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An Integrated One-step Equation for Solving Duct/Pipe Friction Loss by Hand Calculator

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Abstract: ASHRAE Handbooks are the worldwide reference books for HVAC engineers. When we tried to develop a duct software, we also followed the steps shown in 2013 ASHRAE Handbook. Accidently we found that some friction loss data of a duct design example seemed contrary to the data obtained from duct friction chart. Then we go back to adopt Darcy's and Colebrook's equations that have been used to solve duct/pipe friction loss for decades. However, the calculation process needs to use complicated computer program. After doing huge trial and error processes by computerized program, we obtained one integrated equation that can be used to calculate duct/pipe friction loss by hand calculator. We own an HVAC&R consultancy firm and have the opportunity to contact many real duct/pipe projects. This empirical equation has been successfully applied to dozens of actual duct and pipe design projects. For Reynolds Number (Re) is greater than 10,000 (i.e. turbulent flow), our analysis shows the friction losses obtained from this integrated equation are within ±2.0% of those obtained from Darcy's and Colebrook's equations. The accuracy (±2.0%) is good enough for engineers doing realistic duct/pipe designs. Hence, this one-step equation can be the handy alternative for Darcy's and Colebrook's equations. For the practical duct/pipe designs, engineers can calculate friction loss easily, no need to use iterative method.

Keywords: Darcy Equation, Colebrook Equation, Moody Chart, Friction Loss Chart

1. Introduction

Darcy's and Colebrook's equations always come into mind when discussing friction loss in ducts/pipes. For the fluid flow in duct or pipe, pressure drop due to friction loss can be calculated by Darcy equation [1]. Based on SI unit, Darcy equation can be rewritten as below.

$$\frac{\Delta P_f}{L} = \frac{1000 * f}{D} * \frac{\rho * V^2}{2} \tag{1}$$

where
ΔP_f=friction loss, Pa
L=duct/pipe length, m
f=friction factor, from equation (2)
D=hydraulic diameter, mm
ρ=fluid density, kg/m^3
V=fluid velocity, m/s

For transitional and turbulent flows, friction factor (f) can be calculated by Colebrook equation [2]:

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right] \tag{2}$$

where
ε=absolute roughness factor, mm
Re=Reynolds number

Reynolds number (Re) can be calculated by the following equation [2]:

$$Re = \frac{D * V}{1000 * \nu} \tag{3}$$

where ν=Kinematic viscosity, m^2/s

That means three equations are needed to calculate friction loss. In addition, because friction factor (f) appears on both sides of equation (2), solving equation (2) by hand calculator is almost impossible. Therefore, the Moody chart [3] (Figure 1) is adopted. However, using Moody chart is still tedious to obtain an exact f value. Then, in the real-design-world, engineers prefer to adopt friction chart; Figure 2 [4] for duct with ε=0.09mm, ρ=1.20kg/m^3, and Figure 3 [5] for pipe with 20°C water and SCH40 commercial steel pipe.

