-2 Showed the effect of velocity slip and magnetic field parameters on velocity distribution. It is revealed that the velocity of fluid decreases with increase in both velocity slip parameter ξ and magnetic parameter M. The reason for this phenomenon is that application of magnetic field to an electrically conducting nanofuid gives rise to a resistive force which opposes the fluid motion.

Figures 3-5 Show the effects of various parameters on the temperature distribution. We noticed that the temperature of the flow is continuously decreasing as the value of ξ, β $and\ \gamma$ increase. When the thermalslip parameter increases, the movement of the fluid within the boundary layer will not be very sensitive due to the heating influence of the sheet surface and leads to decrease in thermal boundary layer.

The influences of γ and M parameters on nanoparticle concentration of the fluid are demonstrated in figures6-7. For several increase values of nanoparticleconcentration slip parameter γ, the nanoparticle volume fraction decreases. Opposite effect is observed for magnetic field parameter M, as the value of M increases, the nanoparticle concentration increases and enhancing Lorentz force which retards cause the nanofluid flow and Nusselt number reduces.

We investigated the bioconvection in nanofluid flow over a stretching sheet surface with microorganism. The effect of slip parameters on the microorganism concentration of the fluid and other pertinent parameters are shown in Figures 8-11. We noticed that increase in bioconvection Lewis number Lb, microorganism concentration Ω and bioconvectionPéclet number Pe and slip parameter φ lead to decrease in microorganism density in the boundary layer. The wall motile microorganisms flux decreased with higher motile microorganism density slip, Pe, Lb and Ω effects

5. Conclusion

In this study, the effect of the slip parameters on bioconvection Lewis number Lb, microorganism concentration, Ω and bioconvectionPéclet number, Pe is investigated. The main findings of the study show that velocity profiles decrease with the magnetic field and velocity slip parameter while temperature profile decreases with increase in the temperature slip, velocity slip and nanoparticle concentration slip parameter. The skin friction coefficient decreases as the values of velocity slip parameter increases.

It was also found that nanoparticle concentration profile decreases with increase in the nanoparticle concentration slip but increases with magnetic field parameter. Nusselt number increases with an increase in velocity slip and nanoparticle concentration slip parameter whereas the Sherwood number decreases. Finally, the local microorganism transfer rate is increased with greater values of bioconvection Lewis number Lb, microorganism concentration Ω and bioconvection Péclet number Pe.

Appendix

Nomenclature

ξ	Velocity slip parameter
β	thermal slip parameter
γ	concentration slip parameter
χ	dimensionless number density of motile microorganism
φ	microorganism density slip parameter
C_f	skin friction coefficient
D_b	Brownian diffusion coefficient
D_t	thermophoresis diffusion coefficient
f	dimensionless stream function
K	thermal conductivity
Le	Lewis number
Lb	bioconvention Lewis number
Pe	bioconventionpéclet number
Ω	bioconvention concentration difference
M	magnetic parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number

Nn_x	local motile microorganism density
Pr	Prandtl number
R	thermal radiation parameter
Re_x	local Reynolds number
T	temperature of the fluid inside the boundary layer
Sh_x	local Sherwood number
T_w	uniform temperature over the surface of the sheet
T_∞	ambient temperature
C_w	concentration
C_∞	ambient concentration
C	nanoparticles concentration fraction
u, v	velocity component along x- and y-direction
u_x	stretching velocity of the sheet
B_0	magnetic field strength
σ^*	Stefan–Boltzmann constant
K^*	absorption coefficient

Greek Symbols

η	dimensionless similarity variable
μ	dynamic viscosity of the fluid
σ	electrical conductivity
ν	kinematic viscosity of the fluid
ϕ	dimensionless concentration function
ρ_f	density of the fluid
$(c\rho)_f$	heat capacity of the fluid
$(c\rho)p$	effective heat capacity of a nanoparticle
ψ	stream function
α	thermal diffusivity
θ	dimensionless temperature
τ	parameter defined by $\frac{(c\rho)f}{(c\rho)p}$

Subscripts

∞	Condition at the free stream
w	Condition at the surface

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