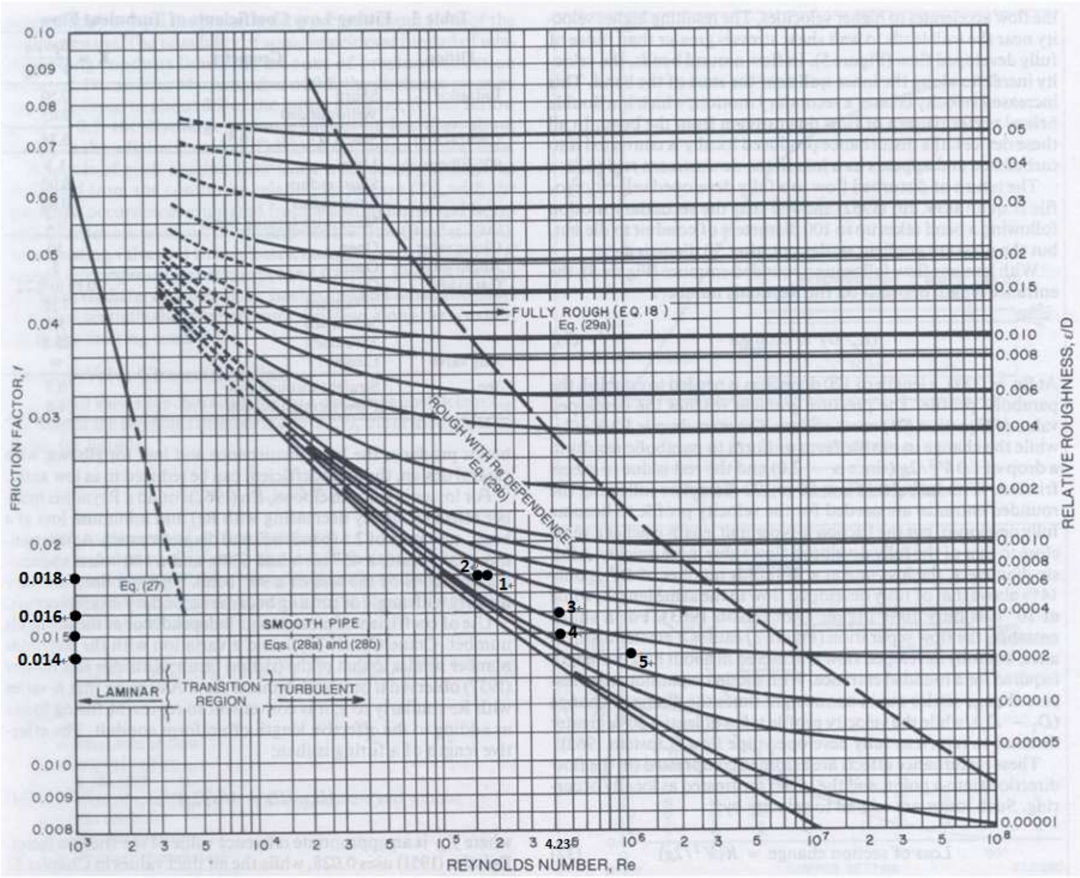


Figure 1. Moody Chart [3].

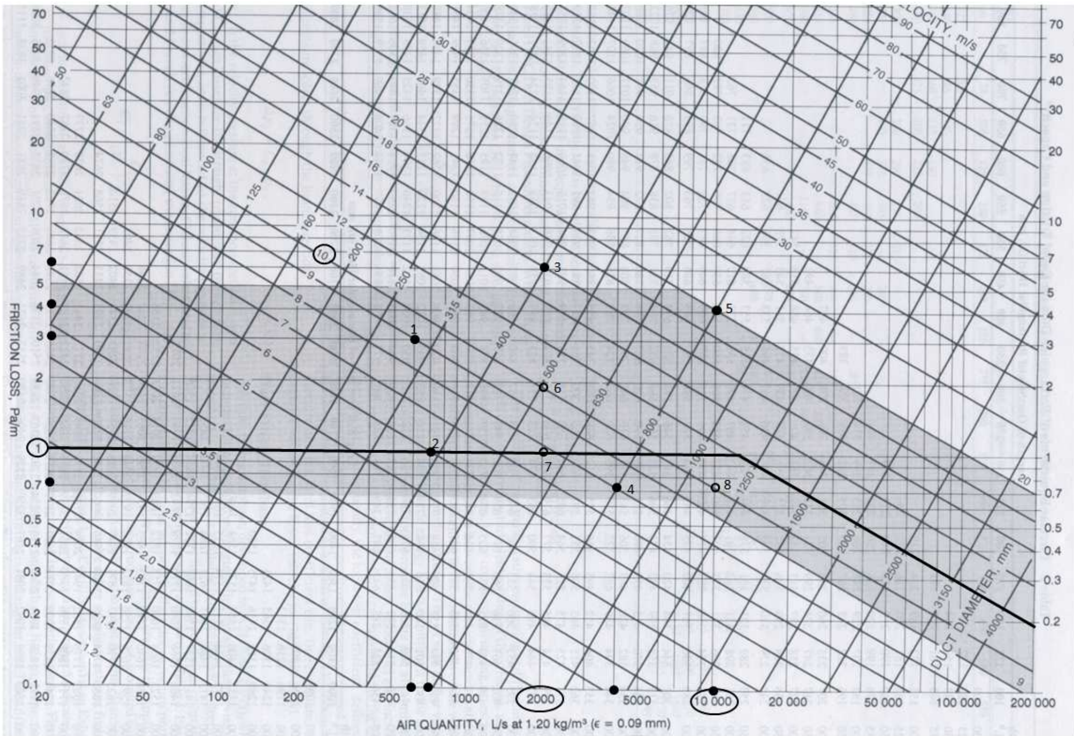
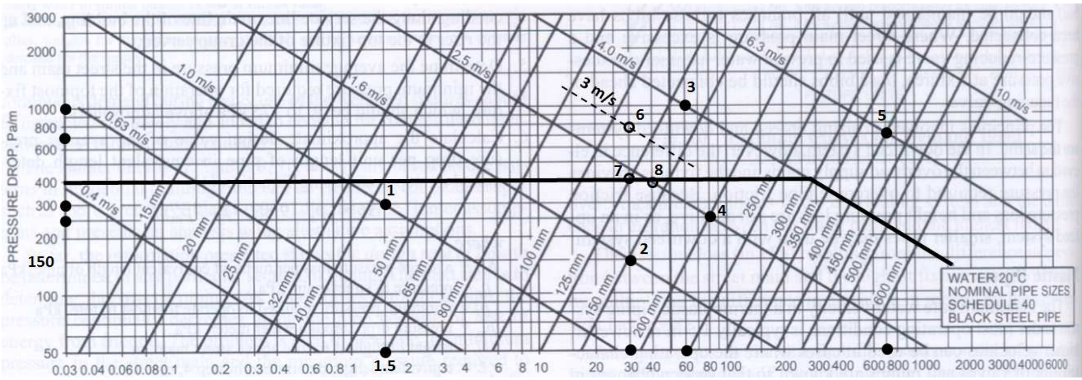


Figure 2. Duct Friction Chart [4].

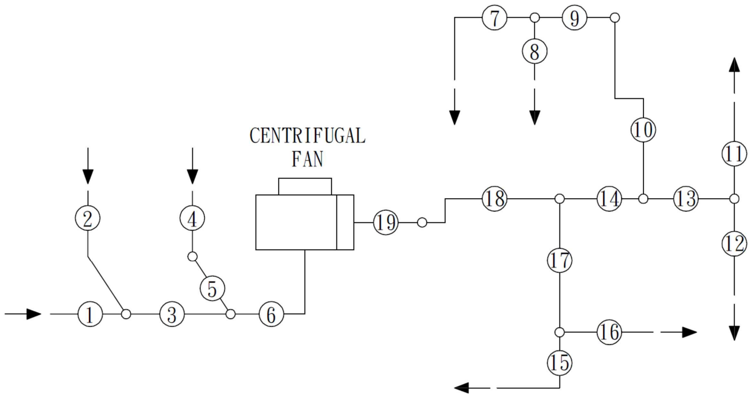


(points 1~5 refer to Table 3) (points 6, 7 refer to Table 5, point 8 refer to Table 6)

Figure 3. *Pipe Friction Chart [5].*

2. Process of Developing an Integrated Equation for Solving Friction Loss

We tried to develop a duct design software in July 2017, and followed the steps as shown in the example 7 in chapter 21 in 2013 ASHRAE Handbook [6], as depicted in Figure 4. This example presented that the straight-duct friction loss (Pa/m) and friction factor (f) were calculated by Darcy equation (1) and Colebrook equation (2). The summarized values were shown in column 1 through column 6 in Table 1. Accidentally, we found that all the friction losses (column 6) were different from the friction losses (column 7) obtained from friction chart (Figure 2) except SN 4 and SN 19. We cannot but help ask "Why".



Original: p21.22 of 2013 ASHRAE Handbook

Figure 4. *Duct System Example [6].*

Table 1. *Different Values Obtained by Figure 2, equation 2 and equation 4.*

SN	copied from table 8 of … [6]						
	ε mm	Q L/s	V m/s	D mm	duct size mm	friction loss, ΔP_6 Pa/m	read from fig.2 Pa/m
	1	2	3	4	5	6	7
1	0.09	700	9.0	315	315ψ	2.2	2.7
2	0.09	250	6.3	224	224ψ	1.8	2.0
3	0.09	950	12.2	315	315ψ	3.9	4.9
4	0.09	950	2.6	656	600*600	0.1	0.1
5	0.09	950	7.6	400	400ψ	1.2	1.5
6	0.09	1900	11.9	450	450ψ	2.5	3.0
7	0.09	275	4.4	273	250*250	0.8	0.82
8	0.09	275	4.4	273	250*250	0.8	0.82
9	0.09	550	4.4	381	500*250	0.6	0.5
10	0.09	550	5.5	343	400*250	0.9	0.93
11	0.09	475	7.6	273	250*250	2.2	2.3
12	0.09	475	7.6	273	250*250	2.2	2.3
13	0.09	950	9.0	351	420*250	2.3	2.3
14	0.09	1500	9.1	414	660*250	1.9	1.8
15	0.09	200	6.7	189	200*150	2.7	2.8
16	0.09	200	6.7	189	200*150	2.7	2.8
17	0.09	400	11.6	202	230*150	7.0	7.3

SN	copied from table 8 of … [6]						
	ε mm	Q L/s	V m/s	D mm	duct size mm	friction loss, ΔP_6 Pa/m	read from fig.2 Pa/m
	1	2	3	4	5	6	7
18	0.09	1900	9.5	470	800*250	2.0	1.8
19	0.09	1900	5.3	649	800*450	0.4	0.4

Table 1. *Continued.*

SN	from column 6 (ΔP6)				from column 11 (ΔP_L)		
	left side of (eq2) 1/√f=	right side of (eq2) -2*log (…)=	Error (%) (9-8)/8	by eq (4) ΔP_L Pa/m	left side of (eq2) 1/√f=	right side of (eq2)-2*log (…)=	Error (%) (13-12)/12
	8	9	10	11	12	13	14
1	8.388	7.447	-11.2	2.748	7.506	7.503	0.0
2	7.698	7.004	-9.0	2.133	7.071	7.052	-0.3
3	8.540	7.585	-11.2	4.891	7.626	7.635	0.1
4	7.876	7.349	-6.7	0.106	7.649	7.370	-3.7
5	8.511	7.556	-11.2	1.493	7.631	7.615	-0.2
6	8.705	7.875	-9.5	3.007	7.937	7.917	-0.3
7	7.305	6.992	-4.3	0.828	7.181	7.003	-2.5
8	7.305	6.992	-4.3	0.828	7.181	7.003	-2.5
9	7.140	7.296	2.2	0.539	7.533	7.263	-3.6
10	7.681	7.296	-5.0	0.935	7.534	7.308	-3.0
11	7.609	7.285	-4.3	2.287	7.463	7.295	-2.2
12	7.609	7.285	-4.3	2.287	7.463	7.295	-2.2
13	7.772	7.579	-2.5	2.278	7.810	7.577	-3.0
14	7.961	7.716	-3.1	1.771	8.247	7.698	-6.6
15	7.277	6.922	-4.9	2.856	7.076	6.937	-2.0
16	7.277	6.922	-4.9	2.856	7.076	6.937	-2.0
17	7.569	7.231	-4.5	7.339	7.392	7.241	-2.0
18	7.603	7.870	3.5	1.707	8.228	7.831	-4.8
19	8.071	7.798	-3.4	0.399	8.080	7.797	-3.5

※For standard air：20°C db, ρ=1.204kg/m3, ν=1.508E-05 m^2/s

In order to solve this problem, we reversed the calculation steps, i.e., using Darcy equation (1) and the friction loss ΔP_6 in column 6 to obtain friction factor (f) first. Then substitute f into Colebrook equation (2), and calculate both side values of Colebrook equation (2). The results were shown in columns 8 and 9 in Table 1. Obviously, there is something wrong with friction factor (f) because the left side values (column 8) do not equal the right side values (column 9).

We thought that maybe the friction factors (f) were just roughly read from Moody chart (Figure 1), not really calculated by Colebrook equation (2). Furthermore, in practice, when doing duct design, engineers still do not know the duct diameter yet. How can we use Colebrook equation to obtain friction factor (f)? Therefore, we tended to think from engineers' viewpoints, and hoped to find out if there is a better method to calculate friction factor (f) without diameter.

The process of solving problems is something like a reverse-thinking logic. We tried Steffensen method, iterative method and some mathematic techniques to solve Colebrook equation (2). After doing huge trial and error processes, we eventually obtained one integrated equation (4) by computerized programs. Equation (4) is a one-step method to calculate duct/pipe friction loss (Pa/m).

$$\Delta P_L=\rho*((0.0769/Q^{0.5}*V^{2.5}) + (12832.5*\nu/Q^2*V^7+0.2559*\varepsilon/Q^2*V^8)^{1/3}) \quad (4)$$

where

ΔP_L=friction loss (Pa/m) Q=flow rate (L/s) V=velocity (m/s)

3. Verify the Validity of Equation (4)

Is Equation (4) correct for practical applications? The following three ways were used for verification.

Way 1: Verification by Darcy's & Colebrook's Equations

Equation (4) was used to calculate friction loss (Pa/m) for the example values in Table 1. The results (ΔP_L) are shown in column 11 in Table 1. These calculated friction losses (Pa/m) are substituted into Darcy equation (1) to obtain friction factor (f). Then, substituted f, Re and ε/D into Colebrook equation (2). The calculated values for both sides of Colebrook equation (2) are shown in columns 12 and 13 in Table 1. Comparing the error (%) shown in column 10 and column 14, it is found that accuracy of Equation (4) is believable. Especially for the round duct sections (SN 1, 2, 3, 5 and 6), the errors are less than 0.3%.

Way 2: Verification by Moody Chart

Equation (4) does not need friction factor (f) for calculating friction loss (Pa/m). Nevertheless, for comparing with the friction factor (f) obtained from Moody chart, we rewrite Darcy equation (1) as the following equation for obtaining friction factor f (ΔP_L).

$$f(\Delta P_L)=0.07136*\Delta P_L *Q^{0.5}/\rho/V^{2.5} \quad (5)$$

where

f (ΔP_L)=friction factor based on ΔP_L

ΔP_L=friction loss (Pa/m) based on Equation (4)

Moody chart can be used to verify Equation (4) by four steps. First of all, with given flow rate (Q) and velocity (V) to calculate friction loss (ΔP_L) by Equation (4). Secondly, to calculate friction factor (f (ΔP_L)) by equation (5). Thirdly, to calculate Reynolds number (Re) by equation (3) and relative roughness (ε/D), and plot the junction point on Moody chart (Figure 1). Finally, to check if f (ΔP_L) equals friction factor (f) on Moody chart (Figure 1).

For galvanized steel duct (ε=0.09) and standard air, the calculated f (ΔP_L) values are shown in column 10 in Table 2. Although you can only read an approximate f value from Moody chart, it is obvious to see the contrast between the f (ΔP_L) values (column 10) and the friction factor (f) on Moody chart (Figure 1 points 1~5). You can clearly see that f (ΔP_L) values (column 10) and f values (column11) in Table 2 are identical.

By the same token, for commercial steel pipe SCH40 (ε=0.065) and 20°C water, using the ε/D (column 8) and Re values (column 9) in Table 3, you can see both f(ΔP_L, column 10) and f (column 11) read from Moody chart are almost the same values. That means, Equation (4) is suitable for calculating pipe friction loss (Pa/m) also.

Table 2. *Duct Verification Example by Moody Chart.*

Inputs						Outputs						
SN	ε mm	ρ kg/m^3	ν m^2/s	Q L/s	V m/s	ΔP_L Pa/m	D mm	by 1/7 ε/D	by eq (3) Re	by eq (5) f(ΔP_L)	Read from Figure 1 f	Read from Figure 2 Pa/m
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.090	1.204	1.508E-05	600	9.0	3.021	291	3.09E-04	1.74E+05	0.018	≒0.018	≒3.0
2	0.090	1.204	1.508E-05	700	6.0	1.000	385	2.34E-04	1.53E+05	0.018	≒0.018	≒1.0
3	0.090	1.204	1.508E-05	2000	16.0	6.158	399	2.26E-04	4.23E+05	0.016	≒0.016	≒6.2
4	0.090	1.204	1.508E-05	4000	8.0	0.707	798	1.13E-04	4.23E+05	0.015	≒0.015	≒0.71
5	0.090	1.204	1.508E-05	10000	20.0	4.101	798	1.13E-04	1.06E+06	0.014	≒0.014	≒4.1

※Compare SN1~5 to points 1~5 in Figure 1 and Figure 2

Table 3. *Pipe Verification Example by Moody Chart.*

Inputs						Outputs						
SN	ε mm	ρ kg/m^3	ν m^2/s	Q L/s	V m/s	ΔP_L Pa/m	D mm	by 1/7 ε/D	by eq (3) Re	by eq (5) f(ΔP_L)	Read from Figure 1 f	Read from Figure 3 Pa/m
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.065	998.2	1.004E-06	1.5	1.0	298.1	44	1.49E-03	4.35E+04	0.026	≒0.0255	≒300
2	0.065	998.2	1.004E-06	30.0	1.6	150.8	155	4.21E-04	2.46E+05	0.018	≒0.0180	≒150
3	0.065	998.2	1.004E-06	60.0	4.0	1028.2	138	4.70E-04	5.51E+05	0.018	≒0.0175	≒1000
4	0.065	998.2	1.004E-06	80.0	2.5	257.7	202	3.22E-04	5.03E+05	0.017	≒0.0165	≒255
5	0.065	998.2	1.004E-06	700.0	6.3	743.9	376	1.73E-04	2.36E+06	0.014	≒0.0137	≒720

※Compare SN1~5 to the points 1~5 in Figure 3

Way 3: Verification by Duct and Pipe Friction Charts

Figure 2 is the air friction chart suitable for galvanized steel round duct (ε=0.09mm) and standard air (20°C, ρ=1.204 kg/m^3, ν=1.508*10^{-5} m^2/s). The friction losses (ΔP_L) calculated by Equation (4) are shown in column 6 in Table 2, and the coincident points (1~5) are plotted on air friction chart (Figure 2). You can see the ΔP_L values (column 6) calculated by Equation (4) are almost the same as the friction loss (column 12) obtained by Q and V from Figure 2.

Similarly, Equation (4) can be used for the liquid Pipes. Figure 3 is the water friction chart suitable for 20°C water (ρ=998.2 kg/m^3, ν=1.004*10^{-6} m^2/s). The friction loss (ΔP_L) calculated by Equation (4) is shown in column 6 in Table 3, and the coincident points (1~5) are plotted on water friction chart (Figure 3). You can see the ΔP_L values (column 6) calculated by Equation (4) are almost the same as the friction loss (column 12) obtained by Q and V from Figure 3 also.

Therefore, the reliability of Equation (4) is verified by duct friction chart (Figure 2) and water pipe friction chart (Figure 3).

4. Darcy & Colebrook Equations vs. Equation (4)

Usually HVAC engineers determine flow rate (Q) based on cooling load calculation first. Duct/pipe diameter is still unknown at that time. Hence, it is not practical to use Moody chart (Figure 1) to obtain exact friction factor (f), let alone Colebrook equation (2). The difference between Darcy's & Colebrook's equations and Equation (4) infers that the former is better for studying the relations between friction factor (f), Reynolds number (Re), relative roughness (ε/D) and friction loss (Pa/m), and Equation (4) is better for realistic duct/pipe designs. You can put different ρ, ν, ε, and D values into Colebrook equation (2) to obtain friction factor (f) iteratively, and use Darcy equation (1) to obtain friction loss (Pa/m). On the contrary, equation (4) is a one-step equation to obtain friction loss (Pa/m) by the known flow rate (Q) and velocity (V) that are determined by (HVAC) engineers.

The comparison between Darcy & Colebrook equations and equation (4) is shown in Table 4.

***Table 4.** Darcy & Colebrook Equations vs. Equation (4).*

	Darcy & Colebrook equations	Equation (4)
parameters for getting Pa/m	ρ, ν, ε, D, Re & f	ρ, ν, ε, Q & V
calculating steps	ε/D → Re → f → Pa/m	one step → Pa/m
calculation method	by iterative method	by hand calculator
suitable for	duct and pipe with the most accuracy	duct and pipe with accuracy±2%
fault	can't be done by hand calculator	may not reliable for Re<10,000

5. Equation (4) Apply to Equal Friction Loss Method

Engineers usually use given flow rate to decide duct/pipe diameter by friction chart. The most common design method is equal friction loss method. Actually, Equation (4) can be used for equal friction loss method by simple trial & error process. Here are the steps for applying Equation (4) to equal friction loss method.

Initially, you can assume friction loss 1 Pa/m (Figure 2) for duct design and 400 Pa/m (Figure 3) for pipe design. Then, plot the junction point of the assumed friction loss (Pa/m) and flow rate (Q) to obtain diameter. For duct design, you can try V=10 m/s first to see if friction loss equals 1.0 Pa/m (target). If the first trail friction loss is larger than 1.0 Pa/m, lower the velocity and try again until the target friction loss (1.0 Pa/m) is obtained. Table 5 is the example showing the simple trail & error steps for duct and pipe designs; please refer to points 6, 7 in Figure 2 and Figure 3.

***Table 5.** Method for Applying Equation (4) to Equal Friction Method.*

given	initial trial (point 6)			second trial			third trial			lucky guess (point 7)				
Q (L/s)	V (m/s)	→	Pa/m	V (m/s)	→	Pa/m	V (m/s)	→	Pa/m	V (m/s)	→	Pa/m	→	D (mm)
2,000	10.00	→	1.88	9.00	→	1.45	8.00	→	1.08	7.77	→	1.00	→	572
	↑For duct design			(↓For pipe design)										
30	3.00	→	751.85	2.5	→	470.93	2	→	266.18	2.346	→	400.18	→	128

Normally, the values ρ, vandεcan be found in common fluid mechanics books. Thus, equation (4) can be widely applied to most fluids and materials. Table 6 shows some applications for different fluids (ρ) and materials (ε). You can see that the friction loss (Pa/m, column 6) is variable depending on different fluids and materials (see Column 11).

***Table 6.** Applications for Different Fluids and Materials.*

	Inputs					Outputs				By eq (5)	Remarks
SN	ε mm	ρ kg/m^3	ν m^2/s	Q L/s	V m/s	ΔP_L Pa/m	D mm	by 1/7 ε/D	by eq (3) Re	f	
	1	2	3	4	5	6	7	8	9	10	11
1	0.09	1.204	1.508E-05	10000	10	0.713	1128	7.98E-05	7.49E+05	0.0134	ε=0.09, standard air (20°C, 0%rh)
2	0.12	1.204	1.508E-05	10000	10	0.737	1128	1.06E-04	7.49E+05	0.0138	ε=0.12, standard air (20°C, 0%rh)
3	0.09	1.100	1.757E-05	10000	10	0.660	1128	7.98E-05	6.42E+05	0.0135	ε=0.09, heating air (45°C, 25%rh)
4	0.12	1.100	1.727E-05	10000	10	0.681	1128	1.06E-04	6.42E+05	0.0135	ε=0.12, heating air (45°C, 25%rh)
5	0.09	1.042	1.877E-05	5000	10	0.956	798	1.13E-04	4.25E+05	0.0146	ε=0.09, kitchen hood (50°C, 100%rh)
6	0.12	1.042	1.877E-05	5000	10	1.503	798	1.50E-04	4.25E+05	0.0150	ε=0.12, kitchen hood (50°C, 100%rh)
↑For Air (↓For water)											
7	0.065	998.2	1.003E-06	40	2.5	394.348	143	4.55E-04	3.56E+05	0.0180	ε=0.065,20°C water
8	0.065	999.7	1.300E-06	40	2.5	401.207	143	4.55E-04	2.74E+05	0.0183	ε=0.065,10°C cooling water
9	0.150	983.2	4.740E-07	40	2.5	451.353	143	1.05E-03	7.53E+05	0.0210	ε=0.15,60°C heating water

Note 1: SN1 can refer to point 8 in Figure 2, SN7 can refer to point 8 in Figure 3.
Note 2: duct ε values: galvanized steel round (0.09), galvanized steel spiral (0.12)
Note 3: pipe ε values: commercial steel SCH40 (0.065), galvanized steel (0.15)

6. Conclusions

The emphasis in this article is to verify if Equation (4) is coincident with Darcy and Colebrook equations. We own an HVAC&R consultancy firm and have the opportunity to contact many real duct/pipe projects. Equation (4) has been successfully applied to dozens of actual duct and pipe design projects since 2018. For the Reynolds number (Re) is greater than 10,000 (i.e. turbulent flow), our analysis indicates that the friction losses (Pa/m) obtained from Equation (4) is within ±2.0% of those obtained from Darcy's and Colebrook's equations. Therefore, the accuracy of equation (4) is good enough for engineers doing duct/pipe designs. Normally, engineers use given Q and V to obtain diameter D and friction loss ΔP_f. We are professional engineers (P. E.) and satisfy with equation (4) applications. Besides, in real life duct/pipe applications, the Reynolds number is greater than 10,000 (see the Re in tables 2, 3, 6). We do not have the chance to try the situation with Re≦10,000. Maybe equation (4) is not reliable enough if it is used for the Reynolds number lesser than 10,000. For someone needs to differentiate laminar, transition and turbulent flow regions (see figure 1) when doing fluid dynamics research, you can use EXCEL worksheet to calculate ε/D, Re, f and ΔP_L as we do in Table 6. Then, you can compare these values with the values obtained from equations (1), (2) and (3), or from Moody chart (Figure 1). There are many approximations of Colebrook's equation mentioned in public references, such as references from [7] to [15]. All these equations still need to calculate Reynolds Number (Re) first and just to solve Colebrook equation only. Not a similar equation like equation (4) using flow rate (Q) and velocity (V) to solve both of Colebrook equation and Darcy equation is found. Hence, we decide to release this article and let more engineers share our effort. Equation (4) can be the handy alternative for engineers to do realistic duct and pipe designs.

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